Rule-Based Optimization of Reversible Circuits

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Outline

- Introduction
- Basic Concepts
- Previous Work
- Proposed Methods
- Experimental Results
- Conclusions

Power dissipation

- Rolf Landauer (1961)
  - Every lost bit causes an energy loss
  - Using conventional irreversible logic gates leads to energy dissipation
    - regardless of the underlying circuit

Motivation

- Decrease in power dissipation
- Application in
  - Low-power CMOS design
  - Quantum computing
    - Each unitary quantum gate is intrinsically reversible

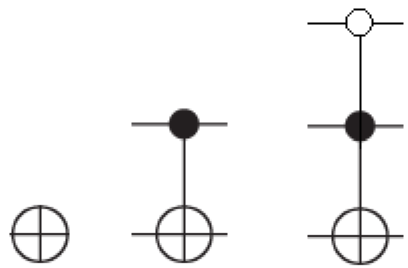
Basic Concepts

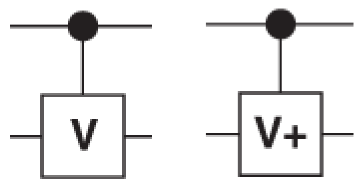
- Boolean reversible functions
  - *n*-input, *n*-output,
  - Unique output assignment
  - Example: a 3-input, 3-output
    - function (2,7,0,1,6,3,4,5)

a_1	a_2	a_3		f_1	f_2	f_3	
0	0	0	0	0	1	0	2
0	0	1	1	1	1	1	7
0	1	0	2	0	0	0	0
0	1	1	3	0	0	1	1
1	0	0	4	1	1	0	6
1	0	1	5	0	1	1	3
1	1	0	6	1	0	0	4
1	1	1	7	1	0	1	5

Basic Concepts

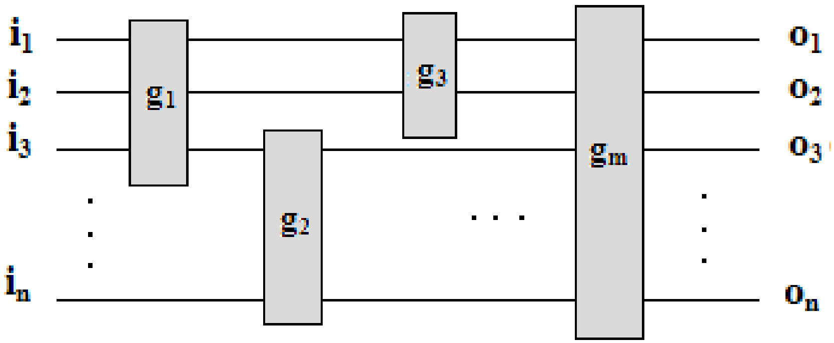
- Reversible gate
- Various reversible gates
  - C^mNOT gates
    - NOT, CNOT, C^2NOT (Toffoli), …
      - **Positive controls**
      - **Negative controls**
  - Controlled-V
  - Controlled-V+





Basic Concepts

- Elementary gates:
  - NOT, CNOT, controlled-V, and controlled-V+ (with positive controls)
- Quantum cost:
  - The number of elementary gates required for simulating a given gate
- Reversible circuit:
  - A set of reversible gates



Reversible Circuits:

Synthesis and Optimization

High-level Description

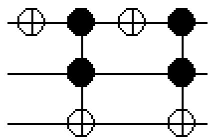
↓

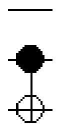
Gate-level circuits

↓

Optimal Gate-level circuits

a_1	a_2	a_3	f_1	f_2	f_3
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0





Previous Work

- A set of local transformation rules [4]
  - Complete Set
    - Change any two equivalence circuits to each other
- Developing a design theory
  - Improving Boolean reversible cost

