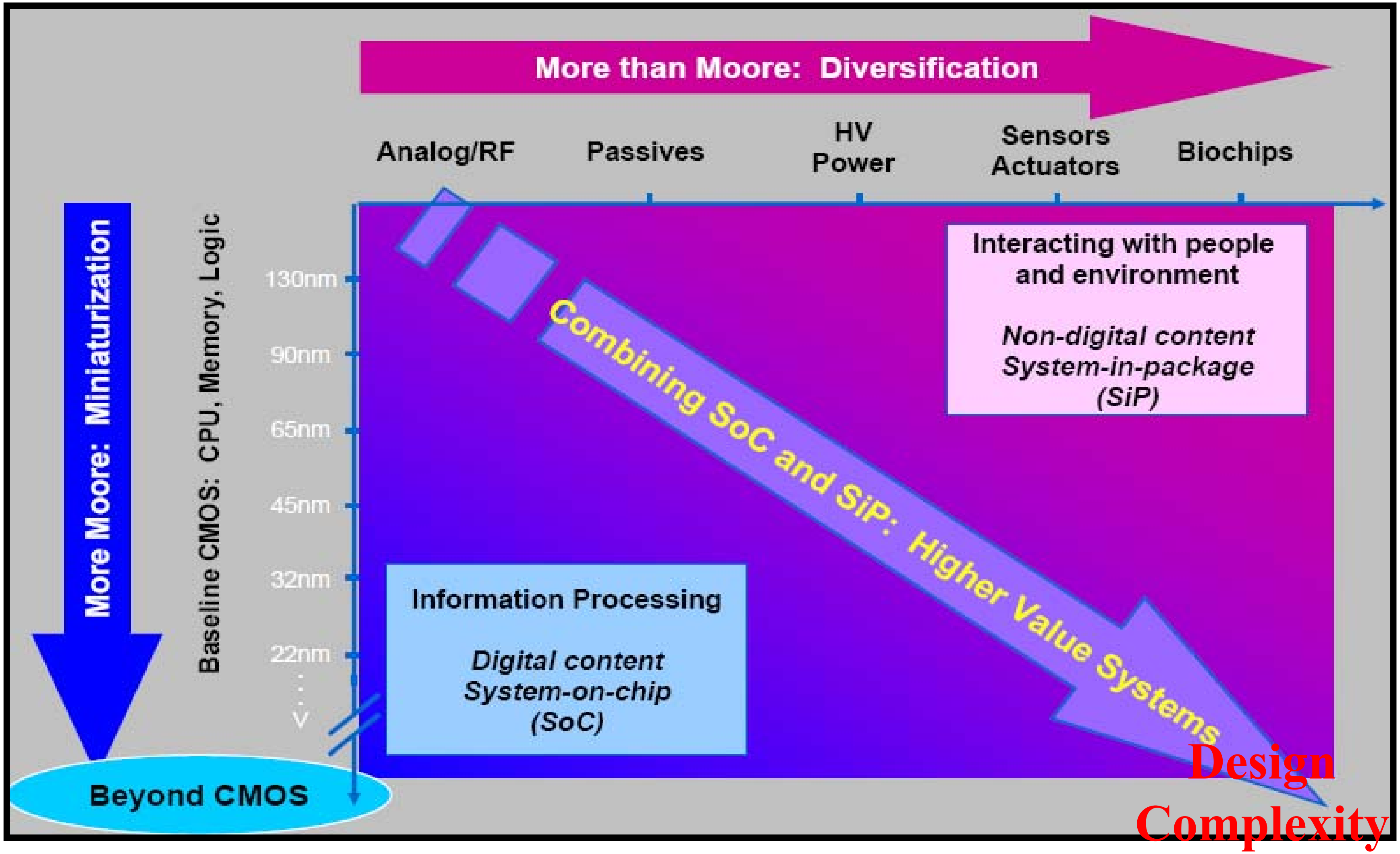
Talk @ ASPDAC'10

A Fast Analog Mismatch Analysis by an Incremental and Stochastic Trajectory Piecewise Linear Macromodel

Hao Yu
School of EEE
Nanyang Technological Univ., Singapore

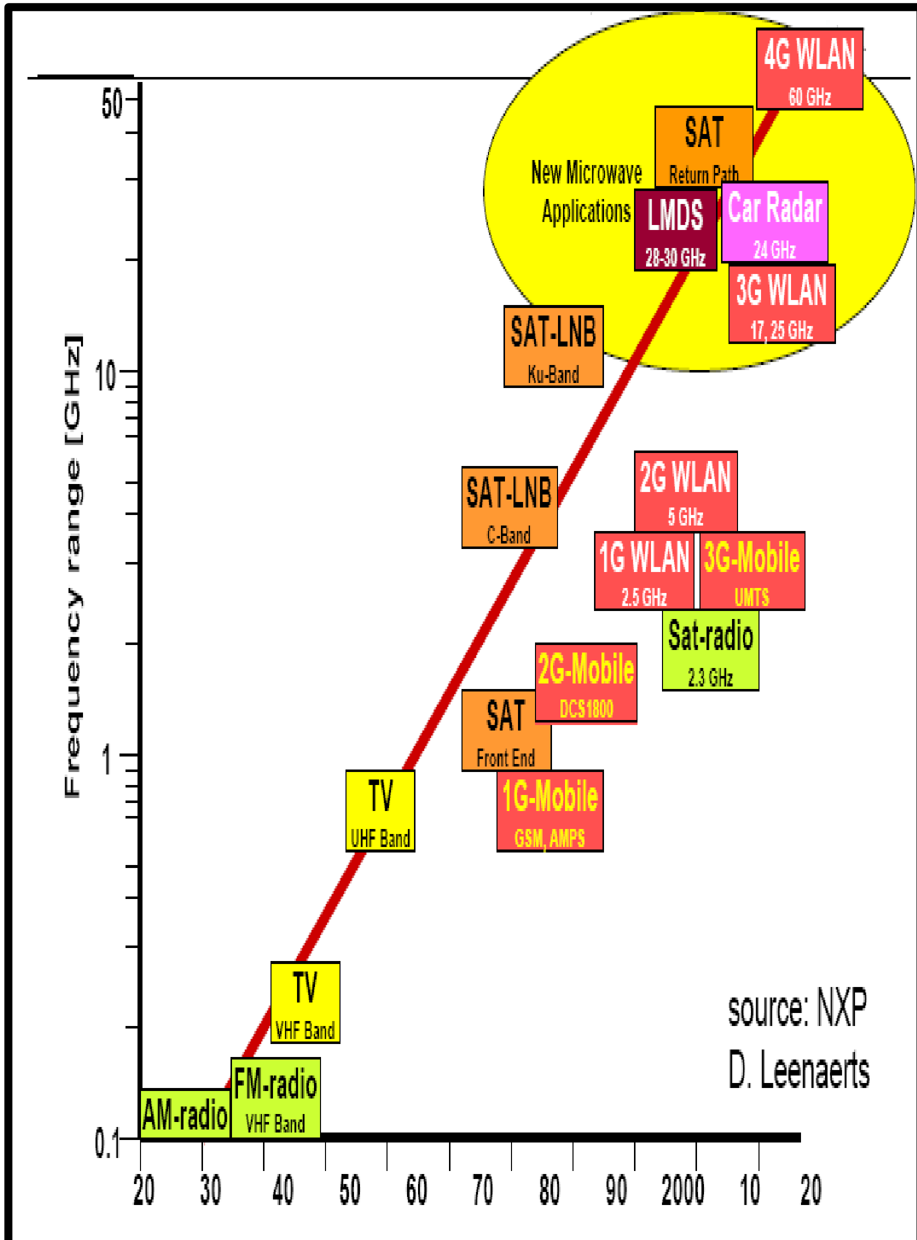
Xuexin Liu, Hai Wang, Sheldon X.D. Tan
EE Department
Univ. of California at Riverside, USA

