A Fast Symbolic Computation Approach to Statistical Analysis of Mesh Networks with Multiple Sources

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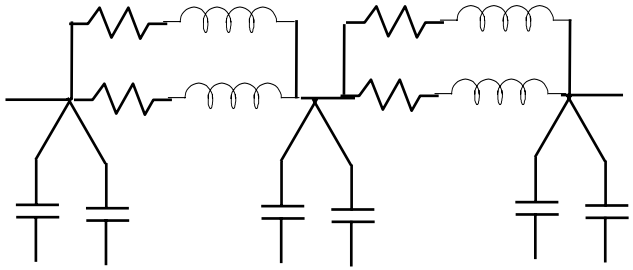
Outline

- Research Background
- Symbolic Moment for Trees
- Symbolic Moment for Mesh
- Symbolic Moment Sensitivity
- Applications
- Conclusion

Bottom-up Modeling

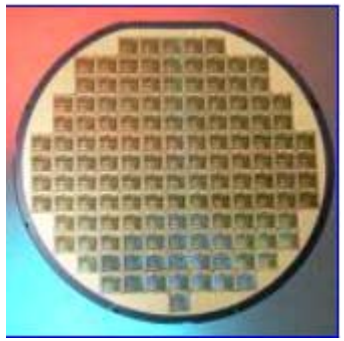
$$C\frac{dx}{dt} + Gx = Bu$$

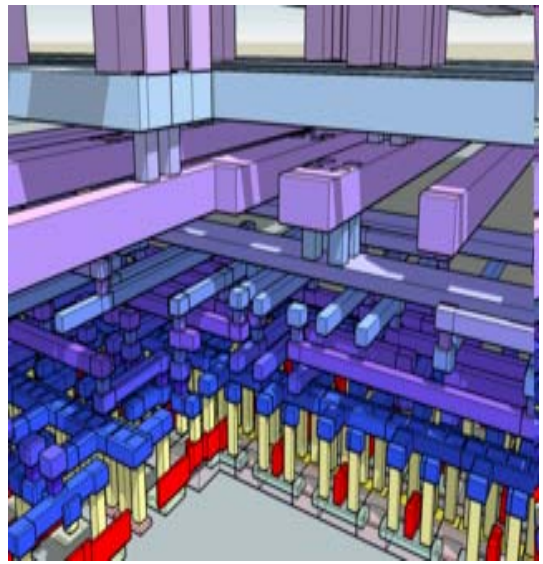


Reduced-order Modeling
Numerical vs. Symbolic

Extraction

SIMULATOR/ SYNTHESIS TOOLS





Source: www.tamaru.kuee.kyoto-u.ac.jp/.../LSI-3D-CG.html

Symbolic Modeling Techniques

- **Parametric** models instead of **numerical** models
- Efficient for **repeated** computation, suitable
  - in an optimization loop
  - in incremental design
  - in design centering (robust design)
  - in statistical timing (Monte Carlo)
  - in yield analysis
- An extension to **symbolic moment calculation** in our latest research

Krylov subspace

$$K_q = \left\{(G^{-1}F), (G^{-1}C)(G^{-1}F), \cdots, (G^{-1}C)^{q-1}(G^{-1}F)\right\}$$

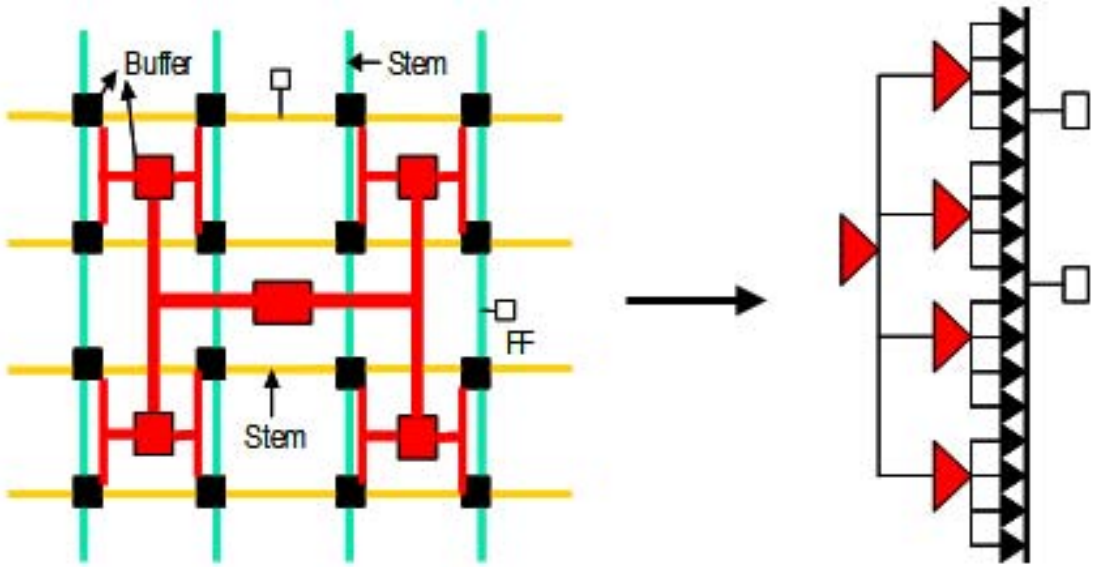
$$= \left\{m^{(0)}, m^{(1)}, \cdots, m^{(q-1)}\right\}$$

Symbolic Matrix Inversion !!!

