Adaptive Admittance-based Conductor Meshing for Interconnect Analysis

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High-Frequency Effects

- Skin effect
  - Current density drops off exponentially from the conductor surface into the interior

- Proximity effect
  - Current crowds to the surface close to or far from nearby conductors, depending on the directions of current flow
High-Frequency Effects

- **Consequences**
  - Non-uniform current distribution across interconnects
  - Variations in resistance and inductance values

- **Solution**
  - Discretization such that the current density in each filament is approximately uniform

- **Challenges**
  - Large lumped circuit model is generated
  - Trade off between the model accuracy and the computational cost
Contributions

- Demonstrate the existence of a quantity that can be used to estimate the model accuracy of a given discretization
- Use this quantity to determine a discretization with the least computation and acceptable model accuracy
Consider a single conductor carrying a signal at a single frequency

Assume a voltage difference of 1V is applied between the two terminals

\[ 1 = (R_i + j\omega L_i)I_i \Rightarrow I_i = (R_i + j\omega L_i)^{-1}1 \]

1 : vector of ones

\( I_i \) : vector of filament currents at discretization \( i \)

\( R_i \) and \( L_i \) : resistance and inductance matrices at discretization \( i \)
The total current at discretization $G_i$ is equal to the conductor-level complex admittance $Y_i$

$$1^T I_i = 1^T (R_i + j\omega L_i)^{-1} 1 = 1^T Y_i 1 = Y_i$$

- Characteristics of $|Y_i|$
  - Increases monotonically with increasing $i$
  - Converges to the true value of the conductor-level admittance $|Y_\infty|$

Discretization with the higher admittance magnitude is more accurate
Quantity Tracking Simulation
Error

- Admittance error: $|Y_\infty| - |Y_i|
  - Equal to the steady-state error in the current amplitude if a unit sinusoidal voltage is applied at terminals of the conductor
  - Can be used as the stopping criterion of meshing schemes if $|Y_\infty|$ is available

- $|Y_\infty|$ is seldom available
  - Use the change in the admittance magnitude
    $|Y_{i+1}| - |Y_i|$
Quantity Tracking Simulation
Error

- Single conductor with cross-section 3×3μm and length 1000μm
- 1V ramp with a rise-time of 10ps as the input signal
- Two quantities are compared
  - $\delta_i = |Y_{i+1}| - |Y_i| @ 100\text{GHz}$
  - $\text{Max}( |I(t) - I_i(t)| )$
Conventional Meshing Schemes

- Uniform meshing (UM)
  - Filaments are all of the same cross-sectional area
  - Width and height of filament are no larger than the skin depth

- Exponential meshing (EM)
  - Dimensions of filaments increase exponentially from outside to inside
  - Width and height of the smallest filament are no larger than the skin depth
Conventional Meshing Schemes

- **EM-1**
  - Width and height of the smallest filament \( \leq \) skin depth
  - Ratio between adjacent filaments = 2
  - Number of filaments is determined automatically
  
  \[
  \delta' = \frac{W}{2(1 + 2) + 2^2} \leq \delta
  \]
  
  \( W \) : width of conductor
  \( \delta \) : skin depth

- **EM-2**
  - Width and height of the smallest filament = skin depth
  - Ratio between adjacent filaments is varying
  - Number of filaments is specified a priori
  
  \[
  W = 2\delta \left( \frac{r^{(N-1)/2} - 1}{r - 1} \right) + \delta r^{(N-1)/2}
  \]
  
  \( N \) : given number of filaments
  \( r \) : ratio of adjacent filaments
Adaptive Exponential Meshing Schemes (AEM)

- AEM-1
  - Width and height of the smallest filament = skin depth
  - Ratio between adjacent filaments = 2
  - Add filaments until the admittance magnitude does not increase appreciably or when there is no room for adding filaments
Adaptive Exponential Meshing Schemes (AEM)

- **AEM-2**
  - Width and height of the smallest filament = skin depth
  - Increase the number of filaments with the ratio between adjacent filaments satisfying the equation

\[
W = 2\delta \left( \frac{r^{(N-1)/2} - 1}{r - 1} \right) + \delta r^{(N-1)/2}
\]
Implementations of Adaptive Meshing Schemes

- Given a discretization that needs refinement
  1. For two candidate discretizations (two more filaments along the width and height), compute the admittance magnitude at the working frequency
  2. Choose the discretization that yields a higher admittance magnitude
  3. Repeat step 1 and 2 until the change in the admittance magnitude for two successive discretizations is below a suitable threshold
Implementations of Adaptive Meshing Schemes

- Example
  - Single conductor of length 1000μm with cross section 10μm×3μm
  - “N×M” mesh: N and M filaments along the width and height, respectively
Non-Effect of Proximity

