Efficient Numerical Modeling of Random Rough Surface Effects in Interconnect Internal Impedance Extraction

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Outline

- Background
- Modeling (Effective Parameters)
- Computation (Modified SIE Method)
- Results
- Conclusion
Surface Roughness in Interconnects
Sources of Surface Roughness

- **Unintentional sources**
  - Technology limitations
  - Process variations

- **Intentional sources**
  - Electronic deposition
  - Chemical etching
  - Annealing

*Enhance the cohesion between metal and dielectric*
Impact of Surface Roughness on Internal Impedance

Interaction between rough surface and current

Current under smooth surface

- Longer current path
- More resistive loss

Current under rough surface

- Larger current loop
- Higher resistance
- Higher internal inductance
High Frequency Effects

• Rough surface effects is insignificant in low frequencies (large skin depth, small roughness)
• It becomes significant in high frequencies (comparable skin depth and roughness)

Skin depth = 6.5 microns when f = 0.1 GHz (From Intel)
Modeling

Model the impact of random rough surface on interconnect internal impedance
Effective Parameters

- Effective Resistivity $\rho_e$ & Effective Permeability $\mu_e$
- Capture the increase of resistance and internal inductance caused by surface roughness

Diagram:
- Layout Information
  - Current Extraction Tools
  - Rough Surface RLC Circuit
- Rough Surface Module
  - Effective Resistivity & Permeability
Analytical Formulation

● For effective resistivity

\[ \rho_e = \rho \left[ 1 + \frac{2}{\pi} \tan^{-1} \left( 1.4 \frac{h}{\delta} \right)^2 \right] \]

- \( h \) – RMS height; \( \delta \) - skin depth
- Widely used in practical design, BUT
- Inaccurate (only \( h \) is considered)

● For effective permeability

- Unavailable
Numerical Formulation of $\rho_e$

Smooth surface power loss

$$P_s = \frac{\rho |H_0|^2 l}{2\delta}$$

Rough surface power loss

$$P_r = \frac{\rho H_0^*}{2} \text{Re} \left\{ \int_{\tilde{S}} dz U(z) \right\}$$

Power loss equivalence $P_s = P_r$

$$\rho_e = \frac{2\delta P_r}{|H_0|^2 l} = \frac{\rho \delta}{H_0 l} \text{Re} \left\{ \int_{\tilde{S}} dz U(z) \right\}$$
**Numerical Formulation of** $\mu_e$

**Smooth surface magnetic energy**

$$W_s = \frac{\mu \delta |H_0|^2 l}{2}$$

**Rough surface magnetic energy**

$$W_r = \frac{\mu \delta^2 H_0^*}{2} \text{Im} \left\{ \int \hat{S} \, dZ U(z) \right\}$$

**Magnetic energy equivalence** $W_s = W_r$

$$\mu_e = \frac{2W_r}{\delta |H_0|^2 l} = \frac{\mu \delta}{H_0 l} \text{Im} \left\{ \int \hat{S} \, dZ U(z) \right\}$$
Governing Equation

\[
\int_{\bar{S}} \, dz G(z, z') U(z) = \frac{1}{2} H_0 + H_0 \int_{cp} \, dz \sqrt{1 + \left( \frac{\partial y(z)}{\partial z} \right)^2} \frac{\partial G(z, z')}{\partial \hat{n}}
\]

Green’s function \( G(z, z') = H_0^{(1)}(k_1 \sqrt{|z - z'|}) \)

Surface unknown \( U(z) = \sqrt{1 + \left( \frac{\partial y(z)}{\partial z} \right)^2} \frac{\partial H(z)}{\partial \hat{n}} \)

Boundary condition \( H(r) = H_0 \quad r \in S \)
Modeling of Random Rough Surface

- Most rough surfaces in reality are random
- Described by stationary stochastic process with Probability Density Function and Correlation Function

\[ P_1(y) = \frac{1}{\sqrt{2\pi h}} \exp\left(-\frac{y^2}{2h^2}\right) \]

\[ C_g(z_1, z_2) = \exp\left(-\frac{|z_1 - z_2|^2}{\eta^2}\right) \]

