

**ASP-DAC : 8A-1**

# **A New Graph-Theoretic, Multi-Objective Layout Decomposition Framework for Double Patterning Lithography**

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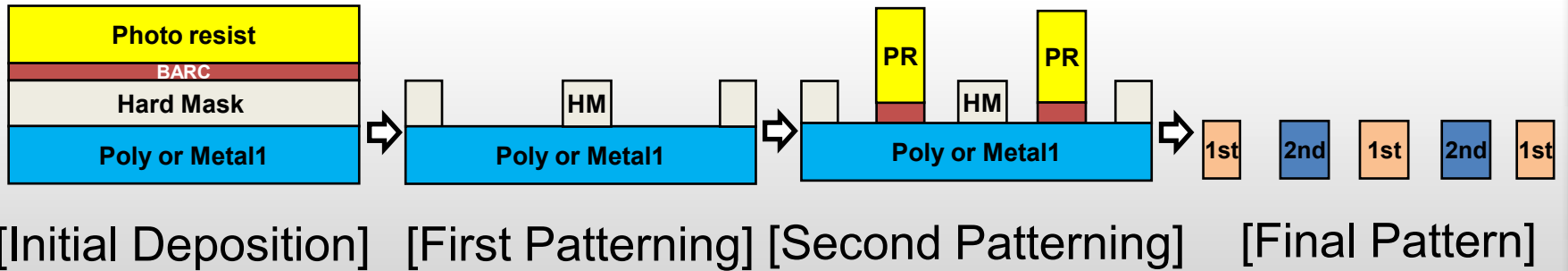
# Outline



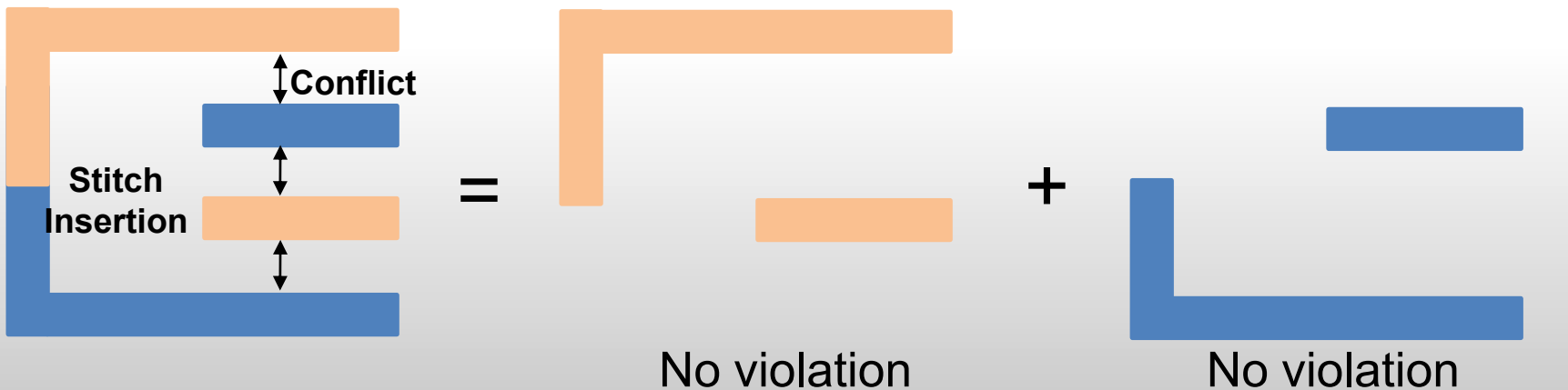
- ◆ **Motivation**
- ◆ **Bi-Partitioning Based Decomposition**
- ◆ **Timing Driven Decomposition**
- ◆ **Experimental Results**
- ◆ **Future Works**

# Layout Decomposition

## LELE DPL Process

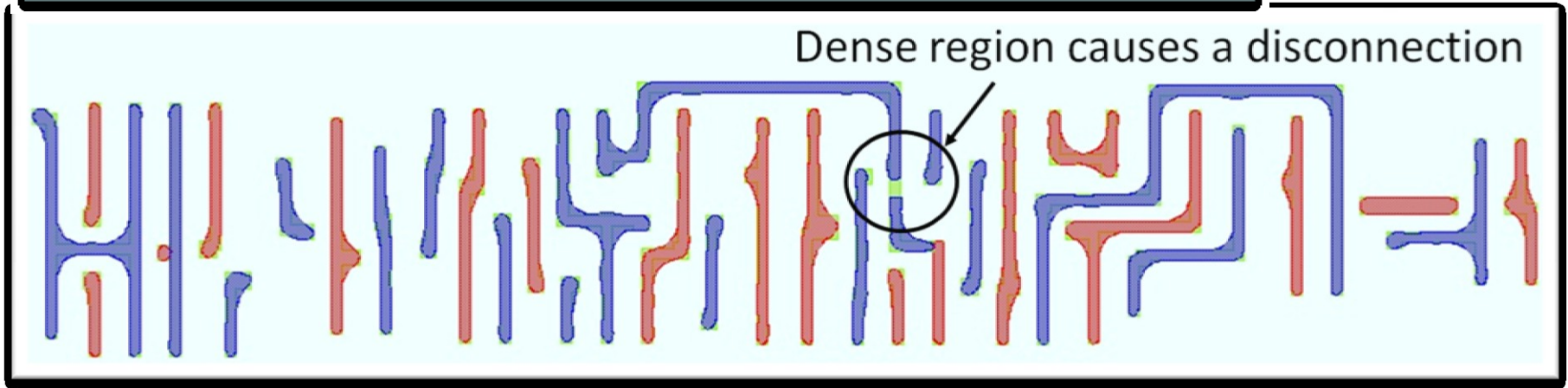


## Decomposition for DPL

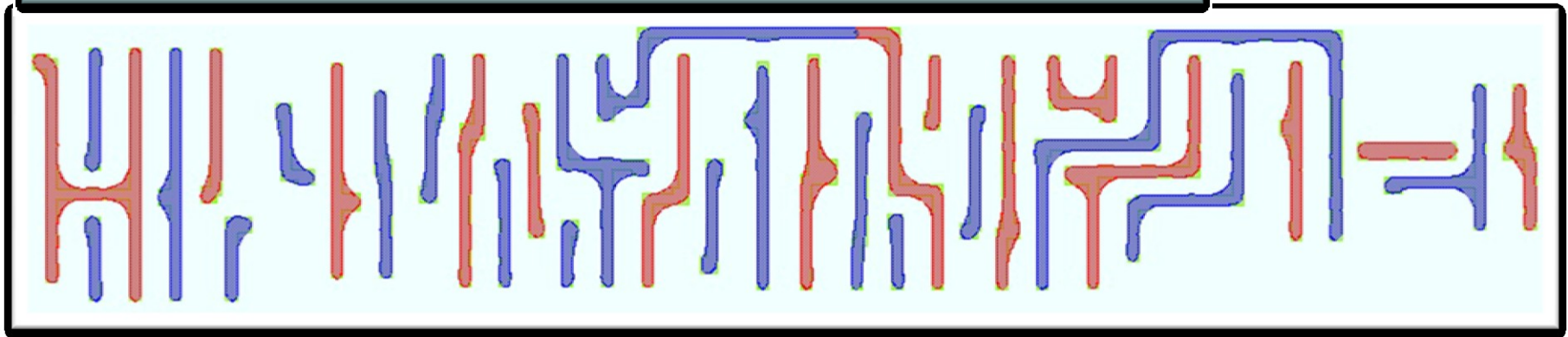


# Balanced Density

Unbalanced decomposition: 38%(Red) and 62% (Blue)



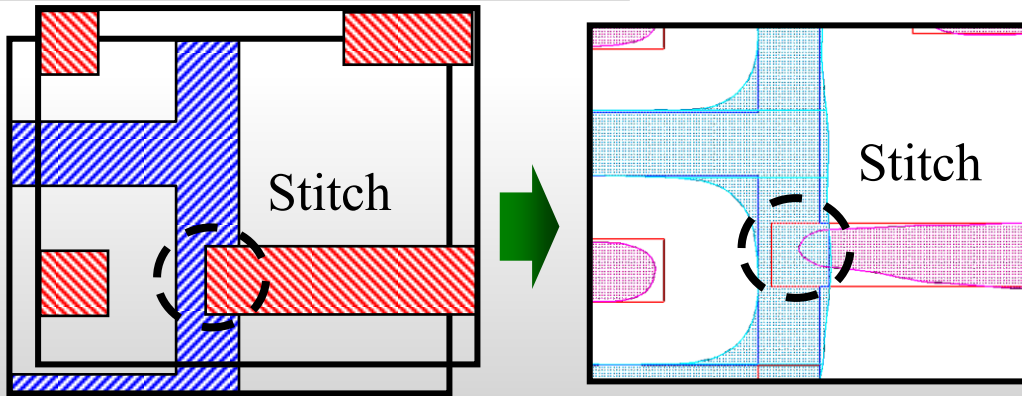
Balanced decomposition: 48%(Red) and 52% (Blue)



**Balanced density is preferred during layout decomposition**

# More Decomposition Requirements

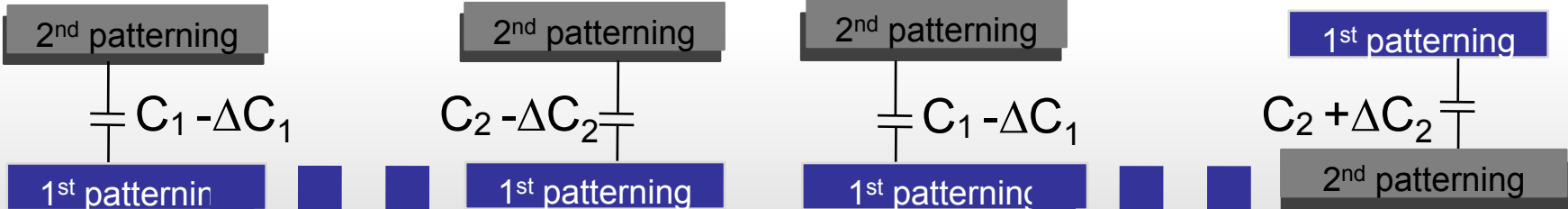
## Minimum Stitch Insertion



[Lucas SPIE'08]

