# Chebyshev Affine Arithmetic Based Parametric Yield Prediction Under Limited Descriptions of Uncertainty

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# Power Leakage

- Exponential rise of IC power dissipation
  - Device dimension scales down.
  - Threshold voltage shrinks.
- Great portion of total power consumption
  - May account for 50% of the total delay
  - Will be further aggravated
  - Significant impact on parametric yield

# Parameter Variations

- Strong dependence of leakage on both process and environmental variations.
- Cause a large spread in leakage current.
- 2 kinds of variations are considered.
  - Process parameters
    - Leff, Vth, and Tox
  - Environmental parameters
    - Vdd, and T

# Parametric Yield Prediction

- Parameter variations reduces the yield of designs.
- Yield prediction methods are required to model the dependency.
- Limited by fundamental features of IC design
  - Incomplete process characterization data.
  - Large uncertainty in statistic metrics.
  - Correlation between parameters.

# Main Purposes

- Uncertainty representations
- Probability representations
- Consider both process variations and environme ntal uncertainty
- Consider correlation between parameters
- Provide reliable probability bounds for leakage c urrent

# Chebyshev Affine Arithmetic

- Affine form:

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots x_n\varepsilon_n$$

  - $\varepsilon_i \in [-1,1]$ and $E[\varepsilon_i] = 0$
  - $x_0$ , central value
  - $\varepsilon_i$ , noise symbols
  - $x_i$ , partial deviations

# Chebyshev Affine Operations

- Can be easily expanded
- 3 cases:

$$\hat{x} \pm \hat{y} = (x_0 + y_0) + (x_1 + y_1)\varepsilon_1 + \cdots (x_n + y_n)\varepsilon_n$$

$$\alpha\hat{x} = (\alpha x_0) + (\alpha x_1)\varepsilon_1 + \cdots + (\alpha x_n)\varepsilon_n$$

$$\hat{x} \pm \zeta = (x_0 + \zeta) + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n$$

- Still in affine form

# Non-affine Operations

- $z = f(\hat{x}, \hat{y}) = f^*(\varepsilon_1, \cdots, \varepsilon_n)$ , $f^*$ is not affine

- Approximations required

$$\hat{z} = f^a(\varepsilon_1, \cdots, \varepsilon_n) = z_0 + z_1\varepsilon_1 + \cdots + z_n\varepsilon_n + z_k\varepsilon_k$$

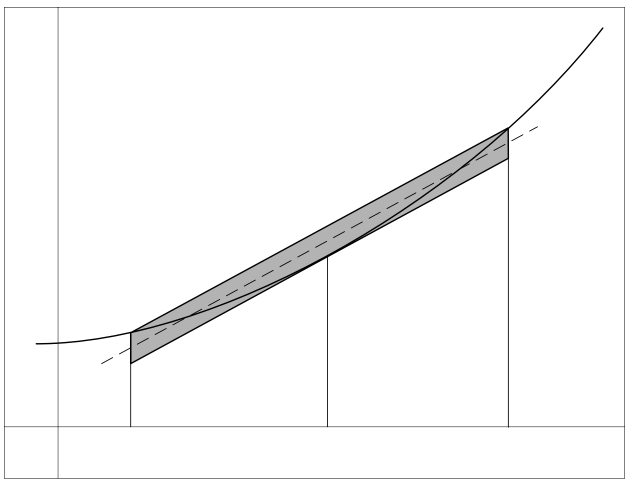
  - $z_k\varepsilon_k$ represents approximation error

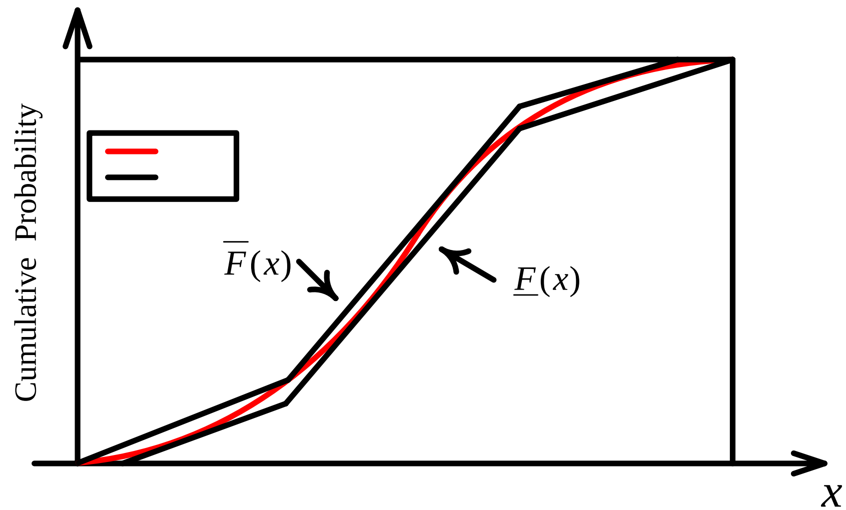
$$e^*(\varepsilon_1, \cdots, \varepsilon_n) = f^*(\varepsilon_1, \cdots, \varepsilon_n) - f^a(\varepsilon_1, \cdots, \varepsilon_n)$$

- Returns an Affine form

# Chebyshev Approximations

- in the form of affine combinations : $\alpha\hat{x} + \beta y + \zeta$
- Optimal: minimizes the maximum absolute error
- Geometric illustration $\hat{z} = \alpha\hat{x} + \zeta + \delta\varepsilon_k$





# Leakage Model

- An empirical model
- Obtained from SPICE simulation
- Model the dependency on parameter variations
  - Leff: quadratic exponential dependency
  - Vth: exponential dependency
  - Tox: exponential dependency
  - Vdd: exponential dependency
  - T: supper linear dependency (approximated as expo nential)

# Analytical Equations

- Mathematical representations of leakage model
  - Subthreshold leakage model

$$I_{sub} = I_{sub,nom} \cdot e^{a\Delta L^2 + b\Delta L + c\Delta V_{th} + d\Delta V_{dd} + e\Delta T}$$

  - Gate leakage model

$$I_{gate} = I_{gate,nom} \cdot e^{h\Delta T_{ox} + k\Delta Vdd}$$

  - Total leakage is the summation

$$I_{total} = I_{sub} + I_{gate}$$

# Parameter Decomposition

- Parameter variations further decomposed into two components.

