



# Statistical Gate Delay Model for Multiple Input Switching

---

Takayuki Fukuoka, Akira Tsuchiya  
and Hidetoshi Onodera  
Kyoto University



# Outline

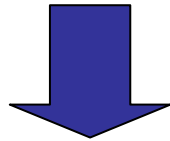
---

- **Motivation**
- Effect of Multiple Input Switching on the gate delay
- Gate Delay Model for Multiple Input Switching
- Experimental results
- Conclusions

# Motivation

---

- Technology scaling
  - Increasing significance of process variations
  - Corner-based timing analysis become too pessimistic

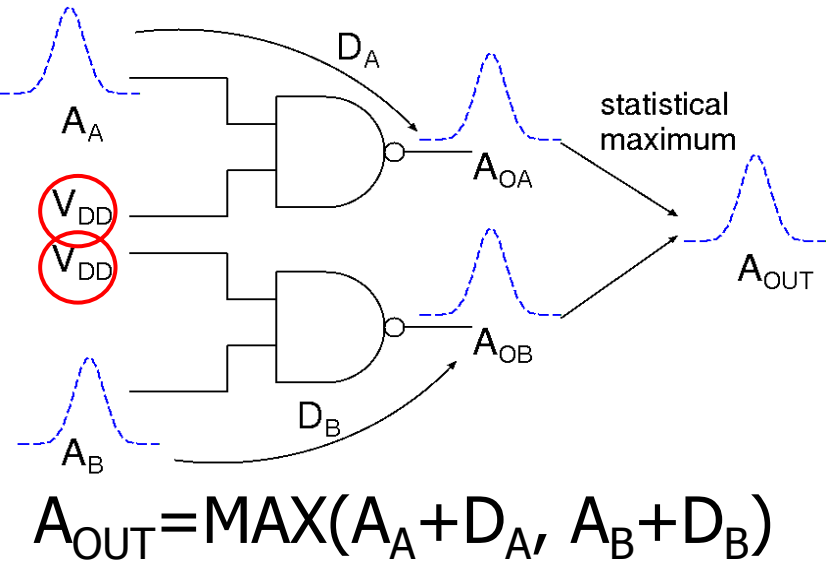


- Statistical Static timing analysis (SSTA)
  - Statistical Max operation
  - Multiple input switching

# Multiple Input Switching

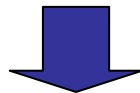
## ■ Single Input Switching (SIS)

- Only one of inputs switches
- SIS assumption is correct when one input transition is away from the other



## ■ Multiple Input switching (MIS)

- Two inputs switch at the same time
- Gate delay is different from that of SIS



We propose gate delay model considering MIS effect



# Outline

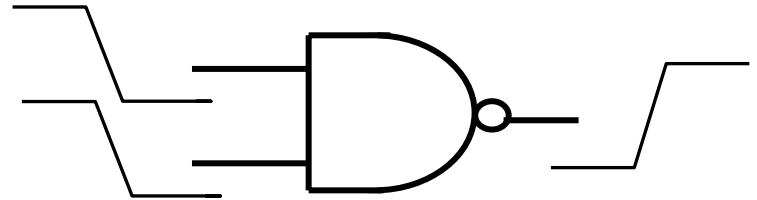
---

- Motivation
- Effect of Multiple Input Switching on the gate delay
- Gate Delay Model for Multiple Input Switching
- Experimental results
- Conclusions

