A New Boundary Element Method for Multiple-Frequency Parameter Extraction of Lossy Substrates

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Outline

- Introduction of Substrate Coupling Problems
- High-Frequency Parameter Extraction Using Direct Boundary Element Method (DBEM)
- Efficient Techniques for Multi-Frequency Extraction
- Numerical Results
- Conclusions
Introduction of Substrate Coupling Problems

- In mixed-signal circuits
  - Digital and analog components are often built on a single lossy substrate (Si)
  - Coupling noises traveling through the substrate severely impact the sensitive analog components

- The quality factor of inductors in RF design is limited by substrate loss
  - Knowledge of substrate coupling is necessary, or even critical for design
Introduction of Substrate Coupling Problems

- Modeling of substrate coupling
  - Substrate resistance among contacts

- At higher frequency, both resistive and capacitive couplings should be considered [TCAD’98]

- Extraction of substrate parameters becomes a challenging task

Numerical Methods for Substrate Extraction

- Volume discretization methods
  - Finite Element Method (FEM)
  - Finite Difference Method (FDM)

- Advantage:
  - Versatile for various kinds of substrate structures
    - Stratified, with multiple parallel horizontal layers
    - More complicated, e.g. those with oxide wells, trenches, sinkers, buried diffusions, shielding for noise reduction

- Disadvantage:
  - Too many unknowns
  - Then, limited to small structures
Numerical Methods for Substrate Extraction

- Green’s function based methods (BEM)
  - Widely investigated

- Advantages:
  - Only discretizes contact surfaces, involves the fewest variables
  - Acceleration techniques proposed, such as DCT (discrete cosine), eigen-decomposition, etc.

- Disadvantages:
  - The derivation of Green’s function involves infinite series, which is expensive to calculate
  - Difficult, or even impossible to find the Green’s function for non-stratified substrates
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Consider both ohmic and displacement current; Each substrate material has a conductivity $\sigma_i$, and a permittivity $\varepsilon_i$. $Z$ denotes the coupling impedance between the contacts (including the plane).
Direct Boundary Element Method (DBEM)

- Discretizes substrate boundary,
  - Contact surface
  - Medium boundary/interface
- Uses the simple free-space Green’s function
- Has no difficulty in handling non-stratified substrates
- Efficient techniques of unknown reduction and matrix sparsification have been proposed for substrate resistance extraction [TCAD’06]

High-Freq. Parameter Extraction using DBEM

- Within each medium
  \[ \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0. \]

- Apply the Green’s identities, with the weighting function of free-space Green’s function
  \[ c_s u_s + \int_{\Gamma_i} q^* u d\Gamma = \int_{\Gamma_i} u^* q d\Gamma \]

  \( u \) is electric potential, and \( q \) is its normal derivative on boundary
  \( s \) is a collocation point;
  \( u^* \) is the free-space Green’s function, and
  \( q^* \) is its normal derivative on boundary

- Get linear equations with unknowns of \( u \) and \( q \) on boundaries
- \( u \) or \( q \) has known values on the outer boundaries
Along the interface of medium a and b:
\[ u_a = u_b, \]
\[ (\sigma_a + j\omega \varepsilon_a)E_{n,a} = (\sigma_b + j\omega \varepsilon_b)E_{n,b}. \]

The linear equations for each medium can be combined together, to get an overall linear system

\[ AX = B \]

A frequency-dependent complex-value system

To get whole impedance matrix, bias voltages are set on contacts, and \( B \) reflects these settings. \( X \) includes the unknowns of \( u \) and \( q \) on elements.

The current flowing through contact \( k \) is

\[ \int_{\Gamma_k} (\sigma + j\omega \varepsilon)E_n d\Gamma = \frac{1}{Z_{mk}}, \text{if contact m is with 1V, and contact k 0V} \]
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- High-Frequency Parameter Extraction Using DBEM
- Efficient Techniques for Multi-Frequency Extraction
  - Extract substrate resistance (freq=0)
    - Perform one time
  - Revise solution of R extraction to freq.-dep. parameters
    - With improving techniques, easy for computation
- Numerical Results
- Conclusions
Multi-Frequency Extraction of Substrate

- When consider both ohmic and displacement current, the substrate coupling parameter \((Z)\) is frequency-dependent.

- In DBEM calculation
  - Interface equation includes frequency and imaginary
    \[
    (\sigma_a + j\omega\varepsilon_a)E_{n,a} = (\sigma_b + j\omega\varepsilon_b)E_{n,b}
    \]
  - Solve a frequency-dependent, complex-valued \(AX = B\)
  - Get freq.-dependent R and C from the complex-valued \(Z\)

- The substrate parameters for many frequencies are necessary for the knowledge of substrate coupling.

- The trivial approach to build and solve \(AX = B\) repeatedly loses its efficiency with the increase of frequency points.

