On-Chip Power Network Optimization with Decoupling Capacitors and Controlled-ESRs

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Outline of Optimization with Decap and Controlled-ESR

- Introduction
  - Existing works add decap to reduce noise
  - Controlled-ESR is shown to be effective to suppress the resonance
- Power network model with controlled-ESR
- Problem statement
  - Power network noise considering overshoot
  - Formulation
- Revised sensitivity computation
  - Sensitivity computation with merged adjoint network
  - Revised sensitivity computation considering voltage overshoot
- SQP based optimization
- Experimental results
- Conclusions
Our Contributions

- We propose to allocate decaps and controlled-ESRs simultaneously to suppress the resonance and reduce SSN of power network.

- We consider both voltage drop and overshoot for voltage violation. Derive revised sensitivity.

- An optimization formulation with the objective function of minimizing the voltage violation area and a constraint of decap budget is presented, and solved with an efficient SQP algorithm.
Power Network Model with Controlled-ESR
Voltage Variation Analysis with Circuit State Equation

From KCL and KVL, we have the circuit state equation:

\[
\begin{bmatrix}
C & 0 \\
0 & L
\end{bmatrix} \begin{bmatrix}
\dot{v} \\
i
\end{bmatrix} = \begin{bmatrix}
-G & -E \\
E^T & -R
\end{bmatrix} \begin{bmatrix}
v \\
i
\end{bmatrix} + BU
\]  \hspace{1cm} (1)

which is denoted to be:

\[C\dot{x} = Ax + Bu\]  \hspace{1cm} (2)

If add extra decap \( \Delta C \) and controlled-ESR \( \Delta A \), solution \( x \) will be updated by \( \Delta x \), so (2) becomes:

\[(C + \Delta C)(\dot{x} + \Delta\dot{x}) = (A + \Delta A)(x + \Delta x) + Bu\]  \hspace{1cm} (3)

By subtracting (2) from (3):

\[(C + \Delta C)\Delta\dot{x} = (A + \Delta A)\Delta x + (\Delta Ax - \Delta C\dot{x})\]  \hspace{1cm} (4)

The solution of (4) is:

\[
\Delta x = e^{\tilde{C}^{-1}\tilde{A}(t-t_0)} \Delta x_0 + \int_{t_0}^{t} e^{\tilde{C}^{-1}\tilde{A}(t-\tau)} \tilde{C}^{-1} \tilde{U}(\tau) d\tau
\]  \hspace{1cm} (5)

where: \( \tilde{C} \equiv C + \Delta C \), \( \tilde{A} \equiv A + \Delta A \), \( \tilde{U} \equiv \Delta Ax - \Delta C\dot{x} \), \( e^\tilde{A} \equiv I + \tilde{A} + \frac{\tilde{A}^2}{2!} + \frac{\tilde{A}^3}{3!} + \ldots \)
Effect of Controlled-ESR on reducing the noise
Power Network Noise Considering Overshoot

\[ g_j = \int_0^T \max(V_{\text{min}} - v_j(t), 0) dt \]

\[ g_j = \int_0^T \max[\max(V_{\text{min}} - v_j(t), 0), \max(v_j(t) - V_{\text{max}}, 0)] dt \]
Problem Formulation

- **Objective function:**
  - Min \( \sum_{j=1}^{N} g_j \)

- **Constraints:**
  - (1) Voltage response satisfies the circuit equation with given stimulus;
  - (2) Total decap budget: \( \sum_{i=1}^{M} c_i \leq Q \)
  - (3) Space constraint for each decap location: \( 0 \leq c_i \leq c_{\text{max}i} \)
  - (4) Space constraint for each controlled-ESR location:
    \[
    0 \leq \text{CtrlESR}_i \leq \text{CtrlESR}_{\text{max}i}
    \]
Sensitivity Computation with Merged Adjoint Network

The sensitivity $s_{ij}$ is defined to be the contribution of decap added at node $i$ to remove violation at node $j$: $s_{ij} = \frac{\partial g_j}{\partial c_i}$

The merged adjoint sensitivity is defined to be the contribution of decap added at node $i$ to remove the violation for all nodes.

The merged adjoint network has a current source $u(t-t_s) - u(t-t_e)$ applied at every node $j$

Merged adjoint sensitivity is calculated with

$$s_i = \sum_{j=1}^{N} s_{ij} = \int_0^T (\tilde{v}_{i,all}(T-t)) \times \dot{v}_i(t) dt, \quad (i = 1, 2, \ldots, M)$$
Revised Sensitivity Computation
Considering Overshoot

We denote the port currents and voltages by vectors $I_p$ and $V_p$. Denote the non-source branch currents and voltages by vectors $I_b$ and $V_b$. From Tellegen’s theorem, we have

$$
\int_0^T \left[ -\dot{i}_p(\tau) \Delta v_p(t) + \dot{v}_p(\tau) \Delta i_p(t) \right]_{\tau = T-t} dt = \int_0^T \left[ \dot{i}_b(\tau) \Delta v_b(t) + \dot{v}_b(\tau) \Delta i_b(t) \right]_{\tau = T-t} dt
$$

We set all voltage sources in the adjoint network to zero and apply a current source for each violation node:

$$
I_s = \sum_{k=1}^{N_v} D_k \left[ u(t-t_{sk}) - u(t-t_{ek}) \right]
$$

$$
D_k = \begin{cases} 
-1, & \text{if } v(t_{sk}) > Vdd \\
1, & \text{if } v(t_{sk}) < Vdd 
\end{cases}
$$