# Multiple Input Switching

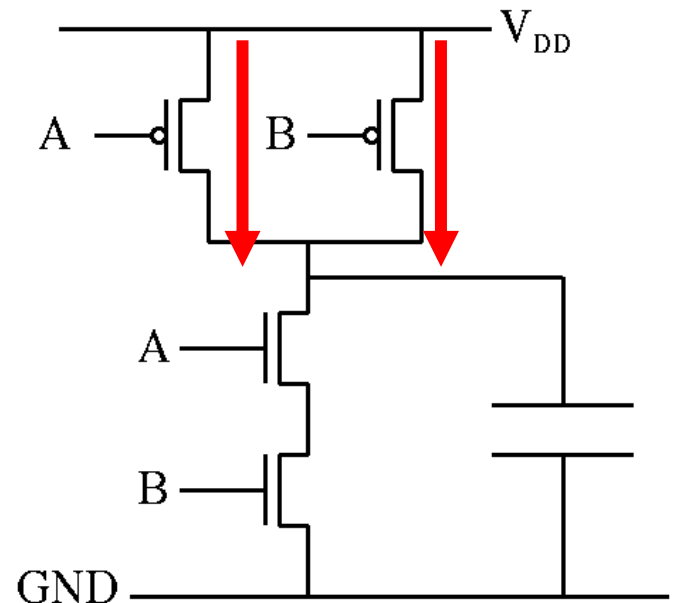
## ■ SIS

- One p-channel transistor turns on



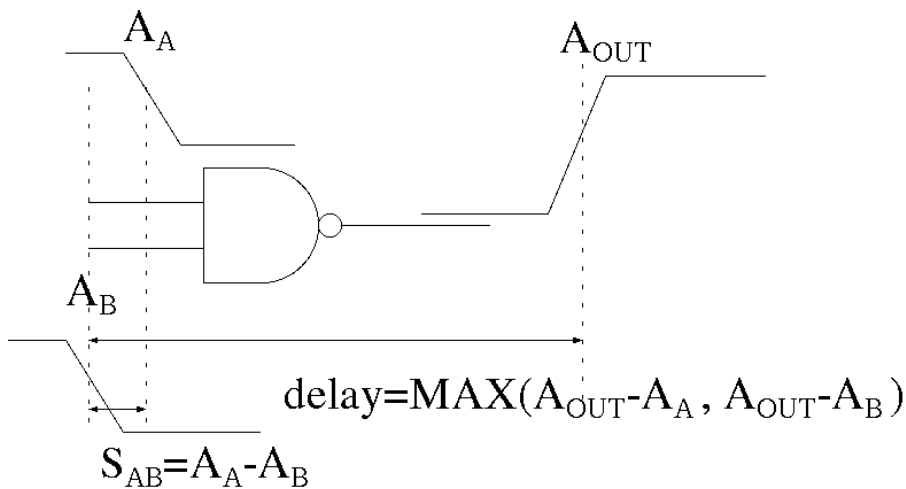
## ■ MIS

- Inputs A and B rise simultaneously
- Two P-channel transistors turn on
- Actual gate delay becomes smaller than that of SIS



