New Block-Based Statistical Timing Analysis Approaches Without Moment Matching

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01/25/2007

Outline

- Review of statistical static timing analysis
- Necessary conditions for statistical MAX operation
- Computation of bounds on timing yield
- Experimental results
- Conclusions

Statistical static timing analysis

- Process variations getting prominent while feature sizes getting smaller: timing or power yield loss
- Corner-based analysis is either pessimistic or expensive
- Statistical Static Timing Analysis (SSTA) is desperately needed
- Block-based SSTA: Chang et al. ICCAD03, Visweswariah et al. DAC04
  - Key computation: $C = \max(A, B)$, where A and B are random variables.
- max is postponed but still needed in path-based SSTA.

Approaches to statistical MAX operation

- $C = \max(A, B)$, where A and B are random variables.
- If A and B have Gaussian distributions
  - Approximate C has also the Gaussian distribution
  - Moment matching (least squares fitting): Chang et al. ICCAD03, Visweswariah et al. DAC04
- If A and B do not have Gaussian distributions
  - Chang et al. DAC05, Zhan et al. DAC05, Zhang et al. DAC05

How good are these approaches to statistical MAX operation?

- Are they accurate?
- Is the computed delay greater or smaller than the actual delay? at one given yield point? in a given range?

Two necessary conditions for Max operation

- If $C = \max(A, B)$, we should have:
- Dominance relation: $Pr(C \geq A) = 1$ and $Pr(C \geq B) = 1$
- Comparison relation:

$$\begin{aligned} Pr(C > A) &= Pr(B > A), \\ Pr(C > B) &= Pr(A > B). \end{aligned}$$

How good are existing approaches to statistical MAX?

- $A = 30 + \epsilon_1$, $B = 30.5 + 0.5\epsilon_1$, compute $C = \max(A, B)$
- Moment matching:
  - Chang et al. ICCAD03: C is Gaussian, $Pr(C \geq A) =$ 89.46% and $Pr(C \geq B) = 62.57\%$
  - Zhan et al. DAC05: C is not Gaussian, $Pr(C \geq A) =$ 63.43% and $Pr(C \geq B) = 49.17\%$
- Dominance relation does not hold.
- Comparison relation does not hold either.