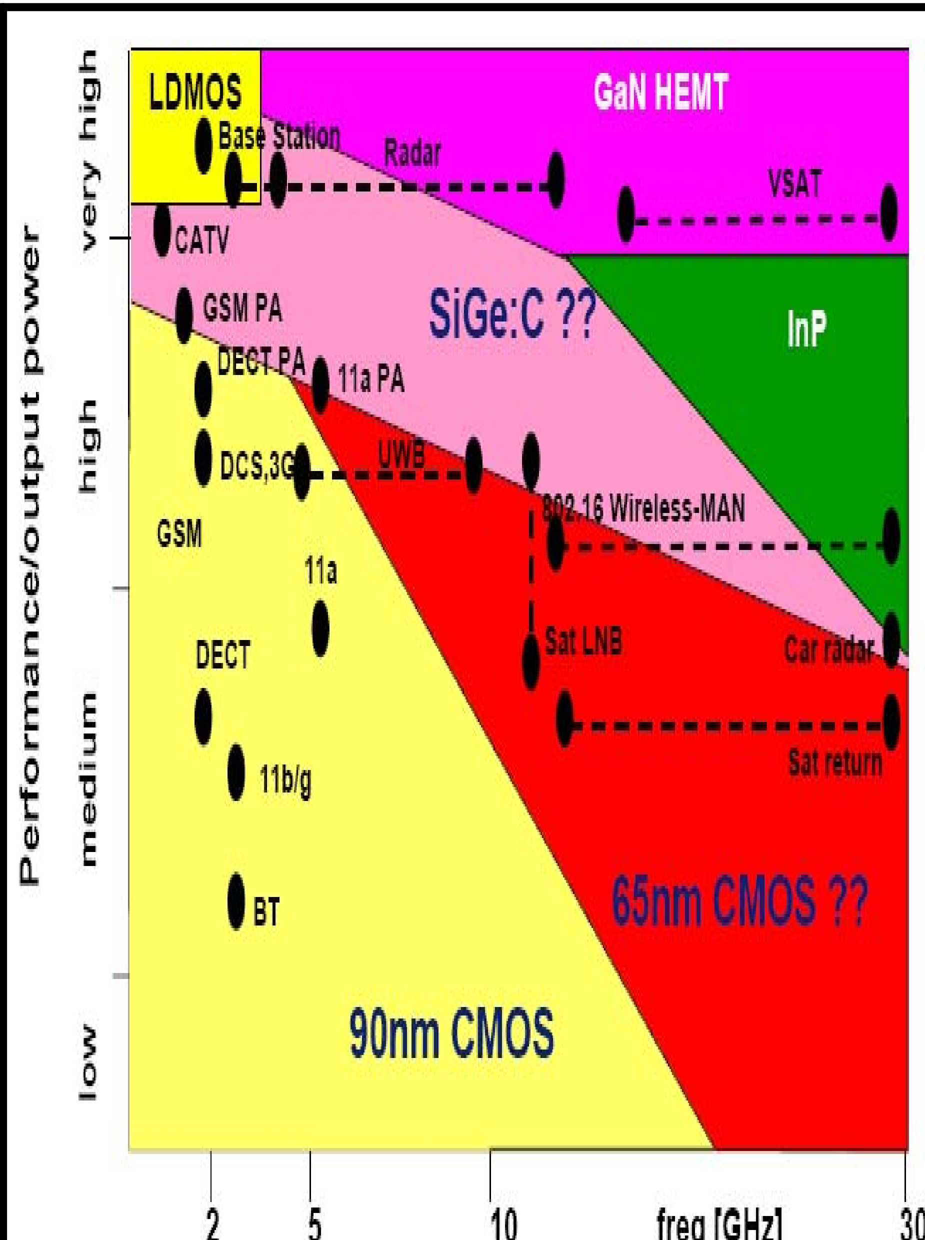
ITRS Roadmap



CMOS Device Scaling and Heterogeneous Integration

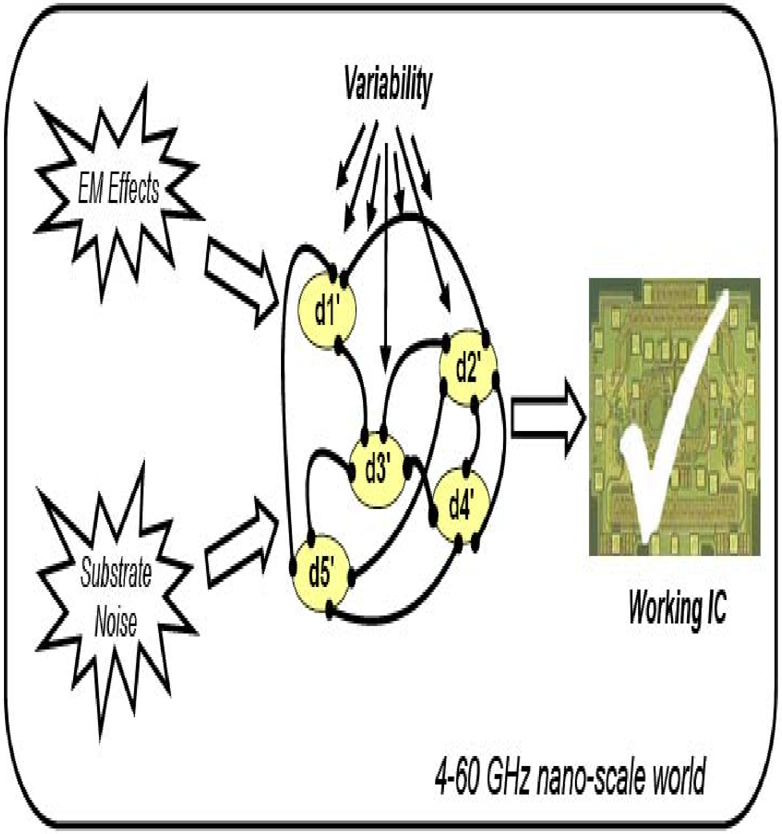
Device Level: Technology Scaling of RF-CMOS





Device Level Complexity: Variability

- **EM-coupling beyond 60GHz leads to deterministic variation**
  - IR-drop, crosstalk and substrate noise
- **Process uncertainty beyond 90nm leads to stochastic variation**
  - Component mismatch of threshold voltage (Vth)
  - Similar mechanism to phase noise (jitter), transistor noise (thermal and flicker)



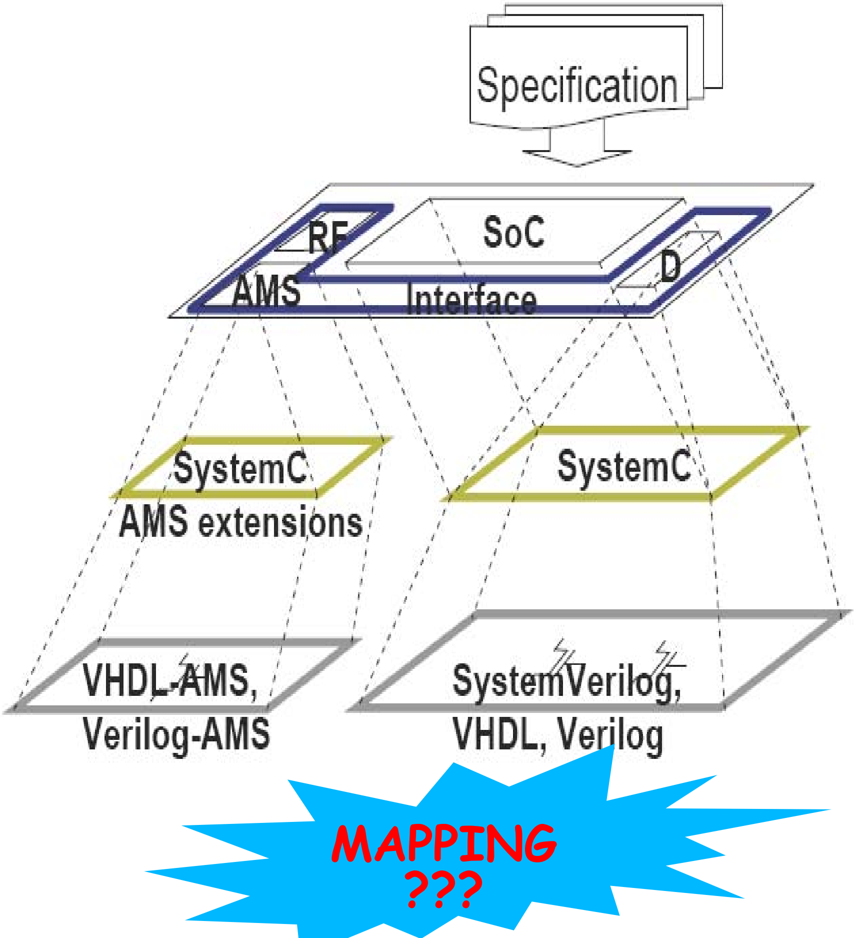
Courtesy from TU

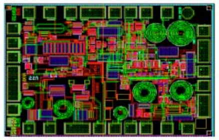
Variation leads to both poor performance and yield rate