- 1) Yield loss with overlay
- 2) Area increase due to overlap margin

## Overlay Compensation



Without Overlay Compensation

With Overlay Compensation

# Comparisons & Complexity

## Previous Approach

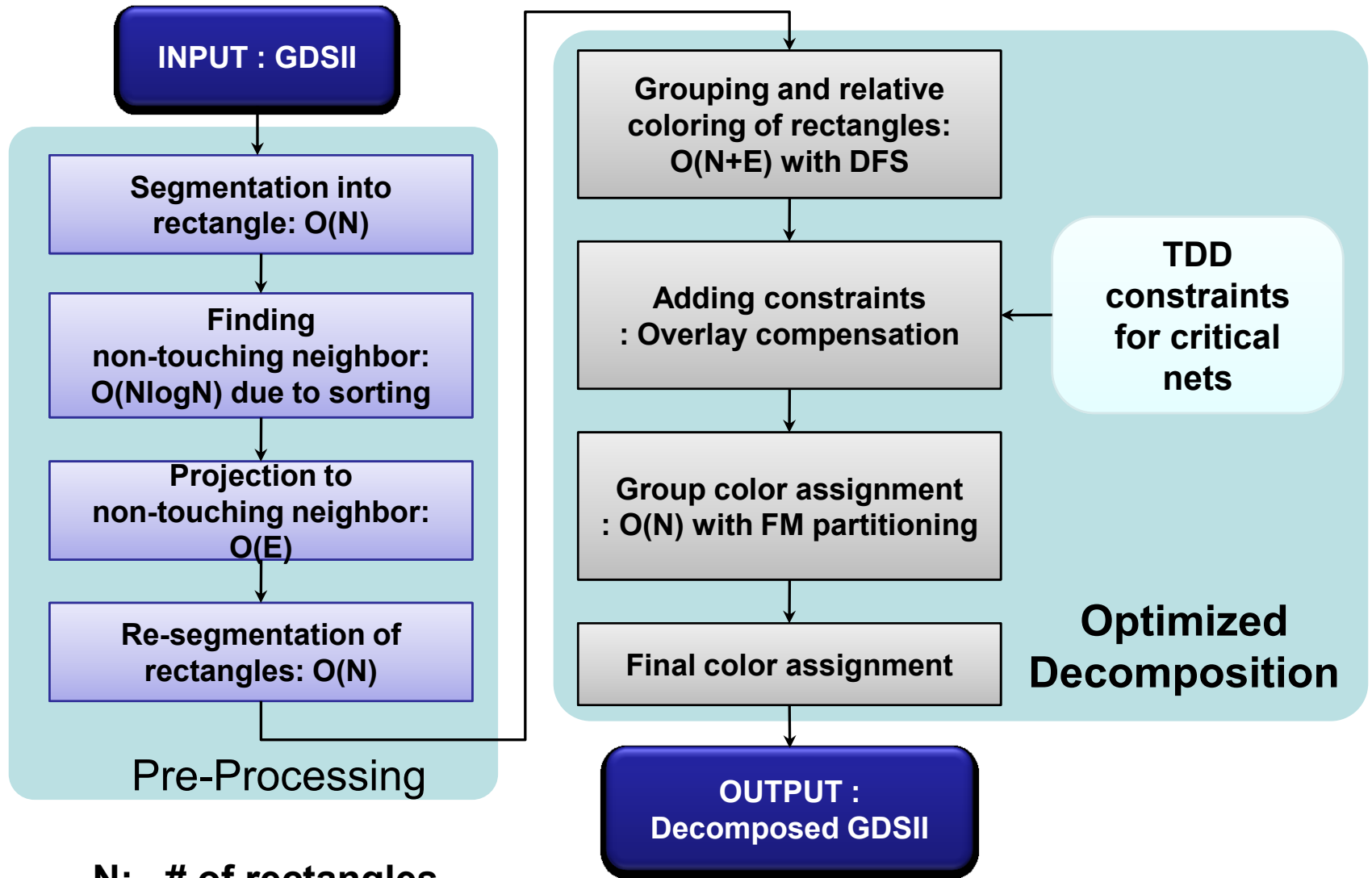
	Balanced Density	Overlay Compensation	Stitch Minimization	Complexity
ICCAD08[1]	No	No	Yes (ILP)	NP-Complete
Our Framework	Yes	Yes	Yes (Bi-Partitioning)	Polynomial Time $O(N \log N)$

## Complexity of Our Decomposition Algorithm

N # of rectangles, and E # of neighboring pairs.

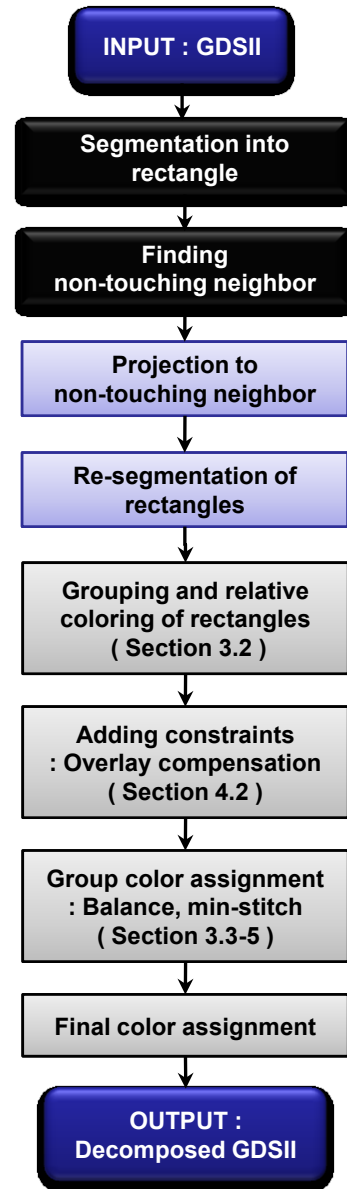
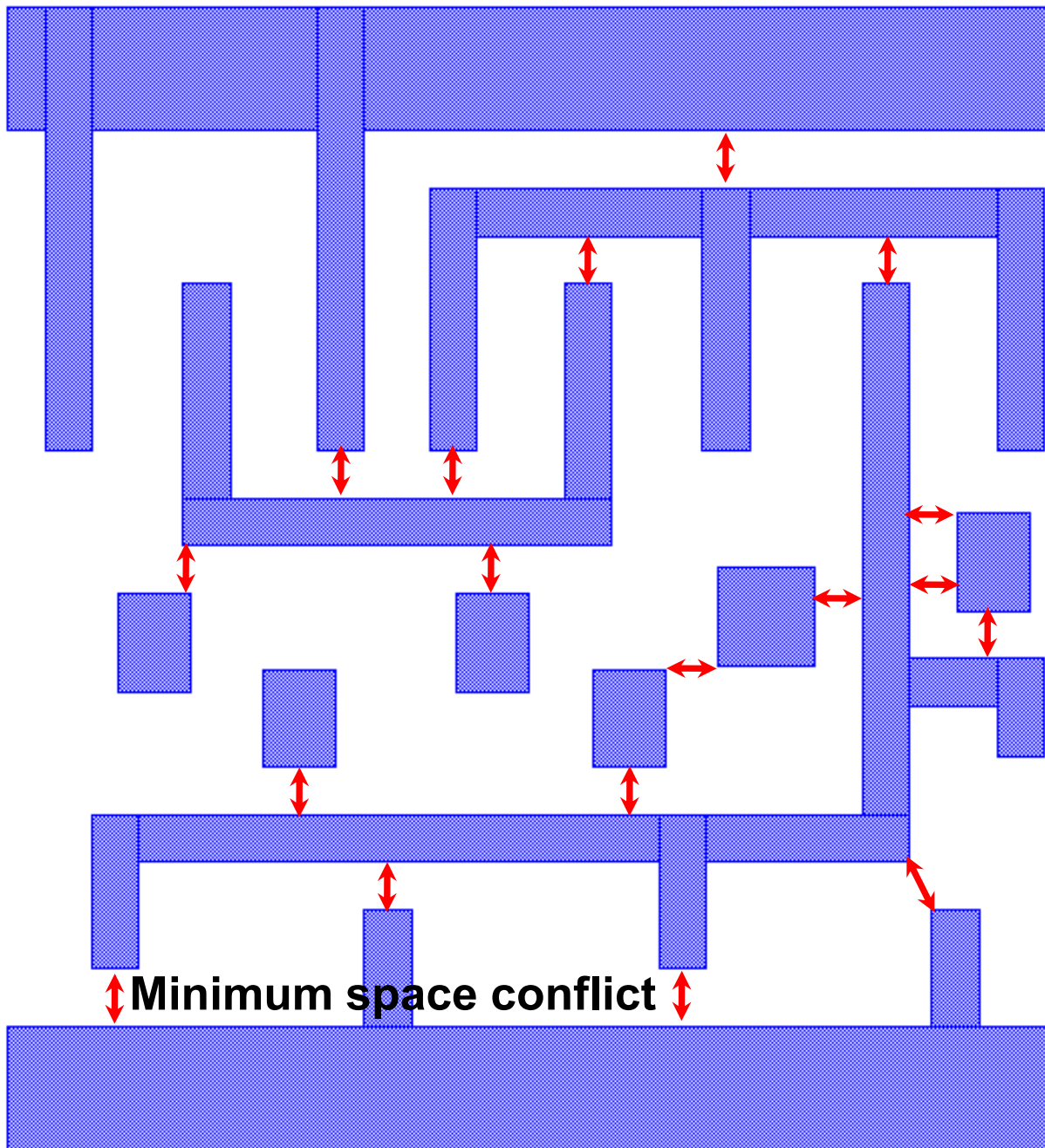
- 1) Segmentation from polygon to rectangles →  $O(N)$
  - 2) Finding neighbors (sorting according to coordinate) →  $O(N \log N)$
  - 3) The complexity of projection to non-touching neighbor →  $O(E)$
  - 4) Grouping and relative coloring using DFS →  $O(N+E)$
  - 5) Group color assignment with min-cut partitioning →  $O(N)$
- Overall complexity** →  $O(N \log N)$

