

# Architecture-level Thermal Behavioral Characterization for Multi-Core Microprocessors



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# Outline

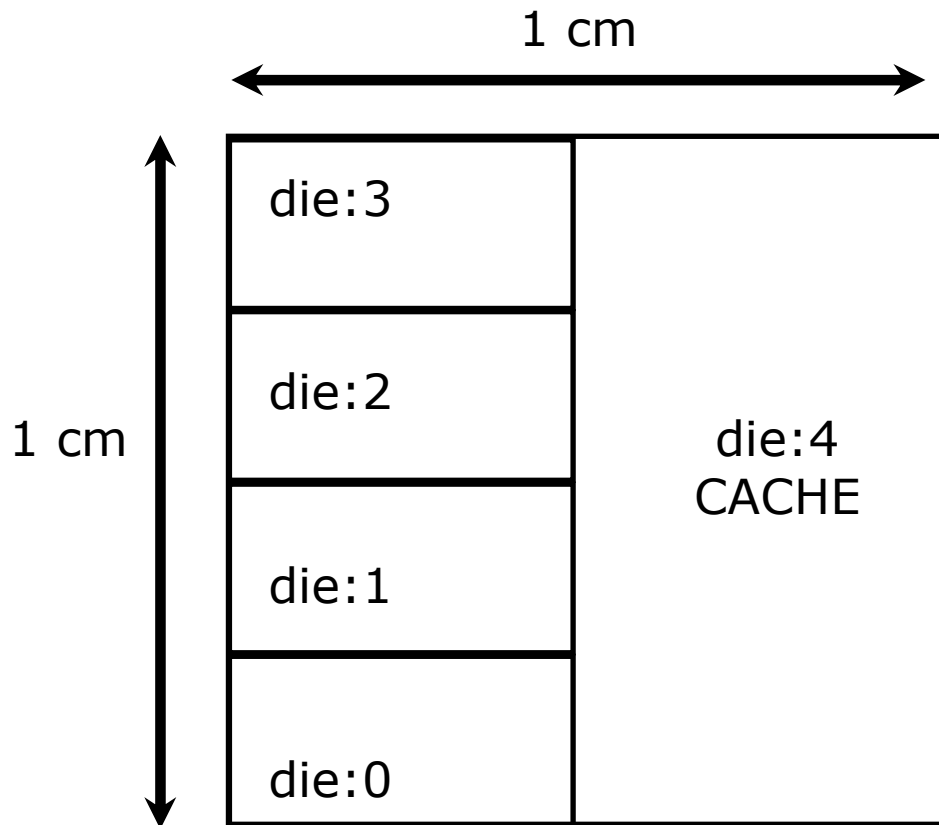
- Introduction and Motivation
  - The need for dynamic thermal management (DTM)
  - Why software thermal sensors
- Power estimation for functional units
- Architecture level thermal modeling
- Summary



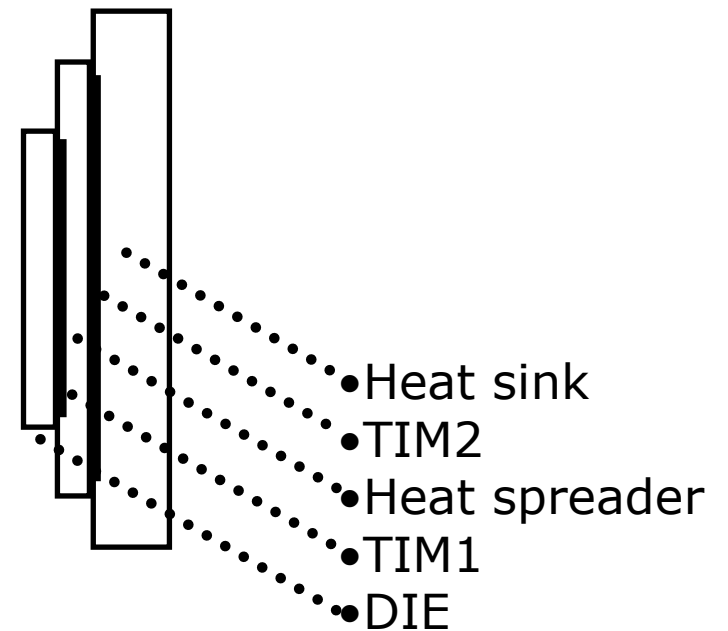
# Outline

- Introduction and Motivation
- Architecture level thermal modeling
  - Intel quad-core structure
  - Transfer function
  - Matrix pencil method
  - Log-sage sampling and stabilization
  - Reduction of thermal models
  - Simulation results
- Summary