$$\Delta P = \Delta P_{global} + \Delta P_{local}$$

  - $\Delta P_{global}$ , the global (inter-chip) variations
  - $\Delta P_{local}$ , the local (intra-chip) variations
  - Assumed to be independent and normal
  - Result in also normal distribution $\Delta P$

# Improved Leakage Model

- Subthreshold leakage model

$$I_{sub} = I_{sub,nom} \cdot e^{a\Delta L_l^{\;2} + (2a\Delta Lg + b)\Delta L_l + c\Delta V_{th,l} + d\Delta Vdd + \Delta T} \cdot e^{a\Delta L_g^2 + b\Delta L_g + c\Delta V_{th,g}}$$

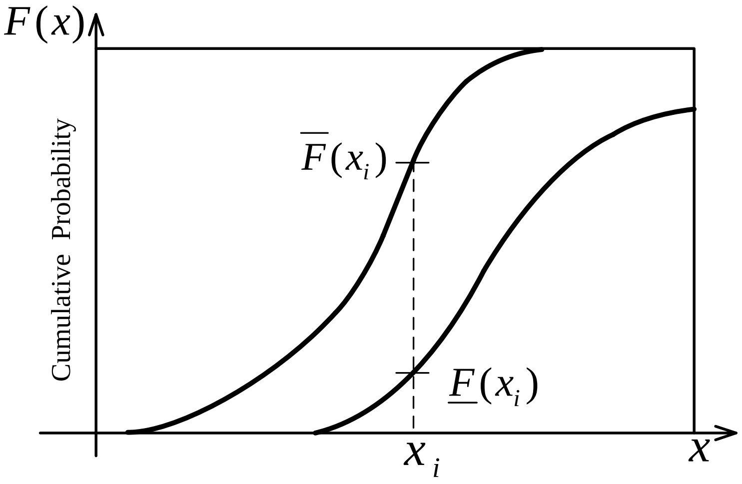
- Gate leakage model

$$I_{gate} = I_{gate,nom} \cdot e^{h\Delta T_{ox,l} + k\Delta Vdd} \cdot e^{h\Delta T_{ox,g}}$$

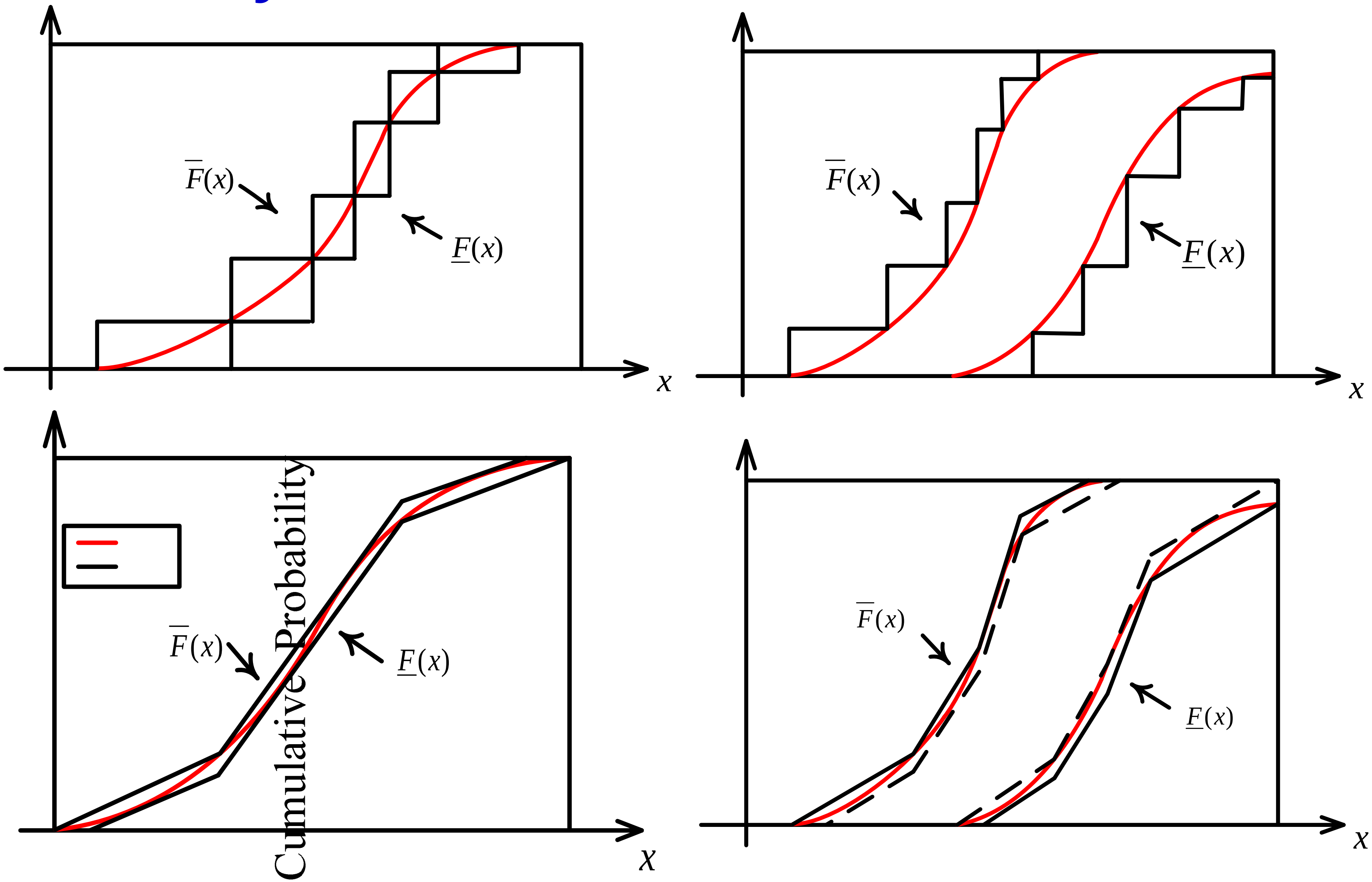
- They are correlated

# Issues with New Technology Nodes

- Parameters are difficult to extract: uncertainty in probability distributions (70 nm and below)
- We use a set of CDFs consisting of a left and a right bound $\underline{F}(x) \leq F(x) \leq \overline{F}(x)$