System Level: Heterogeneous Integration

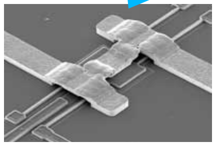
- **Heterogeneous integration**
  - Different physical domains (electrical, mechanical, thermal, biological)
  - Different functionalities (computing, communication, sensing, energy generation)
  - Deferent process technologies (CMOS, biCMOS, SiGe, SOI)

- **A top-down design**
  - Describe specs for each module by system-level code (SystemC, SystemC_AMS)
  - Realize system code by circuit-level behavior model (Verilog/VHDL)
  - Map behavior model to device-level schematic and layout

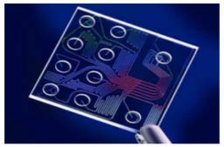




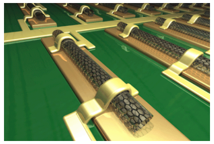
Analog/RF



MEMS



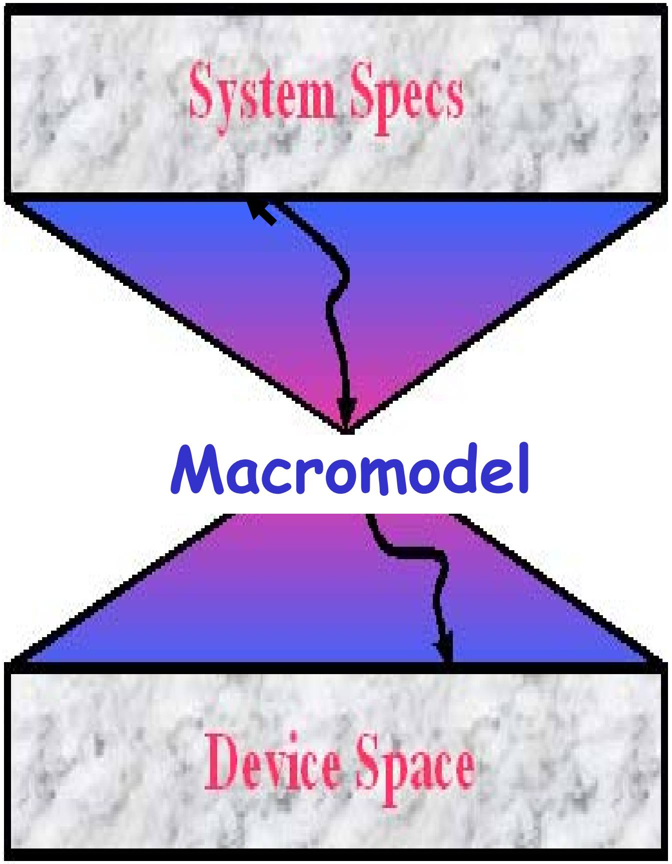
Bio-sensor



Nano tube

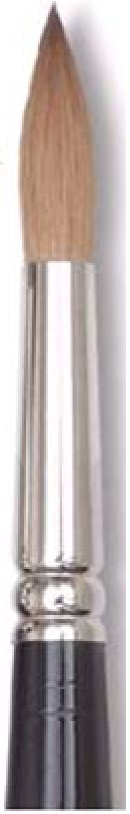
System Level Complexity: Mapping Gap

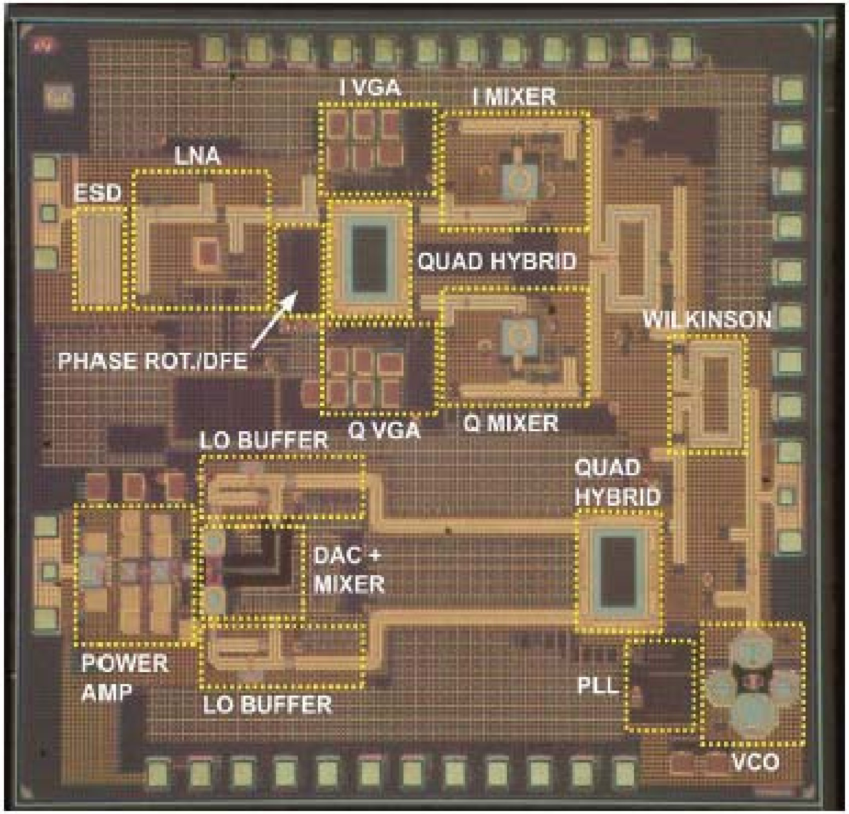
- **Mapping gap is large**
  - Device-level complexity is high (in scale of 10^6-10^{11}) to propagate in system-level
- **Robust design is required**
  - Device-level variation adds robustness (yield) in the list
  - Repeated Monte-Carlo (MC) check is expensive



Mapping gap and robustness validation lead to much longer design cycle

Create Right Tool for Roadmap



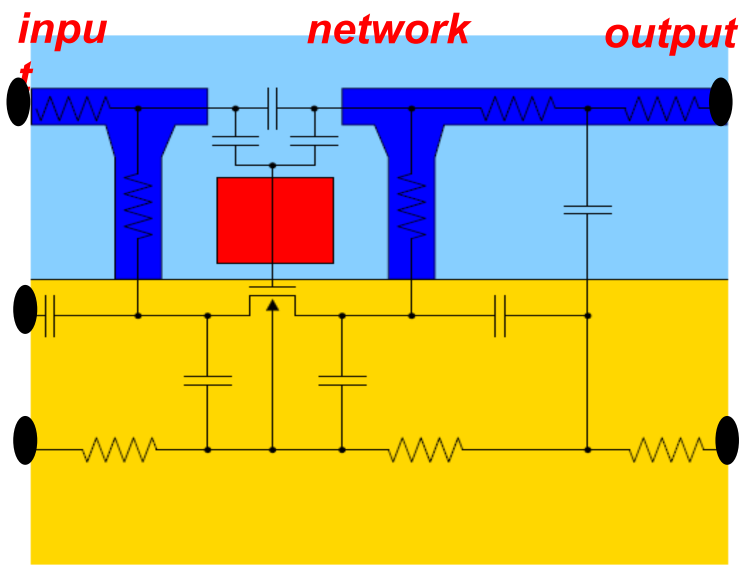


Right design automation tool is needed for robust analog/RF system design ->工欲善其事，必先利其器