G. Shi, B. Hu, C.-J. R. Shi, "On symbolic model order reduction" IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems, vol. 25, no. 7,pp. 1257-1272, July 2006. (**2007 Donald O. Pederson Best Paper Award**)

Clock Mesh & Clock Tree with Cross-links

- **Strong tolerance** for clock skew variation



Our Latest Publications

- Symbolic moment calculation for RCL trees and coupling trees, statistical signal integrity analysis
  - Z. Hao and G. Shi, in Proc. 52nd IEEE MWSCAS, Cancun, Mexico, 2009.
- Symbolic moment calculation for RCL mesh with resistor loops, statistical timing analysis
- Z. Hao and G. Shi, in Proc. 12th IEEE ISIC, Singapore, 2009.
- Symbolic moment calculation for RCL mesh with multiple sources, symbolic sensitivity calculation, statistical timing analysis
- Z. Hao and G. Shi, to appear in 15th IEEE/ACM ASPDAC, Taiwan, 2010.

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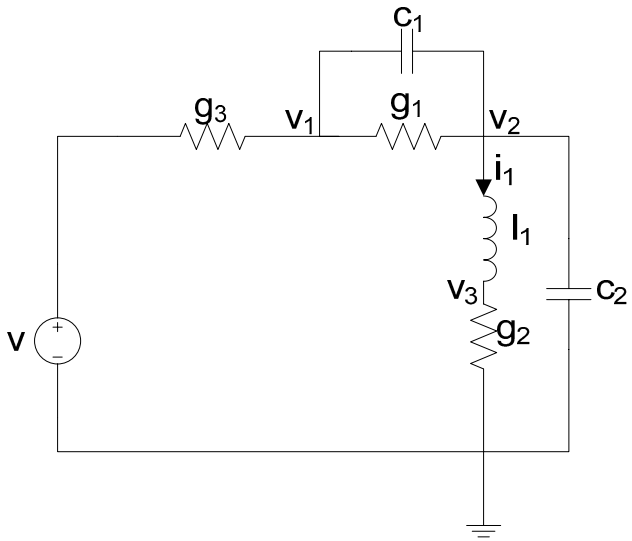
Circuit Moment



KCL & KVL

State-Space Model

$$\begin{cases} C\dot{x}(t) + Gx(t) = Fv_{in}(t) \\ y(t) = L^T x(t) \end{cases}$$

Laplace Transform

Taylor Expansion

$$X(s) = \sum_{k=0}^{\infty} (-1)^k (G^{-1}C)^k (G^{-1}F) V_{in}(s) s^k$$

Applications:

1. model order reduction
2. simple metrics

k^{th} order moment

$$m^{(k)} = (G^{-1}C)^k (G^{-1}F)$$

Representative Moment Metrics

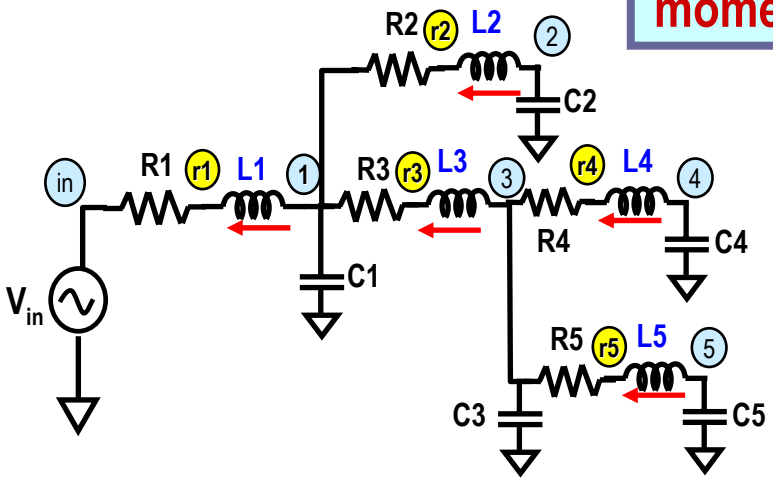
- Elmore delay(1948)
  - The 1st order moment, most widely used timing metric
  - W. C. Elmore, J. Applied Physics, 1948.
- Devgan's crosstalk metric(1997)
  - 1st order moment for capacitive coupling
  - A. Devgan etc., Proc. ICCAD, 1997.
- D2M(2000)
  - An improvement over Elmore using two moments
  - C. J. Alpert etc., ISPD, 2000.
- S2M(2004)
  - A slew metric using two moments
  - K. Agarwal etc., IEEE Trans. on CAD, 2004.
- And lots of other moment metrics in the literature …