Previous Work

- A set of predefined patterns: ***Templates*** [6,8,10]
  - Template ***T*** :
    - A circuit with ***m*** gates
    - Identity function
  - Find the first ***k*** (***k*** > ***m/2***) gates in a circuit
  - A reverse of ***m-k*** gates can be applied instead of the initial ***k*** gates
  - Reduce gate count or quantum cost

Previous Work

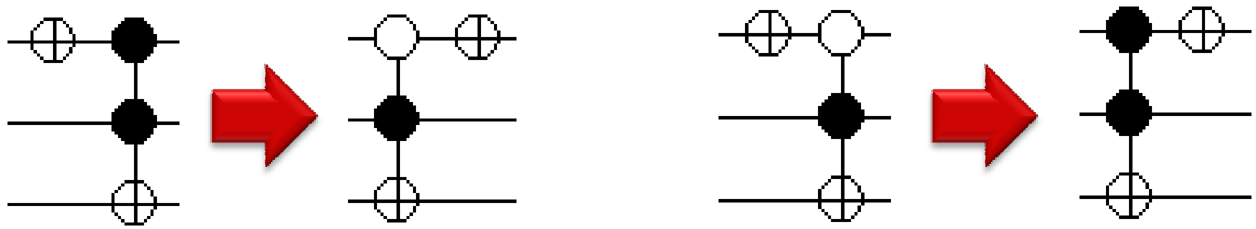
- Developed data structures [9]
  - Generate and store optimal circuits
    - All reversible functions of size 3
    - Many of four inputs circuits
- Examined less than 5 variables sub-circuits
  - The optimal implementation is explored in a pre-constructed library

Proposed Methods

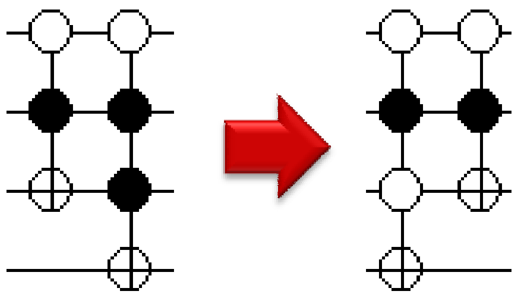
- NOT Reduction
  - Pass Rule (PR)
  - Generalized Pass Rule (GPR)
- Gates with Common Targets
  - Common-Target Rule (CTR)
  - Restricted CTR (R-CTR)

PR & GPR

- Pass Rule (PR)

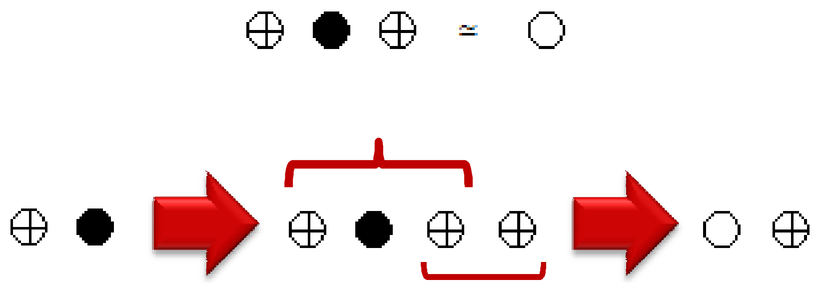


- Generalized Pass Rule (GPR)