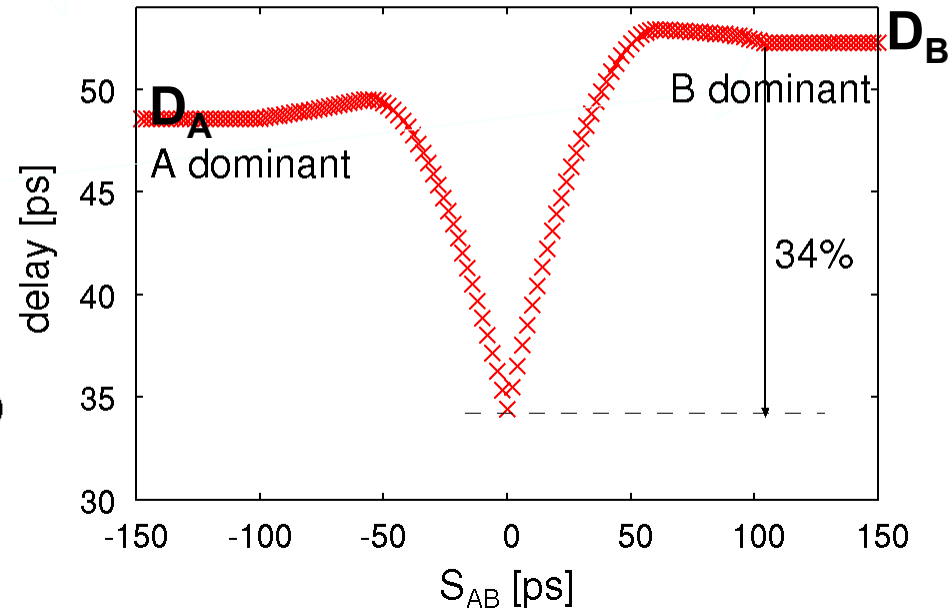
# Example of MIS effect

- Output rise when input A or B fall



$A_A / A_B$ : Arrival time of input A / B

$S_{AB}$ : Separation between  $A_A$  and  $A_B$



Input transition time: 100ps  
Output capacitance: FO4

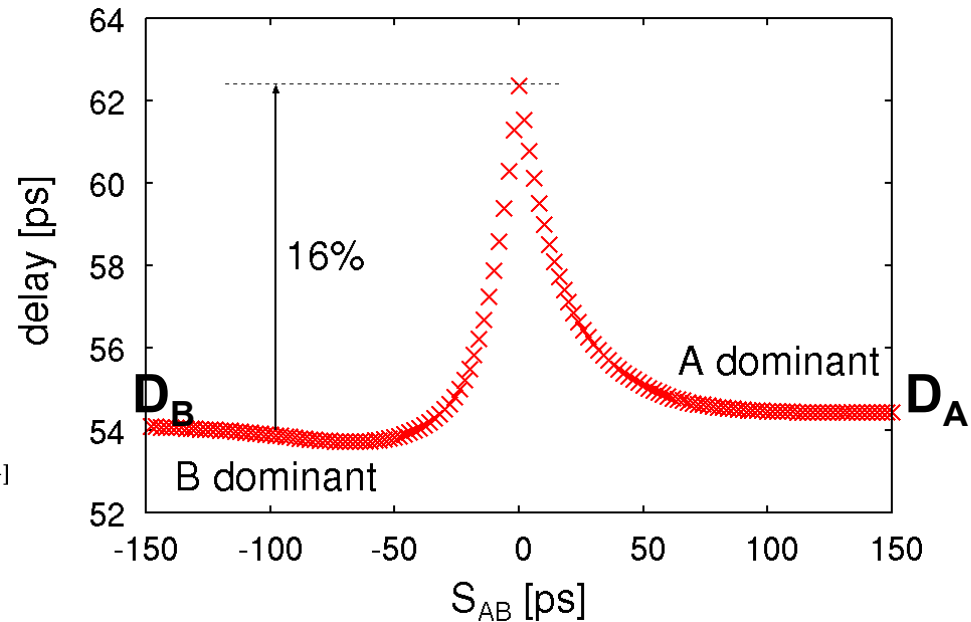
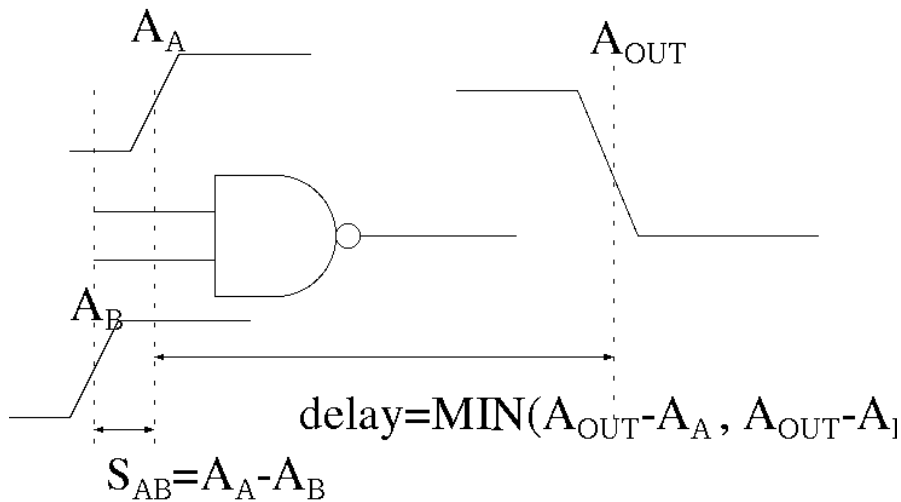
SIS:  $D_A$  49ps  
 $D_B$  53ps



Actual gate delay changes by  $S_{AB}$   
up to 34%

# Example of MIS effect

- Output fall when both inputs A and B rise



- Actual gate delay increases as  $S_{AB}$  gets close to 0
- MIS also affects output transition time



**It is necessary to consider MIS effect**





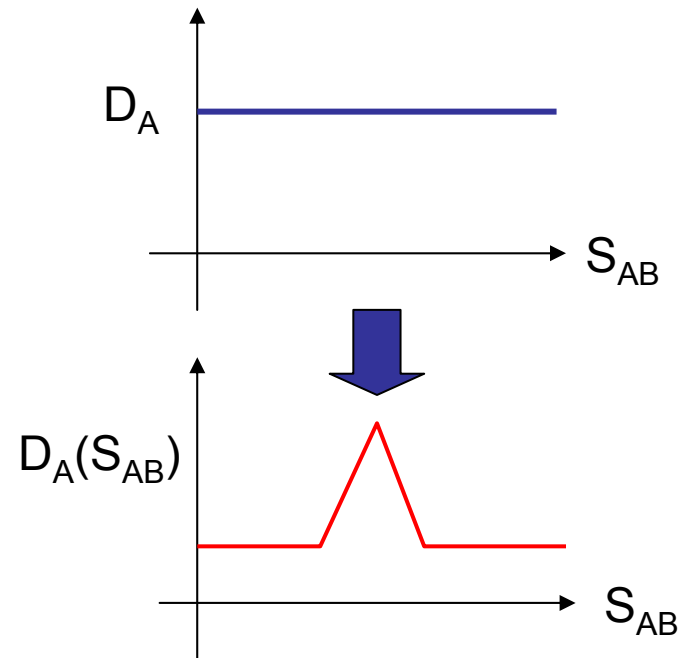
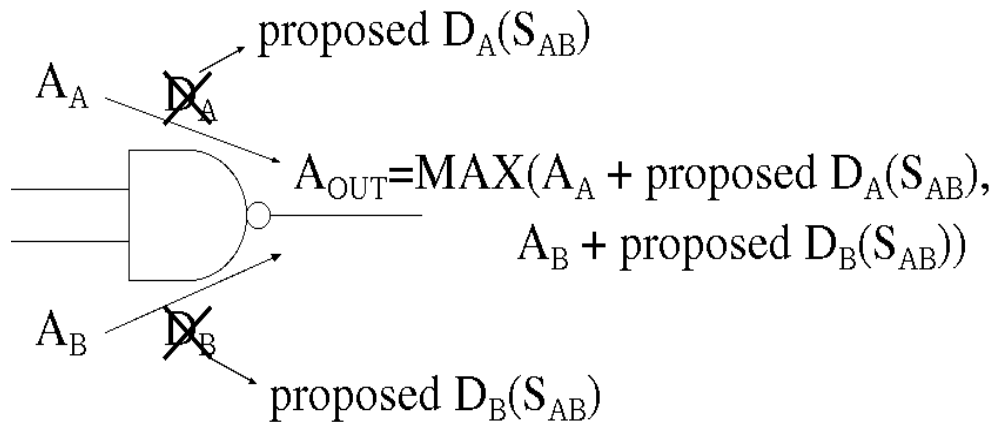
# Outline