\( \eta \) --- correlation length
Conventional Statistical Solver -- Monte-Carlo Method

Input
- Layout information

Random Rough Surface Generator
- $S_1$
- $S_2$
- $S_3$
- $S_{n-1}$
- $S_n$

Direct Integral Equation Solver
- $R_1$
- $R_2$
- $R_3$
- $R_{n-1}$
- $R_n$

Computation

Output
- Mean of the solution

A large number of surface realizations

Corresponding solutions
Limitations of Monte-Carlo method

- **Probabilistic nature**
  - Mean value is also a random variable

- **Slow convergence**
  - More than 2500 runs to converge within 1%
Computation

Efficient Stochastic Integral Equation (SIE) method
- **Stochastic Integral Equation (SIE) method**
  - Use mean value as unknown
  - One-pass solution
  - Deterministic nature

- **Two steps**
  - Zeroth-order approximation
    (Uncorrelatedness assumption)
  - Second-order correction
Zeroth-Order Approximation

Directly applying ensemble average on both sides:

\[
\int_{\tilde{S}} dz \langle G(z, z') U(z) \rangle = \frac{1}{2} H_0 + H_0 \int_{cp} dz \left\langle \sqrt{1 + \left( \frac{\partial y(z)}{\partial z} \right)^2} \frac{\partial G(z, z')}{\partial \tilde{n}} \right\rangle
\]

Ensemble average:
\[
\langle f(x) \rangle = \int_{-\infty}^{+\infty} P_1(f(x)) f(x)
\]

Assuming the Green’s function \( G \) and the surface unknown \( U \) are statistically independent (Uncorrelatedness Assumption):

\[
\int_{\tilde{S}} dz \langle G(z, z') \rangle \langle U(z) \rangle = \frac{1}{2} H_0 + H_0 \int_{cp} dz \left\langle \sqrt{1 + \left( \frac{\partial y(z)}{\partial z} \right)^2} \frac{\partial G(z, z')}{\partial \tilde{n}} \right\rangle
\]

New unknown
Inaccurate zeroth-order approximation

Matrix equation format

\[
\langle G(z, z') \rangle \quad \bar{A} \bar{U} = \bar{L}
\]

\[
\bar{A}_{ik} = \langle G(z_i, z_k) \rangle = \iint dy_i dy_k P_2(y_i, y_k) G(y_i, y_k; z_i, z_k)
\]

Inaccurate zeroth-order approximation

\[
\bar{U}^{(0)} = \bar{A}^{-1} \bar{L} \quad \bar{V}^{(0)} = \bar{L}^T \bar{U}^{(0)}
\]

Cause: uncorrelatedness assumption

\[
\langle G(z, z') U(z) \rangle \neq \langle G(z, z') \rangle \langle U(z) \rangle
\]
Primitive Second-Order Correction

- Improve the accuracy of the mean $V$

$$\bar{V} = \bar{V}^{(0)} + \bar{V}^{(2)}$$

**Zeroth-order approximation term**

$$\bar{V}^{(2)} = \text{trace}(\bar{A}^{-T} D)$$

$$\text{vec}(D) = \left\langle (A - \bar{A}) \otimes (A - \bar{A})^T \right\rangle (\bar{U}^{(0)} \otimes \bar{U}^{(0)})$$

**Second-order correction term**

Limit: Also time-consuming
Computational Bottleneck

- High dimensional infinite integration in $F$

\[ F_{ik}^{nm} = \langle A_{ik} A_{mn} \rangle - \bar{A}_{ik} \bar{A}_{mn} \]

\[ \langle A_{ik} A_{mn} \rangle = \iiint dy_i dy_k dy_m dy_n P_4(y_i, y_k, y_m, y_n) G(y_i, y_k) G(y_m, y_n) \]