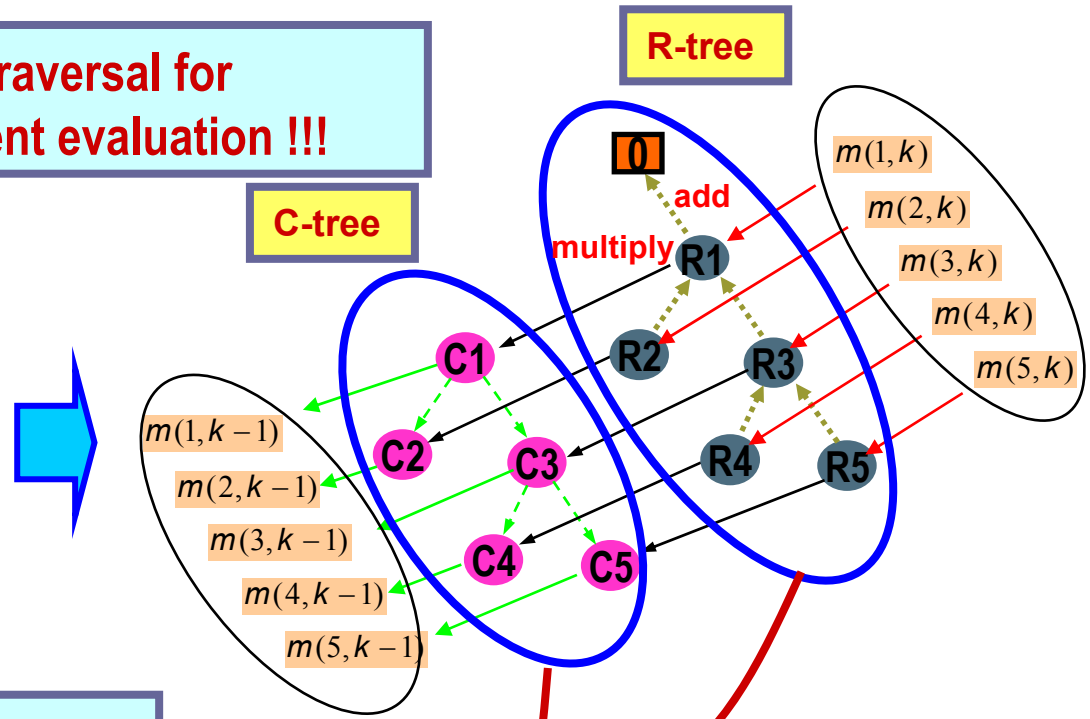
Elmore = m1

D2M = $\ln 2 \left(\frac{m_1^2}{\sqrt{m_2}} \right)$

Tree-BDD Structure

Tree traversal for moment evaluation !!!





Recursive Moments Evaluation

$$m_C^i(j,k) = \sum_{R_\ell^i \in P_\ell^i} R_\ell^i m_L^i(\ell,k) - \sum_{L_\ell^i \in P_\ell^i} L_\ell^i m_L^i(\ell,k-1)$$

$$m_L^i(j,k) = \sum_{n_j^i \in \square^i} C_j^i m_C^i(j,k-1)$$

Linked with tree structures !!!

Q. Yu and E.S. Kuh, "Exact moment matching model of transmission lines and application to interconnect delay estimation," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol.3, no.2, pp.311-322, Jun 1995

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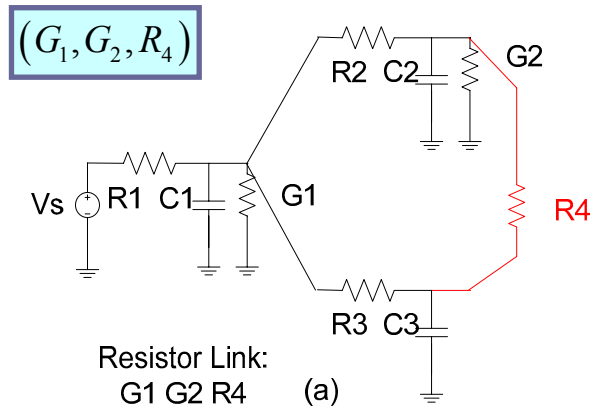
Kron's Branch Tearing

a binary decision process !