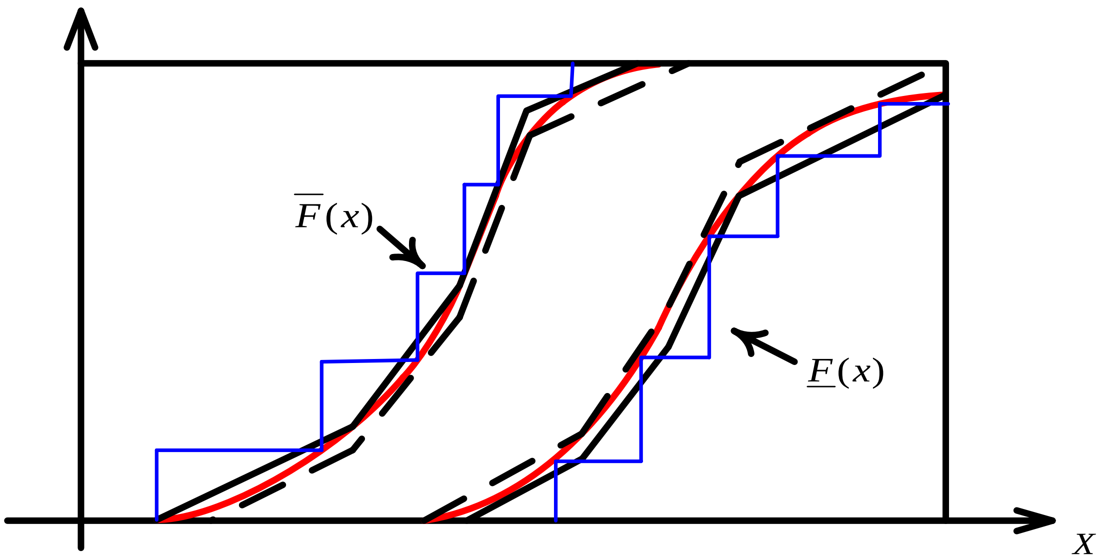


# Chebyshev vs. Discretized Method



# PLPB Representation

- Linearization on CDF: Piece-wise Linear Probability Bounds (PLPB)
- Computation on Parameters' CDF functions



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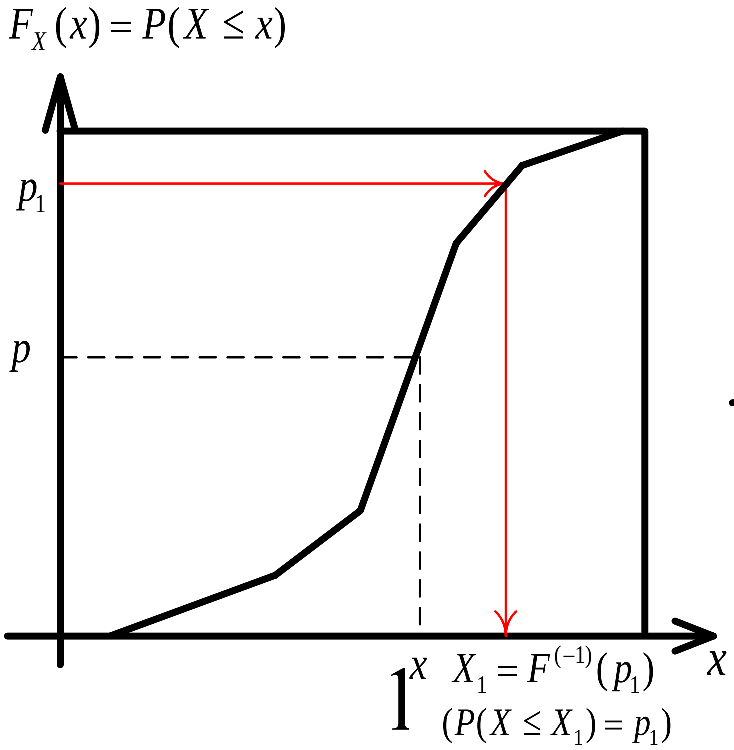
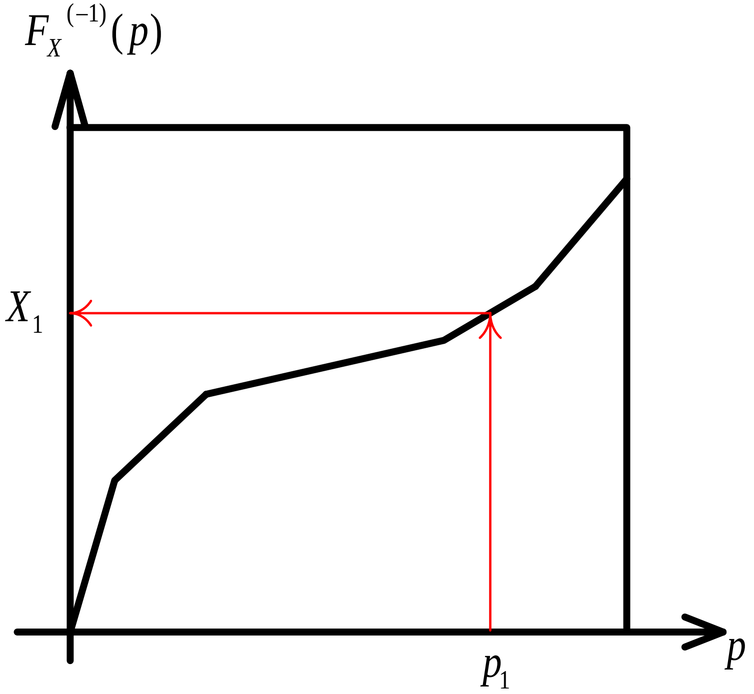
# Piece-wise Linear CDF with its inverse



