Efficient Numerical Modeling of Random Rough Surface Effects in Interconnect Internal Impedance Extraction

CHEN Quan & WONG Ngai

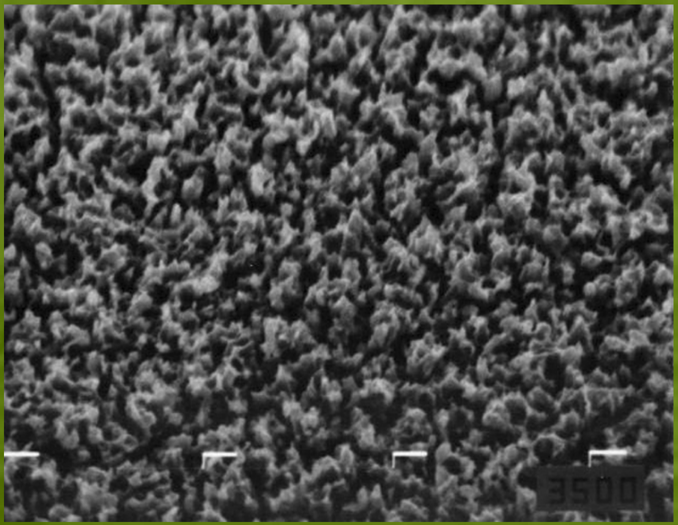
Department of Electrical & Electronic Engineering

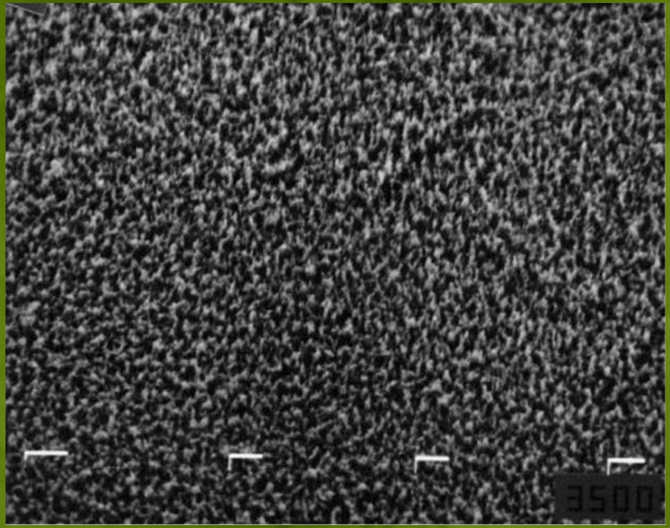
The University of Hong Kong

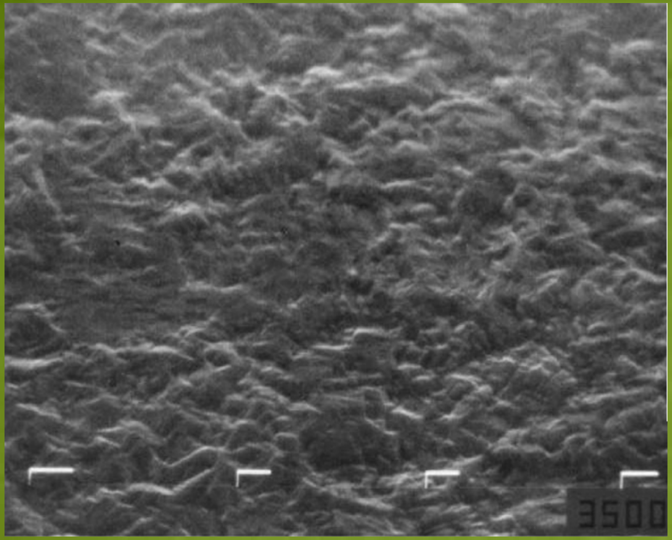
Outline

- Background
- Modeling (Effective Parameters)
- Computation (Modified SIE Method)
- Results
- Conclusion

Surface Roughness in Interconnects

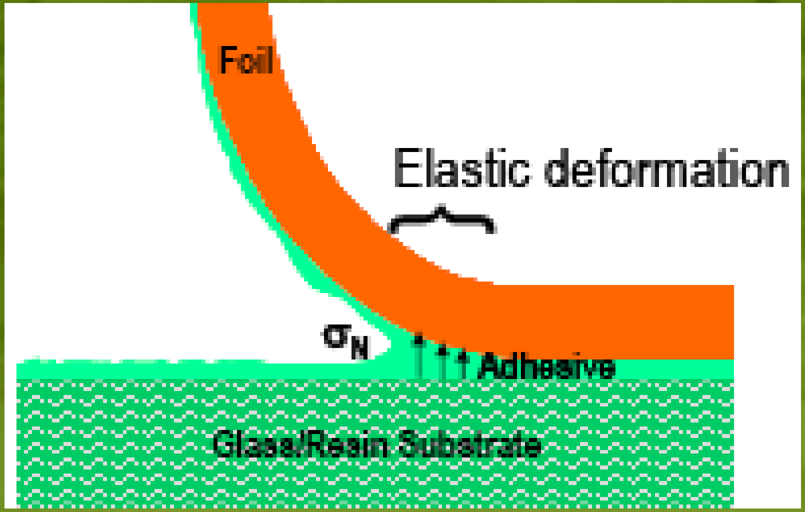






Sources of Surface Roughness

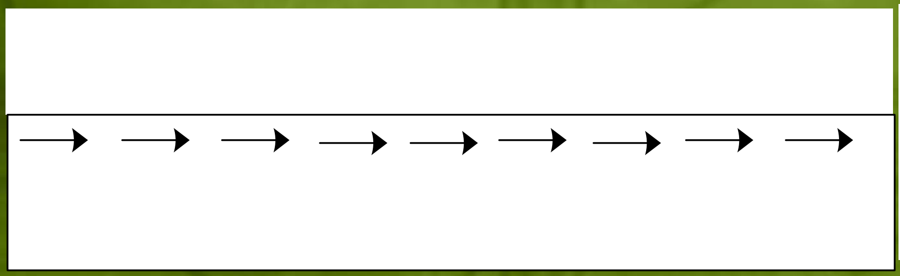
- Unintentional sources
  - Technology limitations
  - Process variations
- Intentional sources
  - Electronic deposition
  - Chemical etching
  - Annealing