# Top view: quad core

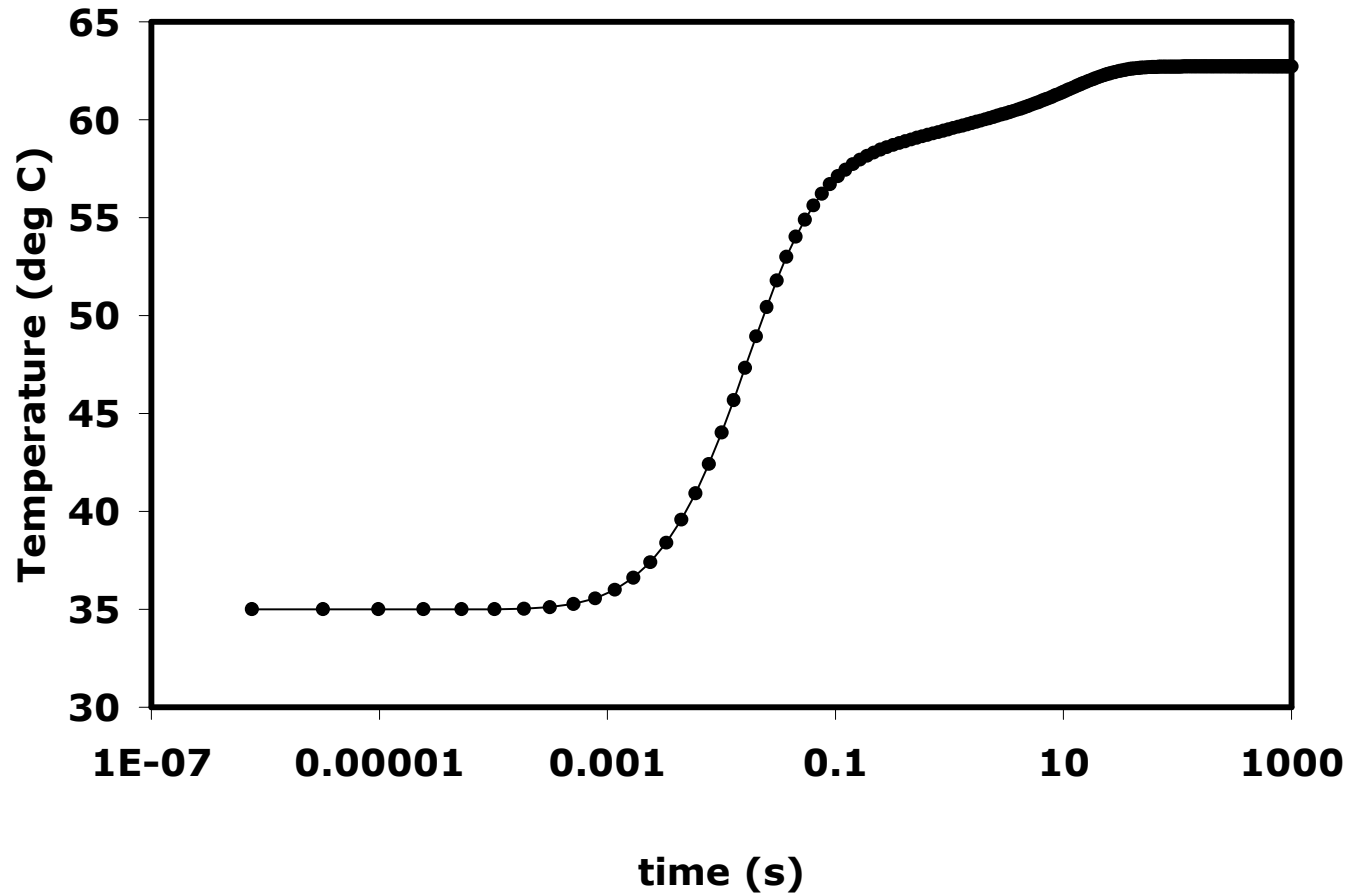


# Lateral view



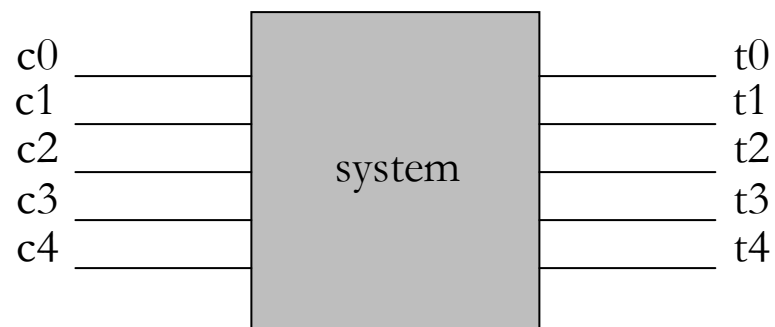
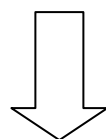
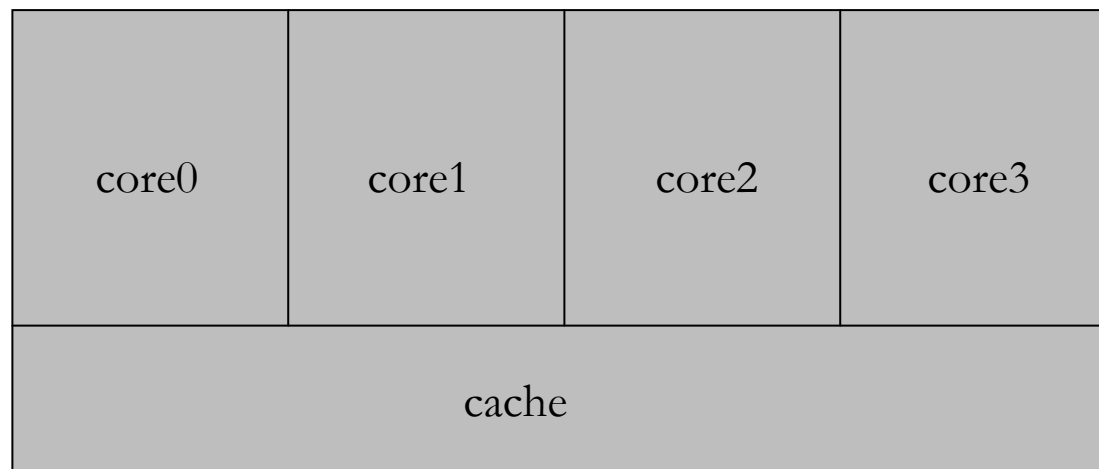
Temperatures reported are on the die bottom face and centered with each die region

# Active core 0 at 20 W: T distribution





# Quad-core





# Transfer function

LTI (linear, time-invariant systems)

input signal  $x(t)$  and output  $y(t)$

$$Y(s) = H(s)X(s)$$

or

$$H(s) = \frac{Y(s)}{X(s)}$$

where  $H(s)$  is the transfer function of the LTI system



# Pole-residue representation

- Pole-zero

$$H(s) = \frac{b_0 + b_1s + \dots + b_ms^m}{1 + a_1s + \dots + a_ns^n}$$

$$H(s) = K \frac{(s - z_1) \cdot (s - z_2) \dots (s - z_m)}{(s - p_1) \cdot (s - p_2) \dots (s - p_n)}$$

- Pole-residue

$$H(s) = \sum_{i=1}^n \frac{k_i}{s - p_i}$$





# Matrix Pencil

- Used for extracting poles and residues.
- Specifically, works for EM transient signal.

$$y_k = \sum_{i=1}^M r_i \exp(p_i \Delta t k)$$

- $k = 0, 1, \dots, N-1$ ,
- $r_i$  are the complex residues,
- $p_i$  are the complex poles,
- $\Delta t$  is the sampling interval.

# General Pencil of Function Method



ALGORITHM: GPOF