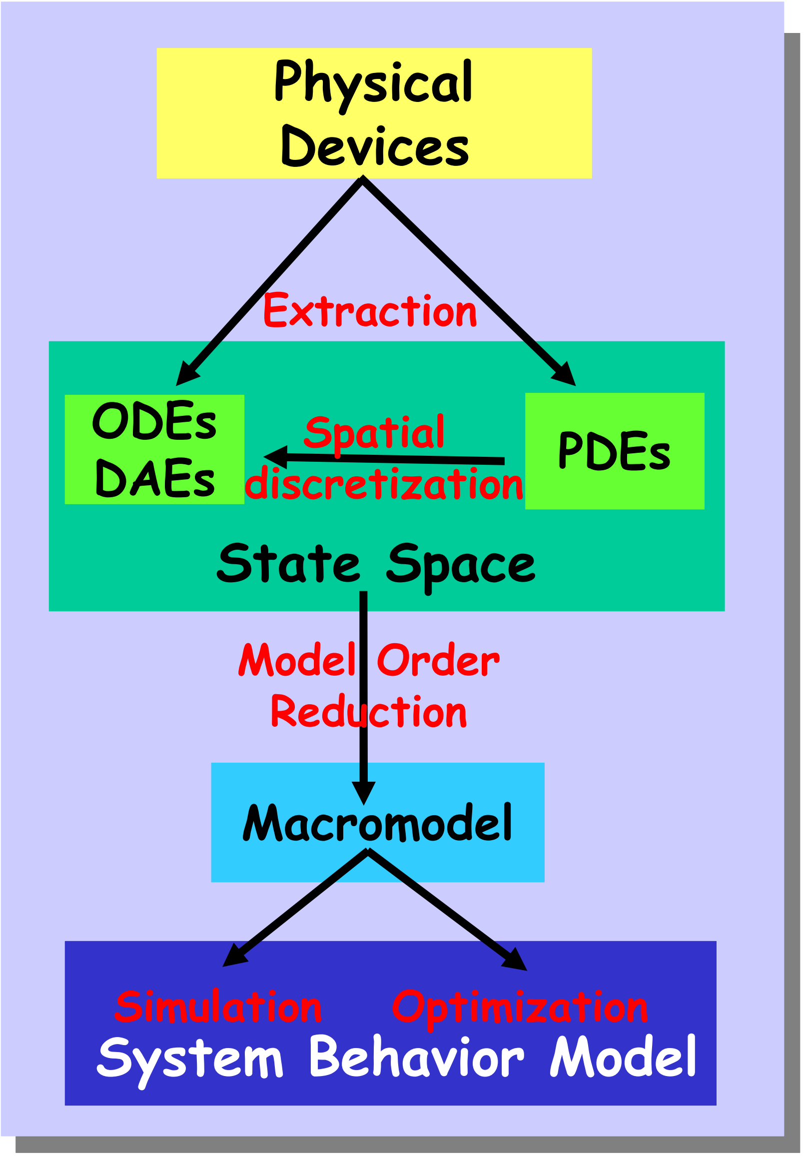
Stochastic Mismatch Analysis and Macromodeling

- **Analog system is described by differential algebra equation (DAE) in state-space**
  - Including stochastic mismatch results in a SDAE
  - Representing full devices leads to millions of variables
- **Robust system-level design requires a right tool**
  - Non-Monte-Carlo stochastic analysis for mismatch
  - Macromodeling to abstract device-level details for system-level use

$$f(x, \dot{x}, t) = \mathcal{B}u(t),$$

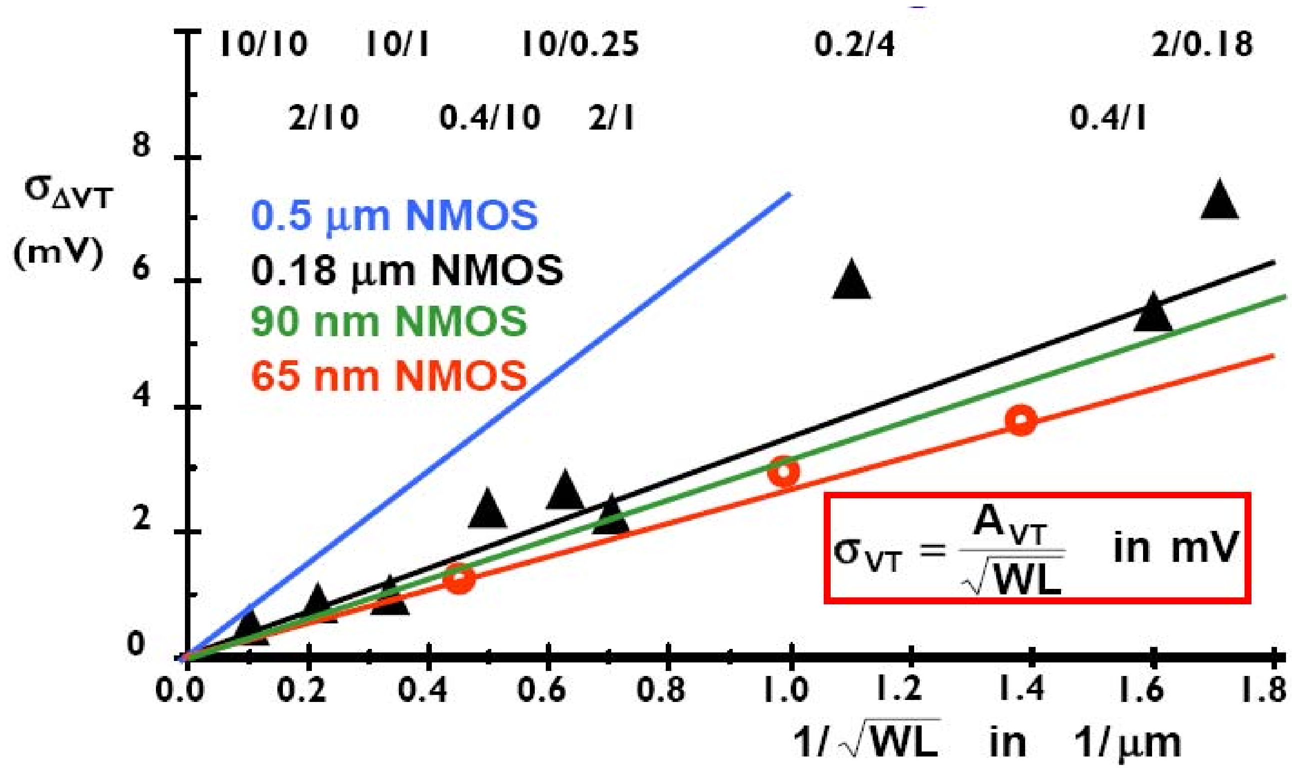


Stochastic Mismatch Analysis



1. **Non-Monte-Carlo Stochastic Mismatch Analysis**
2. Nonlinear Macromodeling

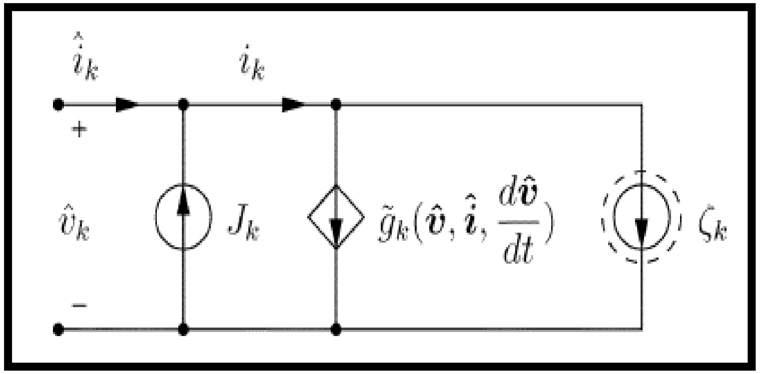
Traditional Mismatch Analysis



- **Pelgrom's model relates one parameter distribution to one CMOS-device geometry, generalized by BVP model**
  - Extract A_{vt} first for each device and then run repeated Monte-Carlo for mismatch response for yield
  - Can obtain mismatch response (nominal + variance) just by one-time run?