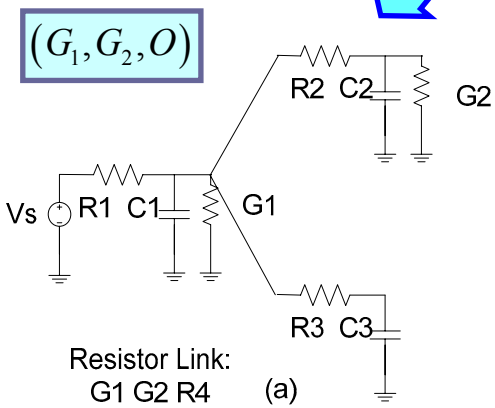
$$V = V^{(O)} - I_R \cdot V^{(A)}$$

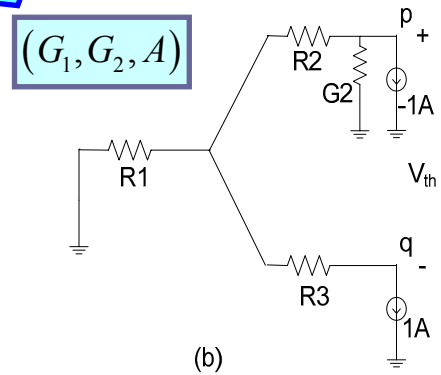
$$I_R = \frac{V_{oc}}{R_{link} + R_{th}}$$



Link resistor R4

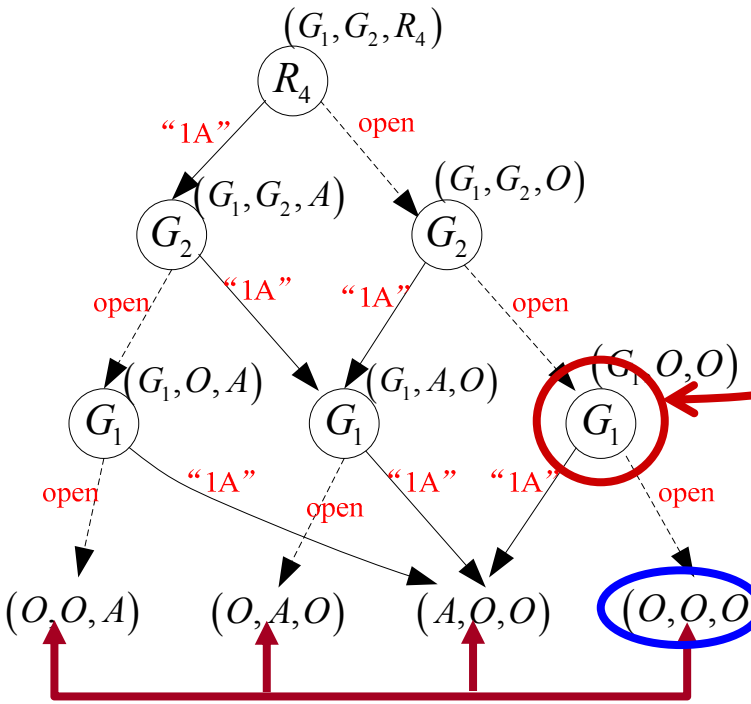
Branch Tearing





R. Rohrer, "Circuit partitioning simplified," *IEEE Trans. Circuits Syst.*, vol. 35, no. 1, pp. 2–5, Jan 1988.

Link-BDD for Multiply Link Resistors



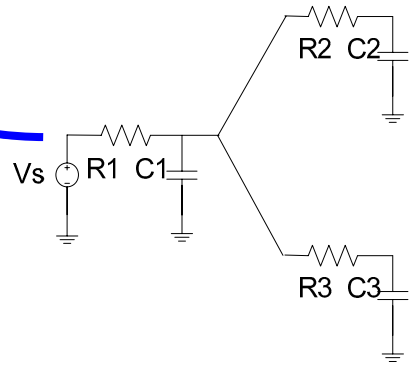
Tree-BDD circuits !!!

Construction Top-down

Decompose by R_4, G_2, G_1

$$m_{i,k} = m_{i,k}^{(O)} - I_R \cdot m_{i,k}^{(A)}$$

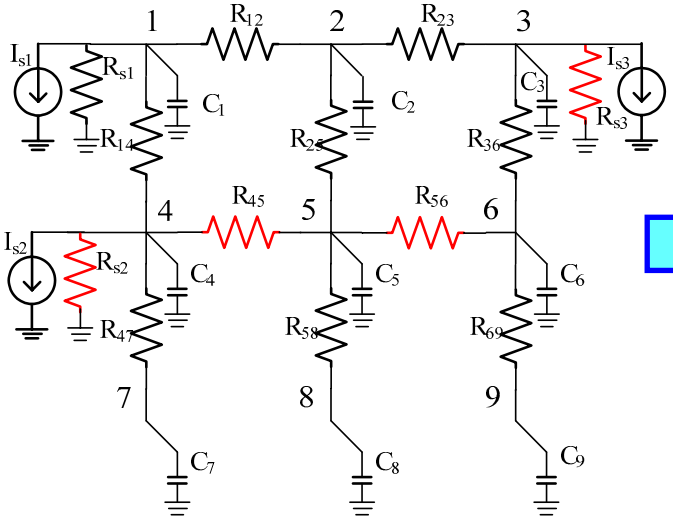
Evaluation Bottom-up.