- We propose a fast method for the multi-frequency extraction.
High Similarity of $A_{res}$ and $A$

- Let $A_{res}$ be the coefficient matrix for R extraction.
- It’s the equation for substrate resistance extraction, if we discard $j\omega\varepsilon$ items in $(\sigma_a + j\omega\varepsilon_a)E_{n,a} = (\sigma_b + j\omega\varepsilon_b)E_{n,b}$.
- The differences between $A_{res}$ and $A$ lie on entries corresponding to $q$ variables on interfaces.
High Similarity of $A_{res}$ and $A$ (Cont.)

- In mathematics,

$$A = A_{res} + UV^T$$

Where $U$ is a submatrix of $A_{res}$, and $V$ is a sparse diagonal matrix.

$$UV^T = A - A_{res} = U \times V^T$$

$$f_{12} = \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1}, \quad r_{12} = \frac{\sigma_2}{\sigma_1}$$

$S_{12}$, $S_{23}$ are matrices with entries calculated from the integrals of $\int_{\Gamma_{ik}} u^* \, d\Gamma$. $I_1$ and $I_2$ are identity matrices.
Revise R to Frequency-Dependent Z

- According to Sherman-Morrison-Woodbury formula,

\[
A^{-1} = (A_{res} + UV^T)^{-1}
= A_{res}^{-1} - A_{res}^{-1}U(I + V^T A_{res}^{-1}U)^{-1}V^T A_{res}^{-1}
\]

\[
X = A^{-1}B = (A_{res} + UV^T)^{-1}B
= A_{res}^{-1}B - A_{res}^{-1}U(I + V^T A_{res}^{-1}U)^{-1}V^T A_{res}^{-1}B
= X_{res} - \{A_{res}^{-1}U(I + V^T A_{res}^{-1}U)^{-1}V^T \} X_{res}
\]

- Thus, freq.-dept. solution \((X)\) can be obtained through revising the solution for R extraction \((X_{res})\)
Efficient Technique for \((I + V^T A_{res}^{-1} U)^{-1}\)

\[
X = X_{res} - A_{res}^{-1} U (I + V^T A_{res}^{-1} U)^{-1} V^T X_{res}
\]

- Usually, difficult to get \((I + V^T A_{res}^{-1} U)^{-1}\)
- \(U\) and \(V\) are sparse matrices, and so is \(I + V^T A_{res}^{-1} U\)
- For the three-medium example:
Algorithm Flow

1. Generate $A_{res}$ in (5). Select some entries to form $U$; Solve the real-valued problem of R extraction; Runs only once.

2. Solve for $X_{res} = A_{res}^{-1}B$ as well as for $A_{res}^{-1}U$;

3. For each frequency point, calculate the corresponding $Z$:
   a) Create such factors as $(f_{12}/r_{01}^{-1})I_1$ and $(f_{23}/r_{02}^{-1})I_2$, so as to form matrix $V$;
   b) Compress $I + V^T A_{res}^{-1}U$ into small matrix $M$;
   c) Inverse matrix $M$, and get $W$;
   d) Refill $W$ to get matrix $(I + V^T A_{res}^{-1}U)^{-1}$;
   e) Get $X = X_{res} - A_{res}^{-1}U(I + V^T A_{res}^{-1}U)^{-1}V^TX_{res}$.
   f) Get the desired $Z$ parameter through (9).

The order of $M$ is # of interface elements, which is much less than total # of unknowns.
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Numerical Results -- Accuracy

Case from [TCAD’98]

The discrepancy between our results and those from [TCAD’98] < 1.0%

Since displacement current increases with frequency, the Z’s magnitude decreases with frequency.

Numerical Results -- Efficiency

Case with 52 contacts

Our method is compared with ASITIC, a shared program using DCT-accelerated Green’s function method.

For each frequency, our method solves a $587 \times 587$ matrix to calculate $Z$ from the precalculated $X_{\text{res}}$.

Our method is much faster than ASITIC.

<table>
<thead>
<tr>
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<th>DBEM</th>
<th>ASITIC</th>
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<tr>
<td># Contacts</td>
<td>52</td>
<td>52</td>
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<td># Variables</td>
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</table>
| Pre-process (seconds) | **420.9** | **360** **
| Extraction (for 1 freq.) | 9.0 | 9000 (Not obtained) |

*: Calculate resistance
**: Calculate Green’s function
Numerical Results -- Versatility

- An example with non-stratified substrate
  - The central block region has a distinct resistivity, called LVB (lateral variation block)
  - LVB’s resistivity is 1000 times larger
  - Size of LVB is set to be 0, 20, and 70\(\mu\)m
- Green’s function based method is not able to handle it, while our DBEM has no difficulty
- Right figure shows the plot of Magnitude vs. frequency, for three settings

Since LVB obstructs ohmic current flow, the Z for L=70 is larger than those corresponding to other two settings
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- For frequency-dependent substrate modeling, we propose an efficient method for multi-frequency extraction
  - Based on DBEM, has ability to handle non-stratified structure
  - Do not directly solve the complex linear system, but firstly solve the R extraction problem with real linear system
  - For each frequency, the result of R extraction is revised to Z parameter through solving a smaller linear system

- The proposed method
  - Is efficient
  - Does not sacrifice accuracy
  - Has large versatility
Thank you!

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