Input: sampling vectors  $y_i = [y_i, y_{i+1}, \dots, y_{i+N-L-1}]^T$

Output: poles vector  $\mathbf{p}$  and residues vector  $\mathbf{r}$

1. Construct matrices  $Y_1$  and  $Y_2$ .

$$Y_1 = [y_0, y_1, \dots, y_{L-1}] \quad Y_2 = [y_1, y_2, \dots, y_L]$$

2. Singular value decomposition (SVD) of  $Y_1$ .  $Y_1 = UDV^H$

3. Construct matrix  $Z$ .  $Z = D^{-1}U^H Y_2 V$

4. Eigen-decomposition of  $Z$ .  $Z_0 = eig(Z)$

find poles vector:  $p_i = \frac{\log(z_i)}{\Delta t}$

5. Solve  $R_1$  and  $R_2$  from  $Y_1 = Z_1 R Z_2$  and  $Y_2 = Z_1 R Z_0 Z_2$ .

$$Z_1 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_M \\ \vdots & \vdots & \dots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \dots & z_M^{N-L-1} \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 1 & z_1 & \dots & z_1^{L-1} \\ \vdots & \vdots & \dots & \vdots \\ 1 & z_M & \dots & z_M^{L-1} \end{bmatrix}$$

find residues vector:  $\mathbf{r} = \frac{R_1 + R_2}{2}$



# How to choose M and L

- M is model order number.
- L is sampling window size.
- N is the number of total sampled points.
- For GPOF,  $M \leq L \leq N-M$ . Allow different window sizes and pole numbers.
- Typically, choosing  $L = N/2$  can yield better results.

