Geometry Variations Analysis of TiO$_2$ Thin-Film and Spintronic Memristors

M. Hu$^1$, H. Li$^1$, Y. Chen$^2$, X. Wang$^3$ & R. E. Pino$^4$

$^1$ Dept. of ECE, Polytechnic Institute of NYU
$^2$ Dept. of ECE, University of Pittsburgh
$^3$ Seagate Technology LLC
$^4$ Air Force Research Laboratory, Advanced Computing

Miao Hu, January 17, 2011
WHAT IS MEMRISTOR

Miao Hu, January 29, 2011
WHAT IS MEMRISTOR

- Memristor is a resistor with memory
  - $M(t) = \phi(t)/q(t)$, unit $\Omega$
  - Intrinsic state to remember the history
  - Passive, AC
PROSPECT OF MEMRISTOR

- Memristor features
  - nano-size, non-volatility, reconfigurable

- Potential Applications
  - High density storage technology
    - DRAM 18 Gbits/cm²
    - Memristor 100 Gbits/cm²
  - Reconfigurable computation
  - Neural network
OUTLINE

- Motivation
- Memristor examples
  - TiO$_2$ thin-film memristor
  - Spintronic memristor
- Memristor model with geometry variations
- Statistical analysis
- Performance analysis
- Summary
MOTIVATIONS

- Improve fabrication
  - Measurement
  - Predict margin and actual performance

- More applications
  - Multi-level memory: Levels?
  - Reconfigurable computation: Accuracy?
  - Neural network: Fuzzy operation?
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MEMRISTOR EXAMPLES

- TiO$_2$ thin-film memristor

\[ M(a) = a \cdot R_L + (1 - a) \cdot R_H \]

\[ \frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)} \]
MEMRISTOR EXAMPLES

- TMR based spintronic memristor

\[ M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1 - a)} \]

\[ \frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} J(t) = \frac{V(t)}{M(a) \cdot l \cdot z}, & J(t) \geq J_{cr} \\ 0, & J(t) < J_{cr} \end{cases} \]
DIFFERENCES

- Typical
- Many differences
  - Principles
  - Equivalent circuits
  - Sizes
- Make model a general solution

<table>
<thead>
<tr>
<th></th>
<th>Length (L)</th>
<th>Width (z)</th>
<th>Thickness (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TiO₂</td>
<td>50 nm</td>
<td>50 nm</td>
<td>10 nm</td>
</tr>
<tr>
<td>Spintronic</td>
<td>200 nm</td>
<td>10 nm</td>
<td>7 nm</td>
</tr>
</tbody>
</table>
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GEOMETRY VARIATIONS

- Line Edge Roughness (LER)
  - Beyond the capability of analytic model
  - The most difficult part

- Thickness fluctuation (TF)
  - Follows Gaussian distribution

- Random doping (RDD)
  - Not significant, not considered
LER CHARACTERIZATION

- LER’s characteristics*: 
  - Root Mean Square (RMS)
  - Skewness (Sk)
  - Kurtosis (Ku)
  - Power spectral density (PSD)
  - Auto-correlation function (ACF)

LER CHARACTERIZATION

Oscillate around zero

\[ \Delta L = L_{\text{max}} \cdot \sin(f \cdot x) + L_{\text{normal}} \cdot p \]

**LER Noise**, **Low Frequency Noise**, **Gaussian Noise**

*from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV*
LER SIMULATION RESULTS

• 1000 times of simulation with $f=3\text{Mhz}, L_{\text{max}}=1, L_{\text{normal}}=10$

• good set of conditions → higher efficiency

Acceptable Margins

LER dist.  LWR dist.  Sk dist.

Ku dist.  Sk vs Ku
LER CHARACTERIZATION

Left line:
$3 \sigma_{\text{LER}} : 3.52$
Sk: 0.1703
Ku: 2.9458

Right line:
$3 \sigma_{\text{LER}} : 2.65$
Sk: 0.1127
Ku: 3.0212

*from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV

Miao Hu, January 29, 2011
Generation Flow

An Example of Spintronic Memristor
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- Performance analysis
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STATISTICAL ANALYSIS

Spintronic memristor

- Both LER and TF

TiO$_2$ memristor

- TF only
- one side LER and TF
- two side LER and TF

- **Compute state of each filament**
- **Combine them to achieve the overall performance**
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PERFORMANCE ANALYSIS

**TiO₂ Thin-film**
- RH vs. RL
- M vs. t
- v vs. a
- a vs. t
- I vs. t
- V vs. I

**Spintronic**
- RH vs. RL
- M vs. t
- v vs. a
- a vs. t
- I vs. t
- V vs. I

More flux, More variation
PERFORMANCE ANALYSIS

- Main source of process variation
  - TiO$_2$ memristor: TF
  - Spintronic memristor: LER

- Variation estimation of M:
  - TiO$_2$ memristor: -36.5% to 24.1%
  - Spintronic memristor: -16.3% to 21.1%

- Signal type does not affects the variation
  Flux does
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- Memristor model with geometry variations
- Model simplification
- Statistical analysis
- Summary
SUMMARY

• We evaluate the impact of geometry variations quantitatively:
  – TiO2 thin-film and spintronic memristors;
  – Electrical properties of memristors; and
  – Static and memristive parameters.
• A simple LER sample generation algorithm is proposed to speed up the related Monte-Carlo simulations.
• This device modeling methodology can be expended to other materials.
• The process-variation analysis will benefit memristor-based design.
• Model download:
  http://eeweb.poly.edu/hli/index_files/memristors.htm
Q & A?
Supplementary slides
MODEL EQUATIONS

- **Thin-film memristor**
  \[
  M(a) = a \cdot R_L + (1-a) \cdot R_H
  \]
  \[
  \frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)}
  \]