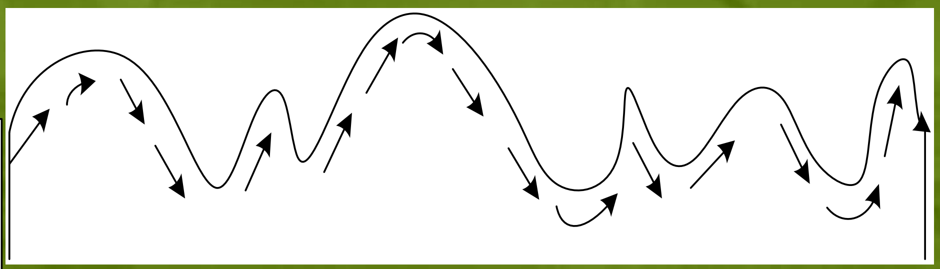
Enhance the cohesion between metal and dielectric

Impact of Surface Roughness on Internal Impedance

Interaction between rough surface and current



Current under smooth surface



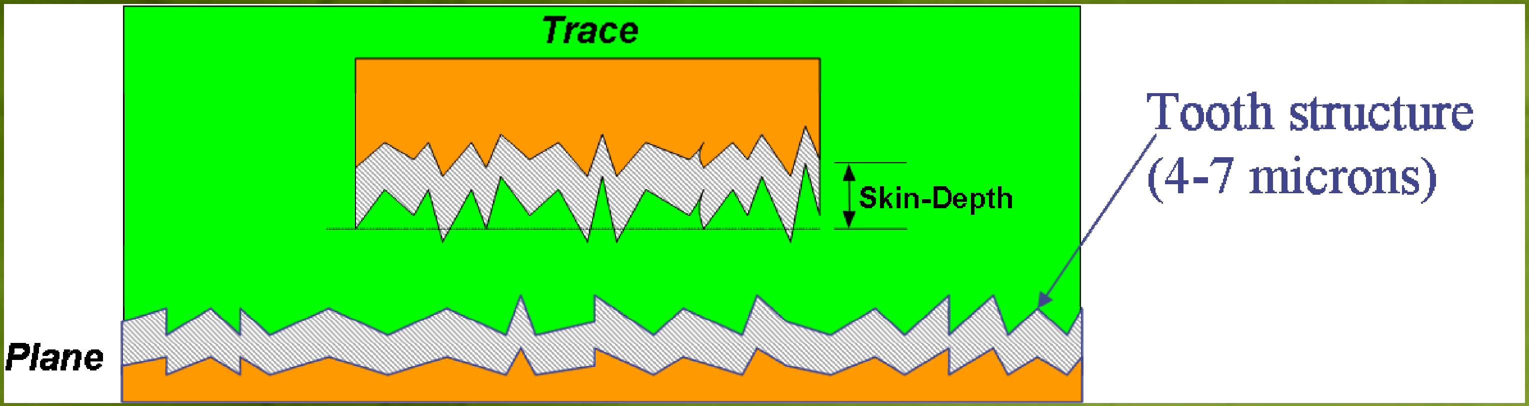
Current under rough surface

Longer current path → More resistive loss → Higher resistance

Longer current path → Larger current loop → Higher internal inductance

High Frequency Effects

- Rough surface effects is insignificant in low frequencies (large skin depth, small roughness)
- It becomes significant in high frequencies (comparable skin depth and roughness)



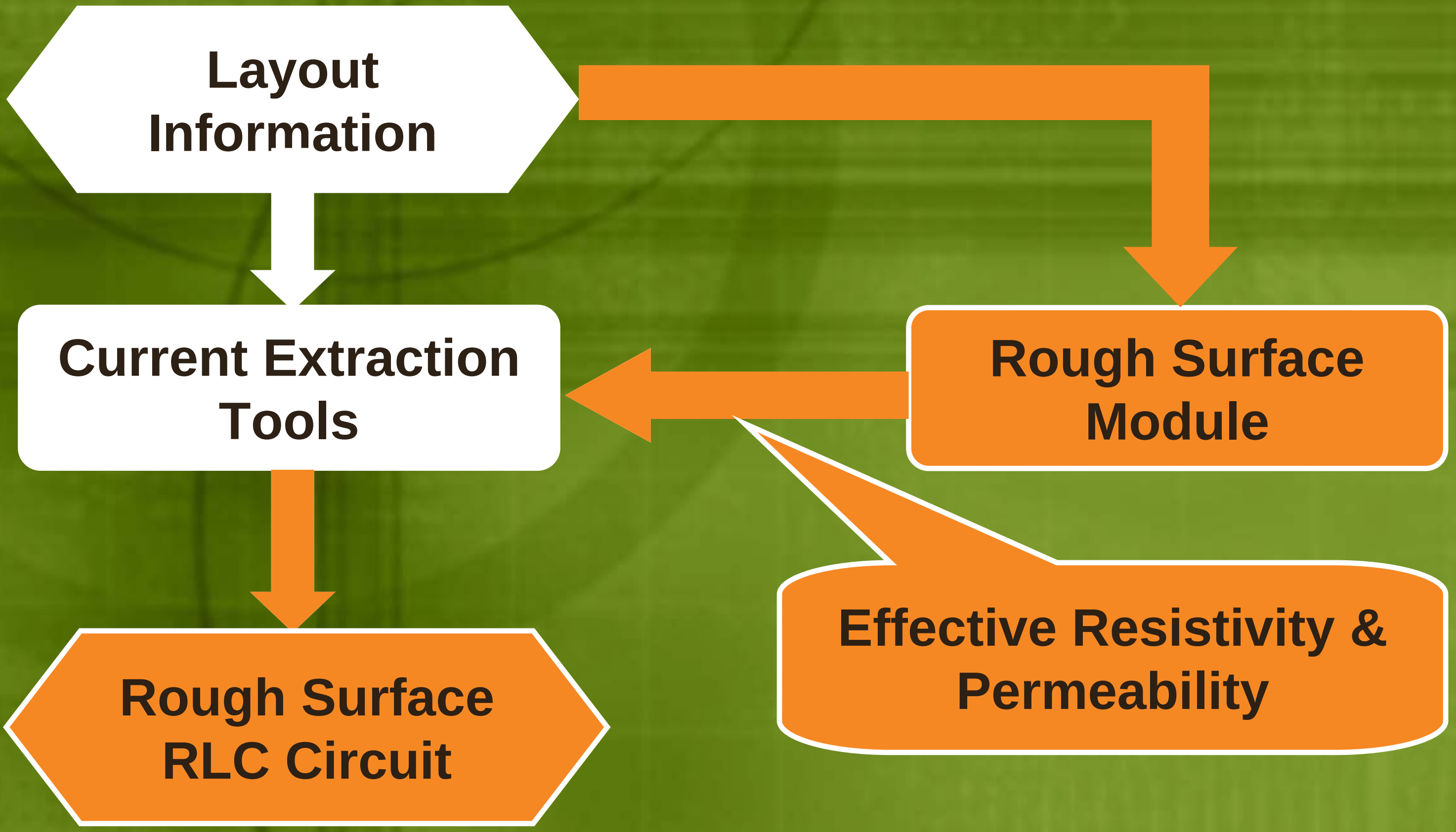
Skin depth = 6.5microns when f = 0.1 GHz (From Intel)