Our investigation

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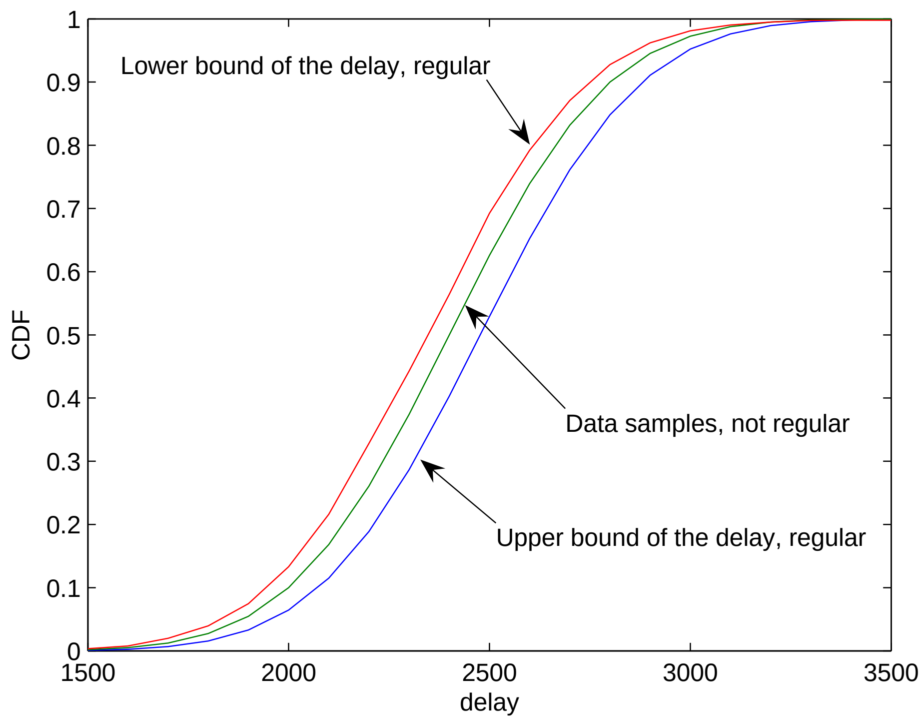
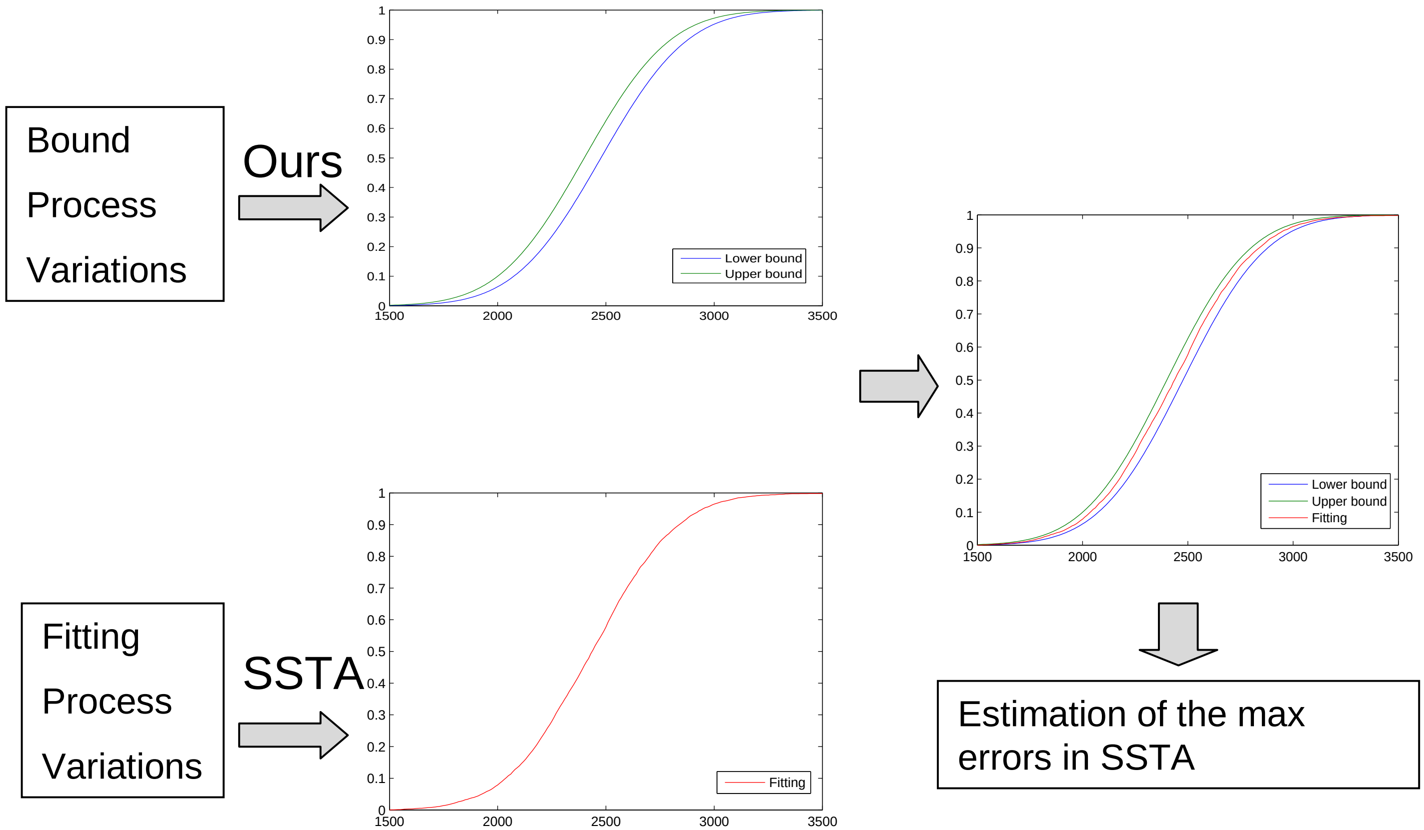
- Can we have an approximation with both conditions?
- Bad news: no!
- Good news: satisfying one of them will give an upper bound; the other a lower bound.