# Dependency Bounds of Z=X+Y

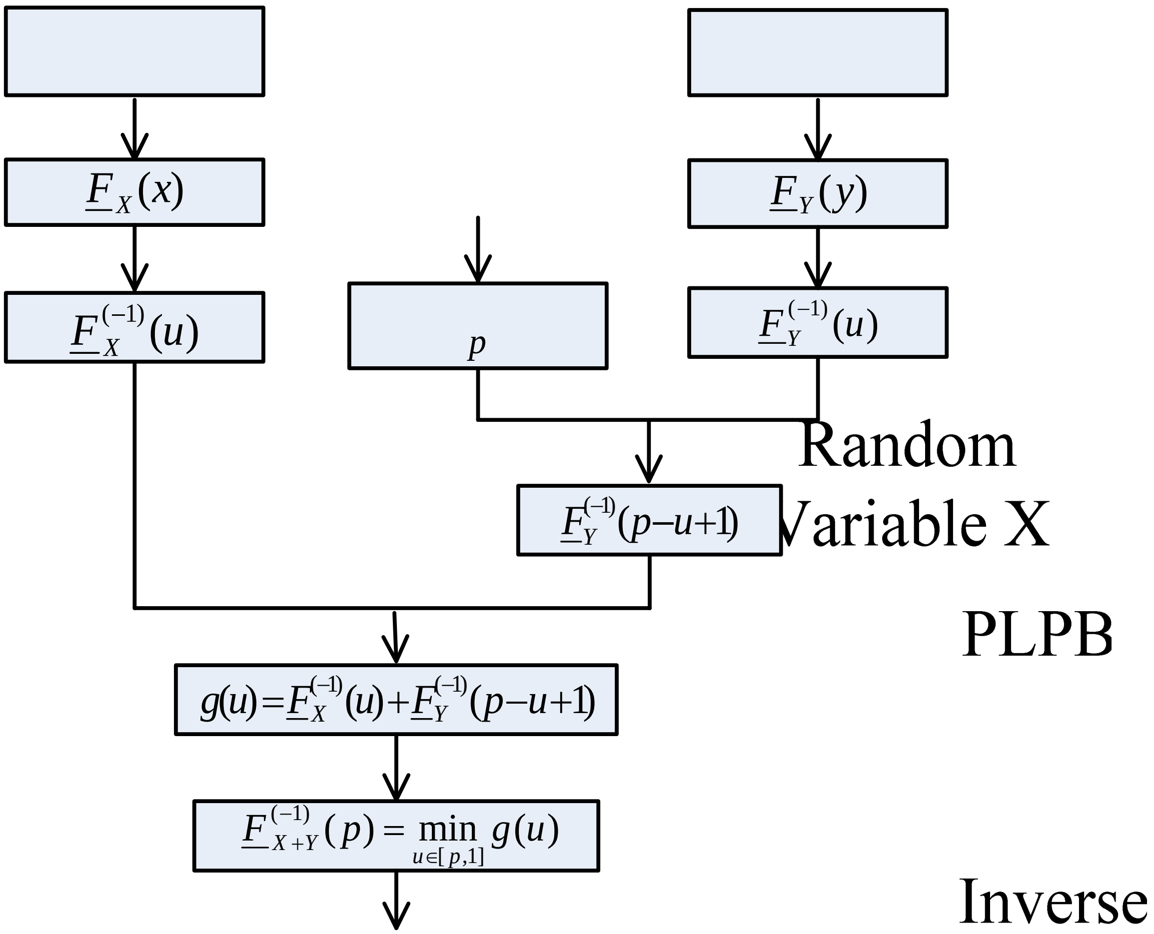
- Upper bound

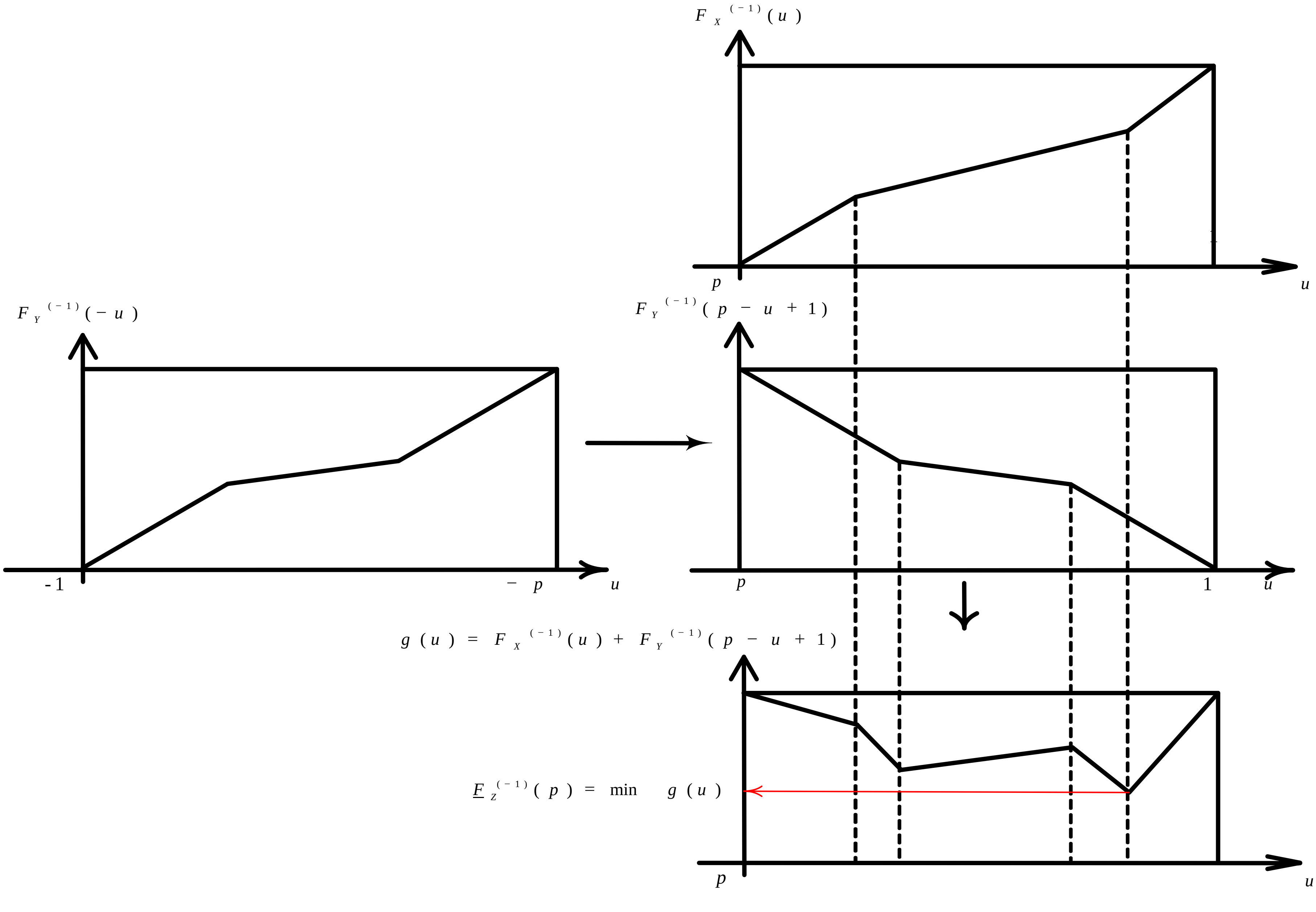
$$\overline{F}^{(-1)}_{X+Y}(p) = \begin{cases} \max\limits_{u\in[0,p]}[\overline{F}^{(-1)}_{X}(u) + \overline{F}^{(-1)}_{Y}(p-u)] & if \quad p \neq 1 \\ \overline{F}^{(-1)}_{X}(1) + \overline{F}^{(-1)}_{Y}(1) & if \quad p = 1 \end{cases}$$