NMC Stochastic Mismatch Analysis I:
RHS Mismatch Source Model

- **Directly adding stochastic mismatch in state-matrix**
  - SDAE derivative (Jacobian) may not be continuous
- **Add noise-source at right-hand-side** [Biagetti:TCAD'04]
  - Map each device mismatch to a noise source
- **Assume noise-source as *ic* condition, not an accurate assumption**
  - Noise-source can have different impact at different time-point



$$f(x, \dot{x}, t) = \mathcal{F}\mathbf{i}(x, \xi) + \mathcal{B}u(t)$$

$$\mathbf{i}(x, \xi) = n(x) \sum_l g^\beta(p_l)\xi_l$$

Assume *l* independent mismatch sources

NMC Stochastic Mismatch Analysis II: Stochastic Perturbation Analysis

- **Assuming small device variation, SDAE is expanded at nominal trajectory $x^{(0)}$**
- **Transient mismatch response x_m is solved from linearized SDAE with a similar correlation analysis for transient noise** [Demir:TCAD'96]
- **Correlation analysis is still inefficient for large-scale, and only variance is needed for yield estimation**

$$f(x^{(0)},\dot{x}^{(0)},t)+\frac{\partial f(x,\dot{x},t)}{\partial x}(x-x^{(0)})+\frac{\partial f(x,\dot{x},t)}{\partial \dot{x}}(\dot{x}-\dot{x}^{0)})$$
$$=\mathcal{F}\mathrm{i}_n(x^{(0)},\xi)+Bu(t).$$

$$G(x^{(0)},\dot{x}^{(0)})x_m+C(x^{(0)},\dot{x}^{(0)})\dot{x}_m=\mathcal{F}\mathrm{i}_n(x^{(0)},\xi),$$

where

$$G(x^{(0)},\dot{x}^{(0)})=\frac{\partial f(x,\dot{x},t)}{\partial x}\Big|_{x=x^{(0)},\dot{x}=\dot{x}^{(0)}}$$
$$C(x^{(0)},\dot{x}^{(0)})=\frac{\partial f(x,\dot{x},t)}{\partial \dot{x}}\Big|_{x=x^{(0)},\dot{x}=\dot{x}^{(0)}}$$

NMC Stochastic Mismatch Analysis III:
SoP Expansion of Stochastic Source

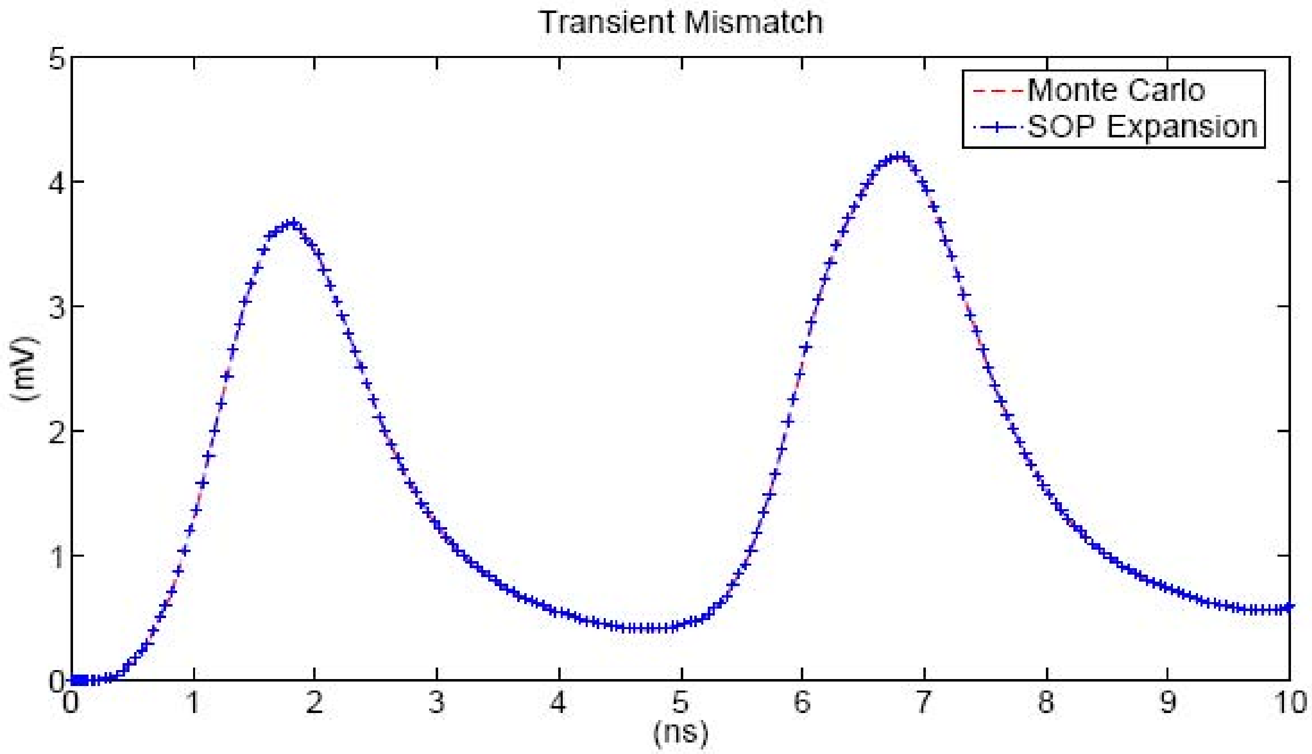
$$\boldsymbol{i} : \mathcal{F}n(x^{(0)}) \sum_l g^{\beta}(p_l)\xi_l \qquad \boldsymbol{x_m} : \sum_i \alpha_i(t)\Phi_i(\xi)$$

1. Represent both RHS mismatch-source and state-variable by a random variable ξ
2. Expand randomized RHS and state-variable by a base of stochastic orthogonal polynomials $\Phi_i(\xi)$
3. Collocate the linearized SDAE by $\Phi_j(\xi)$
4. Solve variance (a_1) just in one run

$$(G_k + \frac{1}{h}C_k)\alpha_1(t_k) = \frac{1}{h}C_k\alpha_1(t_k - h) + \mathcal{F}\mathbf{i}_k$$

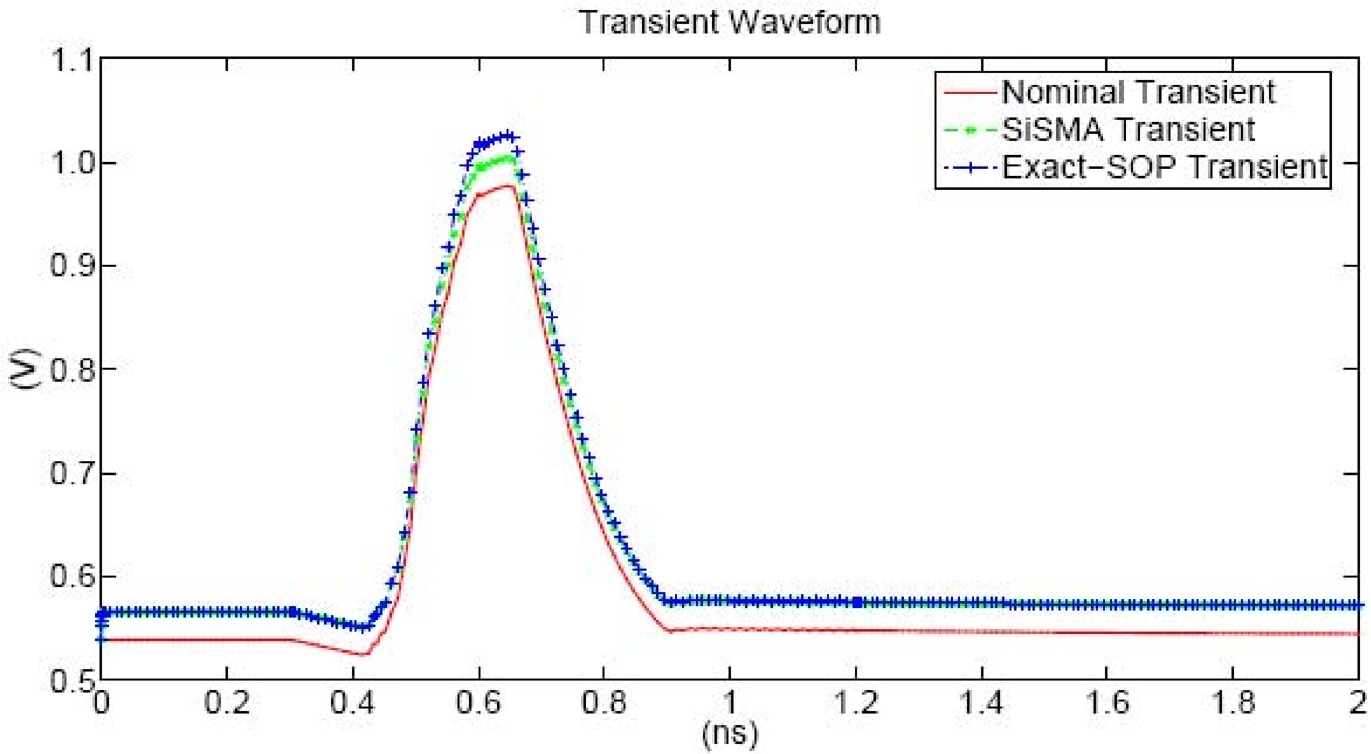
Result of NMC Stochastic Mismatch Analysis I