---

- Motivation
- Effect of Multiple Input Switching on the gate delay
- Gate Delay Model for Multiple Input Switching
- Experimental results
- Conclusions

# Proposed Gate Delay Model

- Replace conventional gate delay with the proposed gate delay
- Proposed gate delay model as a function of  $S_{AB}$



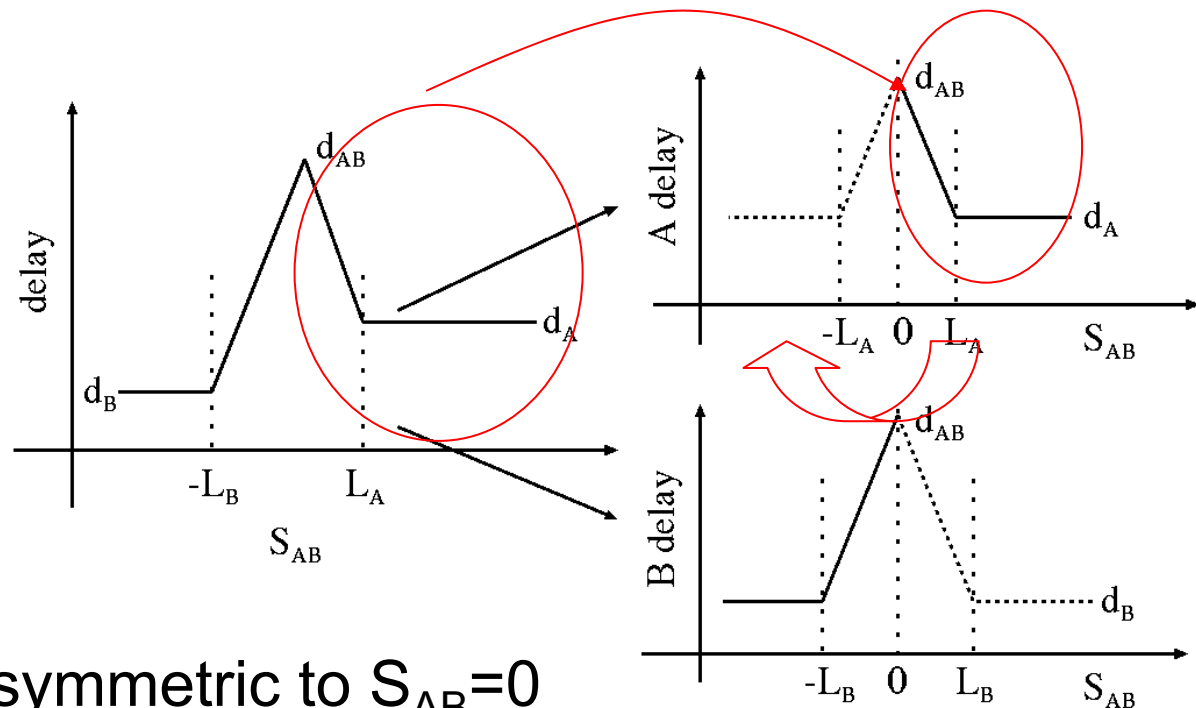
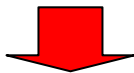
- Other parts are not changed (e.g. Max operation)

# Estimation of proposed gate delay

- Proposed gate delay is a function of  $S_{AB}$
- We estimate the proposed gate delay from SPICE results
- SPICE results:  $A_{OUT} = \text{MAX}(A_A + D_A, A_B + D_B)$

When  $A_A + D_A > A_B + D_B$ ,  
 $D_A = A_{out} - A_A$

When  $A_A + D_A < A_B + D_B$ ,  
 $D_A = ?$



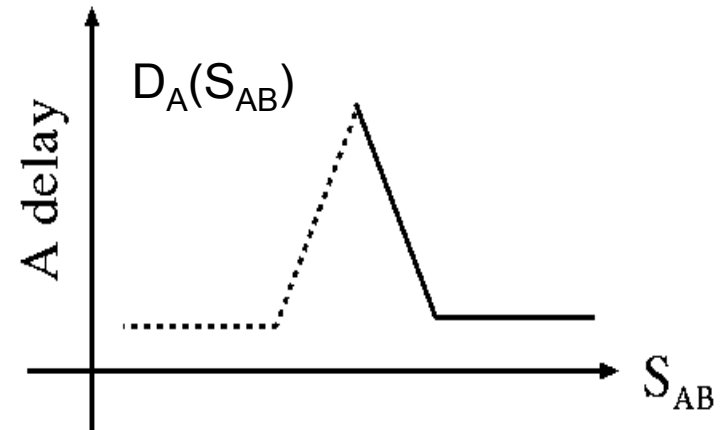
We assume  $D_A(S_{AB})$  is symmetric to  $S_{AB} = 0$