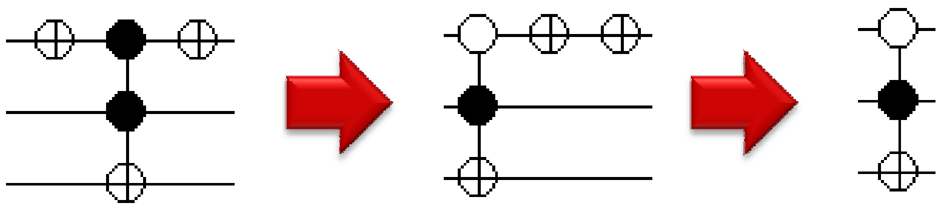


PR & GPR

- Proof



- Example



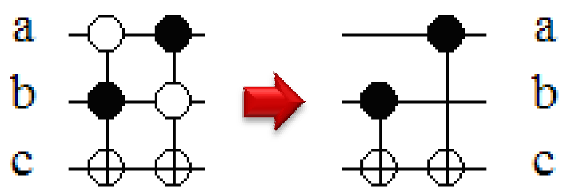
Gates with Common Targets

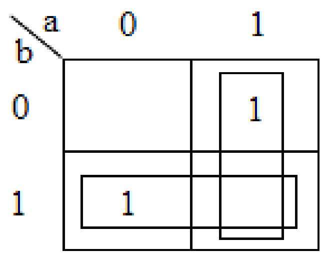
- Using Kmap for optimization
  - For sub-circuits with common targets
  - A $C^{n\text{-}1}$NOT gate can be represented by a Boolean expression with n-1 inputs and one output
    - Gate controls => Inputs
    - Gate target => Output
  - Each group in Kmap defines a gate with n-p controls
    - n => Sub-circuit size
    - 2^p => Group size

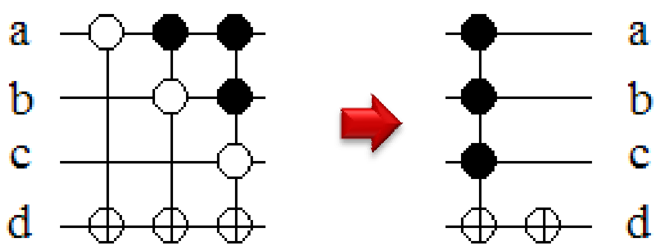
CTR & R-CTR

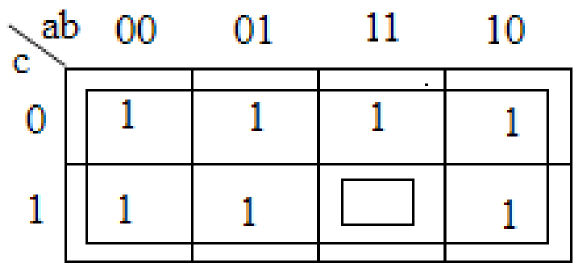
- Common Target Rule (CTR)
  - Each reversible sub-circuit of size *n* with common targets can be optimized by using Kmap
- Restricted CTR (R-CTR)
  - CTR for 2-input sub-circuits