Modeling

Model the impact of random rough surface on interconnect internal impedance

Effective Parameters

- Effective Resistivity ρ_e & Effective Permeability μ_e
- Capture the increase of resistance and internal inductance caused by surface roughness



Analytical Formulation

- For effective resistivity

$$\rho_e = \rho\left[1 + \frac{2}{\pi}\tan^{-1}\left(1.4\frac{h}{\delta}\right)^2\right]$$

  - h – RMS height; δ - skin depth
  - Widely used in practical design, BUT
  - Inaccurate (only h is considered)
- For effective permeability
  - Unavailable

Numerical Formulation of ρ_e

Smooth surface power loss

$$P_s = \frac{\rho |H_0|^2 l}{2\delta}$$

Rough surface power loss

$$P_r = \frac{\rho H_0^*}{2} \mathrm{Re}\left\{ \int_{\tilde{S}} \mathrm{d}z U(z) \right\}$$

Power loss equivalence $\boldsymbol{P_s = P_r}$

$$\rho_e = \frac{2\delta P_r}{|H_0|^2 l} = \frac{\rho\delta}{H_0 l} \mathrm{Re}\left\{ \underbrace{\int_{\tilde{S}} \mathrm{d}z U(z)}_{V} \right\}$$

Numerical Formulation of μ_e

Smooth surface magnetic energy

$$W_s = \frac{\mu\delta |H_0|^2 l}{2}$$

Rough surface magnetic energy

$$W_r = \frac{\mu\delta^2 H_0^*}{2} \mathrm{Im}\left\{\int_{\tilde{S}} \mathrm{d}z U(z)\right\}$$

Magnetic energy equivalence $W_s = W_r$

$$\mu_e = \frac{2W_r}{\delta |H_0|^2 l} = \frac{\mu\delta}{H_0 l} \mathrm{Im}\left\{\underbrace{\int_{\tilde{S}} \mathrm{d}z U(z)}_{V}\right\}$$

Governing Equation

$$\int_{\tilde{S}} \mathrm{d}z G(z,z')U(z) = \frac{1}{2}H_0 + H_0 \int_{cp} \mathrm{d}z \sqrt{1+\left(\frac{\partial y(z)}{\partial z}\right)^2} \frac{\partial G(z,z')}{\partial \hat{n}}$$

Green's function $G(z,z') = H_0^{(1)}(k_1\sqrt{|z-z'|})$

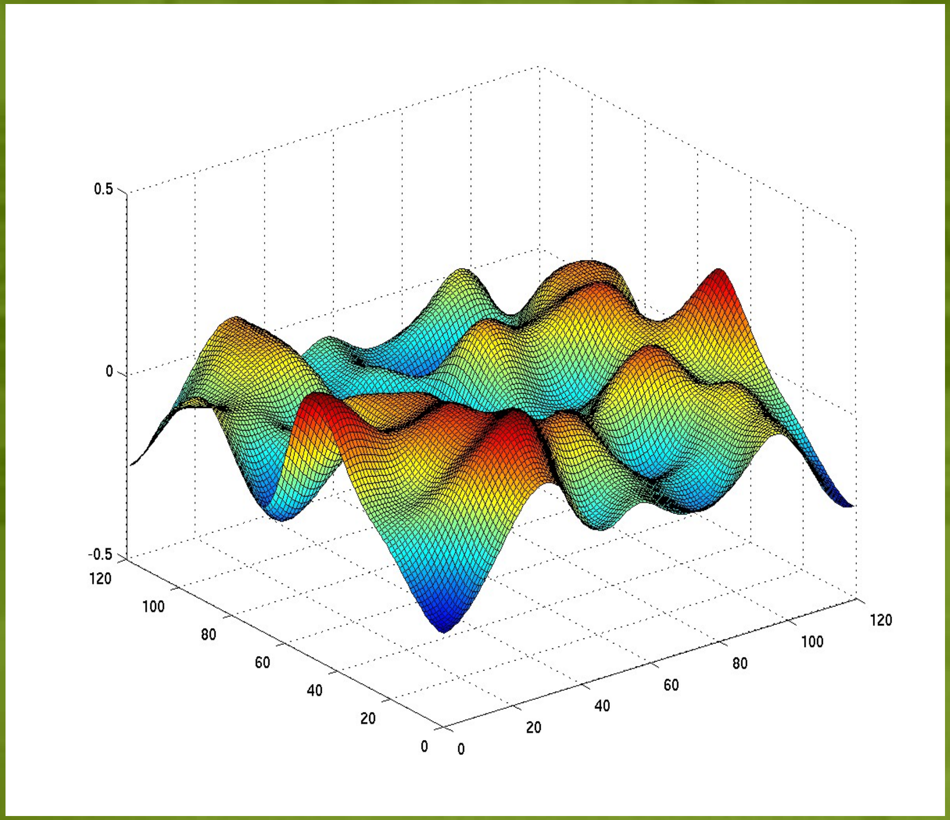
Surface unknown $U(z) = \sqrt{1+\left(\frac{\partial y(z)}{\partial z}\right)^2} \frac{\partial H(z)}{\partial \hat{n}}$