# $\mu$ and $\sigma$ of Gate delay model

- Assume that all variations are normally distributed
- Calculate the mean and variance of  $D_A(S_{AB})$ ,  $D_B(S_{AB})$

$$\mu(D_A) = \int D_A(S_{AB}) p(S_{AB}) dS_{AB}$$

$$\sigma^2(D_A) = \int (D_A(S_{AB}) - \mu(D_A))^2 p(S_{AB}) dS_{AB}$$



Probability density function of  $S_{AB}$ : 
$$p(S_{AB}) = \frac{1}{\sqrt{2\pi}\sigma_{AB}} \exp\left(-\frac{(S_{AB} - \mu_{AB})^2}{2\sigma_{AB}^2}\right)$$

- $A_{OUT} = \text{MAX}( A_A + \text{proposed } D_A , A_B + \text{proposed } D_B )$ 
  - MAX is calculated by Clark's method

# Proposed method under gate length variations, etc.

- Canonical gate delay model can handle the effect of gate length variations, etc

Conventional gate delay  $D_A = \mu_A + \underbrace{\sum_j \alpha_{A,j} r_j}_J$   $\mu_A$  : mean of  $D_A$   
 $r_j \sim N(0,1)$

Gate length variations, etc

Proposed gate delay  $D_A' = \mu_A' + \underbrace{\sum_j \gamma_{A,j} r_j}_j + \underbrace{\sum_j \beta_{A,j} r_j}_j$   
 $S_{AB}$  variations

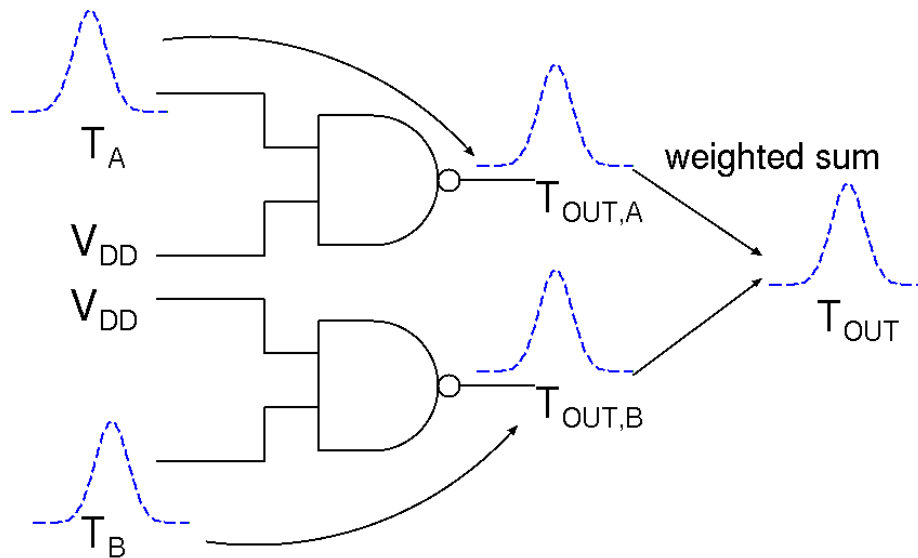
$$\gamma_{A,j} = \frac{\mu_A'}{\mu_A} \alpha_{A,j}$$

# Estimation of output transition time

## ■ Output transition time

- $T_{OUT} = p T_{OUT,A} + (1-p) T_{OUT,B}$

$$p = P(A_A + D_A > A_B + D_B)$$



## ■ Proposed method

- Similar way to the proposed gate delay model
- $T_{OUT,A} \rightarrow$  proposed  $T_{OUT,A}(S_{AB})$
- $T_{OUT,B} \rightarrow$  proposed  $T_{OUT,B}(S_{AB})$



# Outline

---

- Motivation
- Effect of Multiple Input Switching on the gate delay
- Gate Delay Model for Multiple Input Switching
- **Experimental results**
  - **Result1 : only  $S_{ab}$  is a statistical parameter**
  - **Result2 :  $S_{ab}$ , Gate length, input transition time are statistical parameters**
- Conclusions

# Experimental setup

- Comparison with SPICE-based Monte Carlo Sim.