- **For spintronic memristor**
  \[
  M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1-a)}
  \]
  \[
  \frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} 
  J(t) = \frac{V(t)}{M(a) \cdot l \cdot z}, & J(t) \geq J_{cr} \\
  0, & J(t) < J_{cr}
  \end{cases}
  \]
# RESULTS

**Table IV. 3σ min./max. of TiO2 Memristor Parameters**

<table>
<thead>
<tr>
<th>Sinusoidal Voltage</th>
<th>LER only</th>
<th>Thickness only</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3σ</td>
<td>+3σ</td>
<td>-3σ</td>
</tr>
<tr>
<td>( R_H ) &amp; ( R_L )</td>
<td>-5.4%</td>
<td>4.1%</td>
<td>-5.5%</td>
</tr>
<tr>
<td>( M(\alpha) )</td>
<td>-5.4%</td>
<td>4.1%</td>
<td>-37.1%</td>
</tr>
<tr>
<td>( \alpha(t) )</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-13.3%</td>
</tr>
<tr>
<td>( v(\alpha) )</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-9.3%</td>
</tr>
<tr>
<td>( i(\alpha) )</td>
<td>-4.7%</td>
<td>5.7%</td>
<td>-9.3%</td>
</tr>
<tr>
<td>Power &amp;</td>
<td>-4.7%</td>
<td>5.7%</td>
<td>-8.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Square wave Voltage</th>
<th>LER only</th>
<th>Thickness only</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3σ</td>
<td>+3σ</td>
<td>-3σ</td>
</tr>
<tr>
<td>( R_H ) &amp; ( R_L )</td>
<td>-5.3%</td>
<td>3.7%</td>
<td>-6.2%</td>
</tr>
<tr>
<td>( M(\alpha) )</td>
<td>-5.3%</td>
<td>3.7%</td>
<td>-17.8%</td>
</tr>
<tr>
<td>( \alpha(t) )</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-12.1%</td>
</tr>
<tr>
<td>( v(\alpha) )</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-11.6%</td>
</tr>
<tr>
<td>( i(\alpha) )</td>
<td>-4.0%</td>
<td>5.2%</td>
<td>-11.7%</td>
</tr>
<tr>
<td>Power &amp;</td>
<td>-4.0%</td>
<td>5.2%</td>
<td>-7.7%</td>
</tr>
</tbody>
</table>
# RESULTS

**Table V. 3σ min./max. of spintronic memristor parameters**

<table>
<thead>
<tr>
<th>Voltage</th>
<th>LER only</th>
<th>Thickness only</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-3\sigma$</td>
<td>$+3\sigma$</td>
<td>$-3\sigma$</td>
</tr>
<tr>
<td>$R_H$ &amp; $R_L$</td>
<td>-15.3%</td>
<td>22.9%</td>
<td>-6.1%</td>
</tr>
<tr>
<td>$M(\alpha)$</td>
<td>-15.1%</td>
<td>23.3%</td>
<td>-11.0%</td>
</tr>
<tr>
<td>$\alpha(t)$</td>
<td>-9.7%</td>
<td>8.1%</td>
<td>-8.4%</td>
</tr>
<tr>
<td>$v(\alpha)$</td>
<td>-10.7%</td>
<td>22.1%</td>
<td>-9.1%</td>
</tr>
<tr>
<td>$i(\alpha)$</td>
<td>-18.5%</td>
<td>18.5%</td>
<td>-8.9%</td>
</tr>
<tr>
<td>Power</td>
<td>-18.4%</td>
<td>18.6%</td>
<td>-8.3%</td>
</tr>
</tbody>
</table>

<table>
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<th>Voltage</th>
<th>LER only</th>
<th>Thickness only</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-3\sigma$</td>
<td>$+3\sigma$</td>
<td>$-3\sigma$</td>
</tr>
<tr>
<td>$R_H$ &amp; $R_L$</td>
<td>-15.8%</td>
<td>22.0%</td>
<td>-5.3%</td>
</tr>
<tr>
<td>$M(\alpha)$</td>
<td>-15.6%</td>
<td>21.8%</td>
<td>-8.5%</td>
</tr>
<tr>
<td>$\alpha(t)$</td>
<td>-13.1%</td>
<td>13.8%</td>
<td>-7.5%</td>
</tr>
<tr>
<td>$v(\alpha)$</td>
<td>-16.5%</td>
<td>20.7%</td>
<td>-10.0%</td>
</tr>
<tr>
<td>$i(\alpha)$</td>
<td>-19.5%</td>
<td>17.1%</td>
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</tr>
<tr>
<td>Power</td>
<td>-19.4%</td>
<td>17.1%</td>
<td>-7.6%</td>
</tr>
</tbody>
</table>
MODEL SIMPLIFICATION

- Compute state of each filament
  Combine them to get the overall performance
    - Spintronic memristor:
      • each filament is in either $R_{iL}$ or $R_{iH}$ states (under geometric variation)
      • the whole device can be regarded as serial connection of all the filaments
    - TiO$_2$ thin-film memristor:
      • each filament is a smaller memristor
      • TiO$_2$ memristor is the parallel connection of them

Miao Hu, January 29, 2011
We use this main function to mimic LER:

\[ \Delta L = L_{\text{max}} \cdot \sin(f \cdot x) + L_{\text{normal}} \cdot p \]

- \( \Delta L \): LER noise per nm
- \( L_{\text{max}} \): weight of low frequency noise
- \( L_{\text{normal}} \): weight of normal distribution
- \( f \): Frequency for low frequency noise
- \( x \): random number of uniform distribution (0.5-1.5)
- \( p \): random number of standard normal distribution