Generating Stable and Sparse Reluctance/Inductance Matrix under Insufficient Conditions

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Background

VLSI’s

MNA Eqn.

\[
\begin{bmatrix}
G & A^T \\
- & 0 \\
0 & i(t)
\end{bmatrix}
\begin{bmatrix}
v(t) \\
i(t)
\end{bmatrix}
+ 
\begin{bmatrix}
C & 0 \\
0 & L
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
v(t) \\
i(t)
\end{bmatrix}
= b(t)
\]

Fast Simulation

Using Sparse Inductance/Reluctance Matrices
To speed up the transient simulation, we should make the inductance or reluctance matrix sparse.

Positive definiteness of the inductance or reluctance matrix must be guaranteed for stable circuit simulation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift &amp; Truncate</td>
<td>ICCAD1995</td>
</tr>
<tr>
<td>Double Inverse</td>
<td>DAC’01</td>
</tr>
<tr>
<td>INDUCTWISE</td>
<td>TCAD’03</td>
</tr>
<tr>
<td>Wire Duplication</td>
<td>TCAD’04</td>
</tr>
<tr>
<td>Block K</td>
<td>ASP-DAC’04</td>
</tr>
<tr>
<td>Band Matching</td>
<td>ASP-DAC’05</td>
</tr>
</tbody>
</table>
To get a stable and sparse reluctance matrix, off-diagonal elements of reluctance matrix must be negative.

Most previous works with guaranteed stability are based on a fine discretization of conductors.

- Inductance
- Reluctance

[Graphs showing Inductance and Reluctance distributions with column indices and values in Henrys.

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ASP-DAC'08]
Even if the off-diagonal elements are all negative, this does not mean positive definite.

\[
\begin{bmatrix}
  1 & -2 \\
  -2 & 1
\end{bmatrix} = \begin{bmatrix}
  -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
  \sqrt{2} & \sqrt{2}
\end{bmatrix}^T \begin{bmatrix}
  -1 & 0 \\
  0 & 3
\end{bmatrix} \begin{bmatrix}
  -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
  \sqrt{2} & \sqrt{2}
\end{bmatrix}
\]

To get the stable and sparse reluctance matrix using the previous methods, the conductors must be more finely discretized to make the reluctance matrix diagonally dominant.

we present how to obtain the stable and sparse reluctance/inductance matrix under insufficient discretization.
Generating Sparse Reluctance Matrix

Inductance (INDUCTWISE)          \( \text{reluctance} \)
\[
\begin{bmatrix}
1.04 & 0.34 & 0.37 & 0.24 & 0.51 \\
0.34 & 0.45 & 0.09 & 0.06 & 0.27 \\
0.37 & 0.09 & 1.04 & 0.34 & 0.41 \\
0.24 & 0.06 & 0.34 & 0.45 & 0.11 \\
0.51 & 0.27 & 0.41 & 0.11 & 1.69 \\
\end{bmatrix}
\quad
\begin{bmatrix}
1.57 & -0.94 & -0.22 & -0.47 & -0.25 \\
-0.94 & 3.02 & 0.15 & 0.01 & -0.23 \\
-0.22 & 0.15 & 1.42 & -0.93 & -0.24 \\
-0.47 & 0.01 & -0.93 & 3.12 & 0.16 \\
-0.25 & -0.23 & -0.24 & 0.16 & 0.75 \\
\end{bmatrix}
\]

Eigen Value \([0.4, 1.0, 1.4, 3.5, 3.7]\)

Negative off-diagonal elements are found relatively small.

Truncation affects a small change to the eigen values.

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Three methods for generating stable and sparse reluctance matrix are proposed.

- Direct Truncation
  is accurate but check of positive definiteness is necessary.
- Enforcing Positive Definiteness
- Enforcing Diagonally Dominance
  provide a provably positive definite reluctance matrix but are not accurate.
An off-diagonal element is truncated, if the absolute value is less than a threshold.

If the Cholesky decomposition is successfully done, the truncated matrix is positive definite.
Enforcing Positive Definiteness

The off-diagonal element is shifted and truncated and the absolute value is added to the diagonal.
Enforcing Diagonally Dominance

A diagonally dominant matrix can be extracted from the original matrix by removing the negative definite matrices.

