A Delay Model for Interconnect Trees Based on ABCD Matrix

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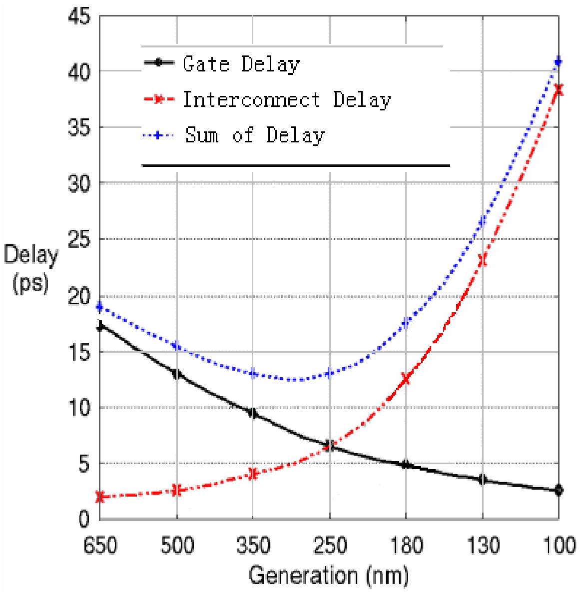
Outline

- Background
- Iterative formulas based on ABCD matrix
- Transport delay and stability tactics
- Experimental results
- Conclusions

Background



- The background of interconnect delay
  - Interconnect delay more roles than gate delay
  - The transport delay can't be ignored in some cases
    - MCM, global interconnect
  - The increasing of system operating frequency

- We need more accurate model with consideration of the transport delay!

Background

- The techniques of interconnect delay analysis
  - Based on circuit simulation
    - Accurate
    - Too expensive to be used in synthesis and layout
  - Based on closed form formulas
    - Elmore delay model based on RC networks
    - Kahng's model based on RLC networks

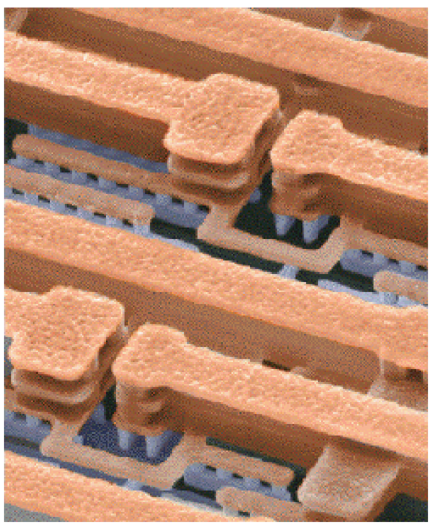
Background

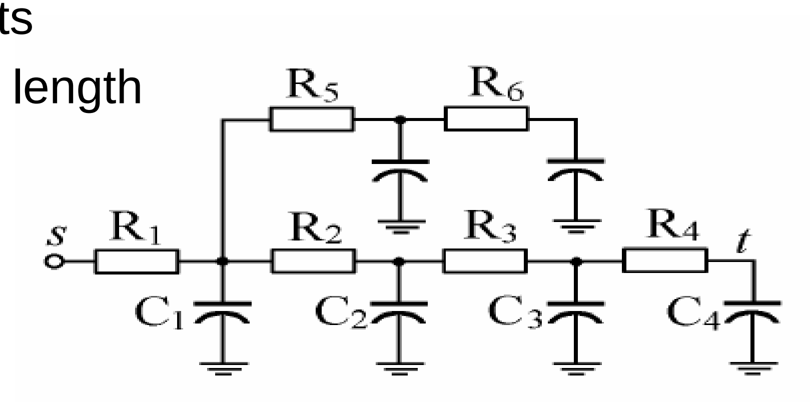
- The summary of pervious works
  - Cut into segments and replaced with lumped circuits
  - Without consideration of the transport delay

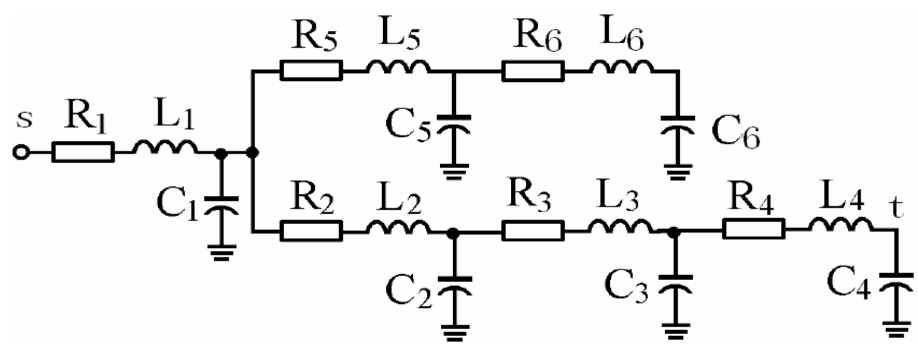
- Quick look of Our works
  - Iterative formulas derived from ABCD matrix
  - The transport delay is considered in our model
  - A tactics is used to keep our model stable