Boundary condition

$$H(r) = H_0 \quad r \in S$$

Modeling of Random Rough Surface

- Most rough surfaces in reality are random
- Described by **stationary stochastic process** with **Probability Density Function** and **Correlation Function**

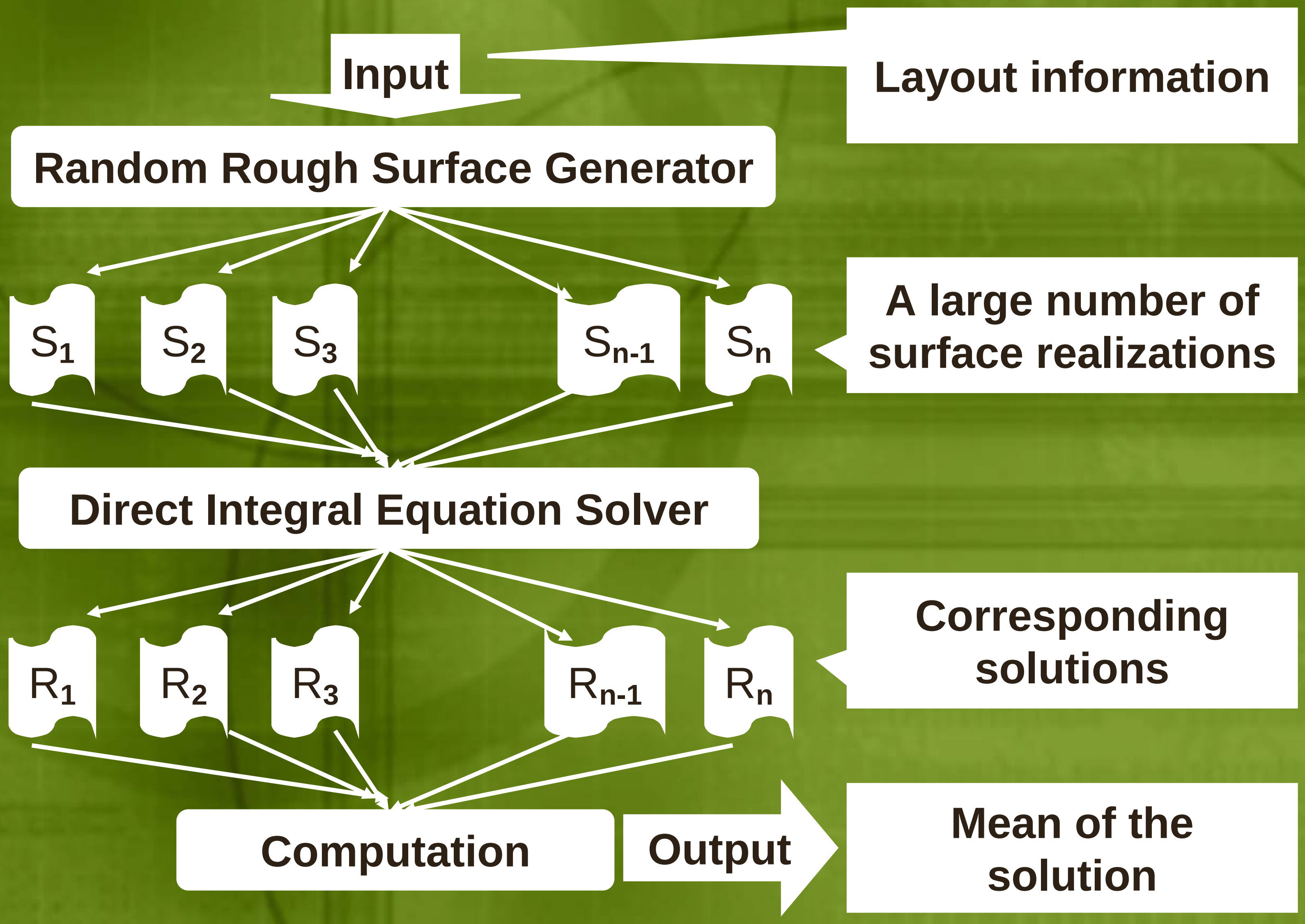


$$P_1(y) = \frac{1}{\sqrt{2\pi h}} \exp\left(-\frac{z^2}{2h^2}\right)$$

$$C_g(z_1, z_2) = \exp\left(-\frac{|z_1 - z_2|^2}{\eta^2}\right)$$

η --- correlation length

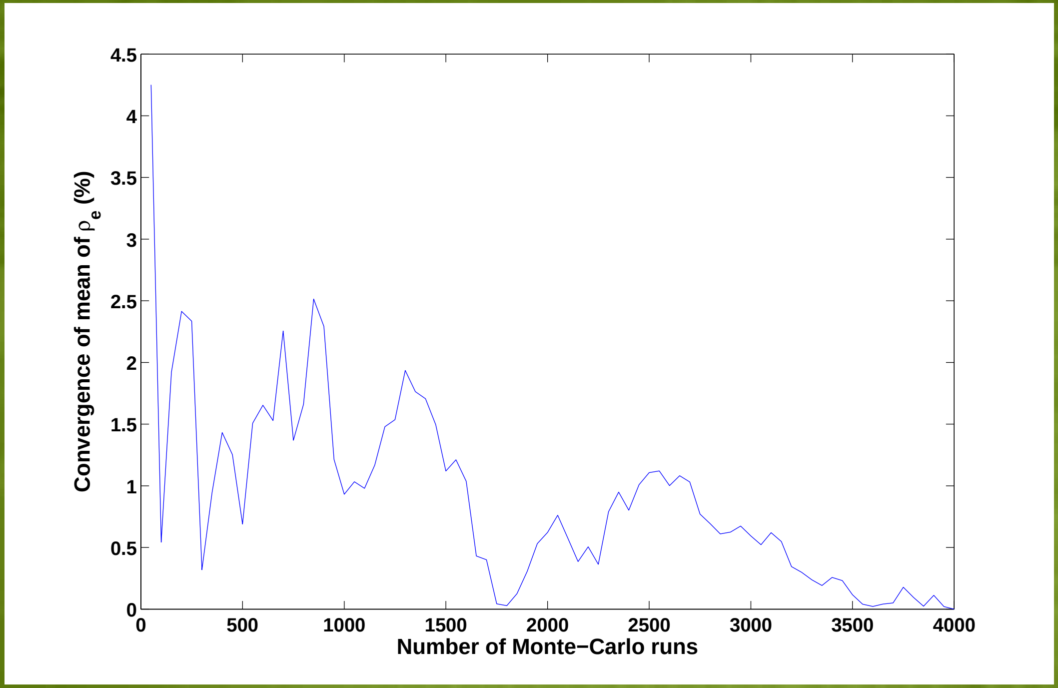
Conventional Statistical Solver -- Monte-Carlo Method



Limitations of Monte-Carlo method

- Probabilistic nature
  - Mean value is also a random variable

- Slow convergence
  - More than 2500 runs to converge within 1%



Computation

Efficient Stochastic Integral Equation (SIE) method

- **Stochastic Integral Equation (SIE) method**
  - Use mean value as unknown
  - One-pass solution
  - Deterministic nature

- **Two steps**
  - Zeroth-order approximation (Uncorrelatedness assumption)
  - Second-order correction

Zeroth-Order Approximation

Directly applying ensemble average on both sides

$$\int_{\tilde{S}} dz \left\langle G(z,z')U(z) \right\rangle = \frac{1}{2}H_0 + H_0 \int_{cp} dz \left\langle \sqrt{1+\left(\frac{\partial y(z)}{\partial z}\right)^2} \frac{\partial G(z,z')}{\partial \hat{n}} \right\rangle$$