- Conventional method

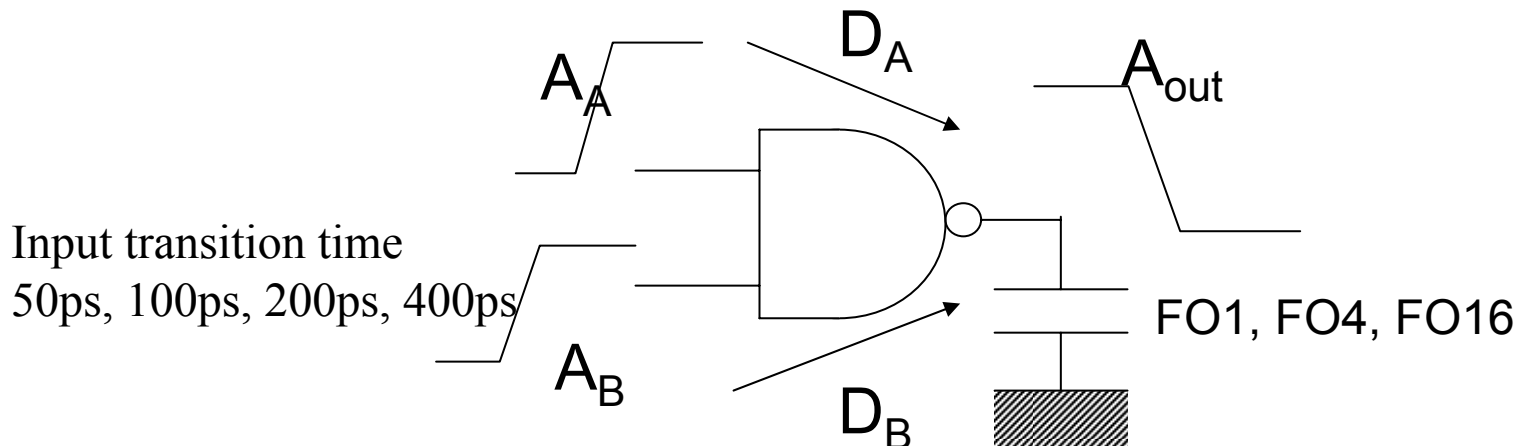
$$A_{OUT} = \text{MAX/MIN}( A_A + D_A , A_B + D_B )$$

- Proposed method

$$A_{OUT} = \text{MAX/MIN}( A_A + \text{proposed } D_A , A_B + \text{proposed } D_B )$$

- $S_{AB}$ : **statistical parameter** with normal distribution

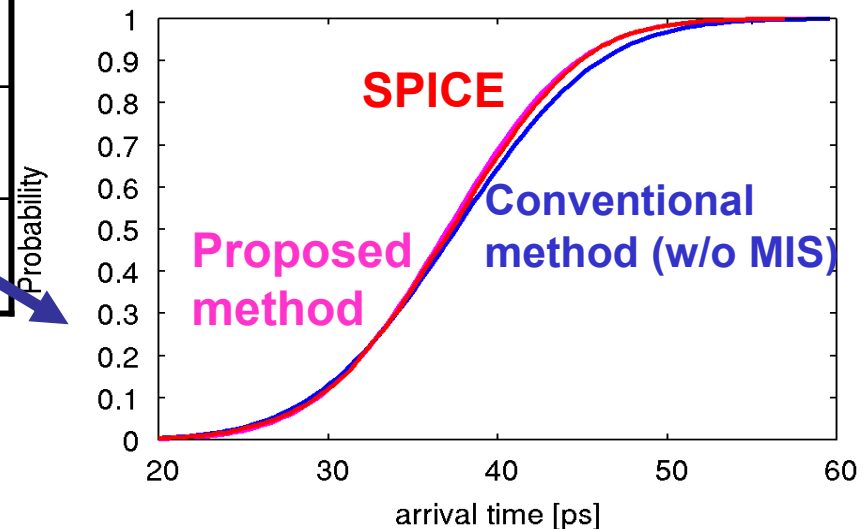
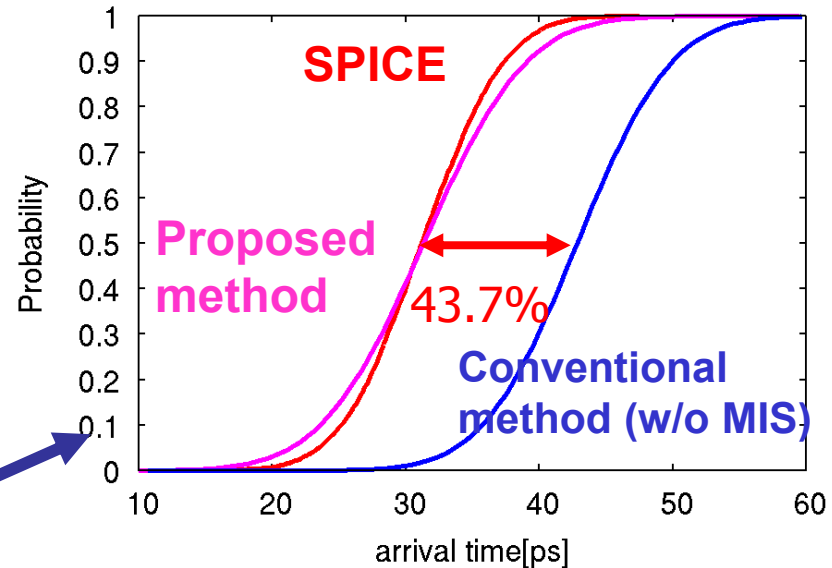
- e.g.  $S_{AB} \sim N(0, 10), N(10,10)$  and so on