# Overall decomposition flow



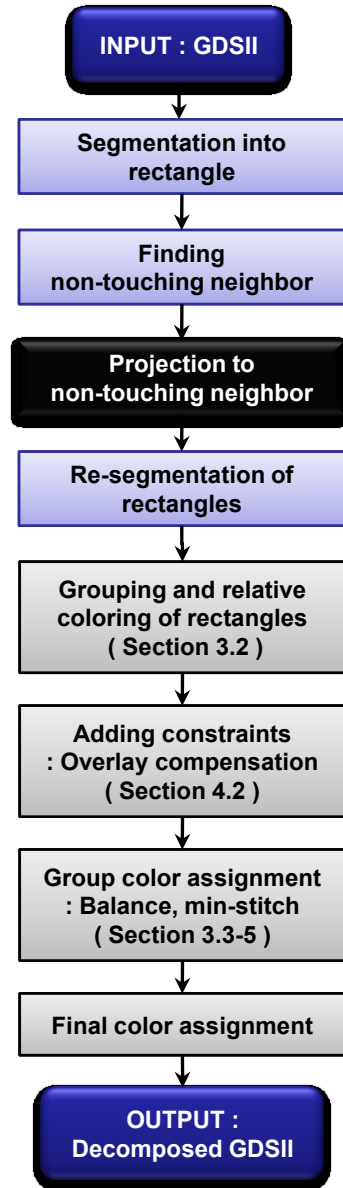
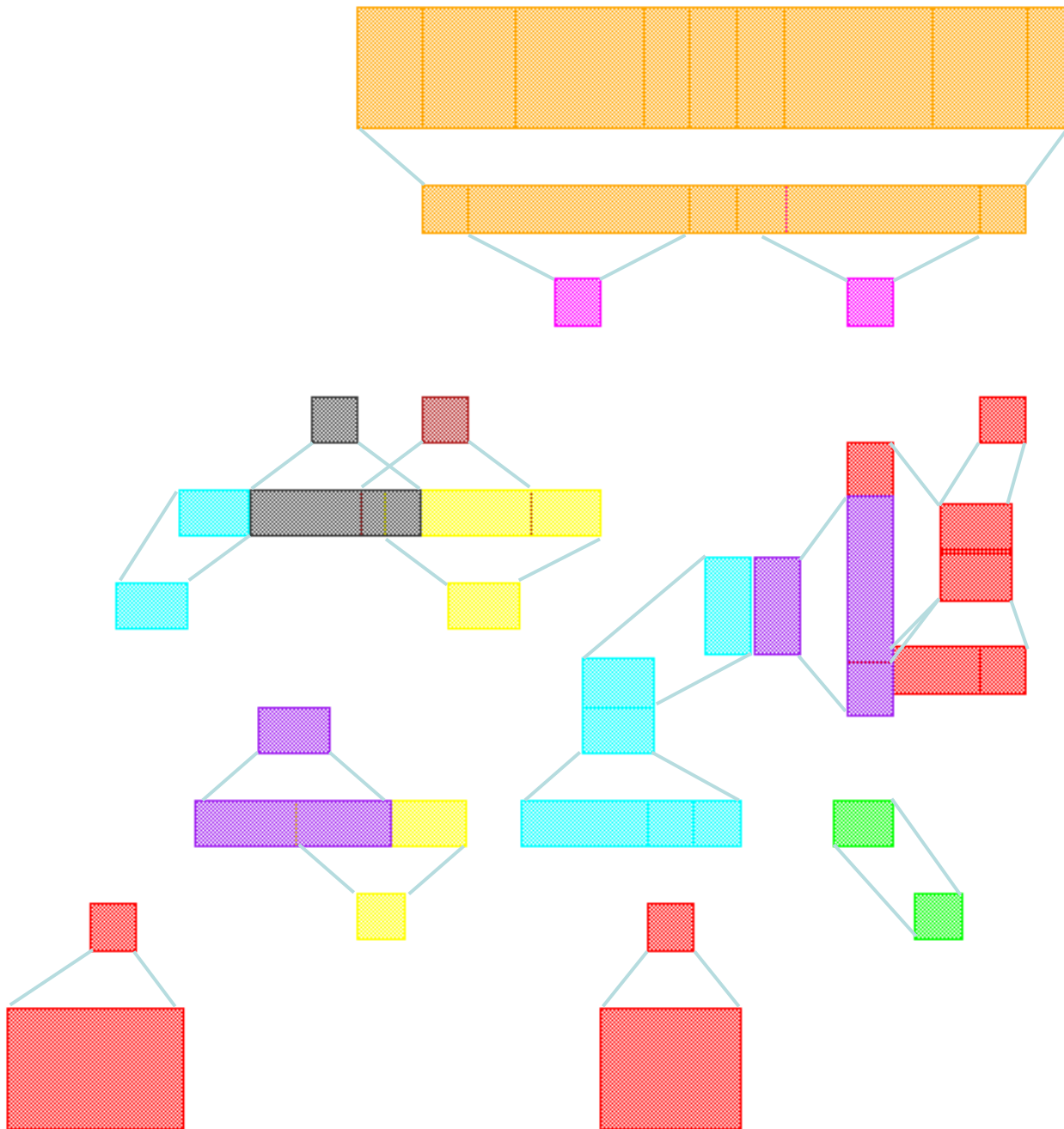
**N:** # of rectangles,  
**E:** # of neighboring pairs

**Overall complexity is  $O(N \log N)$**

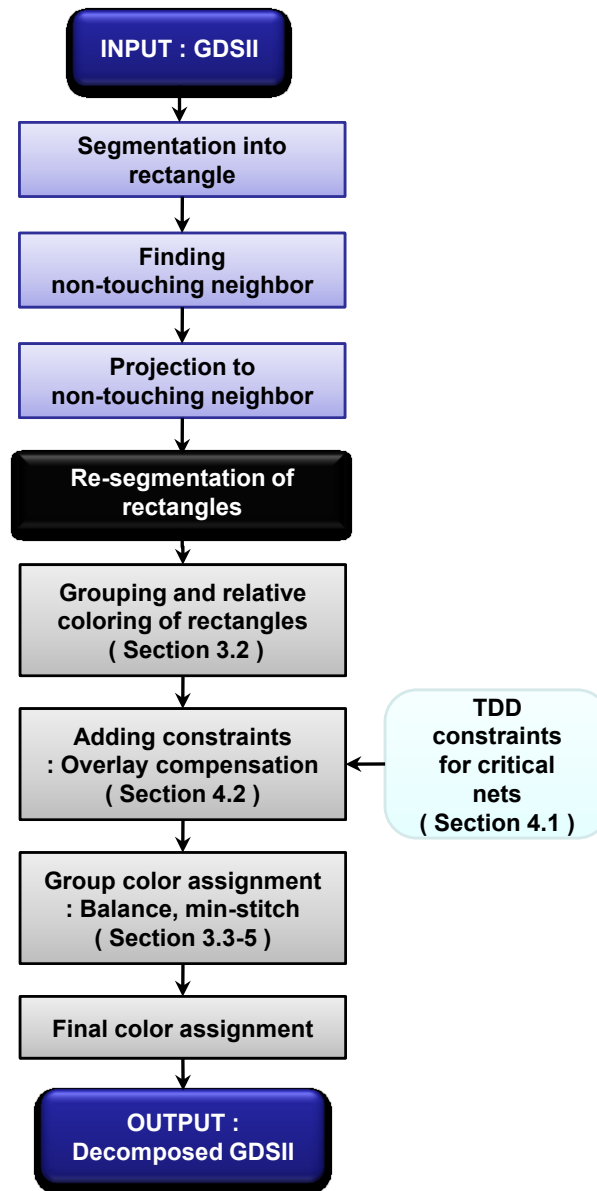
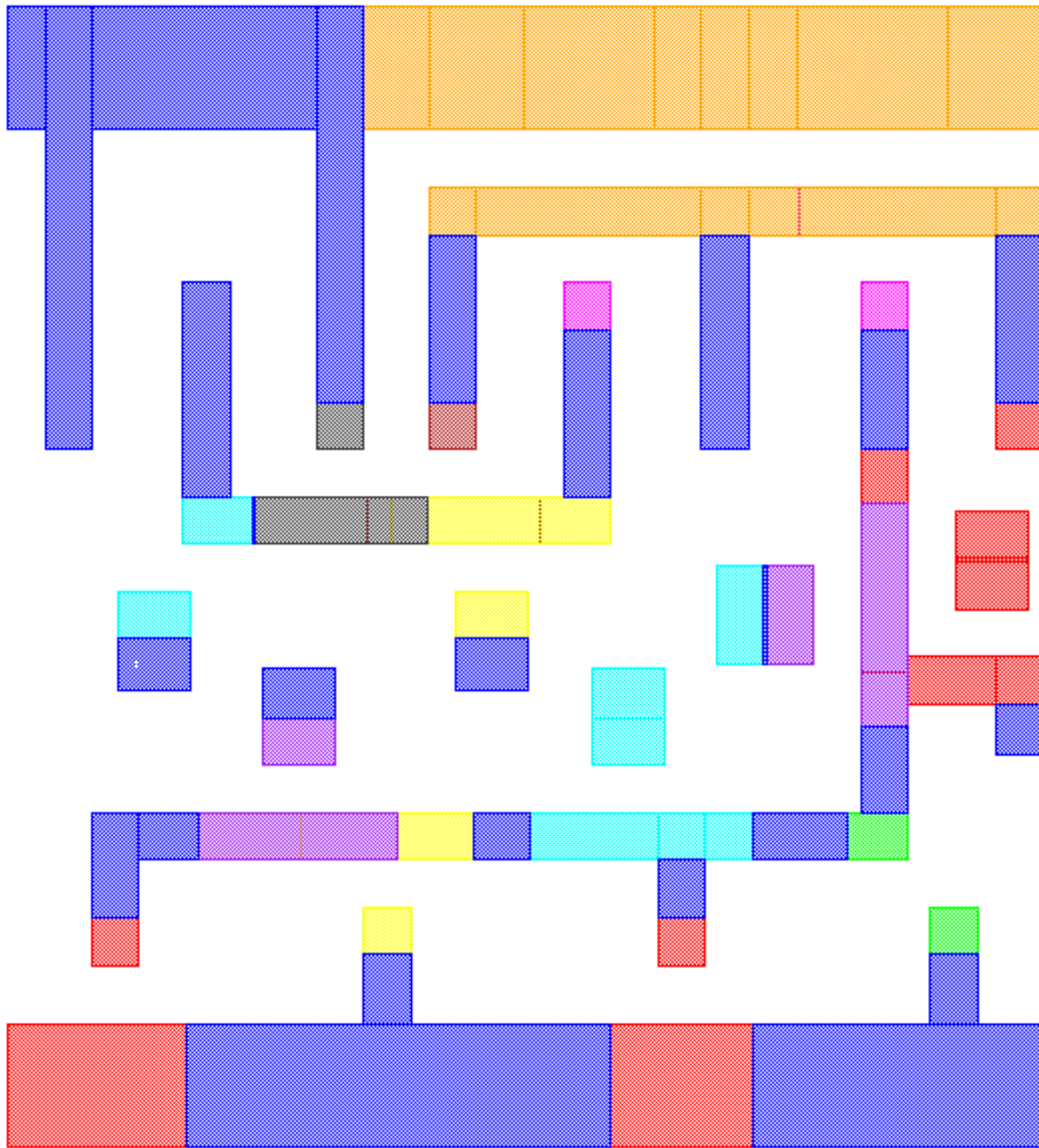