Ensemble average $\langle f(x) \rangle = \int_{-\infty}^{+\infty} P_1(f(x)) f(x)$

Assuming the Green's function G and the surface unknown U are statistically independent (Uncorrelatedness Assumption)

$$\int_{\tilde{S}} dz \left\langle G(z,z') \right\rangle \left\langle U(z) \right\rangle = \frac{1}{2}H_0 + H_0 \int_{cp} dz \left\langle \sqrt{1+\left(\frac{\partial y(z)}{\partial z}\right)^2} \frac{\partial G(z,z')}{\partial \hat{n}} \right\rangle$$

New unknown

Zeroth-order Approximation

Matrix equation format

$$\bar{A}\bar{U} = \bar{L}$$

$\langle G(z,z')\rangle$

$\langle U(z)\rangle$

Deterministic quantities

$$\bar{A}_{ik} = \langle G(z_i, z_k)\rangle = \iint \mathrm{d}y_i \mathrm{d}y_k P_2(y_i, y_k) G(y_i, y_k; z_i, z_k)$$

Inaccurate zeroth-order approximation

$$\bar{U}^{(0)} = \bar{A}^{-1}\bar{L} \qquad \bar{V}^{(0)} = \bar{L}^T \bar{U}^{(0)}$$

Cause: uncorrelatedness assumption

$$\langle G(z,z')U(z)\rangle \neq \langle G(z,z')\rangle\langle U(z)\rangle$$

Primitive Second-Order Correction

- Improve the accuracy of the mean V

$$\bar{V} = \bar{V}^{(0)} + \bar{V}^{(2)}$$

Zeroth-order approximation term

Second-order correction term

$$\bar{V}^{(2)} = trace(\bar{A}^{-T} D)$$

$$vec(D) = \underbrace{\langle (A-\bar{A}) \otimes (A-\bar{A})^T \rangle}_{F_{N^2 \times N^2}} (\bar{U}^{(0)} \otimes \bar{U}^{(0)})$$