- Standard technique: Gauss Hermite quadrature
  - Complexity grows \textbf{exponentially} with the integral dimension (Curse of dimensionality)

  \[
  1 - \text{dimension} \rightarrow O(N) \\
  4 - \text{dimension} \rightarrow O(N^4)
  \]
More than 30 quadrature points for 1D integration (81000 points for 4D)
Improved Formulation of SIE (2D case)

Translation invariance of Green’s function

\[ G(y_i, y_k) = G(y_i, y_i + y_d) = G(0, y_d) \]

Thus

\[ G(y_i, y_j) = \hat{G}(y_d) \]

\[ \bar{A}_{ik} = \langle G(y_i, y_k) \rangle = \int \int dy_i dy_k P_2(y_i, y_k) G(y_i, y_k) \quad (2D) \]

\[ \bar{A}_{ik} = \langle \hat{G}(y_d) \rangle = \int dy_d P_{1d}(y_d) \hat{G}(y_d) \quad (1D) \]

\[ P_{1d} \text{ – Probability density function of } y_d \]
Partial Probability Density Function

\[
P_2(y_i, y_k) = \frac{1}{2\pi h^2 \sqrt{1-c^2}} \exp\left(-\frac{y_i^2 - 2cy_i y_k + y_k^2}{2h^2 (1-c^2)}\right)
\]

\[
\tilde{P}_2(y_i, y_d) = \frac{1}{2\pi h^2 \sqrt{1-c^2}} \exp\left(-\frac{y_i^2 - 2cy_i (y_i + y_d) + (y_i + y_d)^2}{2h^2 (1-c^2)}\right)
\]

\[
P_{1d}(y_d) = \int \mathrm{d}y_i \tilde{P}_2(y_i, y_i + y_d)
\]

\[
= \frac{1}{2h \sqrt{\pi (1-c)}} \exp\left(-\frac{y_d^2}{4h^2 (1-c)}\right)
\]
Improved Formulation of SIE (4D case)

\[ \langle A_{ik} A_{mn} \rangle = \int \int \int \int dy_i dy_k dy_m dy_n P_4(y_i, y_k, y_m, y_n)G(y_i)G(y_k)G(y_m)G(y_n) \]

Let

\[ y_{d1} = y_k - y_i \quad y_{d2} = y_n - y_m \]

\[ \langle A_{ik} A_{mn} \rangle = \int dy_{d1} dy_{d1} P_{2d}(y_{d1}, y_{d2}) \hat{G}(y_{d1})\hat{G}(y_{d2}) \]

Where

\[ P_{2d}(y_{d1}, y_{d2}) = \int dy_{d1} dy_{d2} P_4(y_i, y_i + y_{d1}, y_m, y_m + y_{d2}) \]
Results
Numerical vs Analytical Formulation
(Different Correlation Length)
SIE vs MIE (Mean $\rho_e$ Ratio)

Gaussian surface ($\sigma = 1\mu m$)
SIE vs MIE (Mean \( \mu_e \) Ratio)

Gaussian surface \((\sigma = 1\mu m)\)
## CPU Time Comparison
(Unit: second)

<table>
<thead>
<tr>
<th>Method</th>
<th>$h / \delta = 1$</th>
<th>$h / \delta = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI E (1500run)</td>
<td>7160.3</td>
<td>14484.7</td>
</tr>
<tr>
<td>SI E (Original)</td>
<td>6367.3</td>
<td>6544.7</td>
</tr>
<tr>
<td>SI E (Modified)</td>
<td>198.7</td>
<td>200.3</td>
</tr>
</tbody>
</table>

(Gaussian rough surface --- $\sigma = 1 \mu m$, $\eta = 1 \mu m$)

32X
Conclusion

An efficient numerical approach for modeling the impact of surface roughness on interconnect internal impedance
Numerical effective parameters
- Model the rough surface effects on internal impedance
- Take all statistical information into account

Modified SIE method
- One-pass solution for mean values
- Halve infinite integral dimension by partial PDF formulation
Thank you!