Upper and lower bounds for a random variable

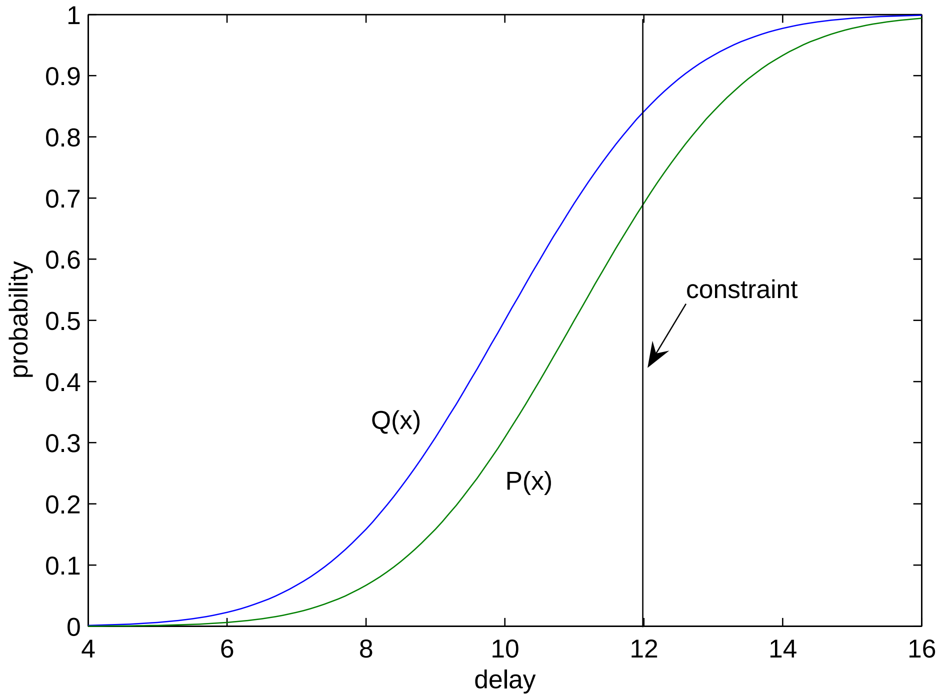


Bound on timing yield is useful



Bounds on timing yield

- $P(x)$ is the lower bound of $Q(x)$
- $Q(x)$ is the upper bound of $P(x)$

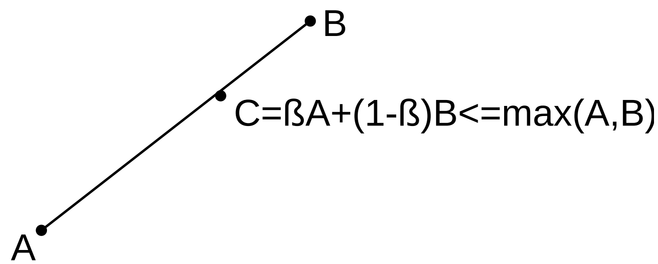


Lower Bounds on timing yield

- Dominance relation: $Pr(C \geq A) = Pr(C \geq B) = 1$
- $\Longrightarrow C \geq \max(A, B)$
- $\Longrightarrow$ Computed delay is always bigger than or equal to the actual delay
- $\Longrightarrow$ Lower bound on timing yield

Upper Bounds on timing yield

- A and B are random variables, and $C = \beta A + (1 - \beta)B$ where $\beta \in [0, 1]$.
- $\Longrightarrow$ Comparison relation holds
- $\Longrightarrow \max(A, B) \geq C$
- $\Longrightarrow$ Computed delay is always less than or equal to the actual delay
- $\Longrightarrow$ Upper bound on timing yield



Lower bound for Gaussian variables: LBDomSSTA

- Objective: find a C for the computation of $\max(A, B)$ such that $Pr(C \geq A) = Pr(C \geq B) = 1$
- For Gaussian variables, $Pr(C \geq A) = 1$ cannot be satisfied unless $C = A + d$ where d is a non-negative deterministic real number.
- Relax it to $Pr(C \geq A) \geq \eta$

Approach in LBDomSSTA

- $A = a_0 + \sum_i a_i \epsilon_i$, $B = b_0 + \sum_i b_i \epsilon_i$.
- $c_i = a_i T_A + b_i(1 - T_A)\ \forall i = 1, 2, \ldots n$ in order to preserve the covariance, where T_A is the tightness probability
- Adjust c_0 such that $Pr(C \geq A) \geq \eta$ and $Pr(C \geq B) \geq \eta$.

Upper bound for Gaussian variables: UBCompSSTA

- $C = T_A A + (1 - T_A) B$
- If the random variables have at most 10% deviation, theoretically, the maximal errors on the mean and the standard deviation between UBCompSSTA and the moment matching are only 2.66% and 1.41%.
- With more positive correlations, the errors become smaller.
- Moment matching is also an approximation approach, so it is possible for UBCompSSTA to have smaller errors than the moment matching approach in reality.

Upper bound for Non-Gaussian variables: UBCompSSTA

- $C = T_A A + (1 - T_A) B$
- Using the quadratic model in Zhang et al. DAC05, theoretically, UBCompSSTA gets exactly the same standard deviation as in Zhang et al. DAC05, and gets the mean very close to that approach (max error$\leq$2.66%).

Experimental results

ISCAS85, 10% deviation, objective yield 90%, $\eta = 90\%$

name	UBCompSSTA				LBDomSSTA				Monte Carlo		
	time (s)	μ	σ	yield	time (s)	μ	σ	yield	μ	σ	yield
c1355	0.01	1580	40	91.15	0.01	1585	40	89.07	1583	40	90.00
c1908	0.01	4000	100	91.92	0.01	4019	101	88.49	4011	100	90.00
c2670	0.01	2918	61	91.15	0.01	2926	61	89.25	2922	61	90.00
c3540	0.03	4700	120	92.22	0.02	4727	120	88.30	4715	119	90.00
c5315	0.03	4900	123	91.47	0.02	4919	123	88.69	4910	125	90.00
c6288	0.03	12400	312	92.36	0.03	12477	314	87.90	12443	313	90.00
c7552	0.05	4300	107	91.47	0.04	4320	107	88.30	4311	107	90.00

Tight bounds.