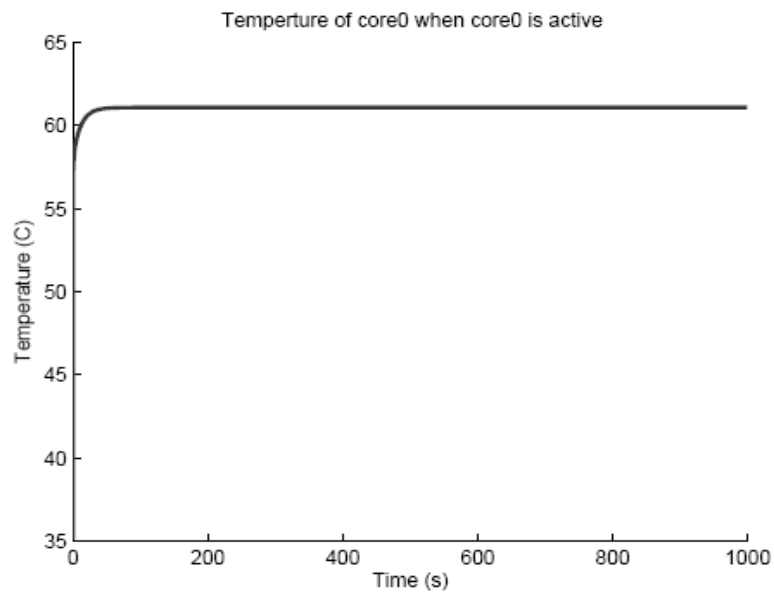


# Sampling issue

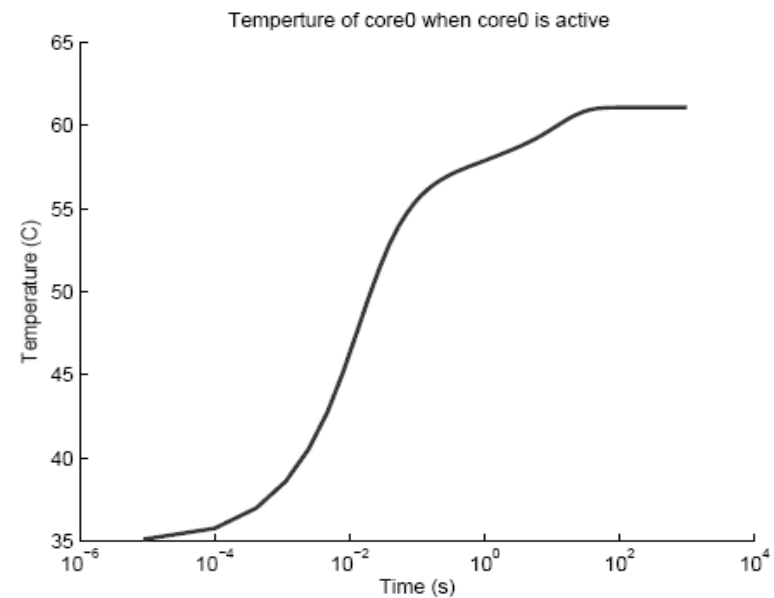
- Traditional MP using constant interval time for sampling.
  - Temperature increase dramatically fast in the first few seconds.
- Log-scale sampling is a good way.
- Numerical differentiation for computing impulse response.
  - Need to compute the impulse response instead of step responses, which are given.



# Linear vs Log-scale



(a) Linear time scale thermal step response.



(b) Logarithmic time scale thermal step response.



# Log-scale sampling

- Temperature increases very fast in a first few seconds.
- Temperature needs a very long time to get steady.
- Offset to make sure it starts at  $t=0$ .
- Get the response back

$$y'(t) = y(\ln(t) - \ln(t_0))$$

$y'(t)$ : response in normal time scale;

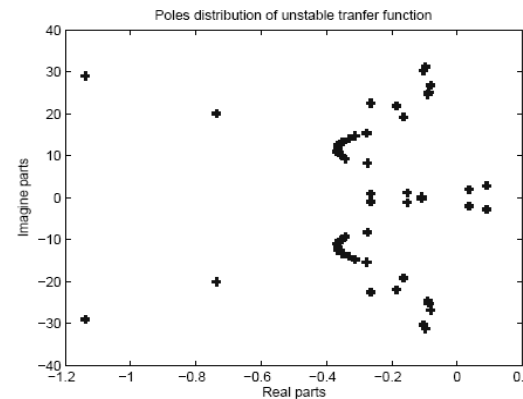
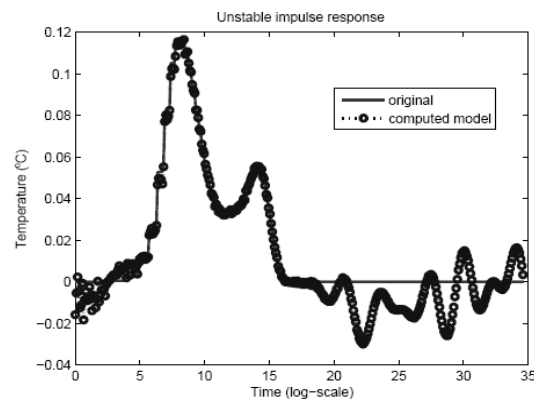
$y(t)$ : response in log-scale;

$t_0$ : offset, usually a very small value.