- **Example 1: a bjt-mixer with distributed inductor**
  - Waveform: transient mismatch (time-varying standard deviation)
  - Comparison: the exact by Monte-Carlo and the exact by SOP-expansion
  - Conclusion: NMC by SoP is 1000X faster yet as accurate as MC

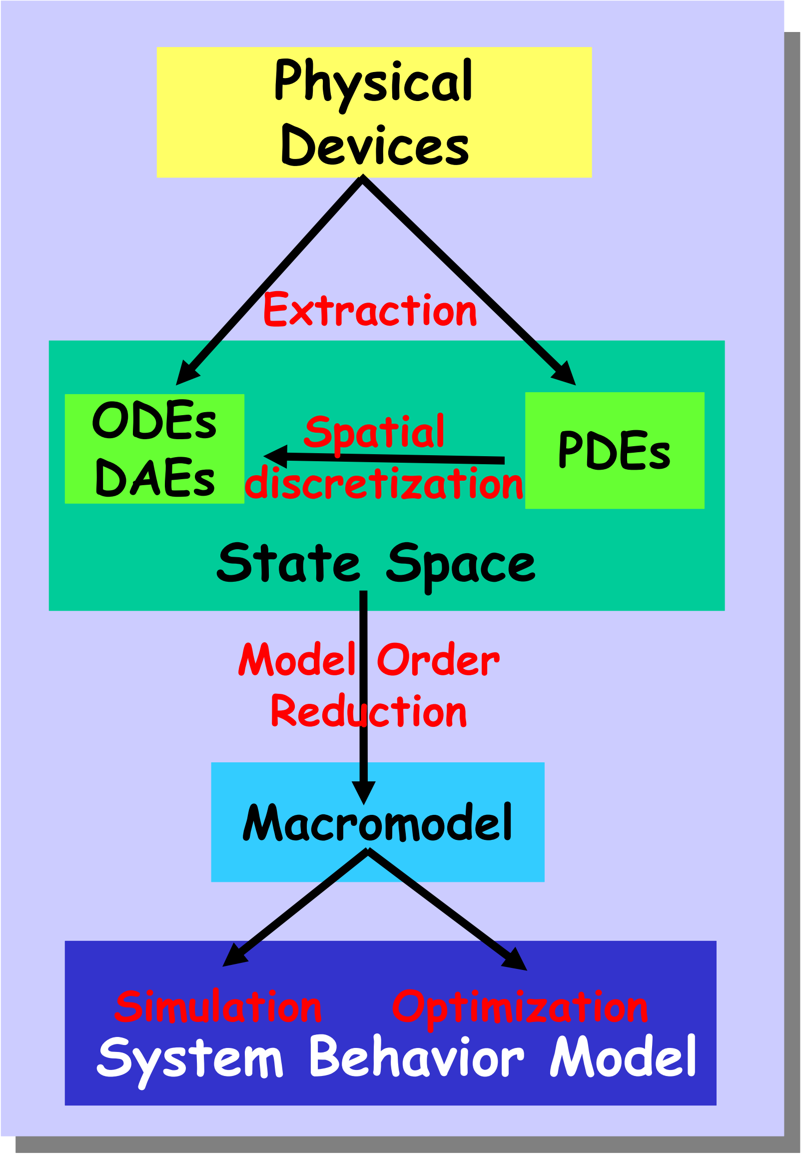


Result of NMC Stochastic Mismatch Analysis II

- **Example 2: a diode-chain**
  - Waveform: transient (nominal + standard deviation)
  - Comparison: Nominal, SOP-expansion, SiSMA with dc-source
  - Conclusion: SiSMA with mismatch-source at ic only is not accurate