Iterative formulas based on ABCD matrix

- The previous works:
  - The accuracy of estimation depend on
    - Number of the circuit moments
    - Number of segments per unit length

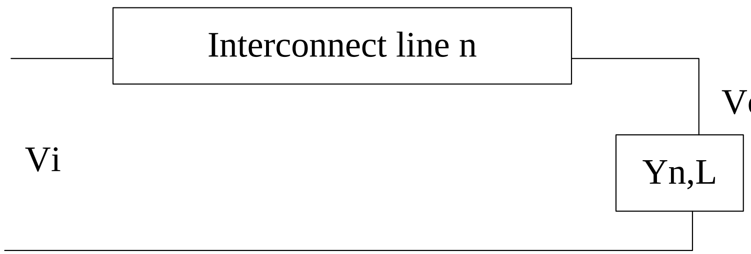




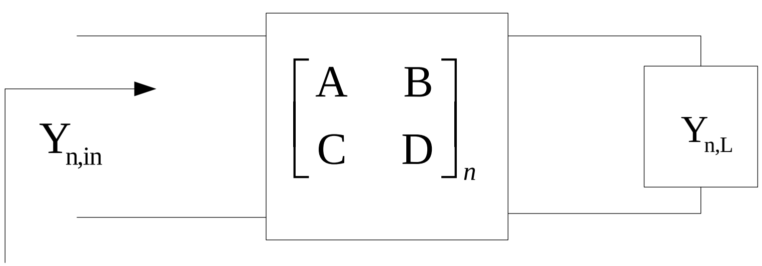


Iterative formulas based on ABCD matrix

- Our works



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_n = \begin{bmatrix} \cosh(\theta_n d_n) & Z_{n,0}\sinh(\theta_n d_n) \\ Y_{n,0}\sinh(\theta_n d_n) & \cosh(\theta_n d_n) \end{bmatrix}$$

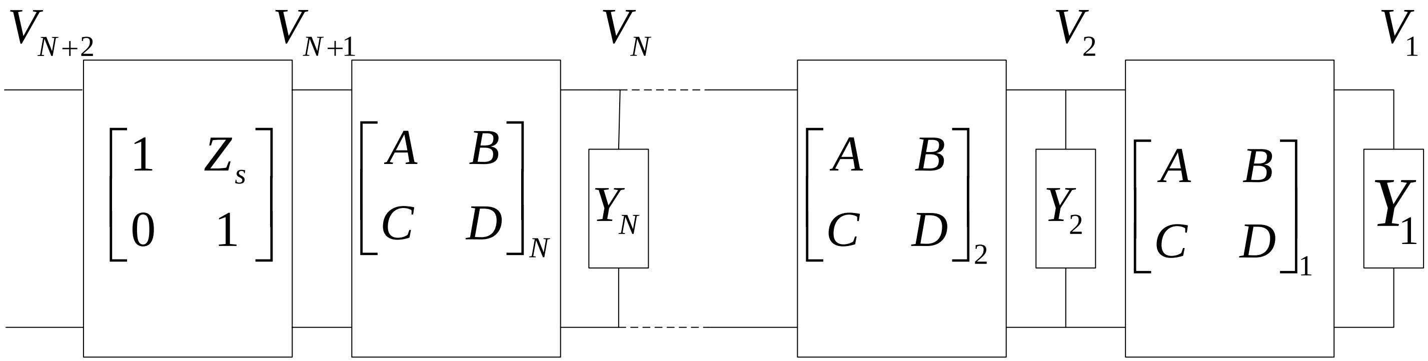


$H_n(S)$

$$H_n(S) = \frac{1}{A + BY_{n,L}}$$

$$Y_{n,in} = \frac{C + DY_{n,L}}{A + BY_{n,L}}$$

Iterative formulas based on ABCD matrix



- Our works
  - The interconnects on the main path between different nodes are described by ABCD matrix
  - Each subtree is replaced by respective admittance

$$H(s) = H_s(s) H_N(s) H_{N-1}(s) \ldots H_1(s)$$

Iterative formulas based on ABCD matrix

- The iterative formulas
  - The first two moments of admittances

$$y_{n,in}^{(1)} = C_n + y_{n,L}^{(1)}$$

$$y_{n,in}^{(2)} = \frac{R_n C_n^{\,2}}{3!} + \frac{y_{n,L}^{(1)} R_n C_n}{2!} + y_{n,L}^{(2)} - (C_n + y_{n,L}^{(1)})\left(\frac{R_n C_n}{2!} + y_{n,L}^{(1)} R_n\right)$$