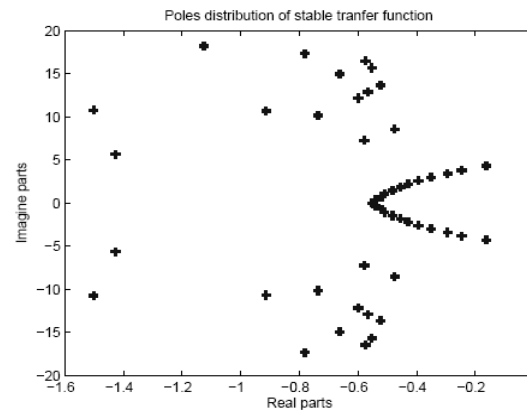
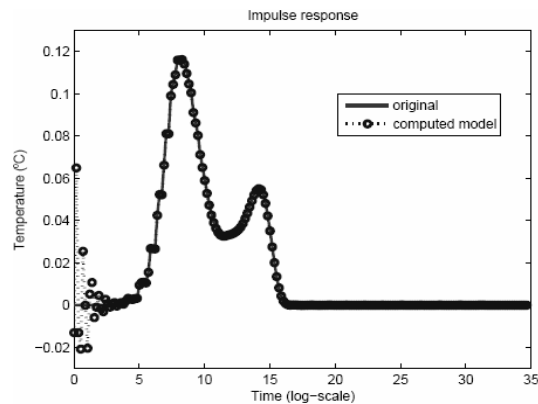
# Numerical Differential and Stabilization (1)



- Stable pole extraction
  - Only negative poles



Impulse responses with some positive poles

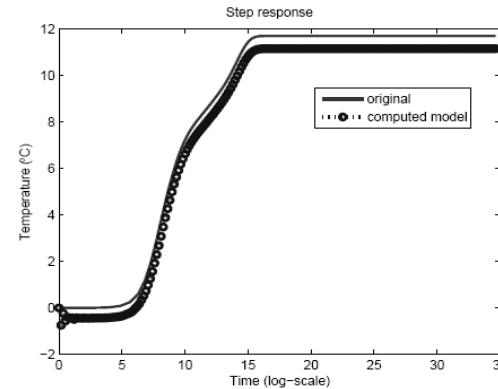
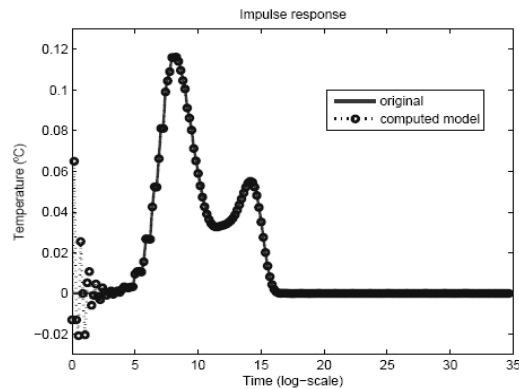


Impulse responses with only negative poles

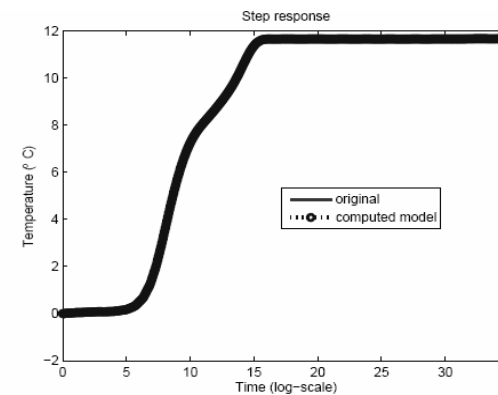
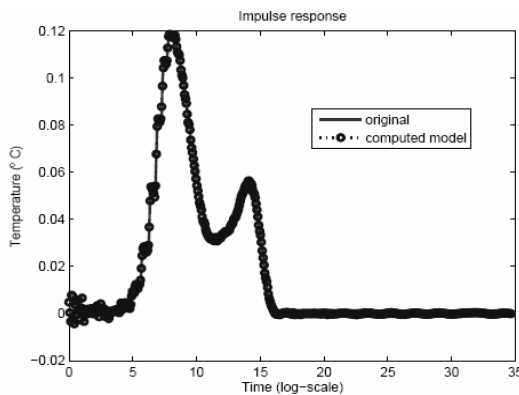
# Numerical Differential and Stabilization (2)