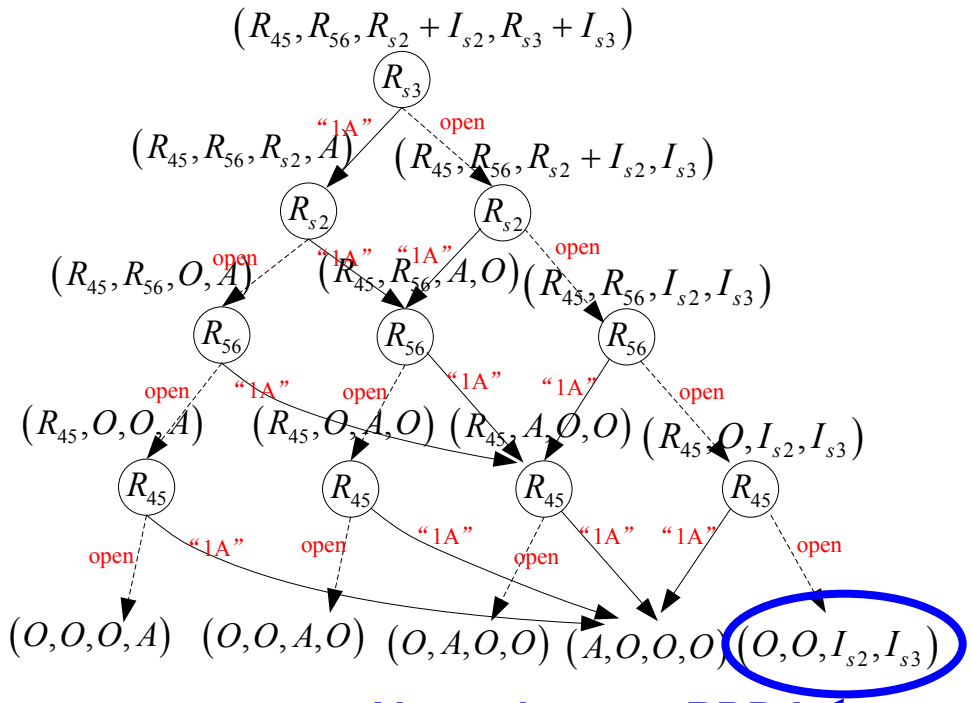


Complexity: $O(NL^2)$

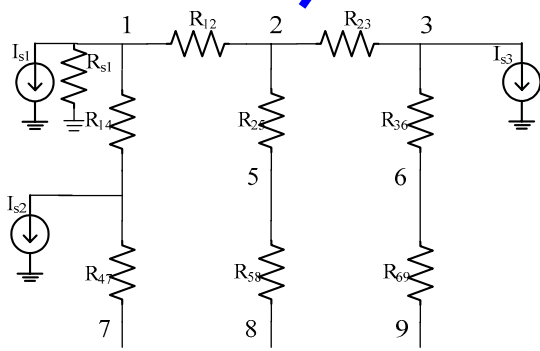
H. J. Lee et al, "Applications of tree/link partitioning for moment computations of general lumped RLC networks with resistor loops," in *Proc. IEEE Int. Symp. Circuits Syst.*, vol. 1, pp. 713–716, May. 2004.

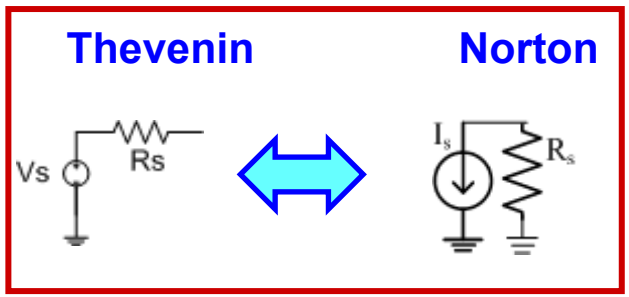
Extension for Multiple Sources





Also using tree-BDD !





All sources need to be calculated only once !!!