**Complexity :  $O(N \log N)$**





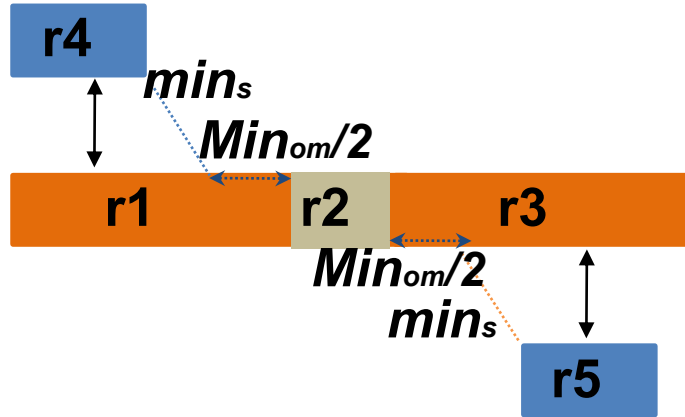
**Complexity :  $O(E)$**



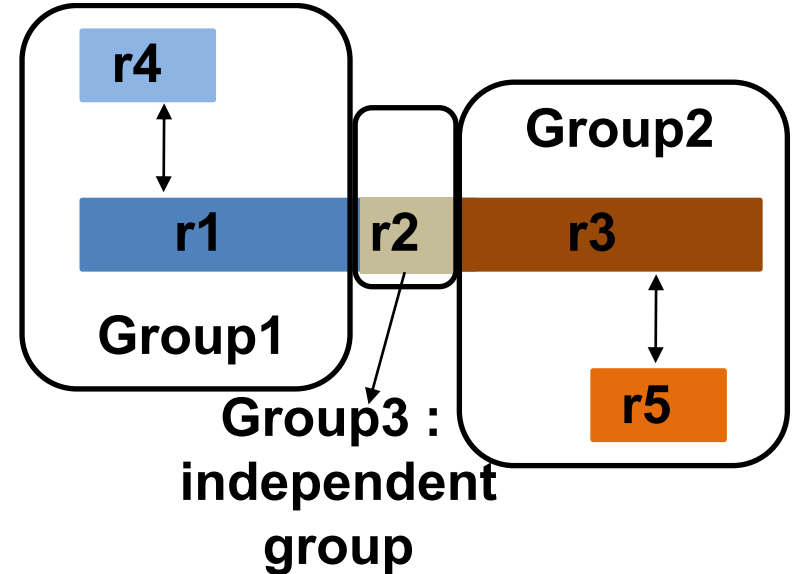
**Complexity :  $O(N)$**

# Grouping and Relative Coloring

After Projection to Neighbors



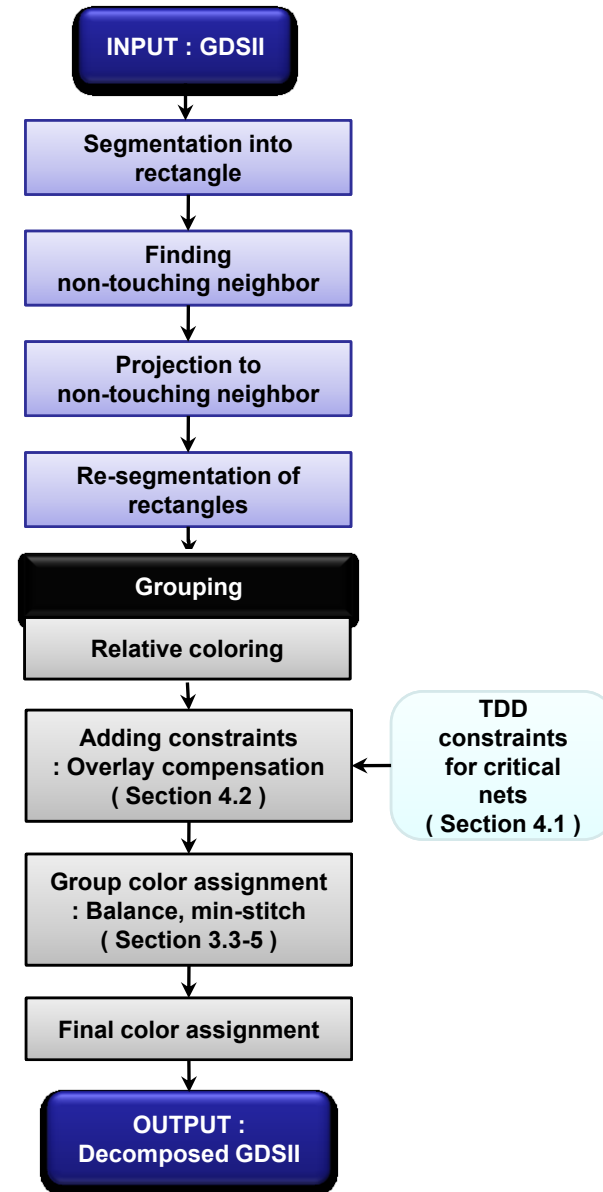
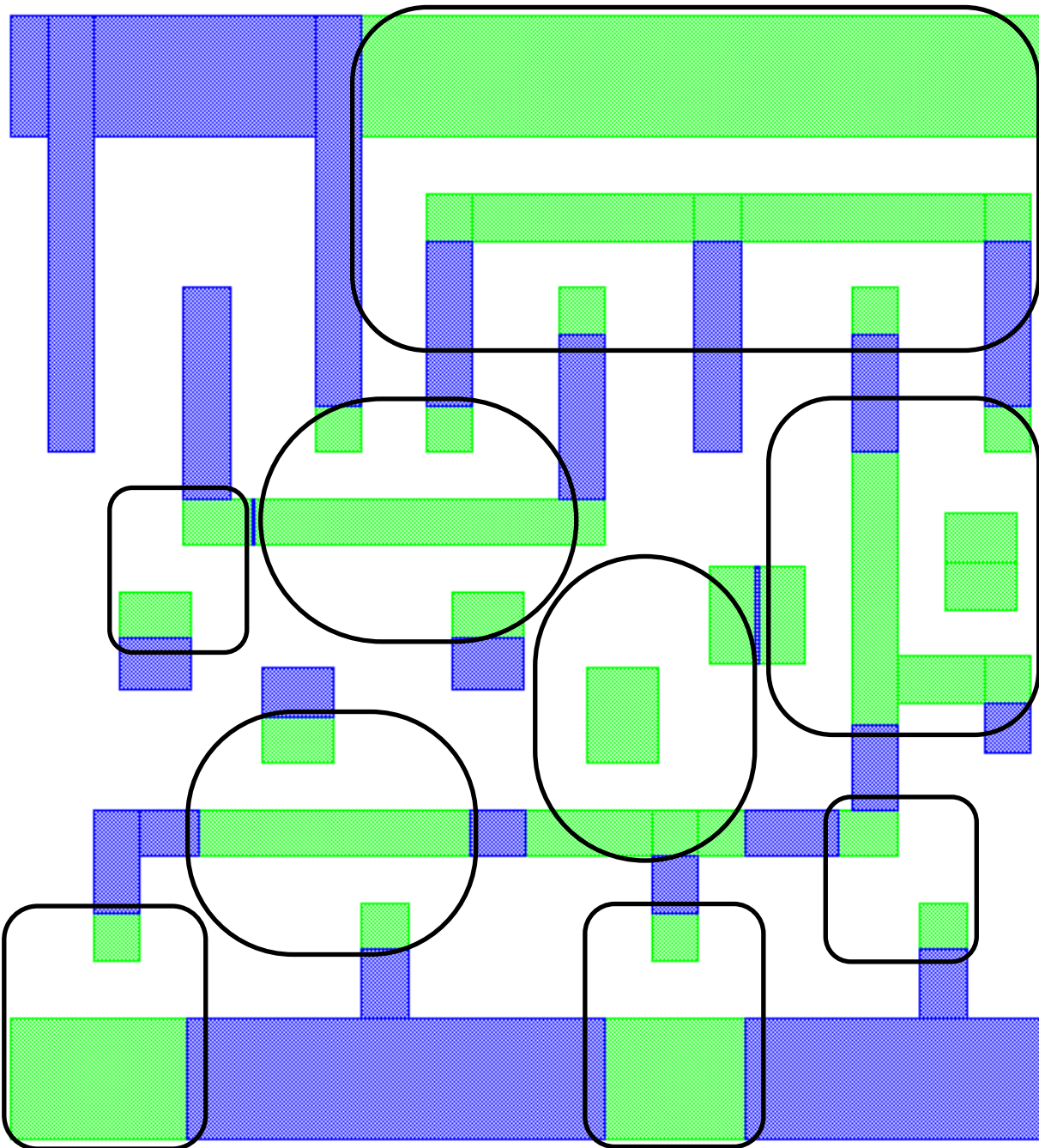
After Grouping

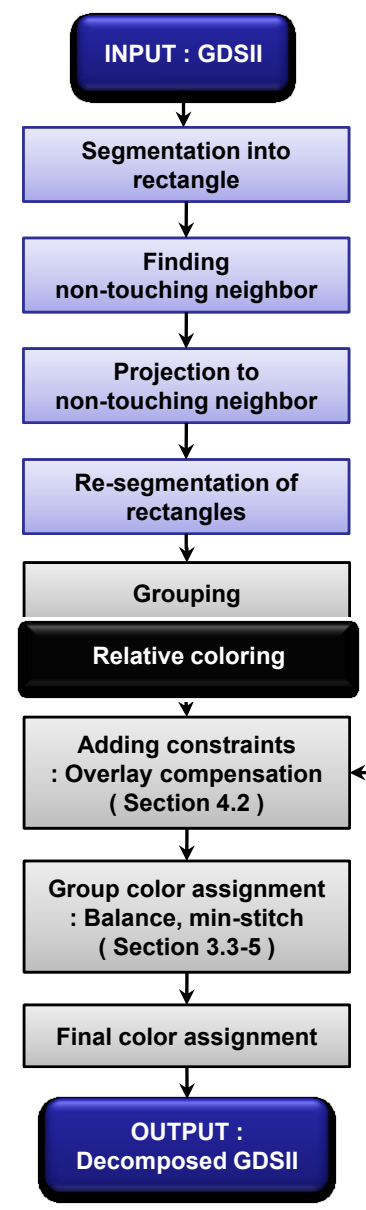
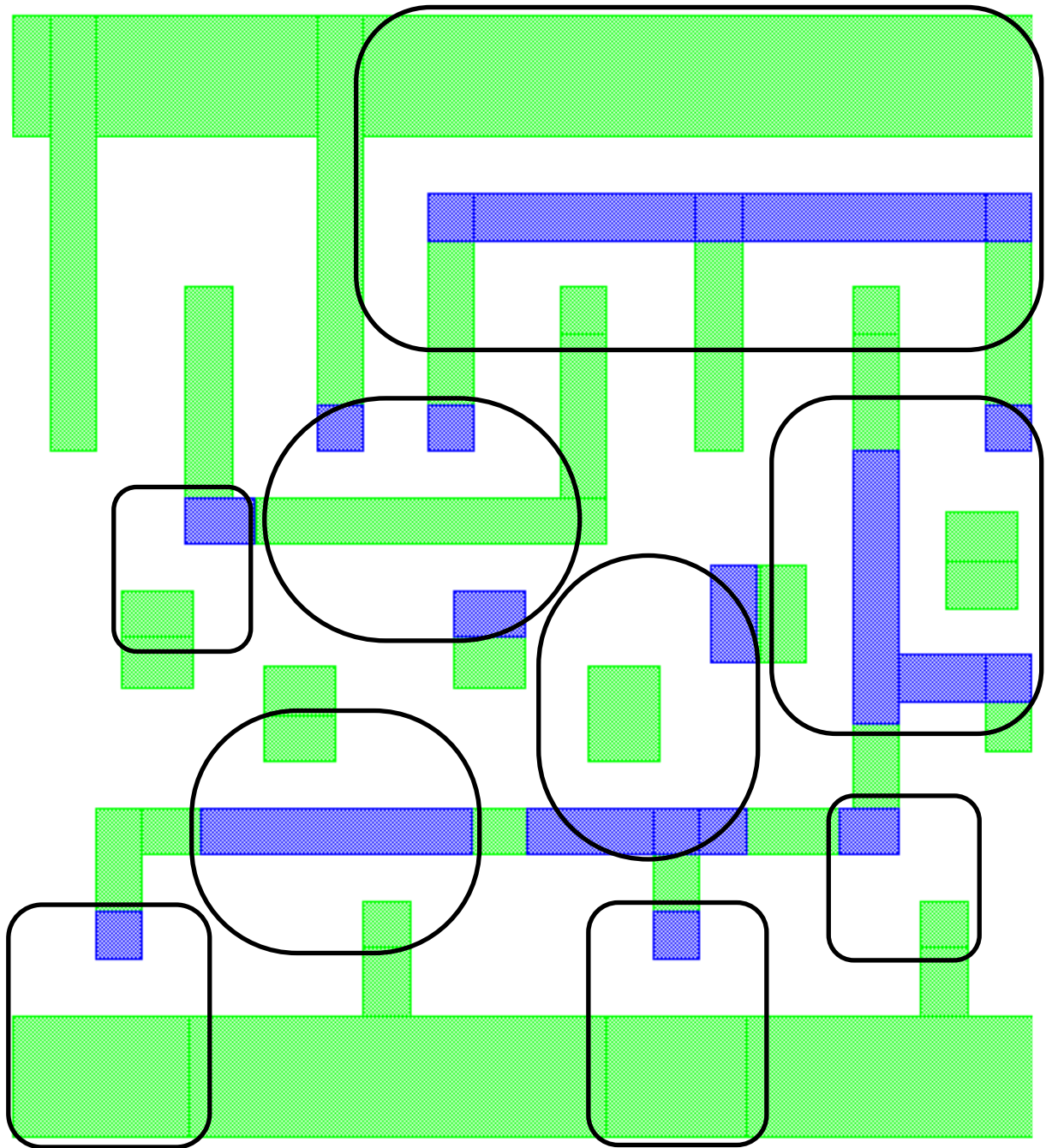