- Stabilizing the starting response
  - Increasing sampling points



Impulse and step responses for  $L = 100$

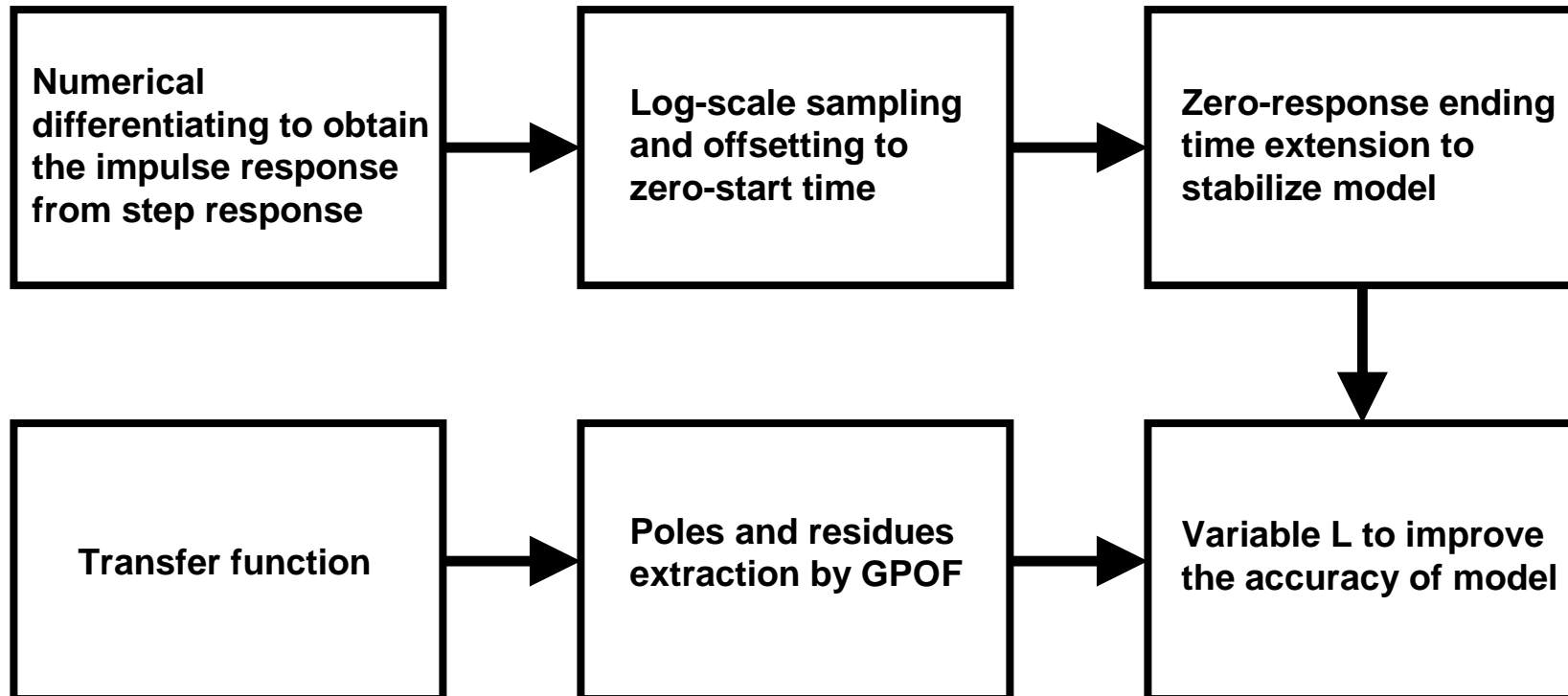


Impulse and step responses for  $L = 200$





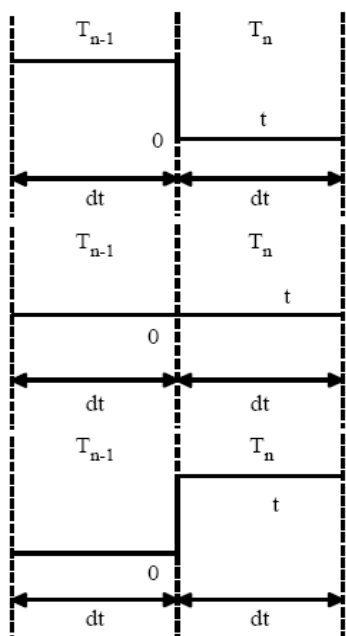
# Thermal modeling flow





# Recursive computation

- Computation complexity is only  $O(n)$ 
  - $n$  is the number of time segments or the number of power traces



$$y_n(t) = y_{n-1}(t + dt) - y_0(t) \quad \mathbf{a)}$$

$$y_n(t) = y_{n-1}(t + dt) \quad \mathbf{b)}$$

$$y_n(t) = y_{n-1}(t + dt) + y_0(t) \quad \mathbf{c)}$$



# Reduction of thermal models

- State-space realization

$$Y(s) = \frac{r}{s-p} + \frac{\bar{r}}{s-\bar{p}} \quad p = a + bj \quad r = c + dj$$

$$\mathbf{A}_i = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \quad \mathbf{b}_i = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \mathbf{c}_i^T = [c \quad d]$$