  - The first two moments of transfer functions

$$h_n^{(1)} = \frac{R_n C_n}{2!} + y_{n,L}^{(1)} R_n$$

$$h_n^{(2)} = \frac{L_n C_n}{2!} + \frac{(R_n C_n)^2}{4!} + \left(L_n + \frac{R_n^{\,2} C_n}{3!}\right) y_{n,L}^{(1)} + y_{n,L}^{(2)} R_n$$

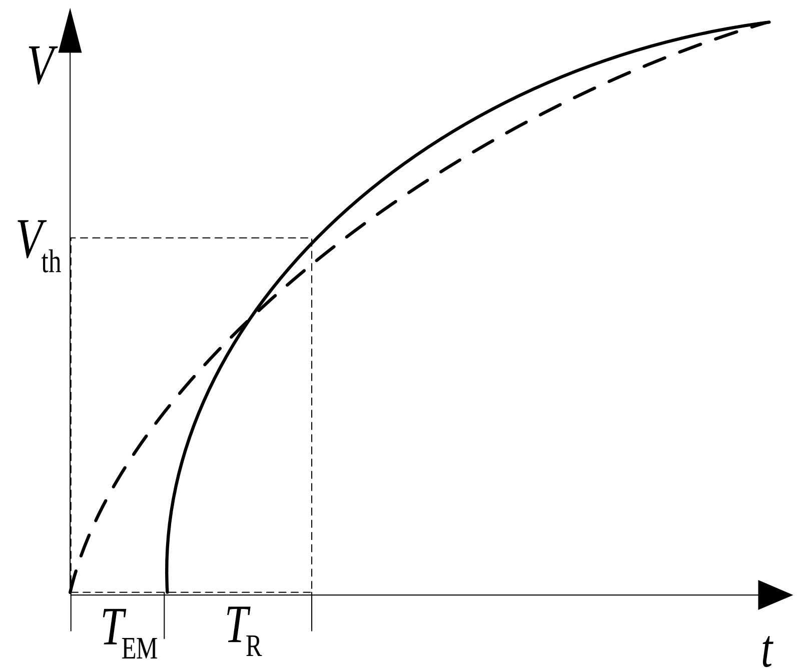
  - Then, we can get the first two moments of total transfer function

$$h_1 = \sum_{n=1}^{N+1} h_n^{(1)} \qquad\qquad h_2 = \sum_{n=1}^{N+1} h_n^{(2)} + \sum_{n=1}^{N+1} \sum_{\substack{m=1 \\ n \neq m}}^{N+1} h_n^{(1)} h_m^{(1)}$$

Transport delay and stability tactics

- Classification delay:
  - The transport delay T_{EM}
  - The rise time delay T_R

- When T_{EM} and T_R are the same order, those traditional lumped model can not match the curve A



Transport delay and stability tactics

- New model with consideration of transport delay

- The first step: calculation of transport delay

  - The transport delay of each interconnect segment on the main path $T_{f,n} = \sqrt{L_n C_n}$

  - Then the total transport delay is the sum of those delays $T_f = \sum_{n=1}^{N} T_{f,n}$

Transport delay and stability tactics

- The second step: the new transfer function
  - We use the formulas $H_n^*(s) = H_n(s)e^{sT_{f,n}}$ to remove the effect of transport delay
  - Expand the numerator and denominator of new transfer function $H^*(s) = \dfrac{1 + k_1 s + k_2 s^2 + o(s^2)}{1 + h_1 s + h_2 s^2 + o(s^2)}$
  - Then, the first two circuit moments of the new transfer function are $M_1 = h_1 - k_1$ $\quad M_2 = k_2 - k_1 h_1 + h_1^2 - h_2$

Transport delay and stability tactics

- The third step: stability tactics
  - The reasons: When we use few moments to describe the system, actually the transfer functions are truncated and the new system may be instable. So some tactics should be used to deal with the situation.
  - We use a two-order stable model to approximate the new transfer function. $$H_{\mathrm{app}}(s) = \frac{1 + a_1 s}{1 + b_1 s + b_2 s^2}$$

where $a_1 = \frac{M_2}{M_1}$ $b_1 = M_1 + \frac{M_2}{M_1}$ $b_2 = M_1^2$

Transport delay and stability tactics

- The forth step: calculation of rise time delay
  - Figure out the unit step response of the new system. Referring to the given threshold voltage, we can obtain the rise time delay
  - Then, the total delay of interconnect is the sum of transport delay and rise time delay !