Grouping and Relative Coloring is done by DFS(Complexity :  $O(N+E)$ )

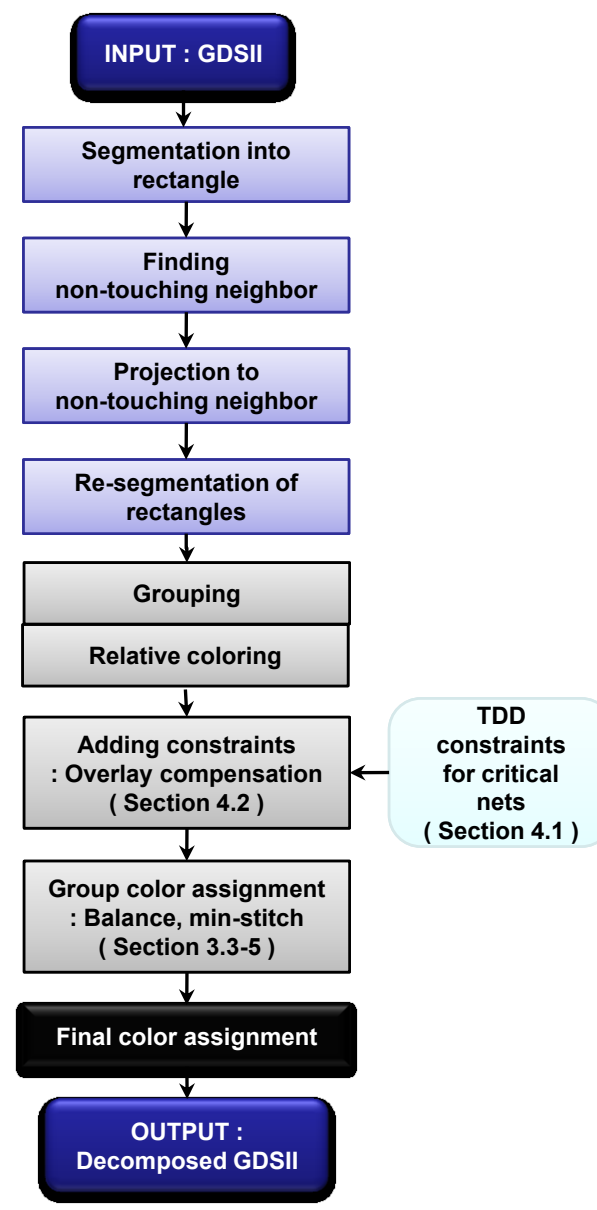
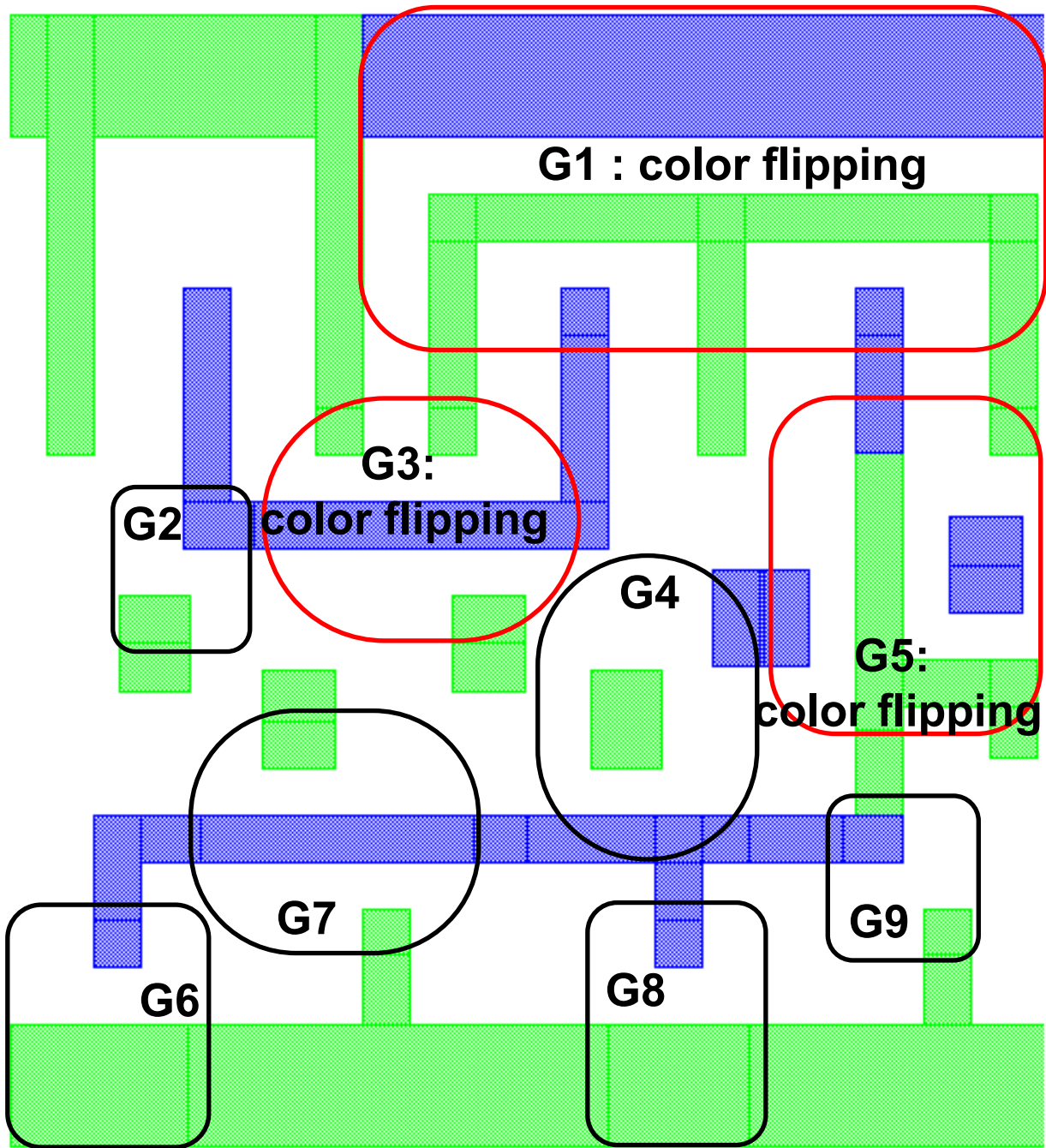
$r1$  and  $r4$  should have different color  
 $r3$  and  $r5$  should have different color  
 $r2$  can have any color

**Relative coloring is a procedure assigning a color to remove conflicts**





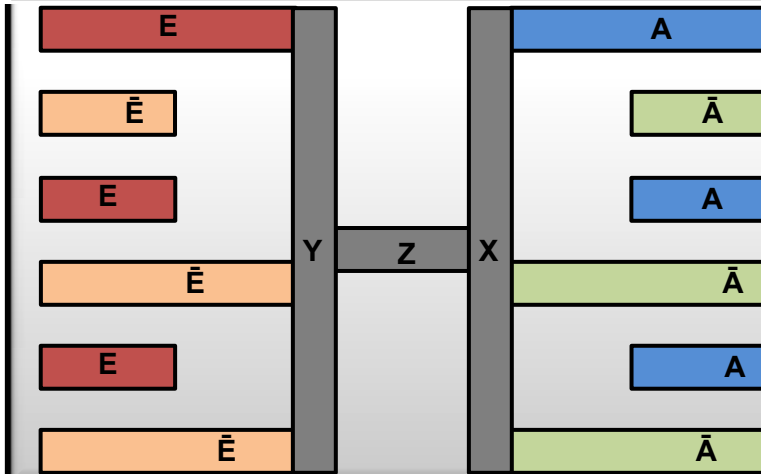
**No conflict, 23 stitches**



**No conflict, 2 stitches**

# Color Assignment – Exact Solution

## Example of Stitch Minimization



Minimize :

$$A \oplus X + X \oplus Z + Y \oplus Z + E \oplus Y + 2(\bar{A} \oplus X) + 2(\bar{E} \oplus Y)$$

Subject to :

$$A \in \{0,1\}, E \in \{0,1\}, X \in \{0,1\}, Y \in \{0,1\}, Z \in \{0,1\}$$

## ILP formulation for Stitch Minimization (Exact Solution)

$$\text{Minimize : } \sum X_w$$

Subject To :

$$: A_w - B_w \leq X_w$$

$$: B_w - A_w \leq X_w$$

**N groups in layout**

**→ 2<sup>N</sup> solution**

**→ NP-Complete**

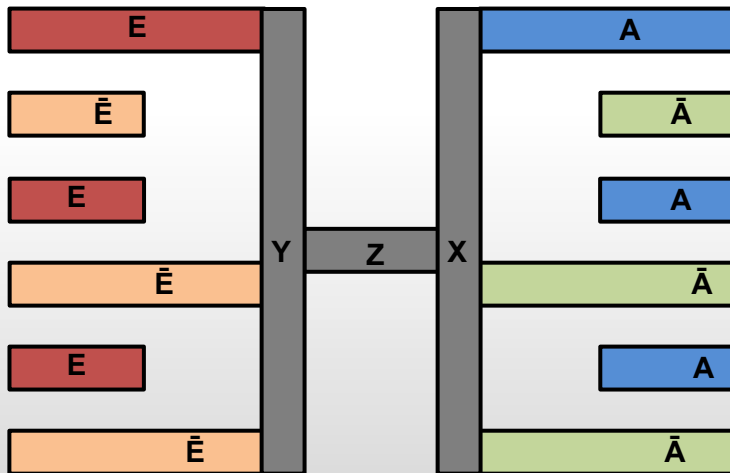
**→ Need a heuristic method**

# Color Assignment – Heuristic Solution

## Theorem 1 : Min-Cut Based Stitch Minimization

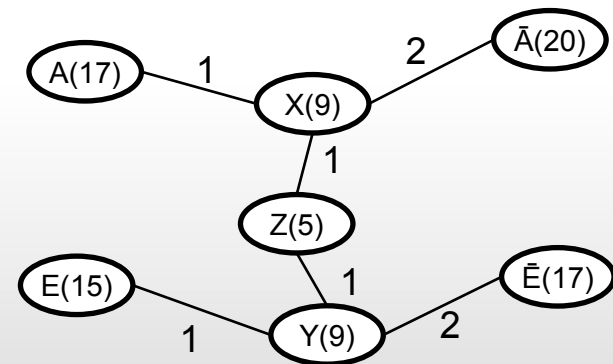
The number of stitches in layout decomposition is equal to the cut size of the bi-partitioning problem in graph theory.