Limit: Also time-consuming

Computational Bottleneck

- High dimensional infinite integration in ***F***

4-D infinite integration

$$F_{nm}^{ik} = \langle A_{ik} A_{mn} \rangle - \bar{A}_{ik} \bar{A}_{mn}$$

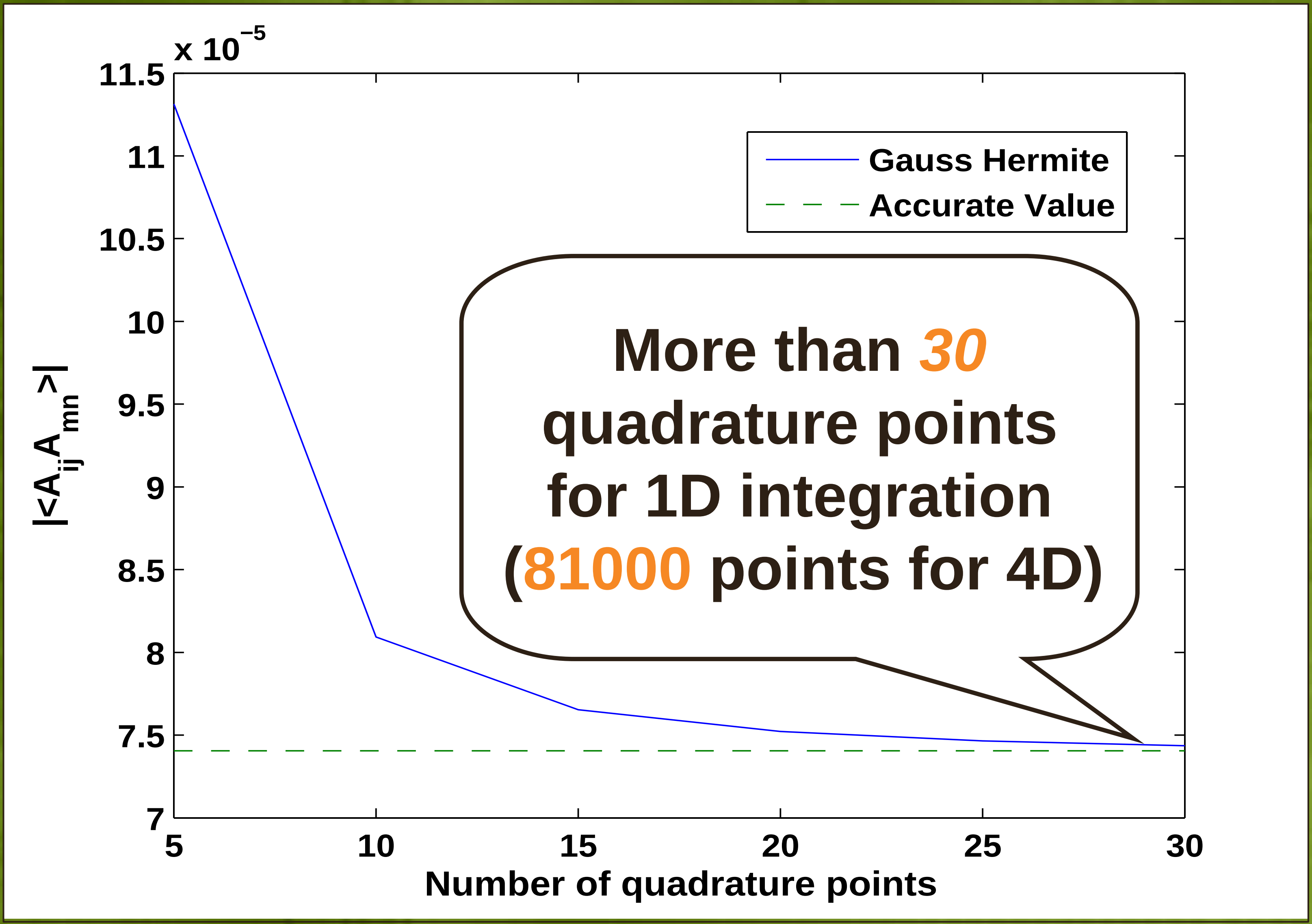
$$\langle A_{ik} A_{mn} \rangle = \iiiint \mathrm{d}y_i \mathrm{d}y_k \mathrm{d}y_m \mathrm{d}y_n P_4(y_i, y_k, y_m, y_n) G(y_i, y_k) G(y_m, y_n)$$

- Standard technique: Gauss Hermite quadrature
  - Complexity grows **exponentially** with the integral dimension (Curse of dimensionality)

$$1-\text{dimension} \rightarrow O\left(N\right)$$

$$4-\text{dimension} \rightarrow O\left(N^4\right)$$

No. of Points for Gauss Hermite Quadrature



Improved Formulation of SIE (2D case)

Translation invariance of Green's function

$$G(y_i, y_k) = G(y_i, y_i + y_d) = G(0, y_d)$$

Thus $\quad G(y_i, y_j) = \hat{G}(y_d)$

$$\bar{A}_{ik} = \langle G(y_i, y_k) \rangle = \iint \mathrm{d}y_i \mathrm{d}y_k \mathrm{P}_2(y_i, y_k) G(y_i, y_k) \quad \text{(2D)}$$

$$\bar{A}_{ik} = \left\langle \hat{G}(y_d) \right\rangle = \int \mathrm{d}y_d \mathrm{P}_{1\mathrm{d}}(y_d) \hat{G}(y_d) \quad \text{(1D)}$$

P_{1d} – Probability density function of y_d

Partial Probability Density Function

$$P_2(y_i, y_k) = \frac{1}{2\pi h^2\sqrt{1-c^2}} \exp\left(-\frac{y_i^2 - 2c y_i y_k + y_k^2}{2h^2(1-c^2)} \right)$$

$$\tilde{\mathrm{P}}_2(y_i, y_d) = \frac{1}{2\pi h^2\sqrt{1-c^2}} \exp\left(-\frac{y_i^2 - 2c y_i (y_i + y_d) + (y_i + y_d)^2}{2h^2(1-c^2)} \right)$$

$$\mathrm{P}_{1\mathrm{d}}(y_d) = \int \mathrm{d}y_i \tilde{\mathrm{P}}_2(y_i, y_i + y_d)$$

$$= \frac{1}{2h\sqrt{\pi(1-c)}} \exp\left(\frac{-y_d^2}{4h^2(1-c)} \right)$$

Improved Formulation of SIE (4D case)

$$\langle A_{ik} A_{mn} \rangle = \iint \iint \mathrm{d}y_i \mathrm{d}y_k \mathrm{d}y_m \mathrm{d}y_n P_4(y_i, y_k, y_m, y_n) G(y_i, y_k) G(y_m, y_n)$$

Let $y_{d_1} = y_k - y_i \qquad y_{d_2} = y_n - y_m$

$$\langle A_{ik} A_{mn} \rangle = \iint \mathrm{d}y_{d_1} \mathrm{d}y_{d_1} \mathrm{P}_{2\mathrm{d}}(y_{d_1}, y_{d_2}) \hat{G}(y_{d_1}) \hat{G}(y_{d_2})$$