Positive Definite and Diagonally Dominant

\[
\begin{bmatrix}
  3 & -2 & 1 \\
-2 & 4 & -4 \\
 1 & -4 & 1 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  2 & -2 & 0 \\
-2 & 2+4 & -4 \\
0 & -4 & 4 \\
\end{bmatrix} + \text{diag} \ (2,0,0)
\]

Negative Definite

\[
\begin{bmatrix}
  -1 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & -1 \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
  0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
\end{bmatrix} + \text{diag} \ (0,-2,-3)
\]
Sparse Reluctance Matrix

exact: no truncation
dt: direct truncation
ep: enforcing positive definiteness
edd: enforcing diagonally dominance
After getting the reluctance matrix, the matrix is again inverted. Then, we can obtain the sparse inductance matrix.
Sparse Inductance Matrix

exact: no truncation
inv: inverse of truncated reluctance matrix
$10^{-10}, 10^{-12}$: threshold of direct truncation
Neumann Series (1/2)

- Inversion & Direct Truncation
- Cholesky Decomposition

Dense

L → L⁻¹ → K → K⁻¹

Sparse

L⁻¹ → K → K⁻¹

Enforcing Diagonally Dominant

Neumann Series
Inversion of the reluctance matrix is done using the Neumann series.

Reluctance Matrix: \( \mathbf{K} = \mathbf{D} - \mathbf{N} \) \( \mathbf{D} \): Diagonal Matrix

Neumann Series: \( \mathbf{K}^{-1} \approx \mathbf{L}_p = \left( \sum_{i=0}^{p} \left( \mathbf{D}^{-1} \mathbf{N} \right)^i \right) \mathbf{D}^{-1} \)
Example

Division into Unit Cells

Inductance and Capacitance Extraction

Reluctance:
- $500 \times 500$: 7.8% positive
- $493 \times 493$: 7.0% positive

Capacitance:
- $568 \times 568$: diagonally dominant
Package Model (R, L Matrices)

PCB Model (L or K, C, R Matrices)

Sparse Approximation

SPICE Transient Analysis
Transient Response (Reluctance) \(1/2\)

Threshold: \(10^9\)

Threshold: \(10^8\)

\[ \text{exact} \quad \text{dt} \quad \text{ep} \quad \text{edd} \]

\[ \text{dt: direct truncation, ep: enforcing positive definiteness, edd: enforcing diagonally dominance} \]

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Transient Response (Reluctance) 2/2

Threshhold: $10^9$

Threshhold: $10^8$

dt: direct truncation, ep: enforcing positive definiteness, edd: enforcing diagonally dominance

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Transient Response (Inductance)

Direct Truncation

- exact
- $dt$ (reluctance)
- $10^{-10}$
- $10^{-12}$

Neumann Series

- exact
- edd (reluctance)
- Neumann

$dt$: direct truncation, edd: enforcing diagonally dominance

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## CPU Time Comparison

### Reluctance

<table>
<thead>
<tr>
<th>Method</th>
<th>Sparsity [%]</th>
<th>CPU times [sec.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Truncation</td>
<td>0.70</td>
<td>61</td>
</tr>
<tr>
<td>Direct Truncation</td>
<td>2.10</td>
<td>1,090</td>
</tr>
<tr>
<td>Enforcing Positive Definiteness</td>
<td>0.70</td>
<td>51</td>
</tr>
<tr>
<td>Enforcing Positive Definiteness</td>
<td>2.10</td>
<td>1,185</td>
</tr>
<tr>
<td>Enforcing Diagonally Dominance</td>
<td>0.68</td>
<td>53</td>
</tr>
<tr>
<td>Enforcing Diagonally Dominance</td>
<td>1.82</td>
<td>867</td>
</tr>
</tbody>
</table>

### Inductance

<table>
<thead>
<tr>
<th>Method</th>
<th>Sparsity [%]</th>
<th>CPU times [sec.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>100</td>
<td>21,108</td>
</tr>
<tr>
<td>Direct Truncation</td>
<td>0.38</td>
<td>10</td>
</tr>
<tr>
<td>Direct Truncation</td>
<td>6.87</td>
<td>847</td>
</tr>
<tr>
<td>Neumann Series</td>
<td>2.01</td>
<td>175</td>
</tr>
</tbody>
</table>
Conclusions

- Generating the sparse and stable reluctance/inductance matrices with insufficient conditions is presented.
- When the discretization of conductors is not fine, we find positive-off diagonal elements.
- Even if the positive-off diagonal elements were found, the positive definiteness was guaranteed after truncation.
- The key techniques are as follows:
  1) check of positive definiteness by Cholesky decomposition
  2) enforcing positive definiteness
  3) enforcing diagonally dominance
  4) Neumann series
SPICE Model of Reluctance (Block K Method)

A row of reluctance matrix is represented with a inductance and voltage control voltage sources.

Relation between current and voltage

\[ V = sLI \rightarrow sI = KV \]

\[ sI_i = \sum_{j=1}^{N} K_{ij} V_j \rightarrow V_i = s \left[ \frac{1}{K_{ii}} I_i - \sum_{j=1, j \neq i}^{N} \frac{K_{ij}}{K_{ii}} V_j \right] \]

inductance voltage control voltage sources