- Lower bound

$$\underline{F}^{(-1)}_{X+Y}(p) = \begin{cases} \min\limits_{u\in[p,1]}[\underline{F}^{(-1)}_{x}(u) + \underline{F}^{(-1)}_{Y}(p-u+1)] & if \quad p \neq 0 \\ \underline{F}^{(-1)}_{X}(0) + \underline{F}^{(-1)}_{Y}(0) & if \quad p = 0 \end{cases}$$

# Dataflow of Computing $\underline{F}_{X+Y}^{(-1)}(p)$




$F_X^{(-1)}(u)$
p
u
$F_Y^{(-1)}(-u)$
-1
$-p$
u
$F_Y^{(-1)}(p-u+1)$
p
1
u
$g(u) = F_X^{(-1)}(u) + F_Y^{(-1)}(p-u+1)$
$\underline{F}_Z^{(-1)}(p) = \min \; g(u)$
p
u

# Dependency bounds of X-Y

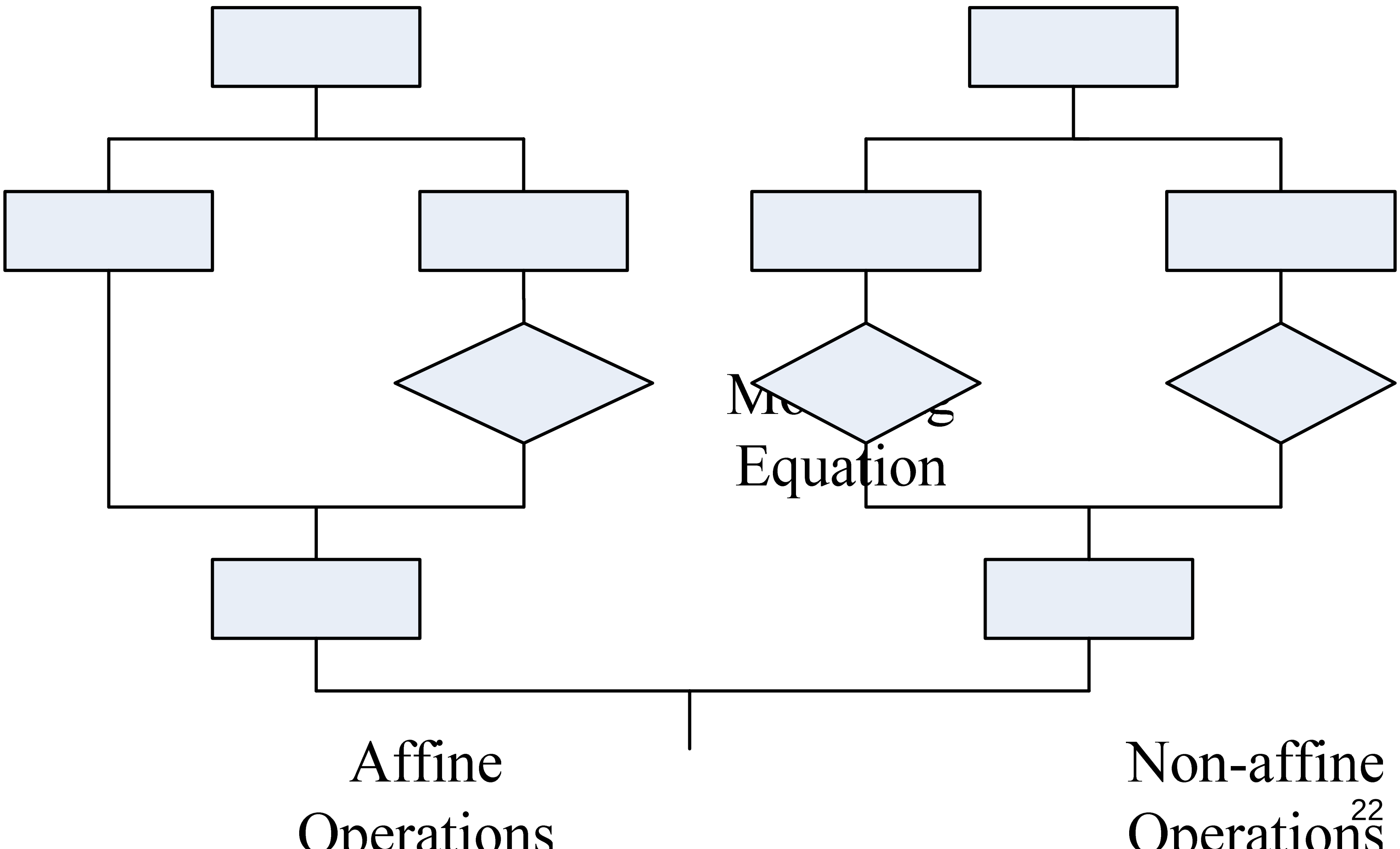
- Upper bound

$$\overline{F}_{X-Y}^{(-1)}(p) = \begin{cases} \max\limits_{u \in [0,x]} [\overline{F}_X^{(-1)}(u) - \underline{F}_Y^{(-1)}(u-p+1)] & \text{if} \quad p \neq 1 \\ \overline{F}_X^{(-1)}(1) - \underline{F}_Y^{(-1)}(0) & \text{if} \quad p = 1 \end{cases}$$