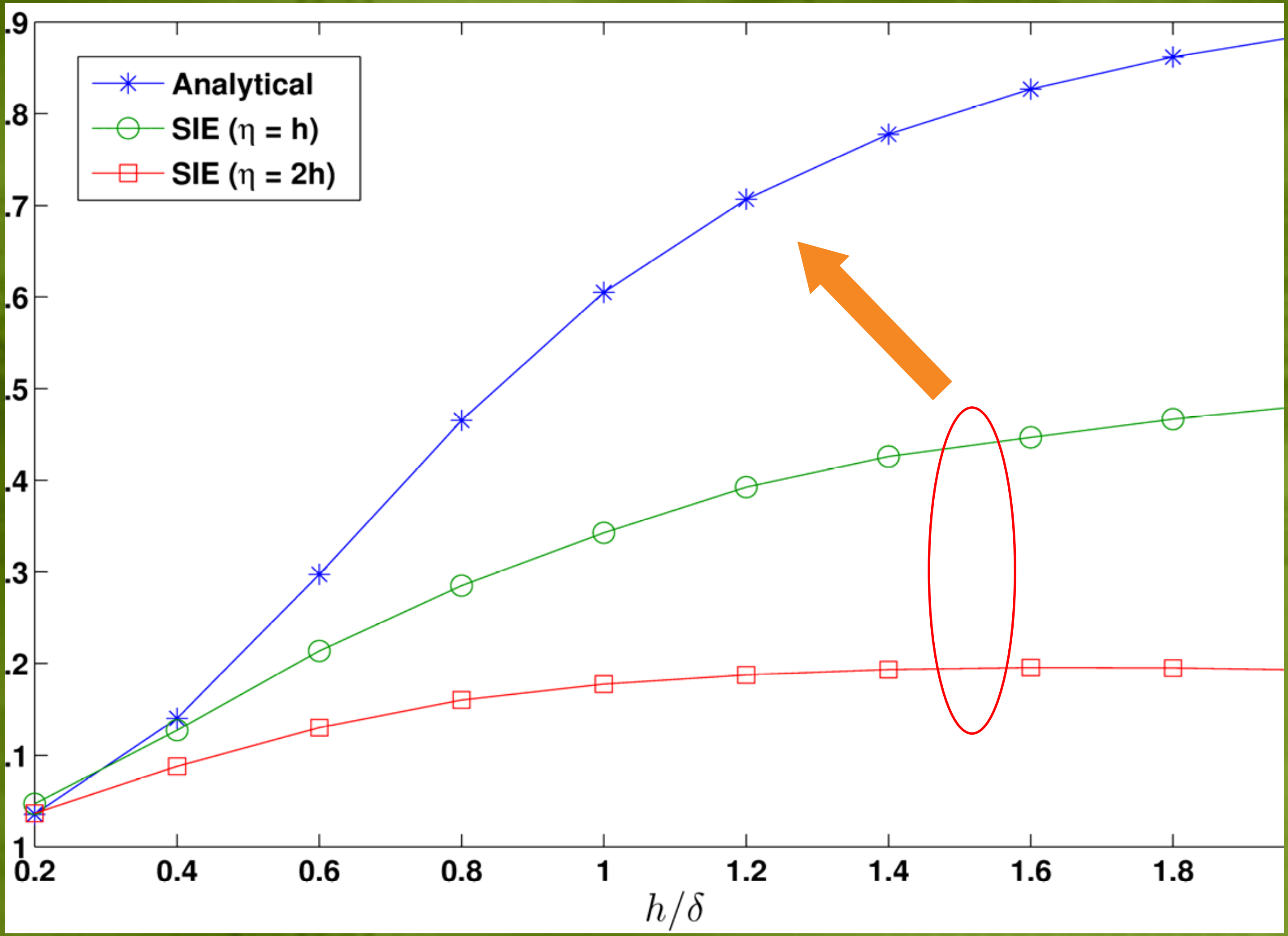
Where

$$\mathrm{P}_{2\mathrm{d}}(y_{d_1}, y_{d_2}) = \int \mathrm{d}y_{d_1} \mathrm{d}y_{d_2} \mathrm{P}_4(y_i, y_i + y_{d_1}, y_m, y_m + y_{d_2})$$