- MOR by PRIMA

$$\mathbf{A}_r = \mathbf{V}^T \mathbf{A} \mathbf{V} \quad \mathbf{b}_r = \mathbf{V}^T \mathbf{b} \quad \mathbf{c}_r^T = \mathbf{c}^T \mathbf{V}$$

- Reduced transfer function

$$Y(s) = \sum_{k=1}^q \frac{\mu_k \cdot v_k}{s - \lambda_k}$$

$$\mathbf{A} = \mathbf{P} \mathbf{\Lambda} \mathbf{P}^{-1}$$

$\lambda_k$  is the  $k$ th diagonal element of  $\mathbf{\Lambda}$ ,  $\mu_k$  is the  $k$ th element of  $\mathbf{c}_r^T \mathbf{P}$ ,  $v_k$  is the  $k$ th element of  $\mathbf{P}^{-1} \mathbf{b}_r$  and  $q$  is the reduced order

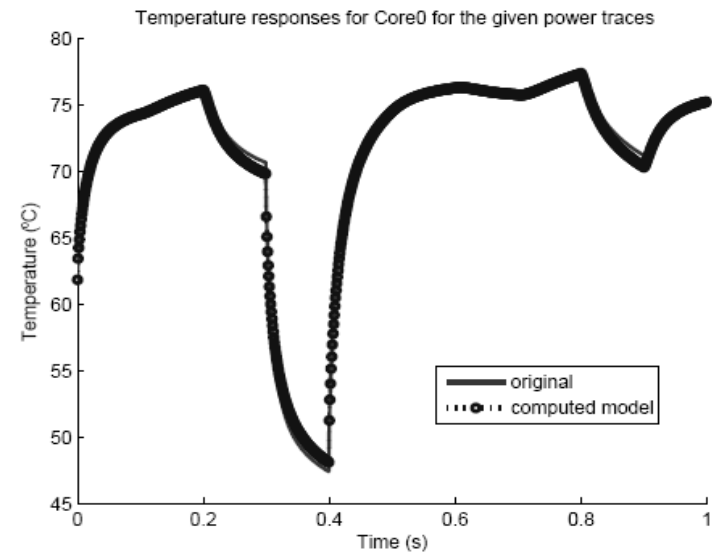
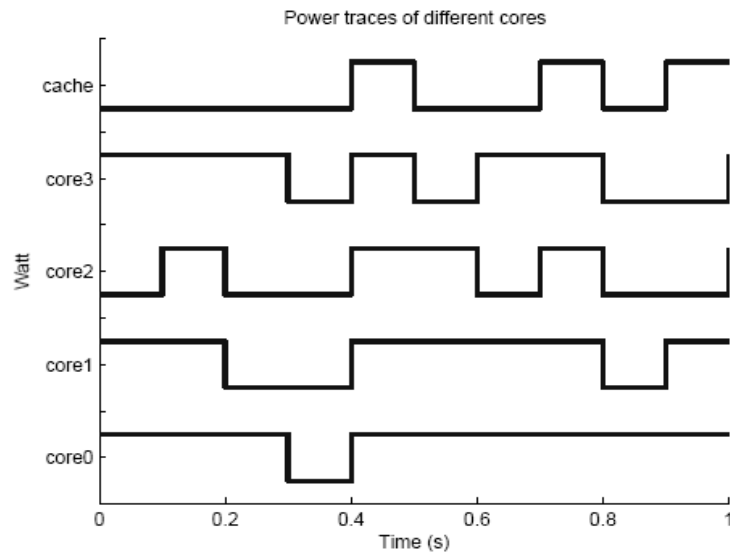


# Training

- Extracting 5 groups of poles and residues using matrix pencil method.
- Obtaining the transfer function of the system.
- Simulating the output of the system (thermal simulation).
- Linear combination
- Benchmark provided by Intel, random power input.



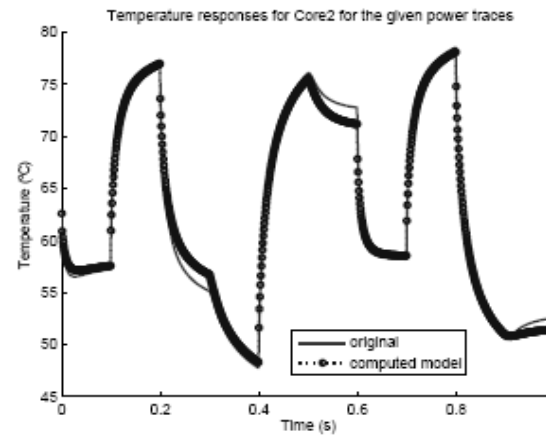
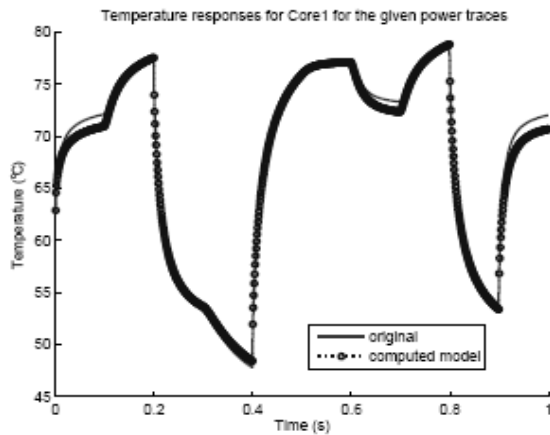
# Simulation result (1)