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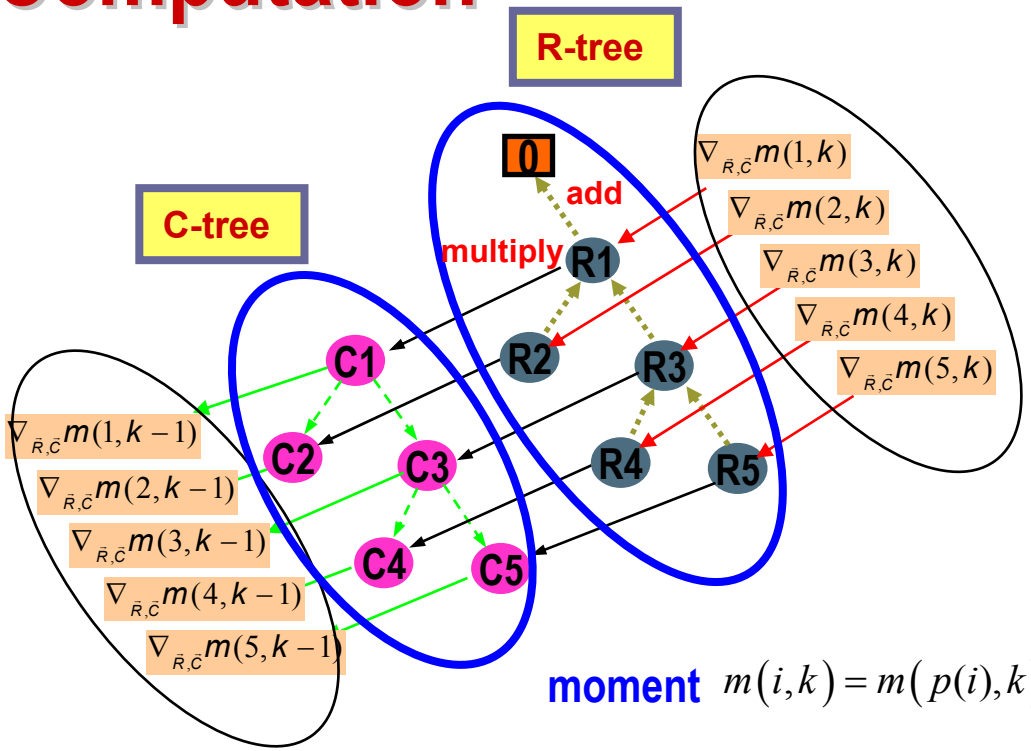
Tree-BDD Moment Sensitivity Recursive Computation



Sensitivity and moment can be evaluated at the same time !!!

moment $$m(i,k) = m(p(i),k) + R_i \sum_{j \in T_i} C_j m(j,k-1)$$

gradient

Intermediate part stored in C node and R node

Recursive sensitivity evaluation for tree-BDD

$$\nabla_{\vec{R}} m(i,k) = \nabla_{\vec{R}} m(p(i),k) + \vec{e}_i \sum_{j \in T_i} C_j m(j,k-1) + R_i \sum_{j \in T_i} C_j \nabla_{\vec{R}} m(j,k-1)$$

$$\nabla_{\vec{C}} m(i,k) = \nabla_{\vec{C}} m(p(i),k) + R_i \sum_{j \in T_i} \vec{e}_j m(j,k-1) + R_i \sum_{j \in T_i} C_j \nabla_{\vec{C}} m(j,k-1)$$

Link-BDD Moment Sensitivity Recursive Computation

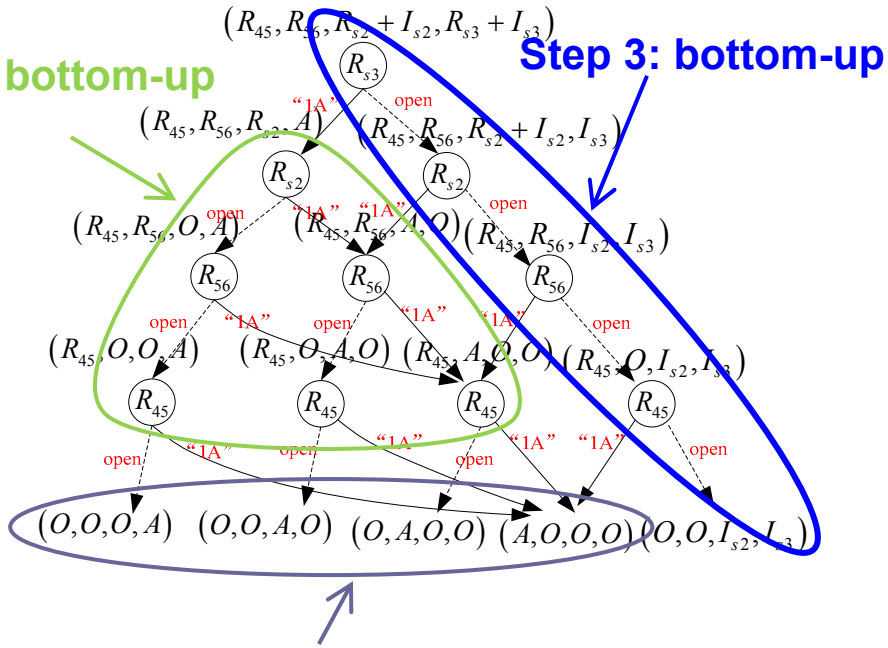
Branch tearing

$$m_{i,k} = m_{i,k}^{(O)} - I_R \cdot m_{i,k}^{(A)}$$

sensitivity

Bottom-up evaluation

$$\nabla_p m_{i,k} = \nabla_p m_{i,k}^{(O)} - \nabla_p I_R \cdot m_{i,k}^{(A)} - I_R \cdot \nabla_p m_{i,k}^{(A)}$$



Step 1 and Step 2 are only evaluated once

Step 3 is evaluated recursively !!!