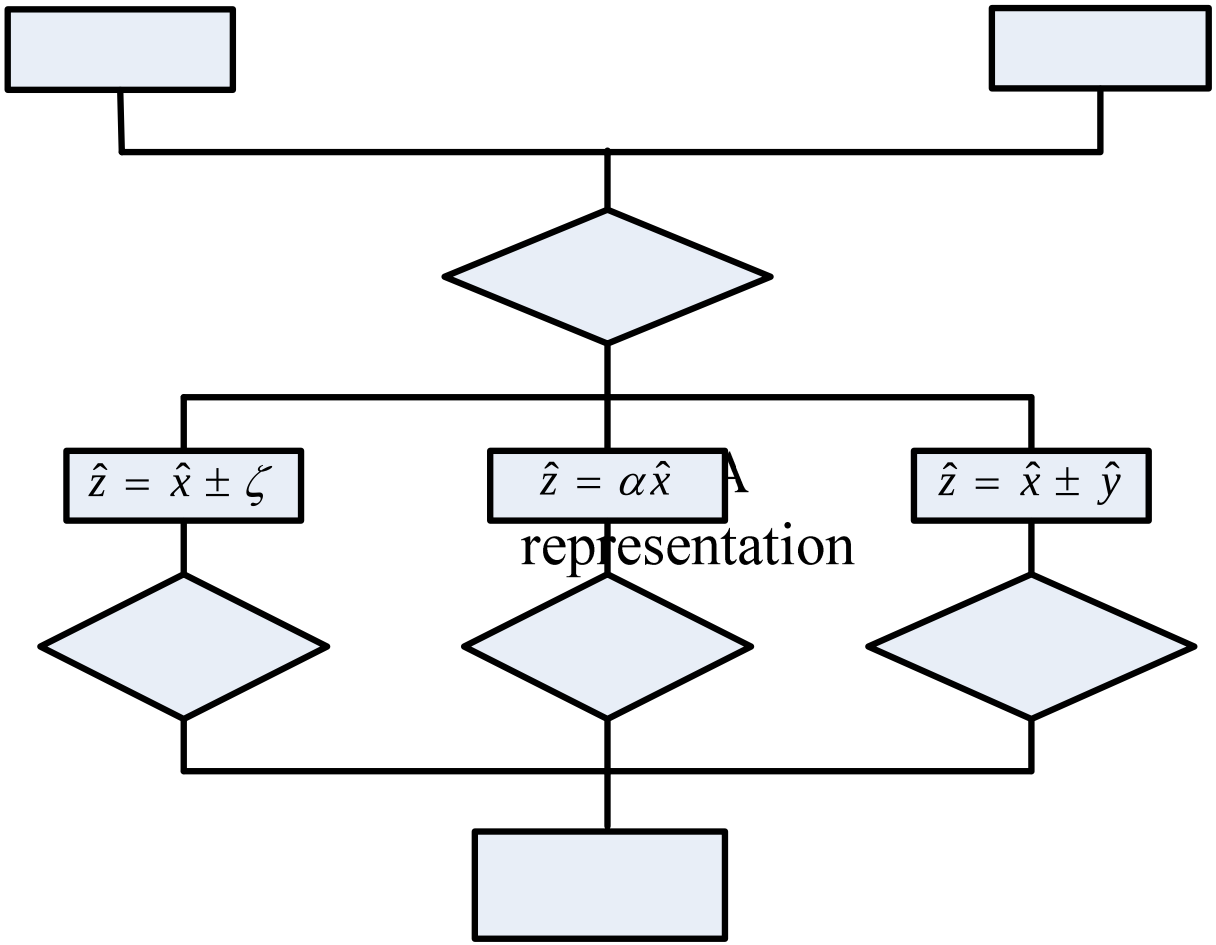
- Lower bound

$$\underline{F}_{X-Y}^{(-1)}(p) = \begin{cases} \min\limits_{u \in [x,1]} [\overline{F}_X^{(-1)}(u) - \underline{F}_Y^{(-1)}(u-p)] & \text{if} \quad p \neq 0 \\ \overline{F}_X^{(-1)}(0) - \underline{F}_Y^{(-1)}(1) & \text{if} \quad p = 0 \end{cases}$$