# Example of experimental result

Mean of $S_{AB}$ [ps]	Proposed method			w/o MIS		
	% error in $\mu$	% error in $\sigma$	% error in $\mu+3\sigma$	% error in $\mu$	% error in $\sigma$	% error in $\mu+3\sigma$
0	0.5	27.6	11.4	43.7	17.6	37.9
10	-0.7	5.9	1.6	34.1	22.3	33.3
20	-1.0	-0.7	-1.0	21.1	32.7	17.2
50	-0.2	-1.9	-0.8	0.9	9.1	3.7



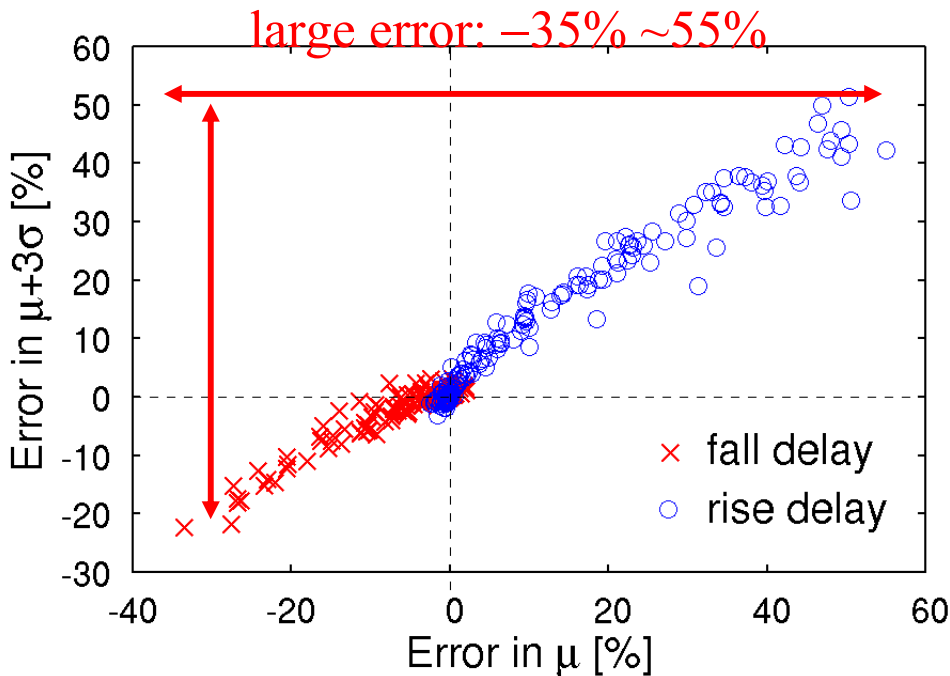
@ Rise delay

Input transition time: 100ps

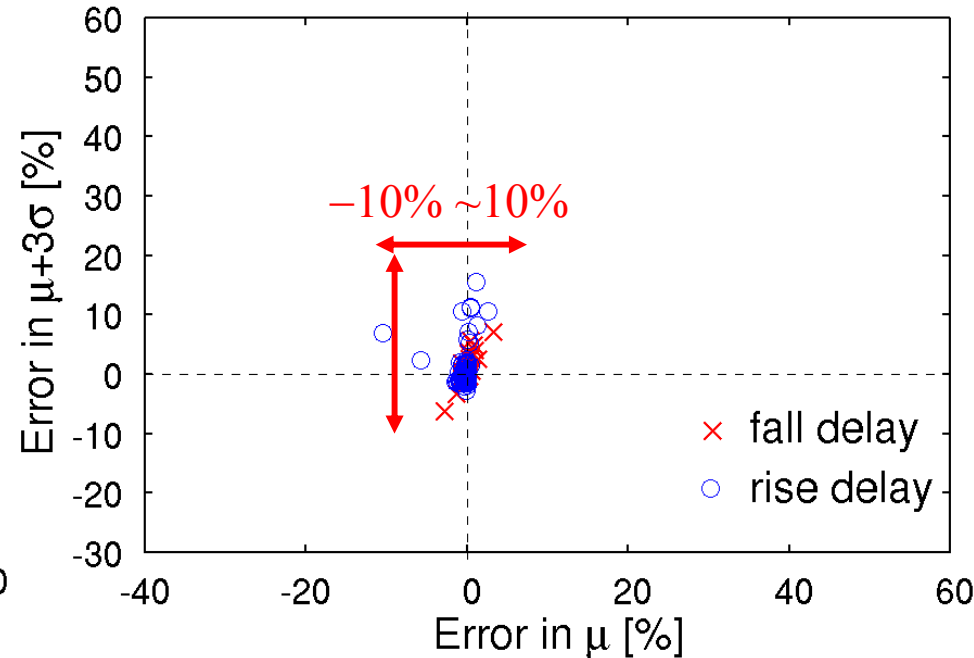
Output capacitance: FO4

# Result1

Input transition time: 50ps, 100ps, 200ps, 400ps  
Output capacitance: FO1, FO4, FO16