CTR Examples









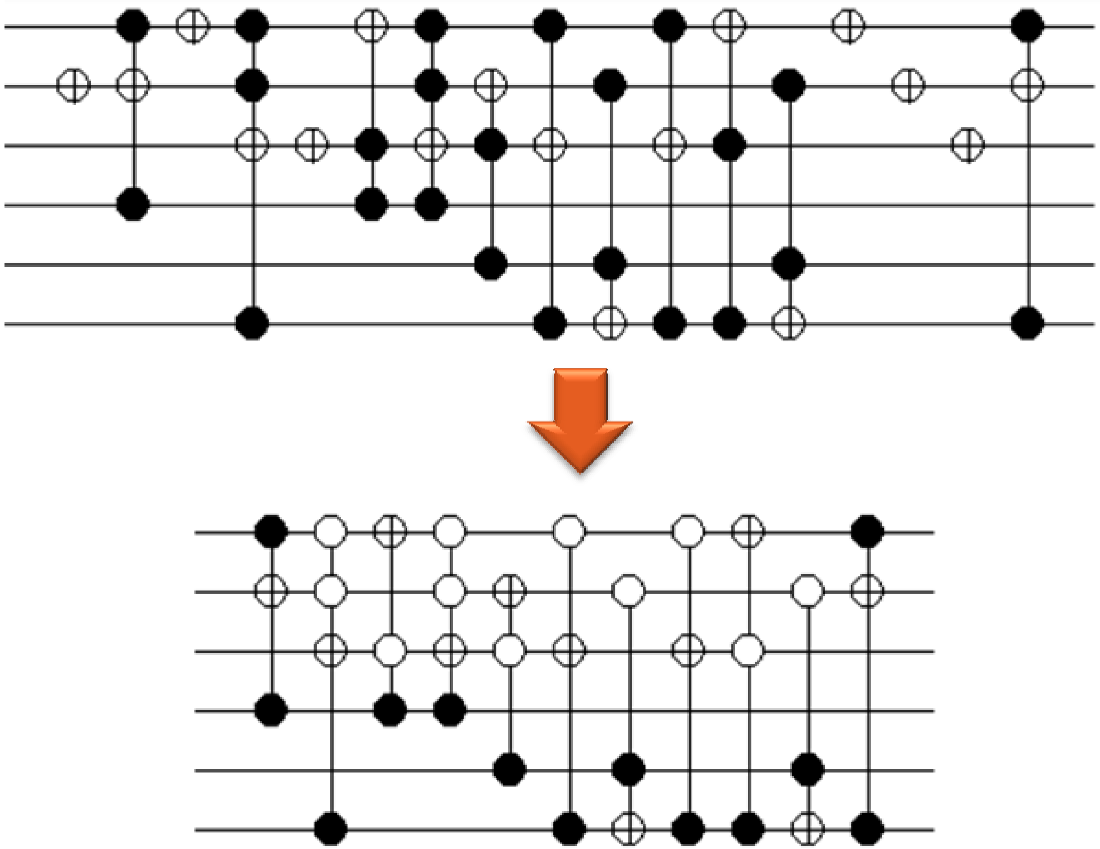
Experimental Results

# Circuits	Number of inputs	specification	Quantum cost		Garbage	Decrease percent
			[15]	Ours		
1	3	(1,0,3,2,5,7,4,6)	18	17	-	5.5%
2	3	(7,0,1,2,3,4,5,6)	7	7	-	0%
3	3	(0,1,2,3,4,6,5,7)	15	15	-	0%
4	3	(0,1,2,4,3,5,6,7)	27	27	-	0%
5	4	(0,1,2,3,4,5,6,8,7,9,10,11,12,13,14,15)	195	131	-	32.8%
6	3	(1,2,3,4,5,6,7,0)	10	7	-	30%
7	4	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,0)	25	20	-	20%
8	4	(0,7,6,9,4,11,10,13,8,15,14,1,12,3,2,5)	12	12	-	0%
9	3	(3,6,2,5,7,1,0,4)	32	29	-	9.3%
10	3	(1,2,7,5,6,3,0,4)	35	26	-	25.7%
11	3	(4,3,0,2,7,5,6,1)	37	29	-	21.6%
12	3	(7,5,2,4,6,1,0,3)	28	19	-	32.1%
13	4	(6,2,14,13,3,11,10,7,0,5,8,1,15,12,4,9)	214	136	-	**36.4%**

Experimental Results (Cont.)

# Circuits	Circuits	Number of inputs	Quantum cost		Garbage	Decrease percent
			[10]	Ours		
1	3_17	3	14	13	-	7.14%
2	4_49	4	32	30	-	6.25%
3	t-add-8	24	322	314	-	2.48%
4	hwb5	5	104	101	-	2.88%
5	hwb6	6	142	140	-	1.40%
6	hwb7	7	2,521	2,516	True	0.20%
7	hwb8	8	6,709	6,687	True	0.33%
8	hwb9	9	20,224	20,207	True	0.08%
9	hwb10	10	52,245	52,225	True	0.04%
10	hwb11	11	121,840	121,830	True	0.008%
11	mod5adder	6	77	71	-	**7.80%**
12	rd53	7	65	62	-	4.61%

Experimental Results (Cont.)



Conclusions

- An optimization approach for reversible circuits
  - A set of rules
  - Both negative and positive control Toffoli gates
- Reduce NOT gates
- Karnaugh map-based optimization method

Thank you for your attention!