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Symbolic Model Order Reduction

Krylov Subspace

$$K_q = \left\{(G^{-1}F), (G^{-1}C)(G^{-1}F), \cdots, (G^{-1}C)^{q-1}(G^{-1}F)\right\}$$
$$= \left\{m^{(0)}, m^{(1)}, \cdots, m^{(q-1)}\right\}$$

Orthogonalization → V_q

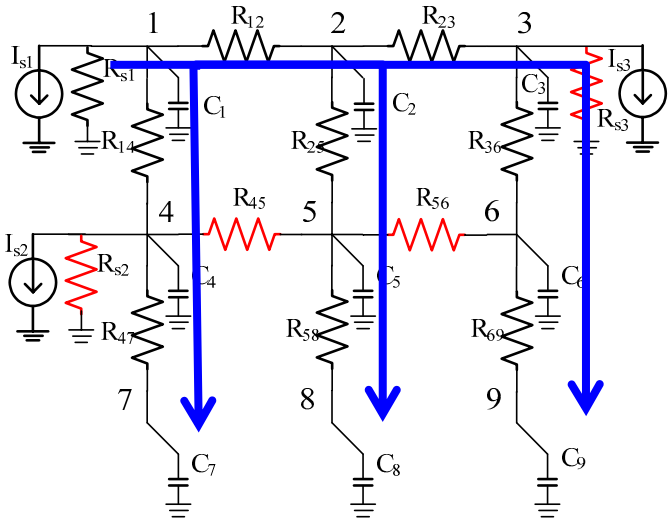
$$x = V_q \xi \qquad C\frac{dx(t)}{dt} + G\,x(t) = F\,v_{in}(t)$$

Projection

$$\left[V_q^T C V_q\right]\frac{d\xi(t)}{dt} + \left[V_q^T G V_q\right]\xi(t) = \left[V_q^T F\right]v_{in}(t)$$

Compute reduced order matrix via spanning tree traversal !!!

Cont.



Obtain reduced order model of all nodes via traversing the spanning-tree once !

Simple and fast !

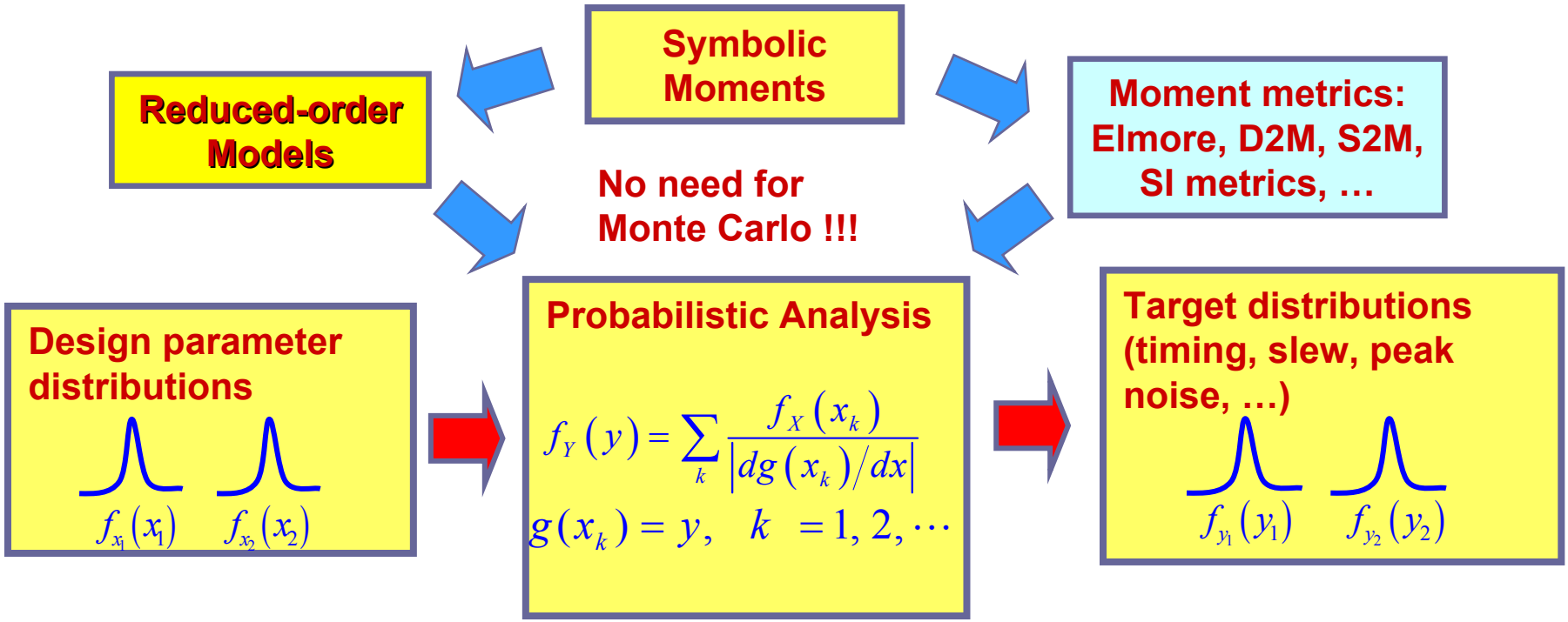
- **Existing methods for model order reduction for massive ports**
  - **Relative gain array, B. Yan, UC Riverside, DAC 2008**
  - **Port sliding, X. Ye, TAMU, ICCAD 2007**