## Example of Graph Based Stitch Minimization



Minimize :

$$A \oplus X + X \oplus Z + Y \oplus Z + E \oplus Y + 2(\bar{A} \oplus X) + 2(\bar{E} \oplus Y)$$



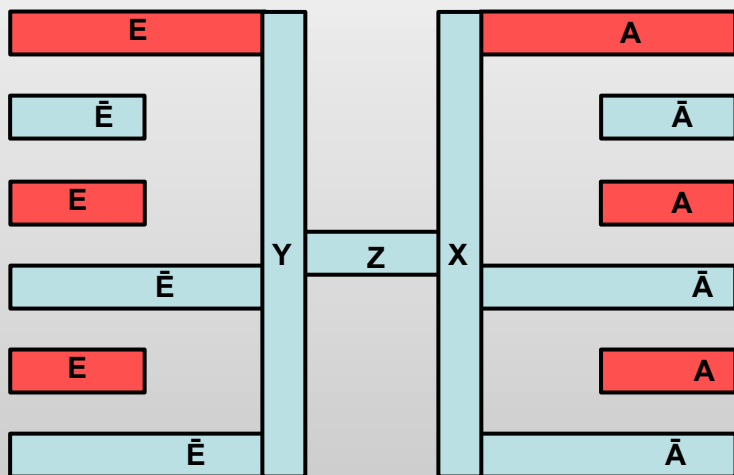
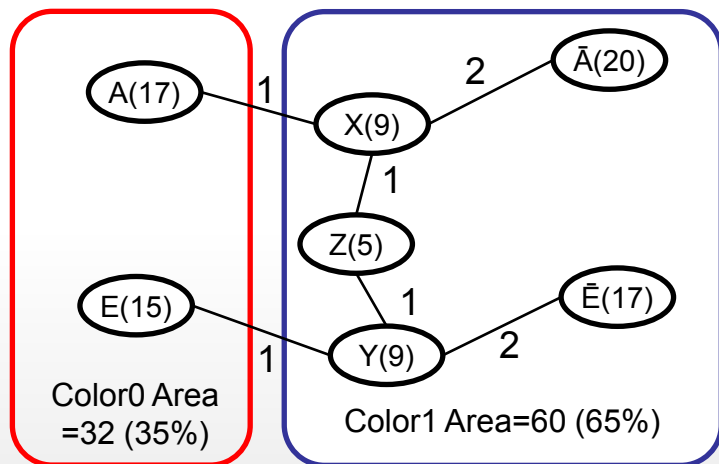
Constraint:

(A,  $\bar{A}$ ) and (E,  $\bar{E}$ ) are repulsive pairs.

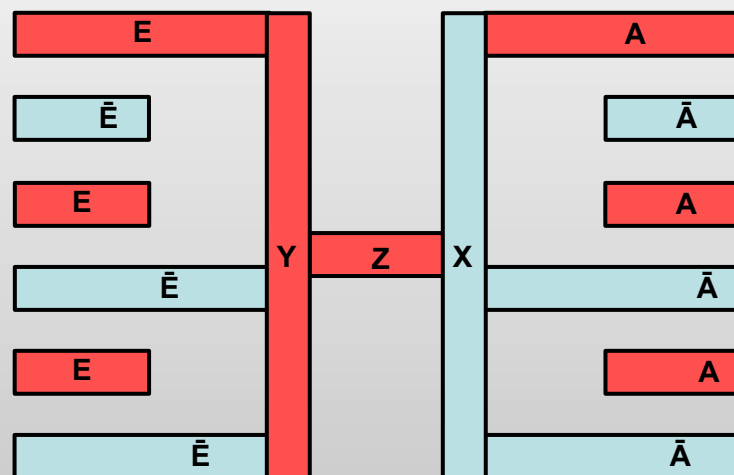
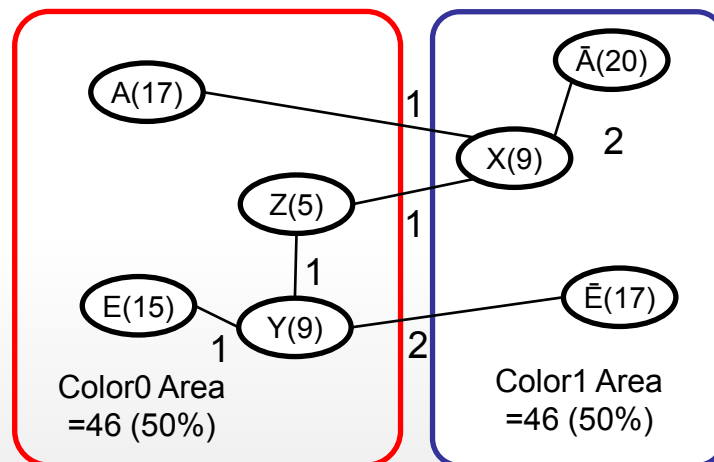


# Graph Partitioning Based Decomposition

## Min-Stitch Coloring

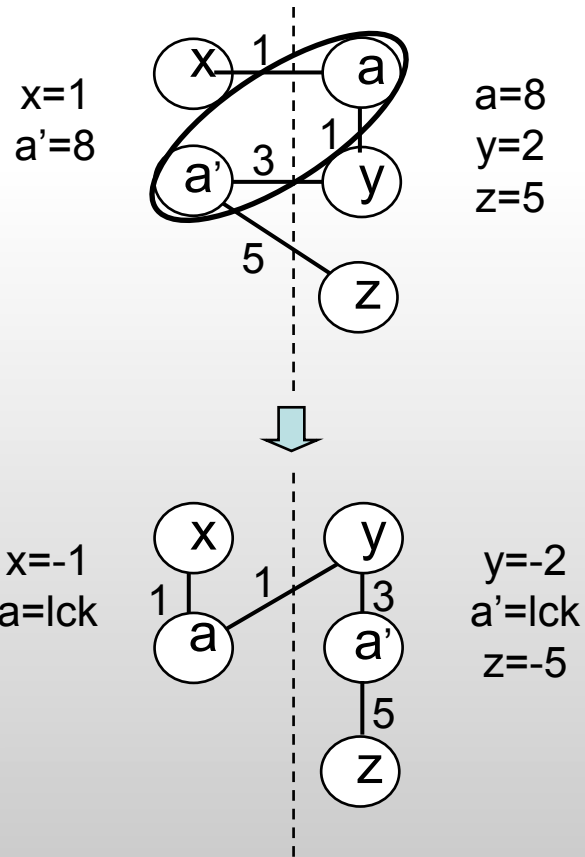


## Balanced Coloring



# Modification of FM partitioning

## Repulsive Pair Consideration



## Local Density Consideration

$R_{11}$	$R_{12}$	$R_{13}$	...	$R_{1i}$
$R_{21}$	$R_{22}$	$R_{23}$	...	$R_{2i}$
$R_{31}$	$R_{32}$	$R_{33}$	...	$R_{3i}$
...	...	...	...	...
$R_{j1}$	$R_{j2}$	$R_{j3}$	...	$R_{ji}$

$$rW_{11} - \text{smax}_{11} \leq |A_{11}| \leq rW_{11} + \text{smax}_{11}$$

$$rW_{12} - \text{smax}_{12} \leq |A_{12}| \leq rW_{12} + \text{smax}_{12}$$

$$\vdots$$

$$rW_{ji} - \text{smax}_{ji} \leq |A_{ji}| \leq rW_{ji} + \text{smax}_{ji}$$

**We implemented FM partitioning with the two new features**

# Minimize $\Delta$ Delay due to Overlay

## 1<sup>st</sup> Order Expression of $\Delta$ Delay

$$\sqrt{\alpha^2 + \beta^2} \sin(\theta + \phi)$$

where,  $\alpha = 2AX^T - \sum_{n=1}^i a_n$ ,  $\beta = 2BY^T - \sum_{n=1}^j b_n$

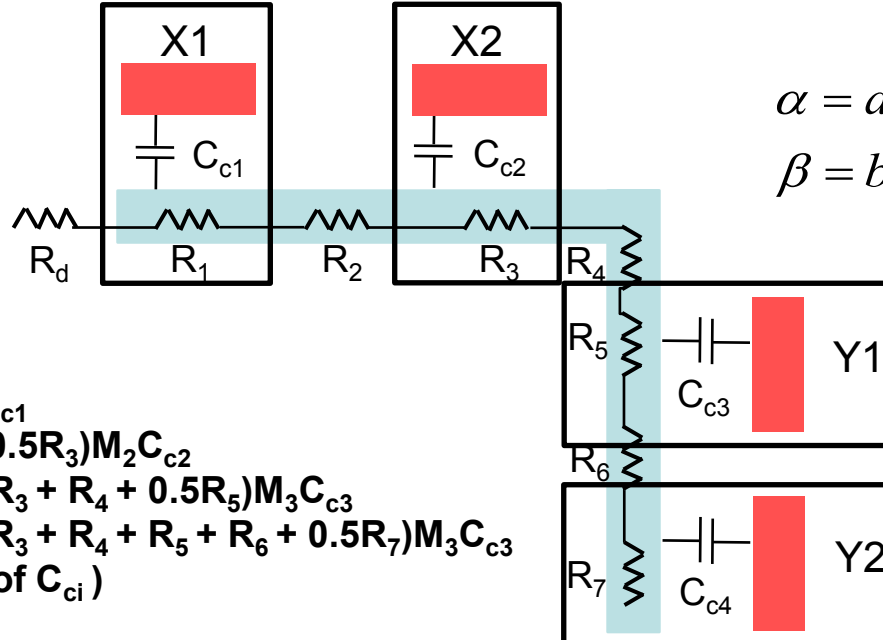
$$X = [x_1, x_2, \dots, x_i], \quad Y = [y_1, y_2, \dots, y_j]$$

$$\phi = \sin^{-1} \left( \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \right)$$

## Theoram2

**Horizontal direction and Vertical direction can be optimized independently to minimize  $\Delta$ Delay due to Overlay**

## Example



$$a_1 = (R_d + 0.5R_1)M_1C_{c1}$$

$$a_2 = (R_d + R_1 + R_2 + 0.5R_3)M_2C_{c2}$$

$$b_1 = (R_d + R_1 + R_2 + R_3 + R_4 + 0.5R_5)M_3C_{c3}$$

$$b_2 = (R_d + R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + 0.5R_7)M_3C_{c3}$$

(  $M_i$  : Miller Factor of  $C_{ci}$  )

$$\alpha = a_1(2x_1 - 1) + a_2(2x_2 - 1)$$

$$\beta = b_1(2y_1 - 1) + b_2(2y_2 - 1)$$

Minimize  $\alpha^2$  and  $\beta^2$  independently to minimize  $\Delta$ Delay due to overlay

# Timing Driven Decomposition(TDD)

ILP  
Formulation  
to minimize  
 $\alpha^2$

$$\text{minimize } 4 \sum_{n=1}^i \left\{ a_n \left( AW_n^T - w_{nn} \sum_{p=1}^i a_p \right) \right\} + \left( \sum_{p=1}^i a_p \right)^2$$

s.t.