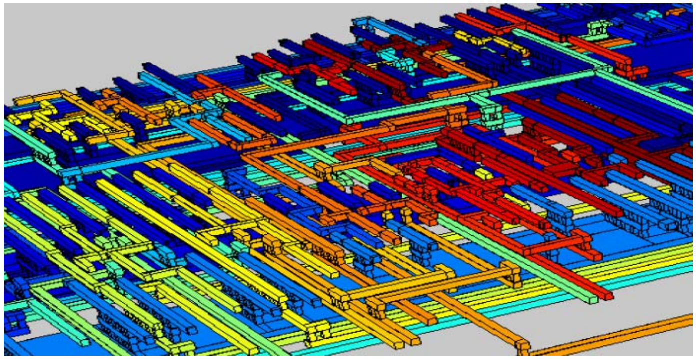
Stochastic Mismatch Macromodeling



1. Non-Monte-Carlo Stochastic Mismatch Analysis
2. **Nonlinear Macromodeling**

Macromdoeling by Model Order Reduction

- **Abstracted representation of dynamical system**

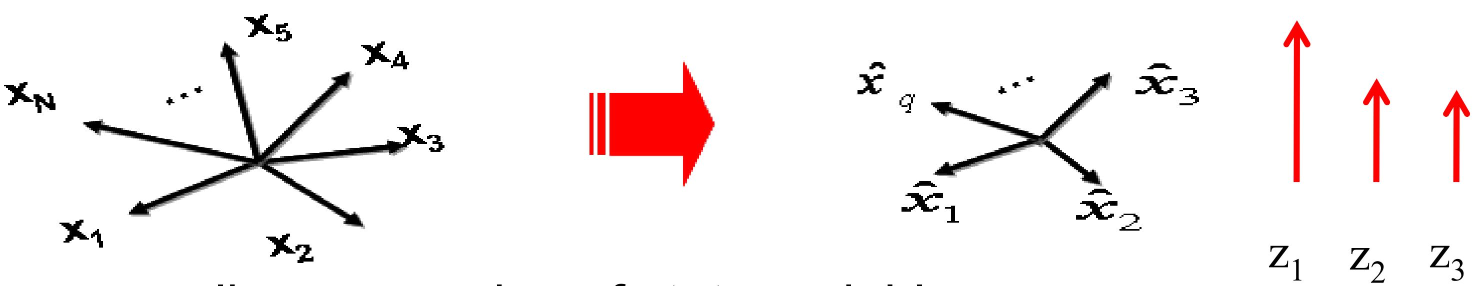


$$\frac{dx_q(t)}{dt} = A_q x_q(t) + b_q u(t)$$
$$y(t) = c_q^{\ T} x_q(t)$$

Complicated geometry and physics with millions of variables

Compact model with ~100 variables and approximated response

- **Essentials in system theory**
  - Identification of dominant state variables

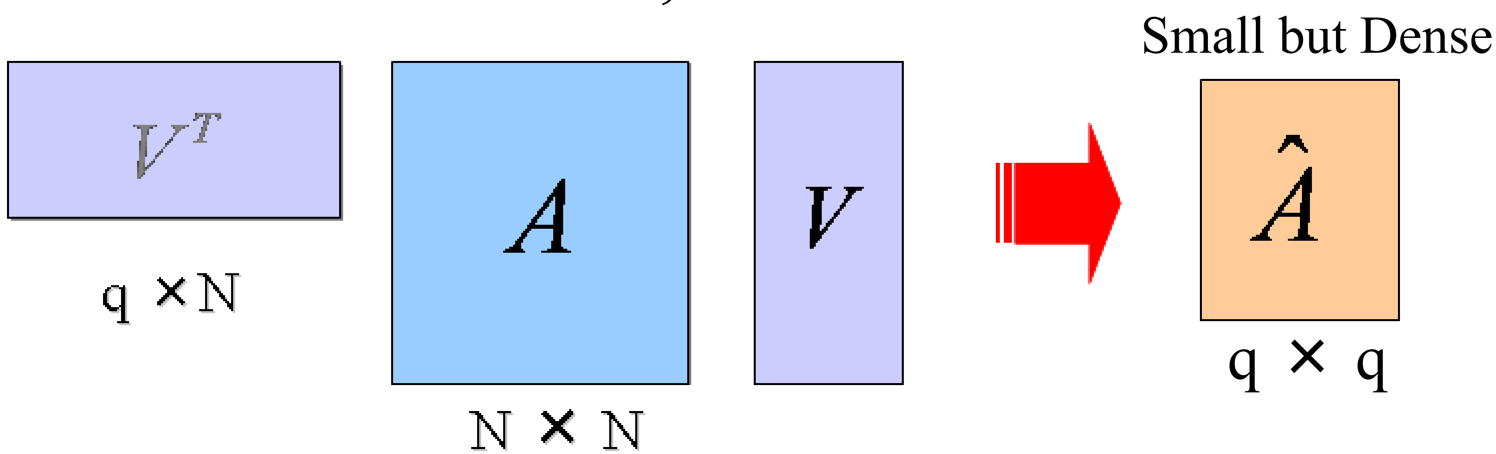


  - Linear or nonlinear mapping of state variables

$$z = V^T x, \quad z = \lambda(x).$$

Linear Reduction by Projection

$$z = V^T x, \quad x = Vz$$



- **The mapping is in form of matrix-projection**
- **Projection matrix is built from orthonormalized subspace**
  - by moments (derivatives of transfer function)
    – PVL, PACT, PRIMA
  - by spectrum (singular-value/eigen vector of transfer function)
    – TBR, POD

Nonlinear Reduction by Projection

- The mapping is a in form of nonlinear function

$$z = \lambda(x), \quad x = \lambda^{-1}(z).$$

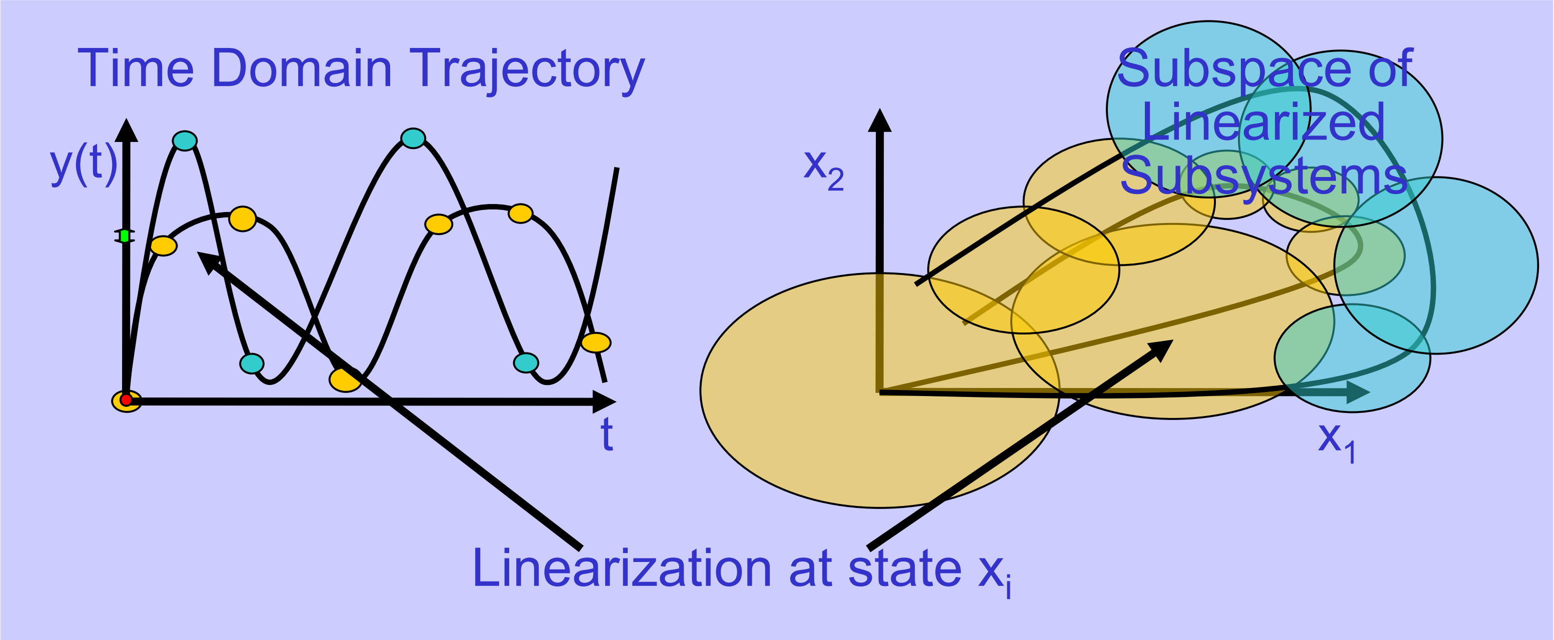
- The reduction becomes a projection by the derivative (tangent) of the nonlinear function, or called manifold

$$\dot{z} = \frac{d\lambda}{dx}\frac{dx}{dt} = \left(\frac{d\lambda}{dx} f(x,t)\right) + \left(\frac{d\lambda}{dx} B\right) u(t).$$

Piecewise Linear Approximation of λ

- **TPWL and maniMOR**
  - Mapping function can be approximated by a piecewise linear function
  - Composed from a number of linearized subsystems sampled along transient trajectory

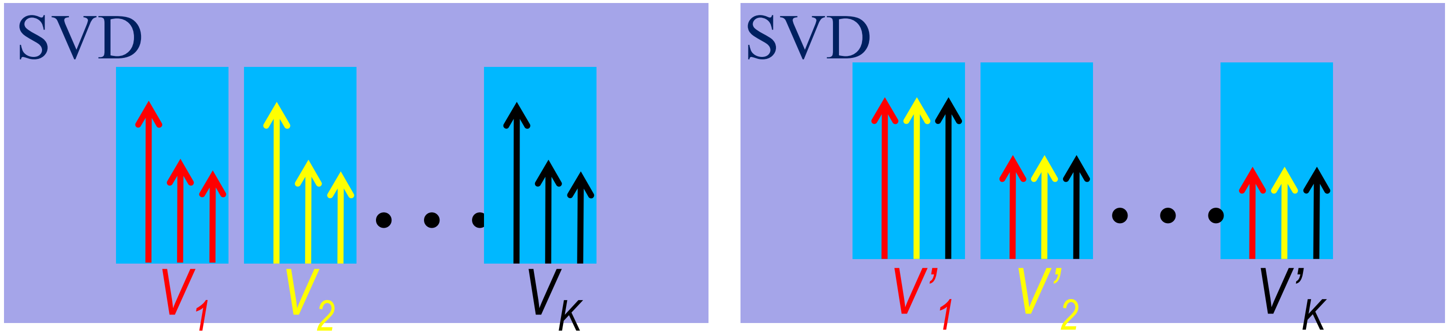
Courtesy from MIT



$$z = \lambda(x) \approx \sum_{k=1}^{K} w_k [z_k + V_k (x - x_k^0)], \quad x = \lambda^{-1}(z) \approx \sum_{k=1}^{K} w_k [x_k + V_k (z - z_k^0)].$$