Left hand:

$$
\Delta g = \int_0^T \left\{ -\sum_{k=1}^{N_v} D_k \left[ u(t-t_{sk}) - u(t-t_{ek}) \right] \Delta v_p(t) \right\}_{\tau = T-t} dt
$$

Right hand:

$$
s_C = \frac{\Delta g}{\Delta C} = \int_0^T \left[ \dot{v}_c(\tau) \Delta v_c(t) \right]_{\tau = T-t} dt
$$

$$
s_R = \frac{\Delta g}{\Delta R} = \int_0^T \left[ \dot{i}_R(\tau) \Delta i_R(t) \right]_{\tau = T-t} dt
$$
Algorithm for the SQP based optimization:
1. Select the intrinsic capacitance and controlled-ESR to be the initial solution $X^{(0)}$.
2. Simulate the power network circuit, and compute the sensitivity as gradient using the revised method.
3. Use the gradient to approximate the problem with a linearly constrained QP subproblem at $X^{(t)}$.
4. Solve for the step size $d^{(t)}$ to move.
5. If meet with termination condition, stop;
   - Else, let $X^{(t+1)} = X^{(t)} + d^{(t)}$.
6. Increase $t$ and return to step 2.
A Simple Case

Initial values:
- \( R = 1 \) ohm and \( L = 1 \) nH. Decap=0.01 nF, controlled-ESR = 1.0e-4 ohm.
- \( Vdd \) is 1V, and the allowable voltage drop is 0.05V.

Constraints:
- Maximum allowable decap at each node to be 0.1 nF,
- Total decap should not exceed 0.2 nF,
- Maximum allowable controlled-ESR at each node is 0.2 Ohm

Without optimization: The overall noise is 193.7 V*ps.

Optimize with decap only:
- Decap at each node are: 0.1 nF, 0.09 nF, and 0.01 nF.
- The noise after optimization is 6.3 V*ps.

Optimize with both decap and controlled-ESR:
- Controlled-ESR values at each node are: 0.2 Ohm, 2.77e-2 Ohm, and 1.0e-4 Ohm.
- The noise is further reduced to be 5.3 V*ps.
- The controlled-ESR improves the noise by 15.9%.
### Experimental Results

#### Table I. Effect of considering voltage overshoot

<table>
<thead>
<tr>
<th>Circuit</th>
<th># Node</th>
<th>Consider voltage drop only</th>
<th>Consider both voltage drop and overshoot</th>
<th>Noise underestimation due to neglecting overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Noise (V*ps)</td>
<td># Violation node</td>
<td>Noise (V*ps)</td>
</tr>
<tr>
<td>CKT1</td>
<td>858</td>
<td>306.7</td>
<td>46</td>
<td>324.8</td>
</tr>
<tr>
<td>CKT2</td>
<td>1794</td>
<td>7406.3</td>
<td>325</td>
<td>8127.8</td>
</tr>
<tr>
<td>CKT3</td>
<td>2006</td>
<td>5111.3</td>
<td>309</td>
<td>5324.9</td>
</tr>
<tr>
<td>CKT4</td>
<td>3634</td>
<td>9770.1</td>
<td>268</td>
<td>10049.6</td>
</tr>
<tr>
<td>CKT5</td>
<td>8330</td>
<td>5608.1</td>
<td>2470</td>
<td>5829.1</td>
</tr>
<tr>
<td>CKT6</td>
<td>14852</td>
<td>31420.8</td>
<td>2243</td>
<td>32477.3</td>
</tr>
</tbody>
</table>

- The noise is the voltage violation area and the number of violation nodes.
- Total noise is on average underestimated by 4.8% due to neglecting the voltage overshoot.
- The number of violation nodes is almost the same for both cases.
## Experimental Results

**Table II. Comparison among three methods for the minimization of power network noise**

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Evenly allocate the decaps</th>
<th>Allocate decaps only with the SQP-based method</th>
<th>Allocate both decaps and controlled-ESRs with the SQP-based method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Noise (V*ps)</td>
<td># Violation node</td>
<td>Noise (V*ps)</td>
</tr>
<tr>
<td>CKT1</td>
<td>229.5</td>
<td>20</td>
<td>113.4</td>
</tr>
<tr>
<td>CKT2</td>
<td>6137.1</td>
<td>156</td>
<td>2538.0</td>
</tr>
<tr>
<td>CKT3</td>
<td>4597.9</td>
<td>141</td>
<td>2308.3</td>
</tr>
<tr>
<td>CKT4</td>
<td>8939.0</td>
<td>235</td>
<td>2212.5</td>
</tr>
<tr>
<td>CKT5</td>
<td>5352.3</td>
<td>1245</td>
<td>1694.3</td>
</tr>
<tr>
<td>CKT6</td>
<td>26916.9</td>
<td>1191</td>
<td>6538.1</td>
</tr>
</tbody>
</table>

- The noise (column 2, 4, 6) and the number of violation nodes (column 3, 5, 7) are reduced.
- The improvement brought by considering the controlled-ESRs is 25% on average.
- With the third method, the average allocated controlled-ESR ranges from 0.038 Ohm to 0.083 Ohm for different cases.
Voltage Waveforms with Different Optimizations

![Graph showing voltage waveforms with different optimizations](image)
Relationship between Decap Budget and Noise

- Larger decap budget leads to smaller noise
- Tradeoff between the noise reduction and the decap investment
Conclusions

- Optimize power network with both decap and controlled-ESR.

- Revised sensitivity computation considering voltage overshoot.

- SQP based optimization
Thank You!

Q & A