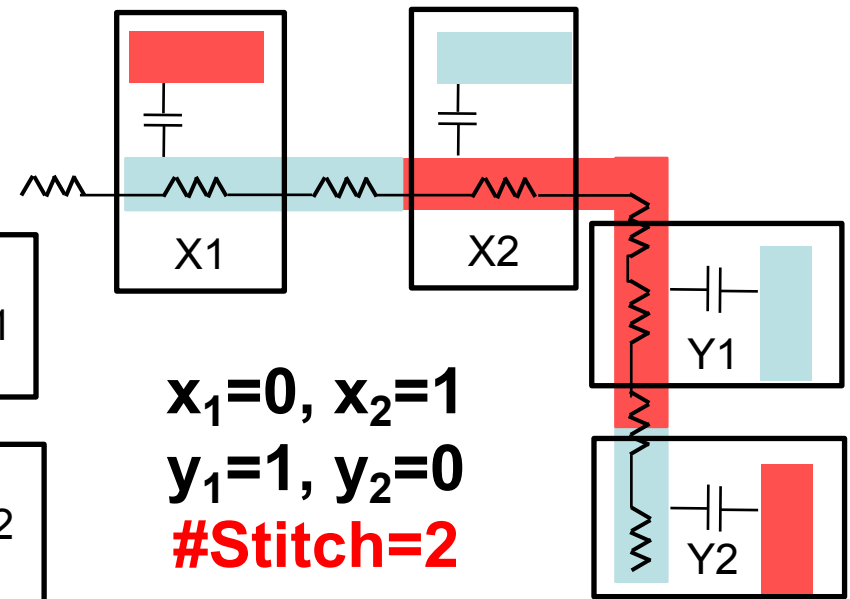
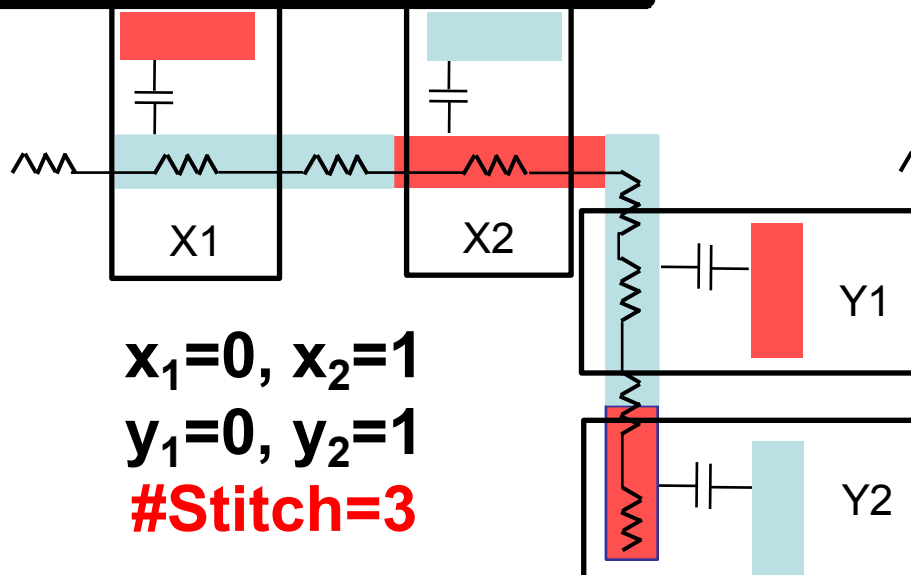
$$w_{ii} = x_i$$

$$1 + w_{ij} \geq x_i + x_j$$

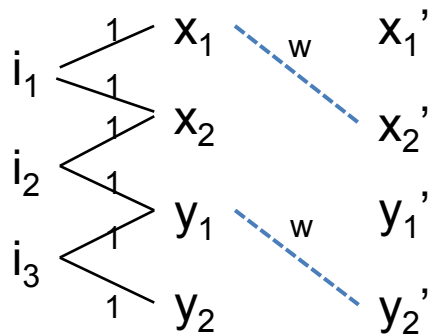
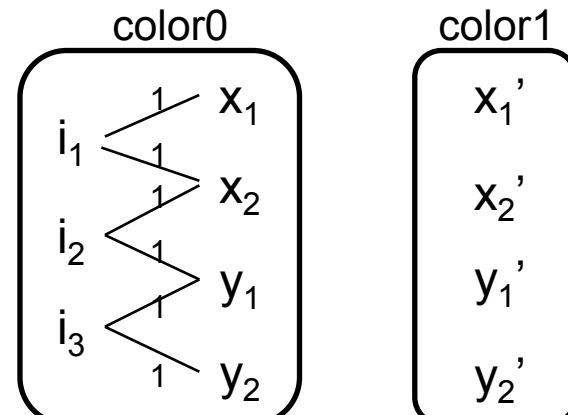
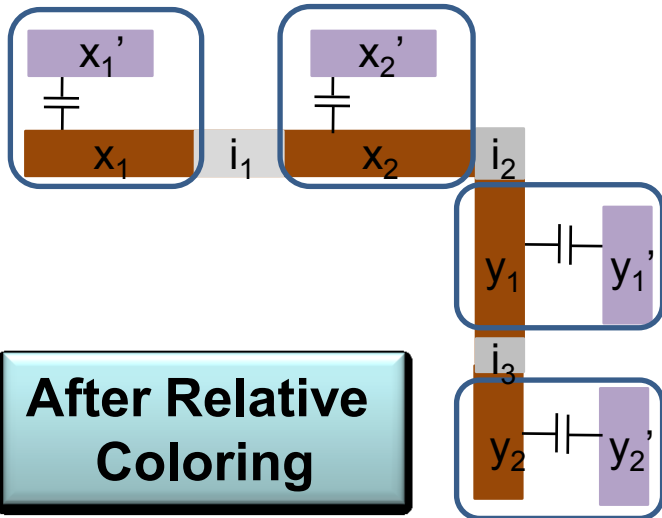
$$x_i \geq w_{ij}$$

$$x_j \geq w_{ij}$$

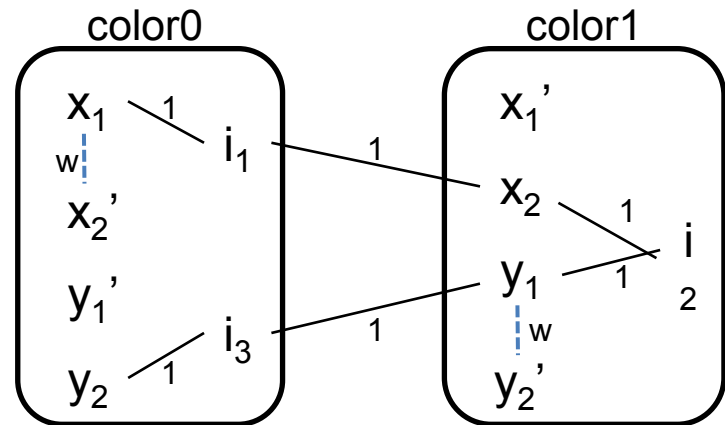
## Two Possible Solutions



# Decomposition with TDD constraint



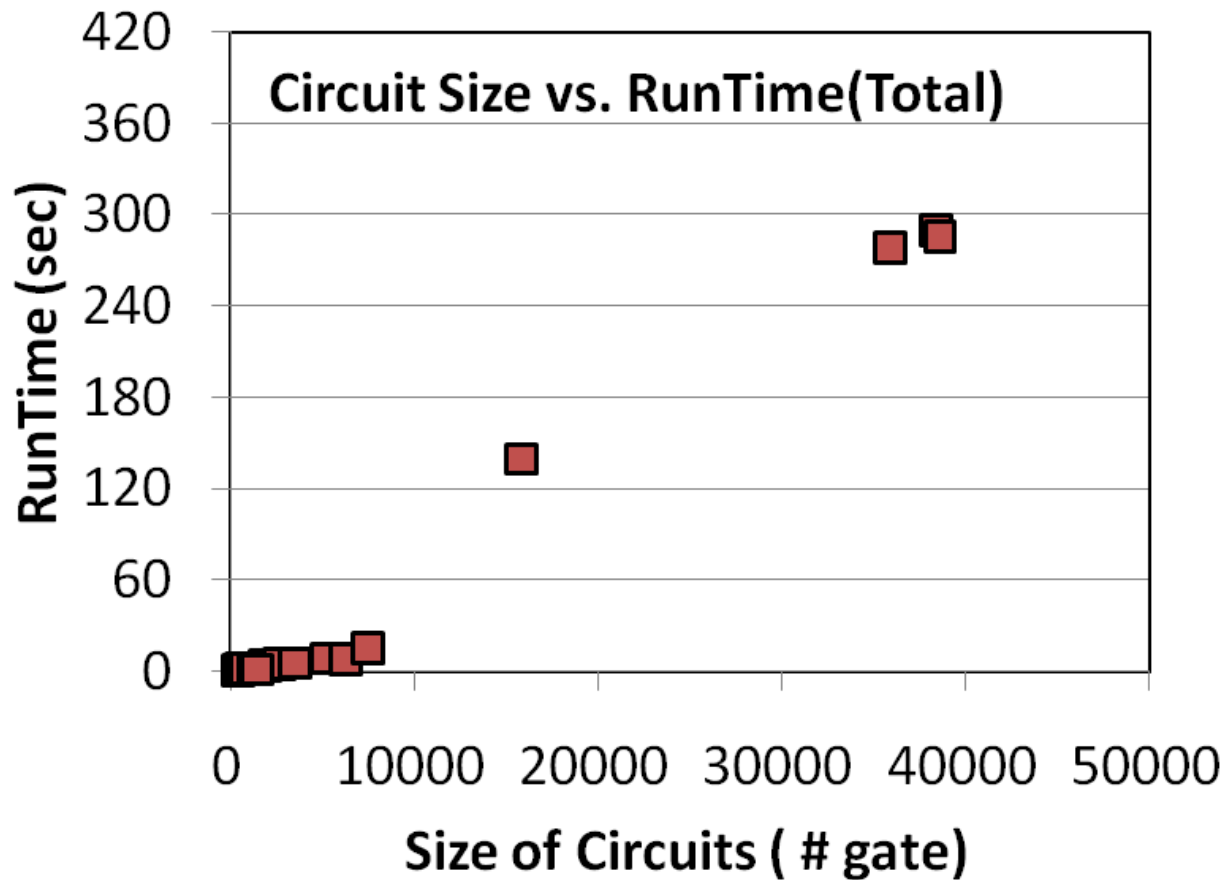
$X_0 = \{x_1, x_2\}$ ,  $Y_0 = \{y_1, y_2\}$  to minimize  $\alpha^2, \beta^2$