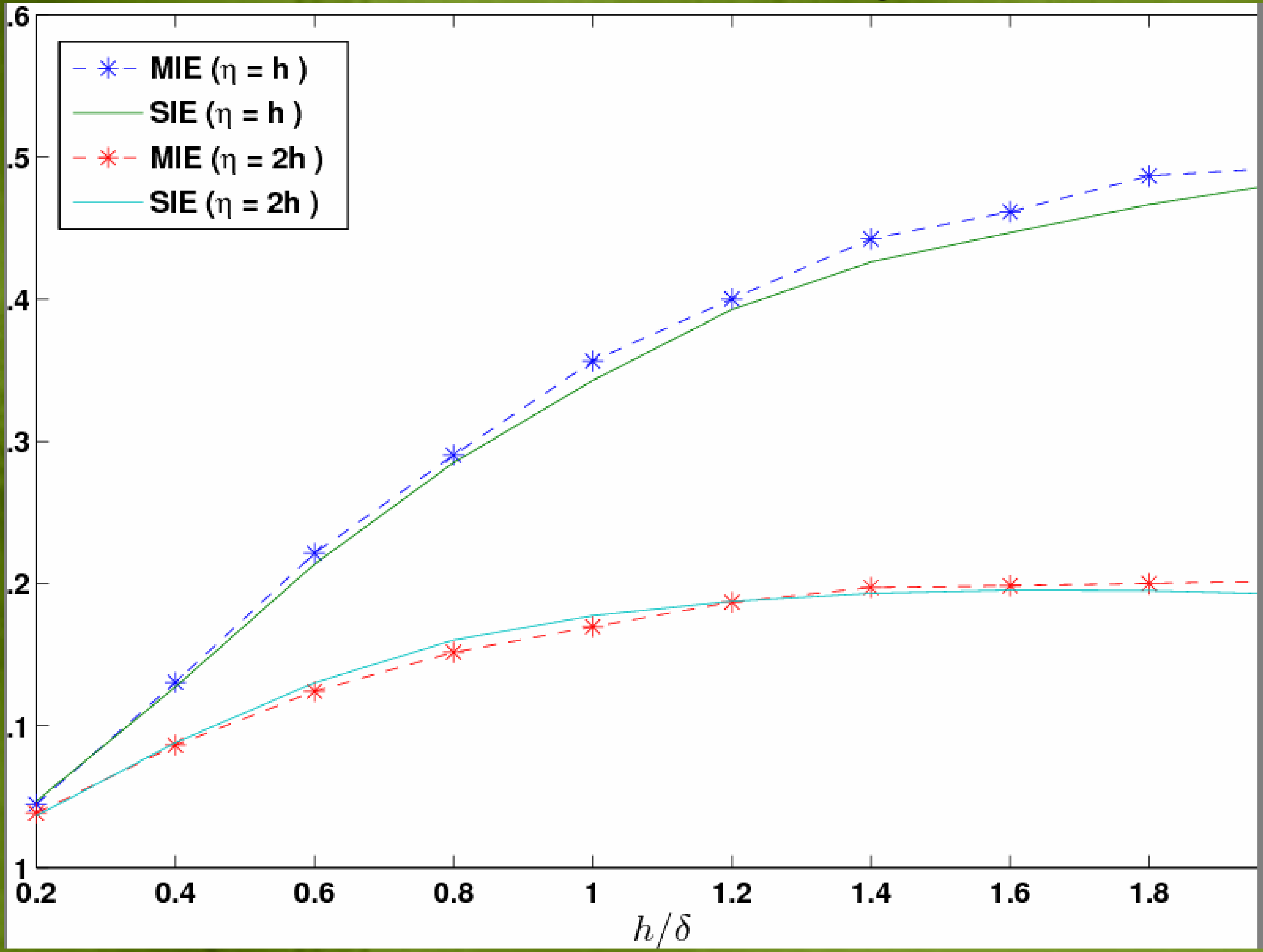
Results

Numerical *vs* Analytical Formulation

(Different Correlation Length)

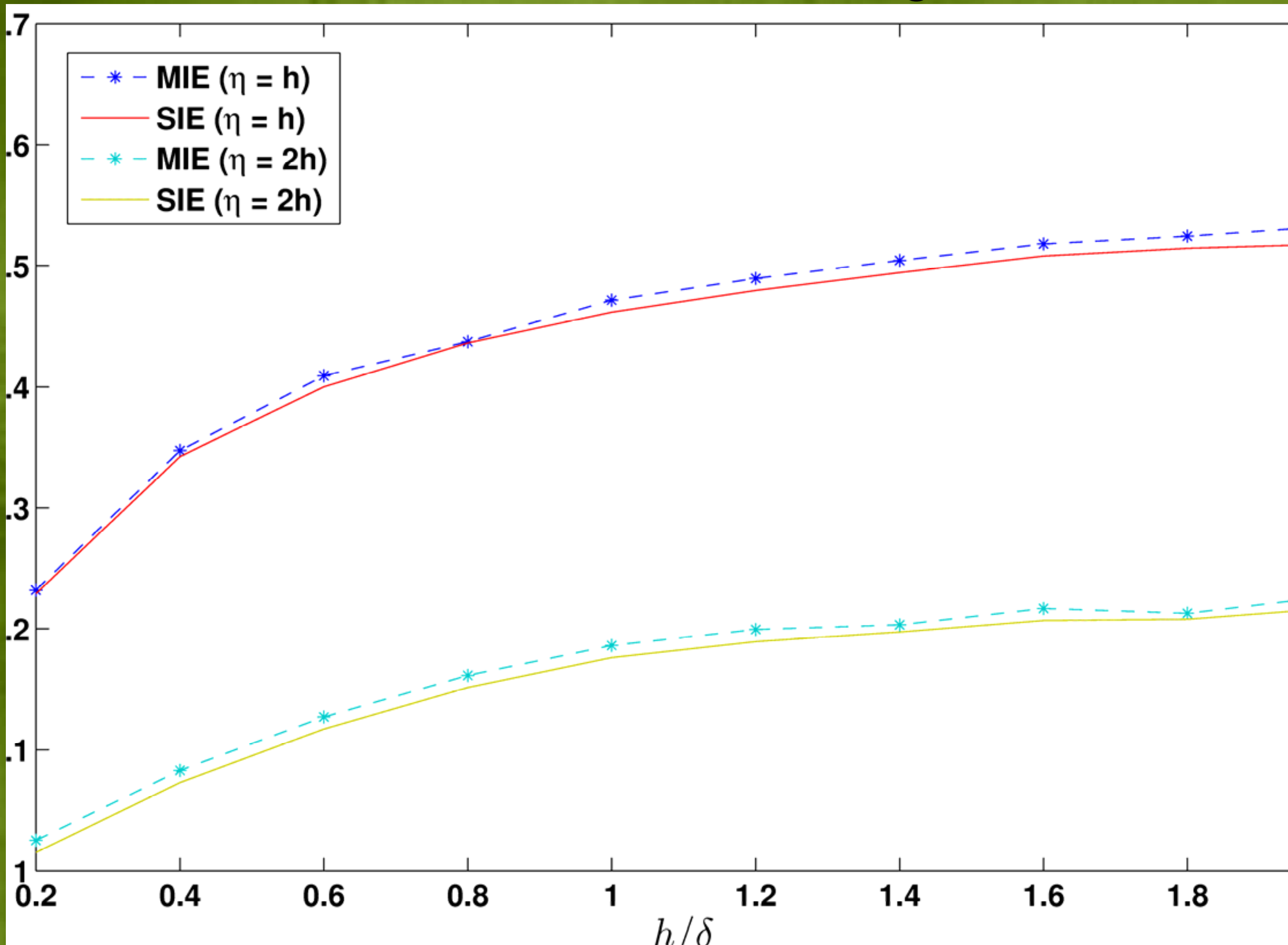


SIE *vs* MIE (Mean ρ_e Ratio)



Gaussian surface (σ = 1μm)

SIE *vs* MIE (Mean μ_e Ratio)



Gaussian surface (σ = 1μm)

CPU Time Comparison
(Unit: second)

Method	$h / \delta = 1$	$h / \delta = 2$
MIE (1500run)	7160.3	14484.7
SIE (Original)	6367.3	6544.7
SIE (Modified)	198.7	200.3

32X

(Gaussian rough surface --- σ=1μm, η=1 μm)

Conclusion

An efficient numerical approach for modeling the impact of surface roughness on interconnect internal impedance

Conclusion

- Numerical effective parameters
  - Model the rough surface effects on internal impedance
  - Take all statistical information into account

- Modified SIE method
  - One-pass solution for mean values
  - Halve infinite integral dimension by partial PDF formulation

Thank you !