Conventional method



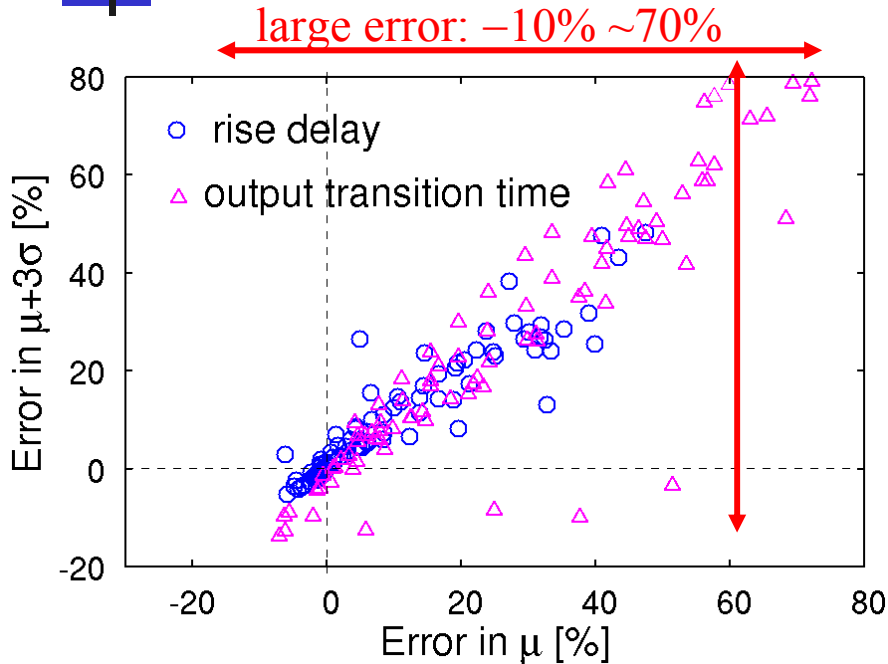
Proposed method

- Proposed method reduces the error

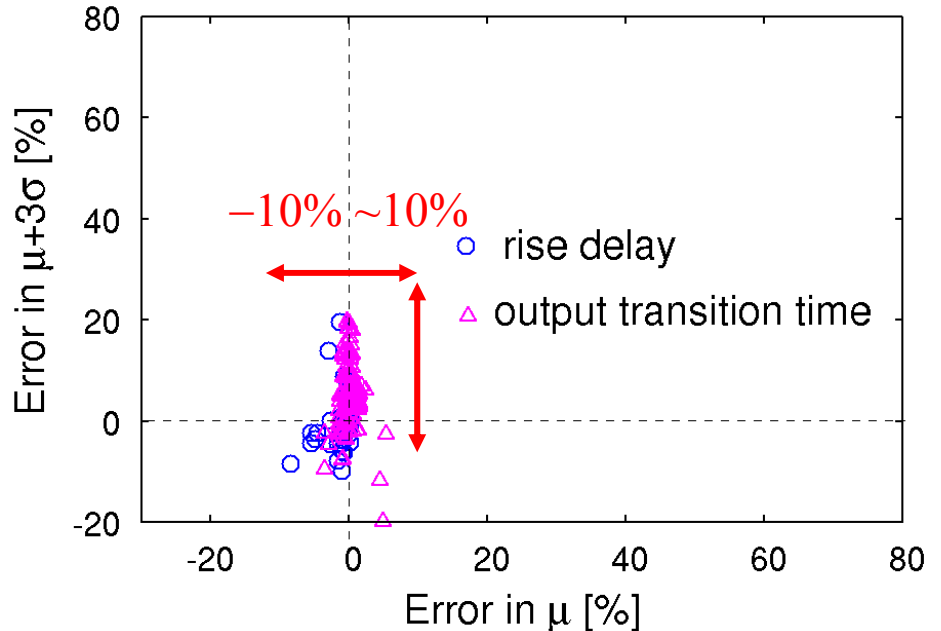
# Result2

Rise delay

Gate length:  $\pm 20\%$ , Input transition time:  $\pm 20\%$



Conventional method



Proposed method

- MIS have a significant effect on gate delay and output transition time
- Proposed method reduces the error by considering MIS effect



# Conclusions

---

- Ignoring MIS causes the large error in statistical maximum operation in SSTA
- We propose the gate delay model considering MIS
- We show the proposed method reduce the error from 70% to 10%
- Future works
  - We need to reduce cell characterization cost