TDD Constraints insertion

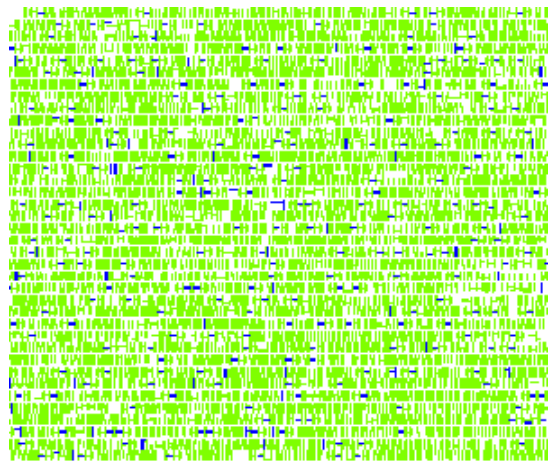
Group color assignment when the edge weight ( $w$ ) is bigger than one: **Two stitches**

# Runtime Result

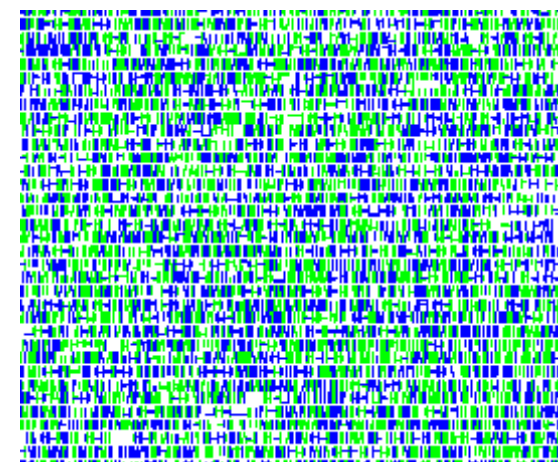


**Complexity:  $O(N \log N)$  → Don't need layout partitioning**

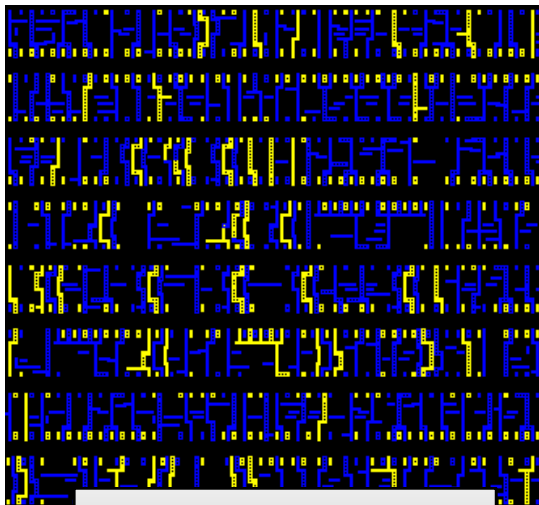
# Balanced Density Result



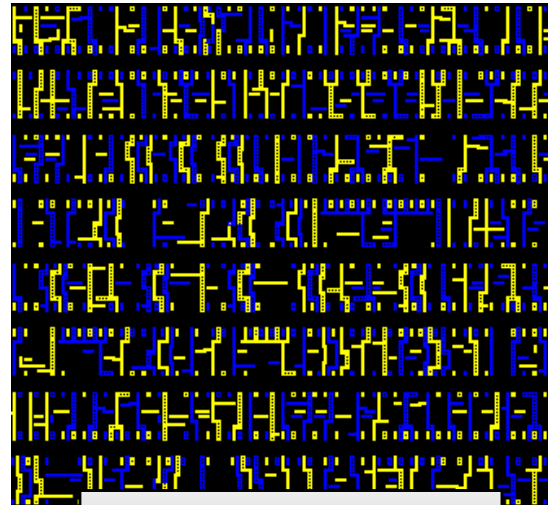
S38584:13% and 87%



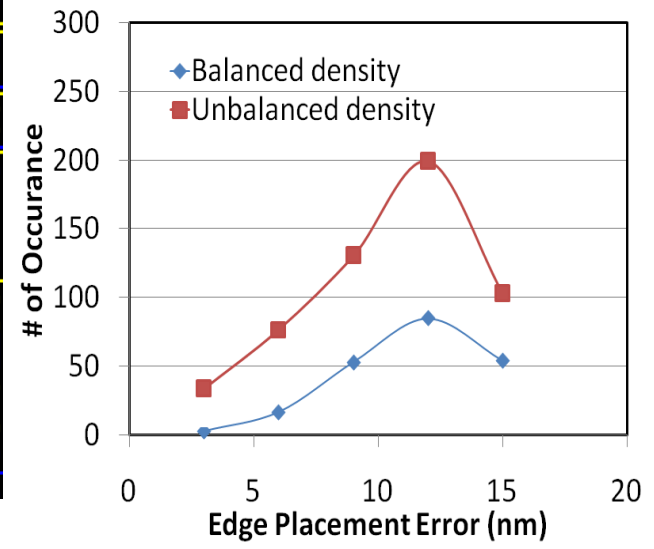
S38584: 50% and 50%



C432:27% and 73%  
(7 stitches)



C432:50% and 50%  
(17 stitches)



# Exact vs. Heuristic comparison



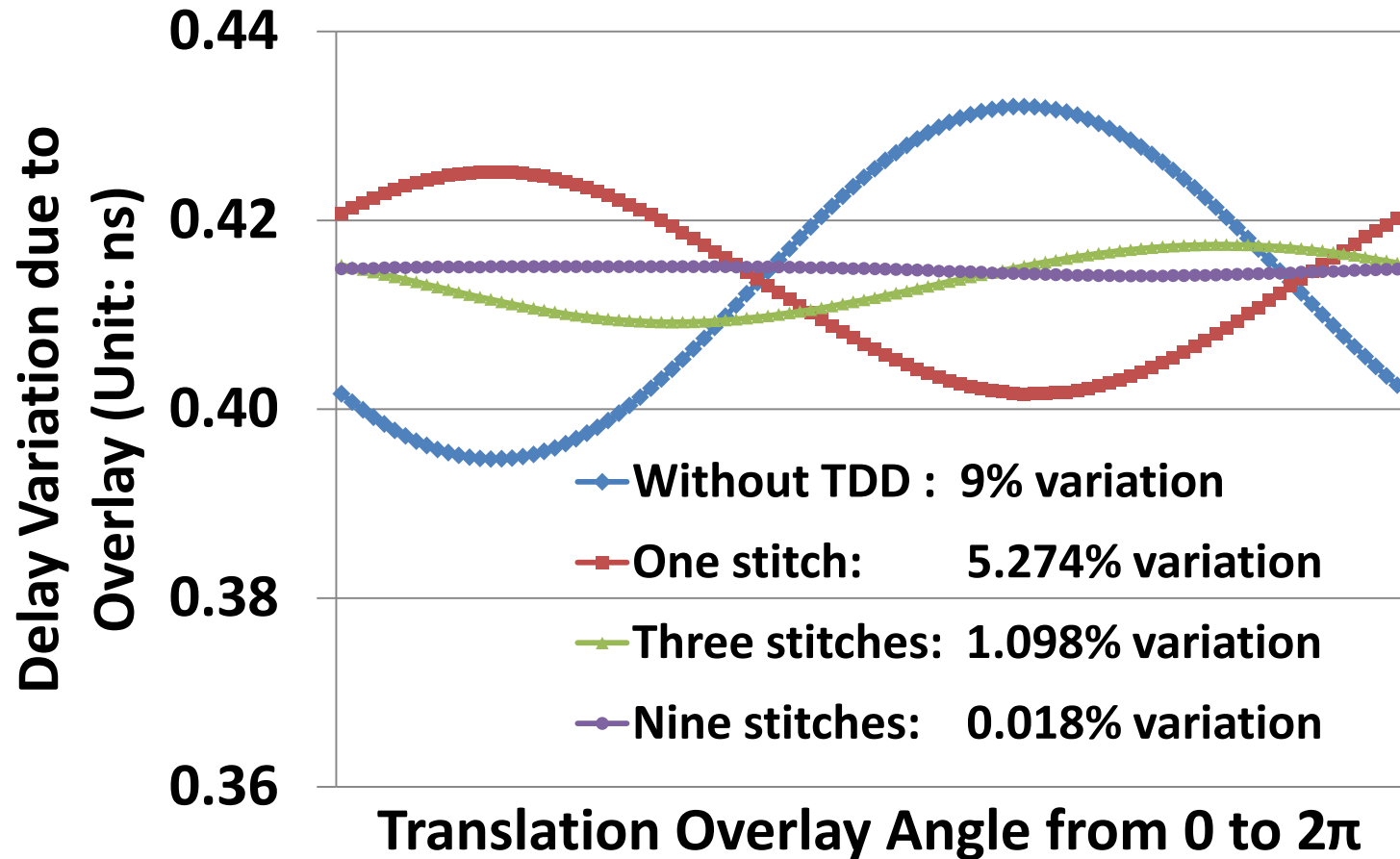
Circuit	#Groups	#Touching neighbors	No balance, ILP (Exact)				No balance, Graph Partition (Proposed heuristic)				48% balance, Graph Partition (Proposed heuristic)			
			# Partitions for ILP	RunTime (total)	Inserted stitches	Balanced ratio(%)	RunTime comparison	RunTime (total)	Inserted stitches	Balanced ratio(%)	RunTime comparison	RunTime (total)	Inserted stitches	Balanced ratio(%)
C432	1512	1098	1	0.63	1	20.35	x1.4	0.46	1	33.60	x1.0	0.65	2	48.12
C499	3103	3280	12	100.85	50	24.01	x49.9	2.02	50	46.47	x49.9	2.02	50	48.50
C880	3758	2631	14	4525.57	198	30.09	x2773.0	1.63	198	47.12	x2807.4	1.61	198	48.87
C1355	4836	3083	18	702.4	114	18.91	x347.4	2.02	114	36.12	x344.0	2.04	114	48.00
C1908	7795	5472	18	37019.7	371	22.09	x9762.6	3.79	372	46.78	x10422	3.55	373	48.66
C2670	12863	9905	-	> 24Hr	-	-	-	6.7	947	43.51	-	6.87	948	49.30
C3540	16638	12021	-	> 24Hr	-	-	-	9.85	1034	41.46	-	10.07	1034	49.39
C5315	24483	18373	-	> 24Hr	-	-	-	17.43	1546	40.87	-	18.5	1549	48.00
C6288	19922	11577	-	> 24Hr	-	-	-	11.57	256	30.81	-	11.25	256	48.13
C7552	34309	24789	-	> 24Hr	-	-	-	30.89	2058	41.97	-	31.52	2060	48.02

**Runtime : Bi-partitioning based decomposition is up to 10K faster than ILP based decomposition.**

**Accuracy : C1908 has two more stitches in our heuristic algorithm. All benchmarks except C1908 have the same #stitches.**



# Overlay compensation result



We could compensate overlay effect on timing  
More stitches → Less overlay effect on timing

# Conclusion & Future Works

- ◆ **Graph-based multi-objective decomposition**
  - › Super linear time complexity :  $O(N \log N)$
  - › Stitch minimization
  - › Balanced density
  - › Constraint insertion : overlay compensation
  
- ◆ **Future work**
  - › Multiple Decomposition for Multiple Patterning
  - › Correlation Aware Decomposition