Incremental Aggregation of Subspace

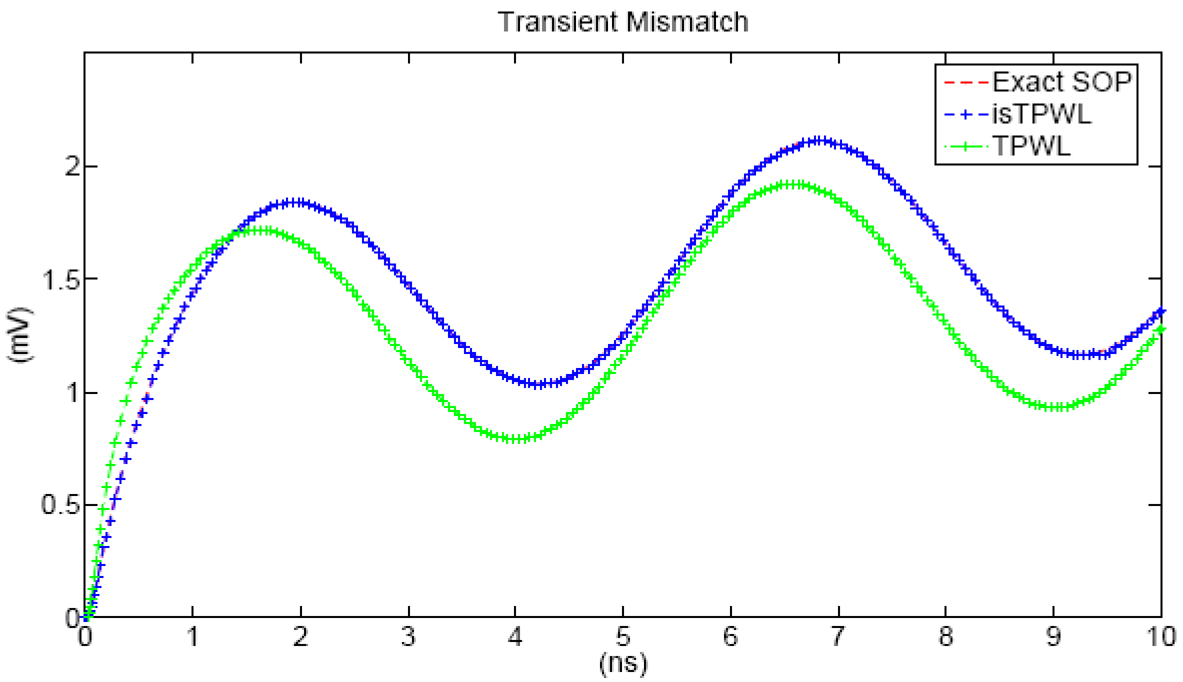
- **Reduction by local subspace projection**
  - Maintain local subspace V_k though accurate may be expensive for wide-range
  - Linear Krylov and nonlinear maniMOR
- **Reduction by global subspace projection**
  - Aggregate (by SVD) local subspace V_k into global subspace is economic but error-prone
  - Linear pmTBR and nonlinear TPWL
- **Key is how to identify a global manifold**
  - How to sample the local subspaces?
  - How to aggregate the sampled local subspaces?
- **Incremental aggregation in this paper (isTWPL)is an initial trial**



Result of Nonlinear MOR I

Example 3: a bjt-mixer with distributed substrate

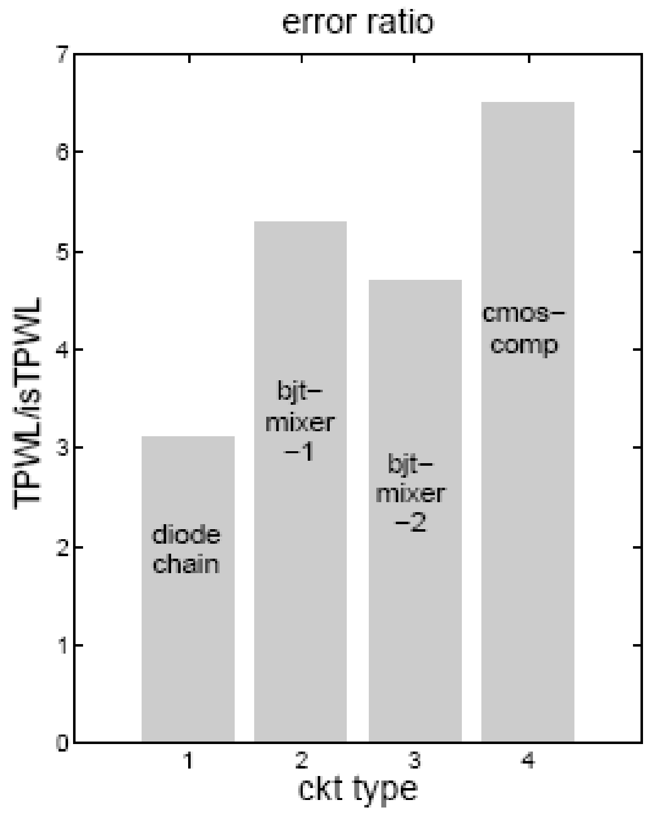
- Waveform: transient mismatch(time varying standard deviation)
- Comparison: TPWL, isTPWL, exact
- Conclusion: isTPWL is more accurate than TPWL

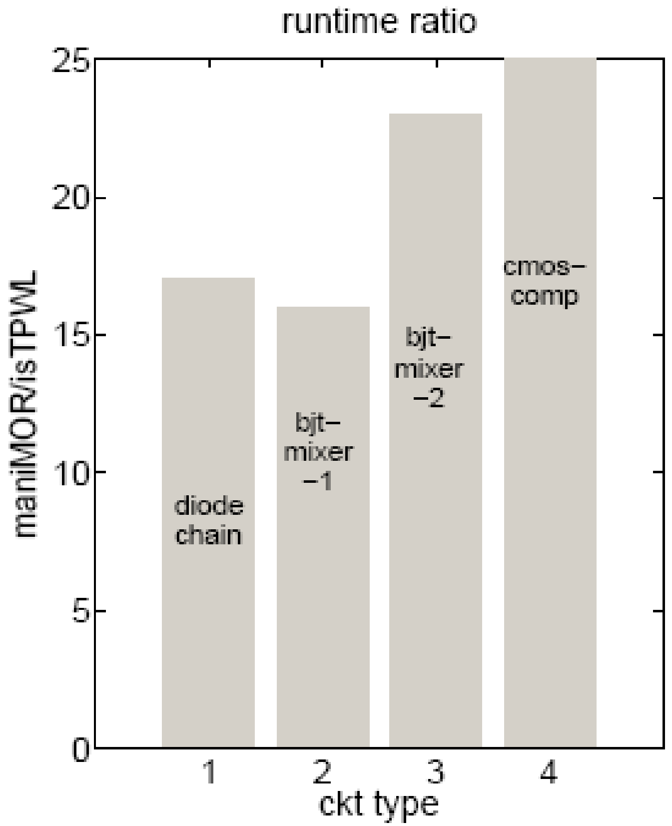


Result of Nonlinear MOR II

Examples: diode-chain, bjt-mixer-1, bjt-mixer-2, cmos-comp

- Error and runtime ration: transient mismatch (time varying standard deviation)
- Comparison: TPWL, maniMOR and isTPWL
- Conclusion: isTPWL is more accurate than TPWL yet faster than maniMOR





Conclusions

- **Robust system-level design requires**
  - a NMC stochastic mismatch analysis
  - an accurate yet fast nonlinear macromodeling
- **RHS source model, perturbation and SoP expansion show a promising NMC stochastic mismatch analysis for small variation**
  - how about large one?
  - **Will** answer in future work
- **Incremental aggregation balances accuracy and speed**
  - How to sample and compress local subspaces for global projection?
  - **NOT** resolved yet for both linear (pmTBR) and nonlinear MORs

Thank you!

Please send comments to haoyu@ntu.edu.sg