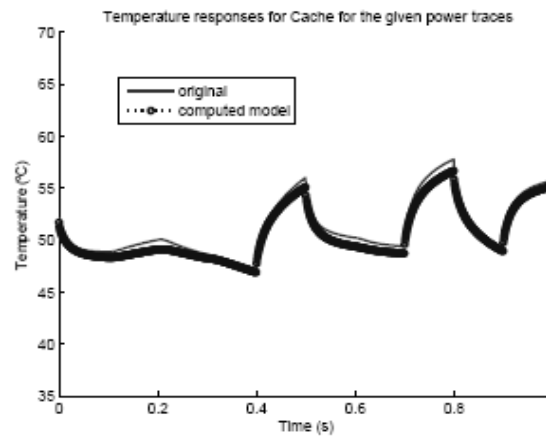
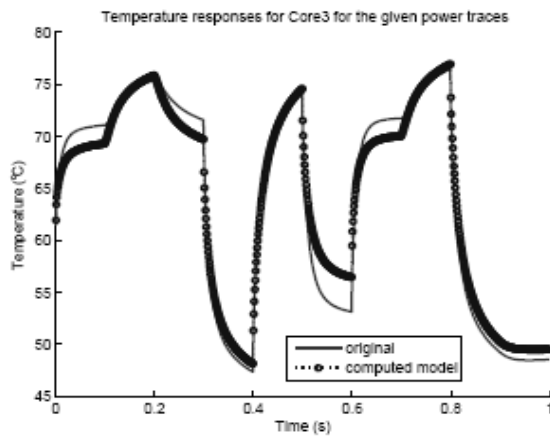
**Random power input on all cores and thermal response for Core0.**



# Simulation result (2)



Thermal response  
of Core1 and  
Core2



Thermal response  
of Core3 and  
Cache



# Simulation result (3)

- Features of the errors between measured and computed temperatures (M = 50)

	Error ( $^{\circ}C$ )			Error percentage	
	Maximum	Mean	Std. deviation	Maximum	On average
Core0	1.05	0.34	0.23	1.56%	0.50%
Core1	1.67	0.53	0.48	2.44%	0.78%
Core2	1.78	0.61	0.47	2.56%	0.98%
Core3	3.33	1.10	0.82	6.09%	1.80%
Cache	1.05	0.63	0.22	1.84%	1.22%

- Errors of the maximal and minimum peaks (M = 50)

	Maximal peak			Minimum peak		
	Measured ( $^{\circ}C$ )	Error ( $^{\circ}C$ )	Percentage	Measured ( $^{\circ}C$ )	Error ( $^{\circ}C$ )	Percentage
Core0	77.27	0.45	0.58%	47.47	0.38	0.79%
Core1	78.86	0.04	0.05%	47.81	0.35	0.73%
Core2	78.55	0.38	0.48%	47.77	0.24	0.51%
Core3	76.48	0.75	0.98%	47.38	0.45	0.95%
Cache	57.80	0.99	1.72%	48.86	0.11	0.23%



# Simulation result (4)

- Reduction of thermal models ( $M = 30$ )

## Errors of the maximal and minimum peaks and means ( $M = 30$ )

	Maximal peak		Minimum peak		Mean	
	Error ( $^{\circ}C$ )	Percentage	Error ( $^{\circ}C$ )	Percentage	Error ( $^{\circ}C$ )	Percentage
Core0	0.40	0.52%	0.46	0.96%	0.36	0.48%
Core1	0.12	0.15%	0.49	1.00%	0.47	0.69%
Core2	0.06	0.07%	0.34	0.70%	0.56	0.88%
Core3	0.76	0.98%	0.53	1.11%	1.11	1.66%
Cache	1.01	1.78%	0.01	0.02%	0.03	1.25%

## Speedup when $M = 30$ compared to $M = 50$

	Run time (s) when $M = 50$	Run time (s) when $M = 30$	Time reduced
Core0	1.31	0.80	38.9%
Core1	1.29	0.78	39.5%
Core2	1.28	0.78	39.1%
Core3	1.28	0.78	39.1%
Cache	1.30	0.79	39.2%





# Conclusion

- Efficient on-chip thermal analysis technique is required for on-chip dynamic thermal management study and run-timing DTM.
- Developed a new estimation method to compute real microprocessor Function Units' power.
- Developed behavioral thermal modeling techniques based on matrix pencil.
- Developed thermal reduction techniques.