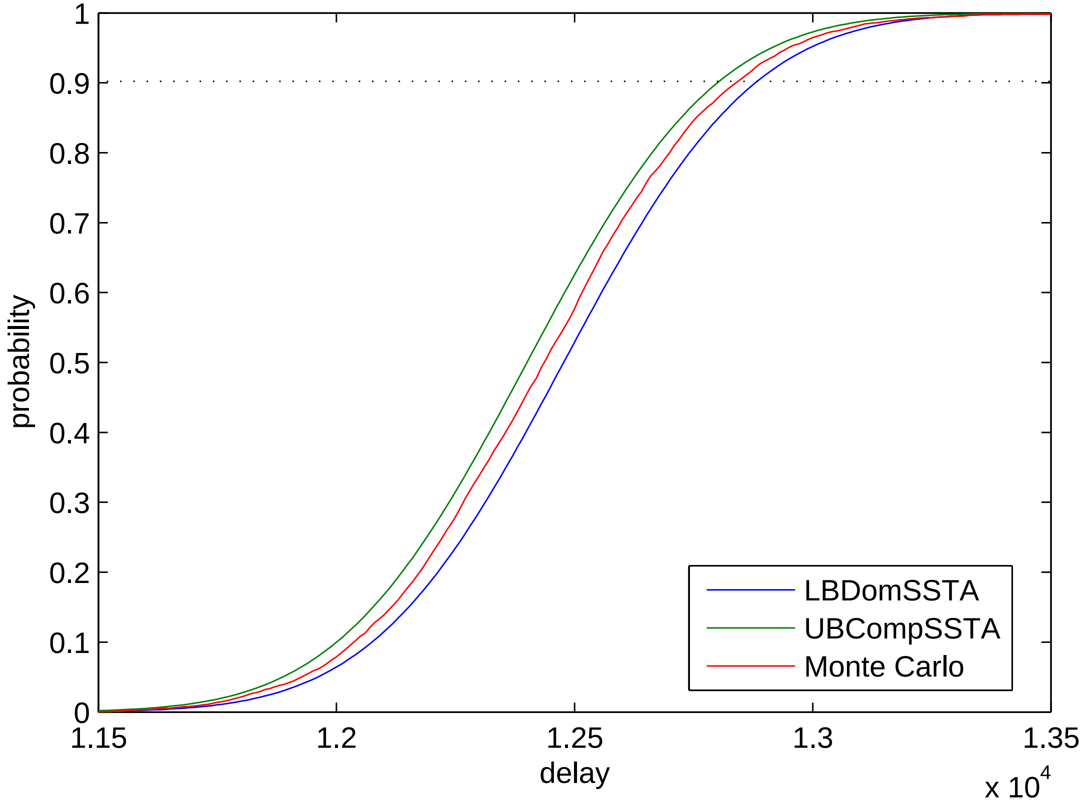
Experimental results

ISCAS85, 10% deviation, objective yield 90%, $\eta = 90\%$

name	UBCompSSTA				LBDomSSTA				Moment-matching		
	time (s)	μ	σ	yield	time (s)	μ	σ	yield	μ	σ	yield
c1355	0.01	1580	40	91.15	0.01	1585	40	89.07	1583	40	89.62
c1908	0.01	4000	100	91.92	0.01	4019	101	88.49	4018	101	88.49
c2670	0.01	2918	61	91.15	0.01	2926	61	89.25	2923	61	89.80
c3540	0.03	4700	120	92.22	0.02	4727	120	88.30	4718	120	89.44
c5315	0.03	4900	123	91.47	0.02	4919	123	88.69	4913	123	89.25
c6288	0.03	12400	312	92.36	0.03	12477	314	87.90	12464	314	88.69
c7552	0.05	4300	107	91.47	0.04	4320	107	88.30	4313	107	89.44

Fast estimation of the maximal errors of moment-matching.

The CDFs from different approaches for "c6288".



Conclusions

- max is an important operation in SSTA.
- Existing approaches do not satisfy two necessary conditions:
  - Dominance: $Pr(\max(A, B) \geq A) = Pr(\max(A, B) \geq B) = 1$
  - Comparison: $Pr(\max(A, B) > A) = Pr(B > A)$
- Enforcement of dominance gives an upper bound
- Enforcement of comparison gives a lower bound
- Both are useful for yield estimation

Thank you