Experimental results

- Experiment 1
  - Table 1: delay of different source and load
  - Table 2: delay in different length

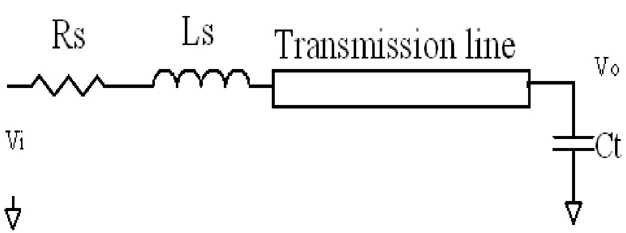


TABLE I
50% delay of different source and load

Source R_s /Ω	Source L_s /pH	Load C_T /pF	Spice /ps	Delay Model: Kahng T_d /ps	Kahng Error /%	New Model T_{total} /ps	New Model Error /%
40	2.46	0.176	33.9	39.9	17.7	36.3	7.07
50	2.46	0.176	38.9	44.1	15.1	40.6	6.01
75	2.46	0.176	51.9	54.8	5.58	52	0.19
40	2.46	1.76	130	133	2.31	130.7	0.54
50	2.46	1.76	145	148	2.07	145.7	0.48
75	2.46	1.76	184	187	1.63	184.7	0.38

TABLE II
50% delay in different length

h /mm	Spice /ps	Delay Model/ps: Kahng T_d /ps	Kahng Error /%	New Model T_f /ps	New Model T_R /ps	New Model T_{total} /ps	New Model Error /%
0.1	7.08	7.1	0.28	0.66	6.41	7.07	0.14
0.5	10.5	11.3	7.62	3.29	7.55	10.8	3.33
1	14.9	17	14.1	6.58	9.32	15.9	6.71
1.5	19.9	23.1	16.1	9.87	11.5	21.4	7.39
2	24.4	29.5	20.8	13.2	14.2	27.2	11
2.5	31.6	36.6	15.8	16.4	17.3	33.7	6.64
3	38.3	44.1	15.1	19.7	18.9	40.6	6.01

Experimental results

- Experiment 2
  - Simulation of interconnect trees under different situations

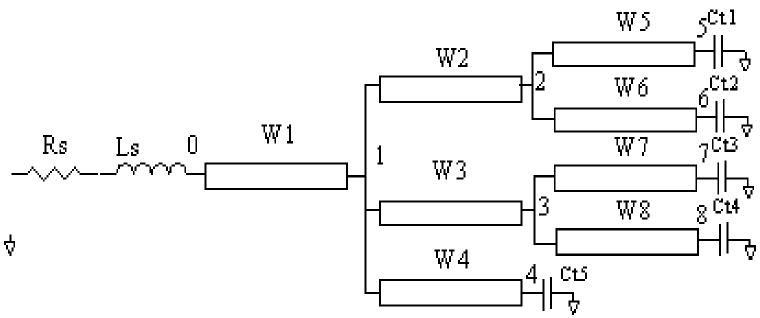


TABLE III
50% delay of interconnect trees

Cond.	node /mm	Spice /ps	Delay Model /ps			
			Kahng	New Model		
			Td /ps	T_f /ps	T_R /ps	T_{total} /ps
Cond1	4	120	116	13.2	103.1	116.3
	5	133	137	19.7	115	134.7
	6	150	163	32.9	119	151.9
	7	130	129	19.7	109	128.7
	8	130	129	19.7	109	128.7
Cond2	4	179	201	32.9	154	186.9
	5	180	195	26.3	163	189.3
	6	194	221	39.4	167	206.4
	7	175	188	26.3	158	184.3
	8	175	188	26.3	158	184.3

Conclusions

- Our model is a stable model. And it has the same accuracy with traditional models when the transport delay is much smaller than the rise time delay ;Our model is more accurate when the two are of the same order.

Thank you for your attentions!