Statistical Interconnect Simulator (SIS) Performance

- Implemented in C++, Intel 2.83G, 4GB memory, HSPICE 2005.09, Matlab 2007a
- Industrial level mesh testing circuits, 0.18um processing technology, PUL R value 0.1Ω, C value 0.41fF

Nodes	Mesh Type	Sources	R-Links	BDD Building(s)	4th Order Moment(s)	4th Order Sensitivity(s)	Obtain Reduced Order Model (s)	Speedup over Superposition
576	5x5	30	54	0.11	0.03	0.23	0.01	29.8
1296	7x7	56	104	0.89	0.09	1.53	0.02	55.5
1720	9x9	63	143	2.28	0.16	3.63	0.02	63.2
3838	13x13	130	298	20.97	0.75	32.88	0.04	130.2
6976	15x15	180	404	70.31	1.83	106.61	0.07	179.8

Sensitivity-based PDF Profiling



1st order approximation for multiple parameters

$$D(W,H) = D_{norm} + f_W(\Delta W) + g_H(\Delta H)$$

$$pdf(D - D_{norm}) = pdf(f_W(\Delta W)) * pdf(g_H(\Delta H))$$

E. Matoglu et al, "Statistical signal integrity analysis and diagnosis methodology for high speed systems," *IEEE Trans. on Advanced Packaging* , vol. 27, no. 4, pp. 611-629, Nov. 2004

50% Delay and Skew Distribution

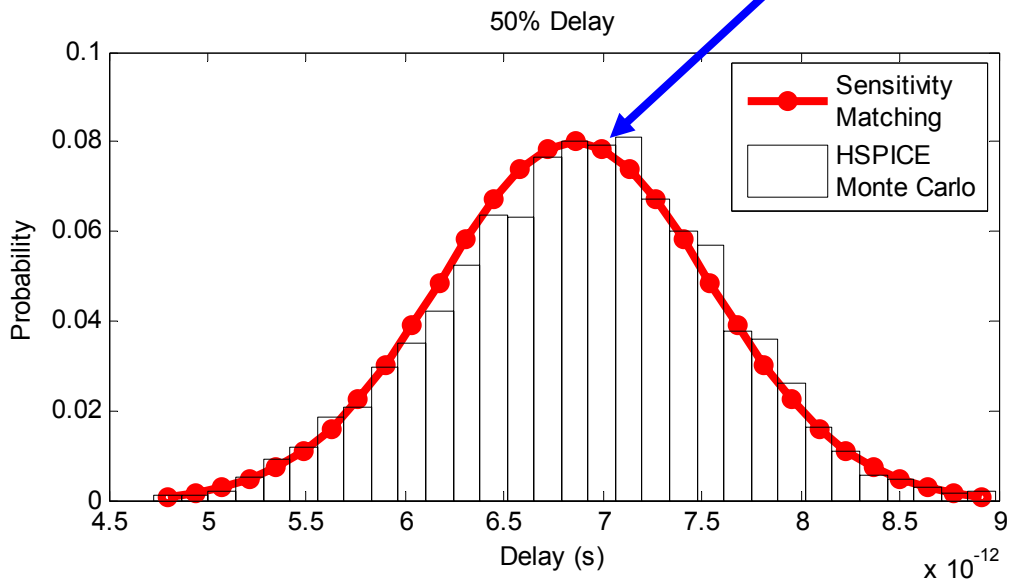
- Per-unit-length R, C value 30% variation corresponding to 3σ, observe the 50% delay and skew distribution of the selected nodes
- 31 samples for pdf profiling, compare with HSPICE Monte Carlo 5000 samples

Mesh Type	All Nodes	Average Error	Speedup to
	mean	std	Monte Carlo
5x5	0.21%	1.27%	1121
7x7	0.09%	1.12%	843
9x9	0.23%	0.09%	658
13x13	0.17%	0.75%	356
15x15	0.13%	0.62%	340

Selected Simulation Result

Delay distribution of one node of mesh 7x7

Only 31 samples for pdf function



Speed up to HSPICE: 843 times !

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Conclusion

- Symbolic moment for trees
- Symbolic moment for mesh
- Symbolic moment sensitivity
- Application to statistical timing analysis
- Future Work
  - Application to clock mesh synthesis
  - Application to interconnect optimization

Questions ?
Thanks for listening !