# Yield Prediction Procedure



# Yield Prediction Procedure (continued)

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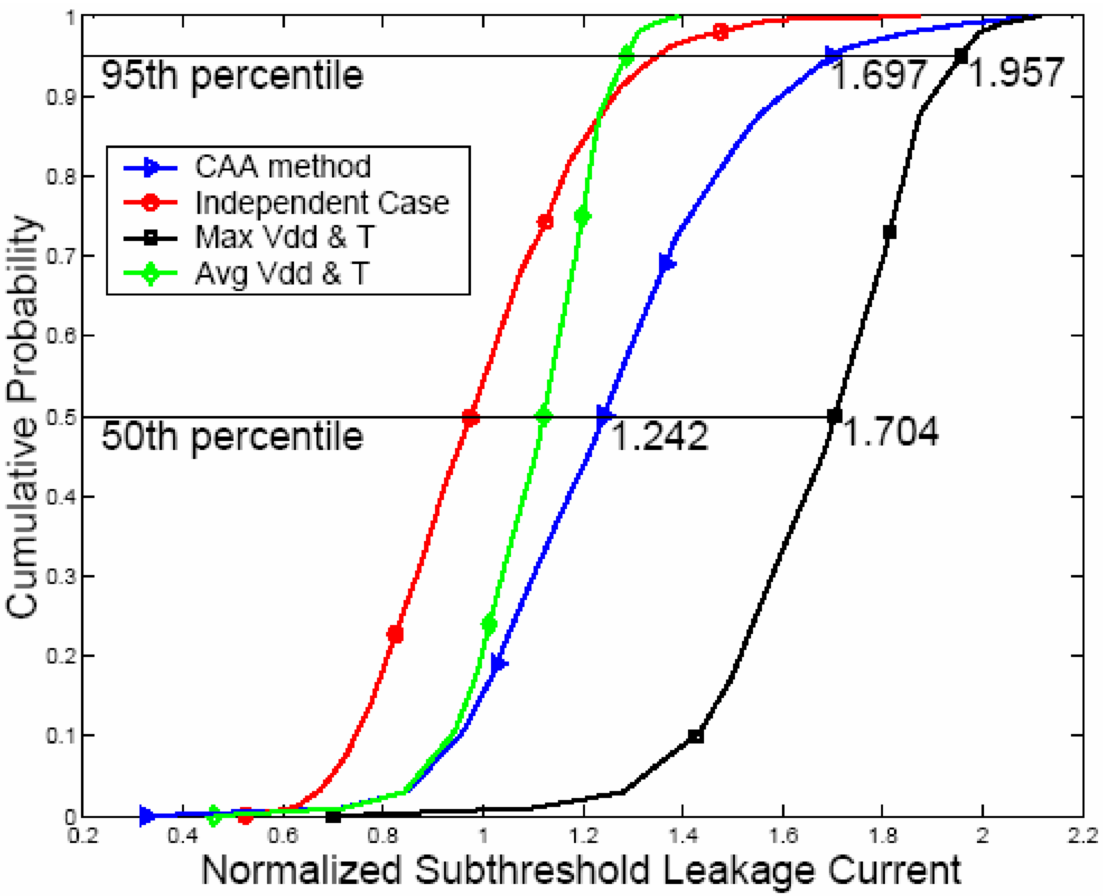
# Experiment Environments

- 65nm Technology node PTM model.
  - Leff = 24.5nm
- Coefficients extracted by SPICE simulations.
- Parameter variations
  - Modeled as truncated Gaussian distributions.
  - Can be well handled if non-Gaussian.
  - Leff: 20% variation, Vth: 10% variation, Tox: 8% variat ion

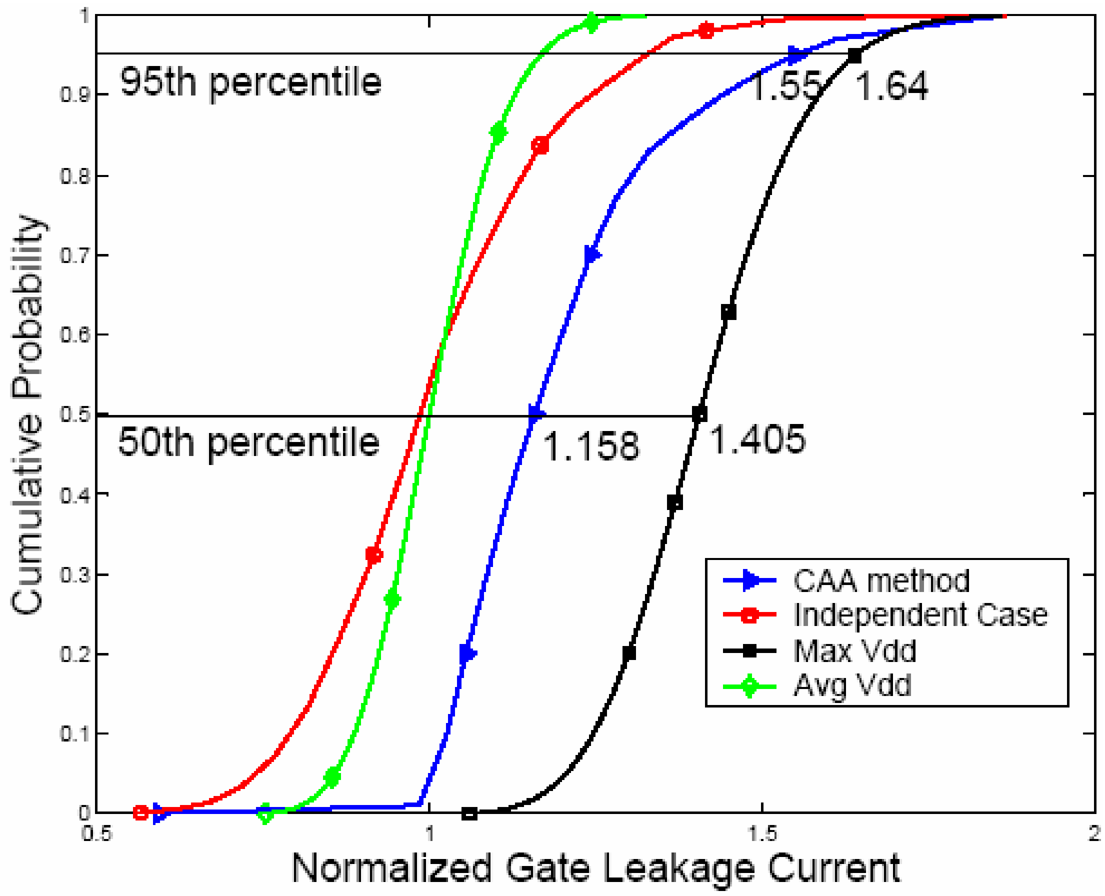
    Vdd: 10% variation, T: 10℃ variation.