- Skin effect dominates the proximity effect
  - Interaction of filaments within a conductor overwhelms any effect from the filaments in any other conductors
  - Example - Three conductors are all of length 1000μm and cross section 10μm×3μm

| Mesh size | $|Y|$ ($\times 10^{-2}$) | error   | $|Y|$ ($\times 10^{-2}$) | error   | $|Y|$ ($\times 10^{-2}$) | error   | $|Y|$ ($\times 10^{-2}$) | error   | $|Y|$ ($\times 10^{-2}$) | error   |
|-----------|-------------------------|---------|-------------------------|---------|-------------------------|---------|-------------------------|---------|-------------------------|---------|
| 5×3       | 2.5094                  | 0.9793% | 1.5437                  | 0.8933% | 1.1063                  | 0.8442% | 0.8973                  | 0.8155% | 0.7724                  | 0.8188% |
| 7×3       | 2.5166                  | 0.8609% | 1.5465                  | 0.7788% | 1.1078                  | 0.7296% | 0.8984                  | 0.6995% | 0.7732                  | 0.8188% |
| 5×5       | 2.5077                  | 0.9375% | 1.5421                  | 0.8622% | 1.1050                  | 0.8196% | 0.8962                  | 0.7946% | 0.7715                  | 0.8188% |
| 9×3       | 2.5176                  | 0.8143% | 1.5470                  | 0.7425% | 1.1082                  | 0.6992% | 0.8987                  | 0.6720% | 0.7734                  | 0.8188% |
| 7×5       | 2.5135                  | 0.9400% | 1.5440                  | 0.8639% | 1.1060                  | 0.8209% | 0.8970                  | 0.7957% | 0.7720                  | 0.8188% |
Experimental Results

- Simulation of wires with uniform cross-section
  - N parallel conductors, each with a cross-section of 5x1μm, length of 1000μm and inter-conductor spacing of 1μm
  - Ramp signals with a rise-time of 10ps as aggressor inputs

<table>
<thead>
<tr>
<th>number of conductors</th>
<th>discretization</th>
<th>EM-1</th>
<th>EM-2</th>
<th>AEM-1</th>
<th>AEM-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>8 x 4</td>
<td>8 x 4</td>
<td>5 x 3</td>
<td>5 x 3</td>
</tr>
<tr>
<td>N</td>
<td>error</td>
<td>runtime</td>
<td>error</td>
<td>runtime</td>
<td>error</td>
</tr>
<tr>
<td>5</td>
<td>0.1492%</td>
<td>1.00</td>
<td>0.1101%</td>
<td>1.07</td>
<td>0.5936%</td>
</tr>
<tr>
<td>7</td>
<td>0.1571%</td>
<td>1.00</td>
<td>0.1160%</td>
<td>1.07</td>
<td>0.6835%</td>
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<tr>
<td>9</td>
<td>0.1581%</td>
<td>1.00</td>
<td>0.1183%</td>
<td>1.03</td>
<td>0.7202%</td>
</tr>
<tr>
<td>11</td>
<td>0.1576%</td>
<td>1.00</td>
<td>0.1184%</td>
<td>1.01</td>
<td>0.8279%</td>
</tr>
<tr>
<td>13</td>
<td>0.1635%</td>
<td>1.00</td>
<td>0.1231%</td>
<td>0.97</td>
<td>1.0085%</td>
</tr>
</tbody>
</table>
Experimental Results

- Simulation of wires with non-uniform cross-sections
  - Consider the cases of five parallel conductors and seven parallel conductors
  - Length = 1000μm, height = 3μm
  - Widths of each conductor are randomly chosen from the set {3μm, 5μm, 7μm}

<table>
<thead>
<tr>
<th>number of conductors</th>
<th>EM-1 error</th>
<th>runtime</th>
<th>EM-2 error</th>
<th>runtime</th>
<th>AEM-1 error</th>
<th>runtime</th>
<th>AEM-2 error</th>
<th>runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.2121%</td>
<td>1.00</td>
<td>0.1263%</td>
<td>1.04</td>
<td>0.5682%</td>
<td>0.42</td>
<td>0.8481%</td>
<td>0.44</td>
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<tr>
<td>7</td>
<td>0.2133%</td>
<td>1.00</td>
<td>0.1311%</td>
<td>1.07</td>
<td>0.5521%</td>
<td>0.14</td>
<td>0.7421%</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Experimental Results

- Extraction of frequency-dependent inductance
  - Spiral inductor composed of segments with a uniform cross-section of $3\mu m \times 1\mu m$ and the longest segment of $20\mu m$ long
  - Separation between any adjacent parallel segments is $1\mu m$
  - Use FastHenry to extract the frequency-dependent inductance
  - Frequency points is taken from the $10GHz \sim 100GHz$ range

<table>
<thead>
<tr>
<th>meshing scheme</th>
<th>EM-1</th>
<th>EM-2</th>
<th>AEM-1</th>
<th>AEM-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>error</td>
<td>0.0913%</td>
<td>0.0885%</td>
<td>0.6063%</td>
<td>0.1726%</td>
</tr>
<tr>
<td>runtime(second)</td>
<td>12.60</td>
<td>12.33</td>
<td>4.01</td>
<td>7.60</td>
</tr>
<tr>
<td>speedup</td>
<td>1.00</td>
<td>1.02</td>
<td>3.14</td>
<td>1.66</td>
</tr>
</tbody>
</table>
Conclusion

- Conductor-level admittance magnitude can be used to predict simulation errors due to discretizations.
- Observations can be applied to determine the coarsest discretization with an acceptable model accuracy by the admittance magnitude.
- Significant savings in computation can be realized with little sacrifice in accuracy.