# Comparison with MC simulation and interval analysis: $I_{sub}$



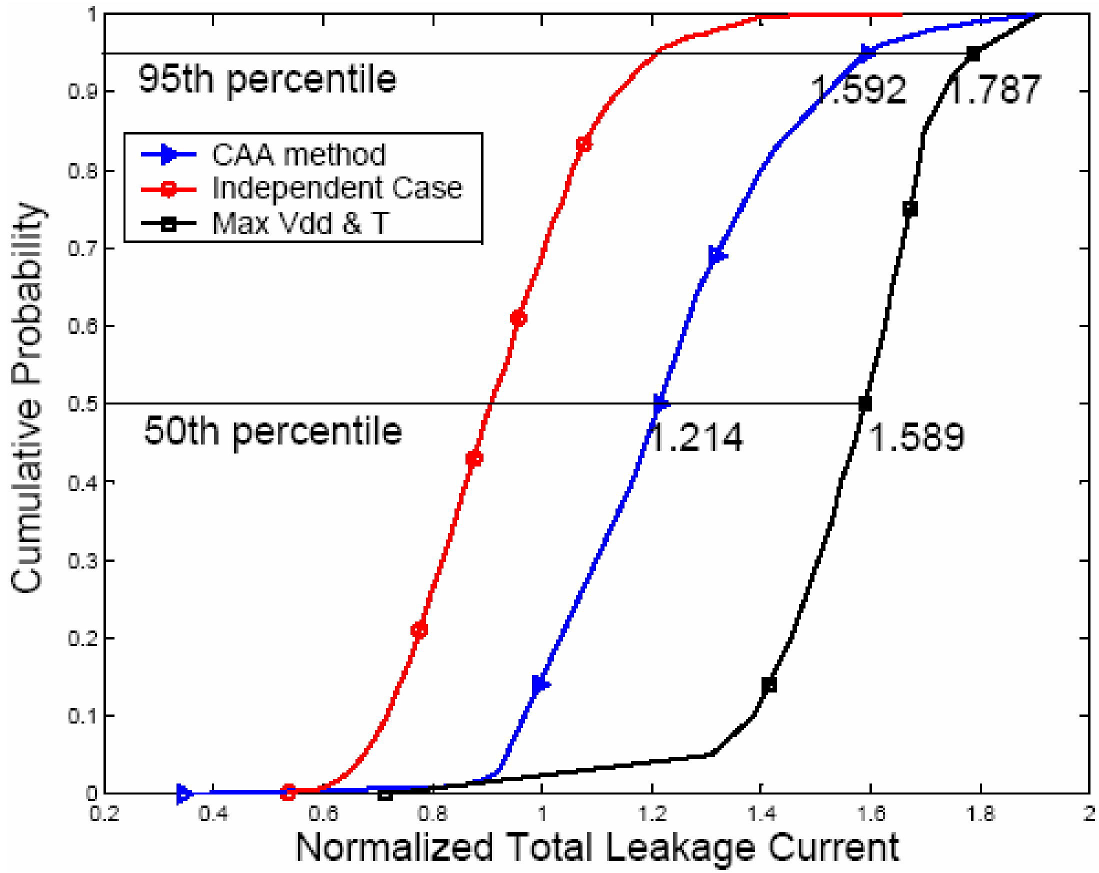

- Improvements:
  - 50% percentile 13.3%
  - 95% percentile 27.1%
  - Mean value 25.1% (1.674->1.254)

# Comparison with MC simulation and interval analysis: $I_{gate}$



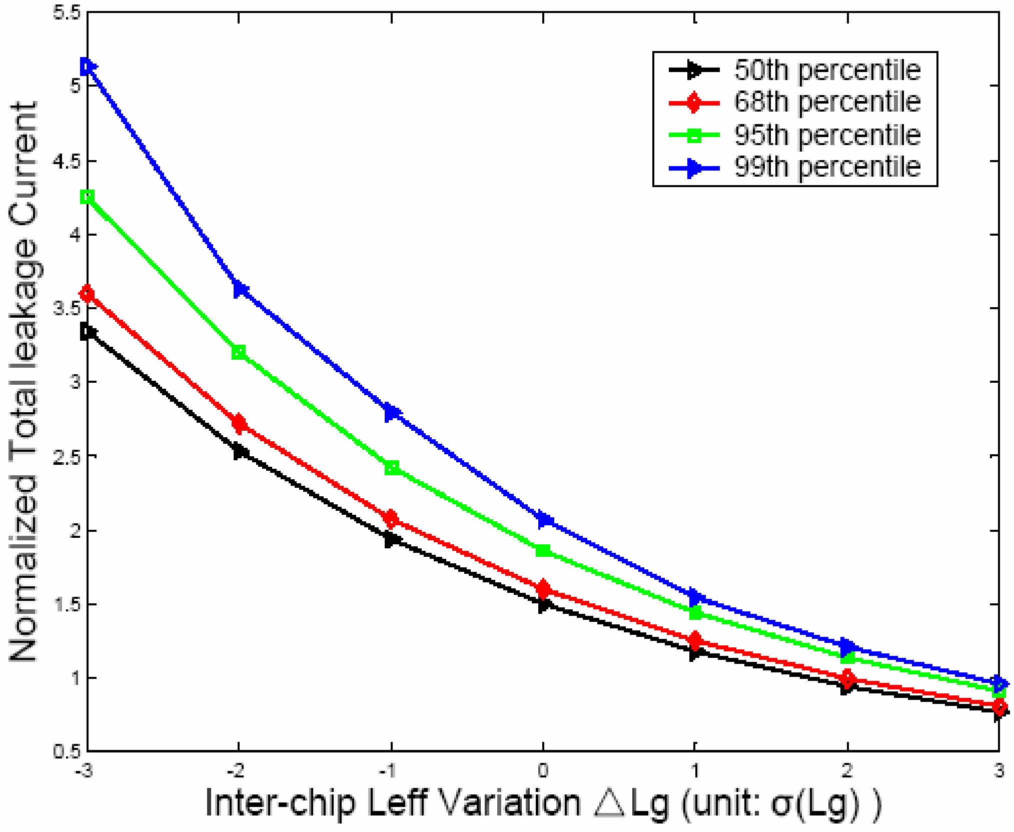

- Improvements:
  - 50% percentile 5.5%
  - 95% percentile 17.6%
  - Mean value 15.3% (1.412->1.196)

# Comparison with MC simulation and interval analysis: $I_{total}$




- Improvements:
  - 50% percentile 10.9%
  - 95% percentile 23.6%
  - Mean value 21.7% (1.566->1.226)

# Contours for inter-chip L Variation




- Shorter channel len gth causes more si gnificant variation of leakage current.

# Conclusion

- Based on Chebyshev affine arithmetic
- Handle uncertainty of distributions
- Deal with correlations among variations
- Efficient and reliable yield prediction