Geometry Variations Analysis of TiO_2 Thin-Film and Spintronic Memristors

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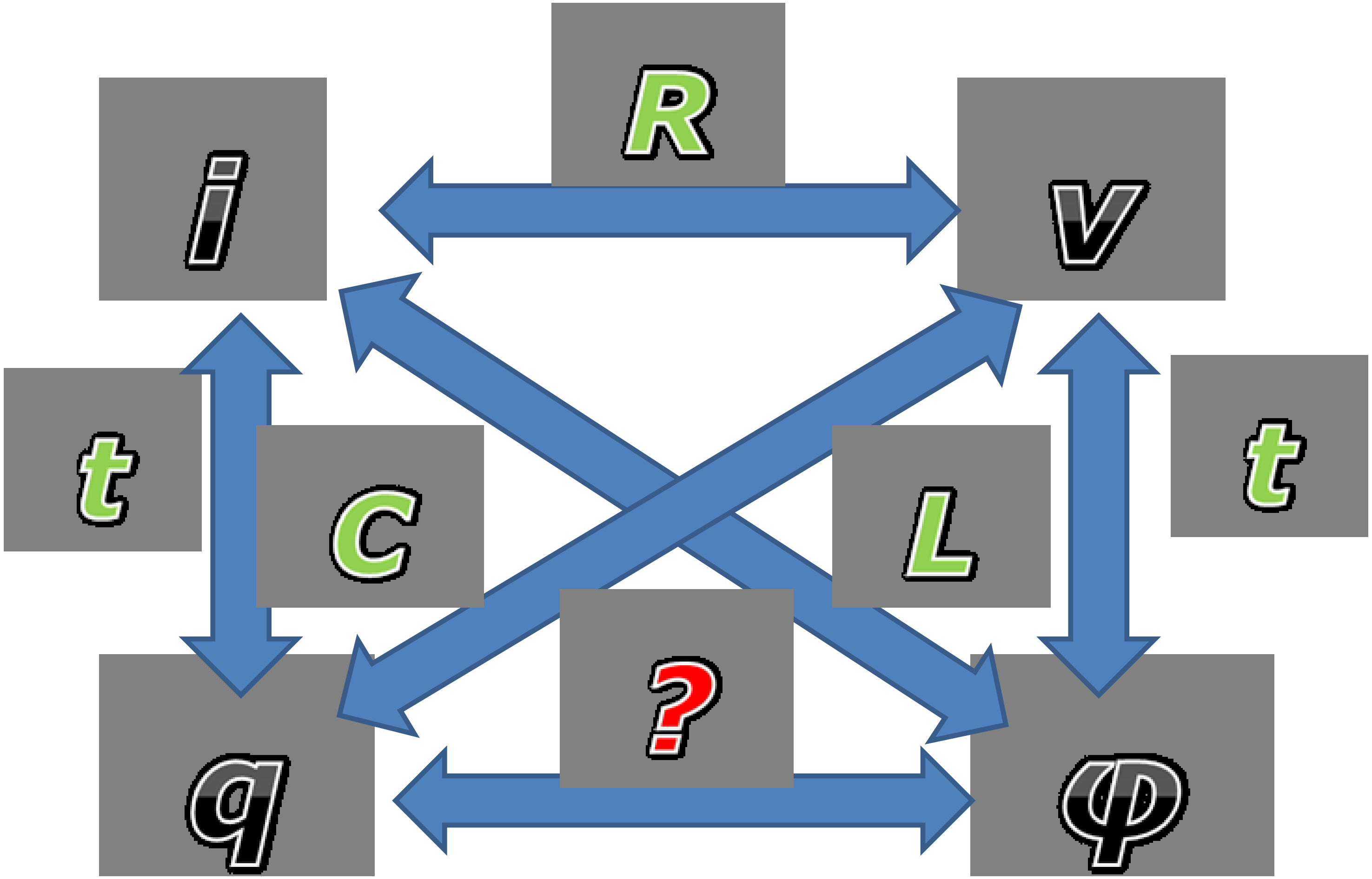
WHAT IS MEMRISTOR



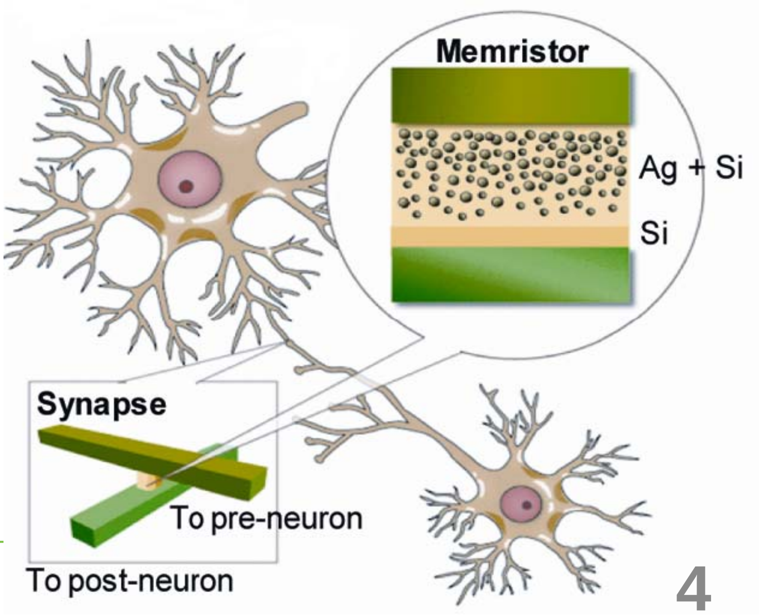
WHAT IS MEMRISTOR

- ❑ Memristor is a resistor with memory
  - $M(t) = \phi(t)/q(t)$, unit Ω
  - Intrinsic state to remember the history
  - Passive, AC
- ❑ Predicted in 1971, Found in 2008.

PROSPECT OF MEMRISTOR

- Memristor features
  nano-size, non-volatility, reconfigurable

- Potential Applications
  - High density storage technology
    DRAM 18 Gbits/cm^2
    Memristor 100 Gbits/cm^2
  - Reconfigurable computation
  - Neural network



OUTLINE

- ☐Motivation
- ☐Memristor examples
  - TiO_2 thin-film memristor
  - Spintronic memristor
- ☐Memristor model with geometry variations
- ☐Statistical analysis
- ☐Performance analysis
- ☐Summary

MOTIVATIONS

❑ Improve fabrication
  - Measurement
  - Predict margin and actual performance

❑ More applications
  - Multi-level memory: Levels?
  - Reconfigurable computation: Accuracy?
  - Neural network: Fuzzy operation?

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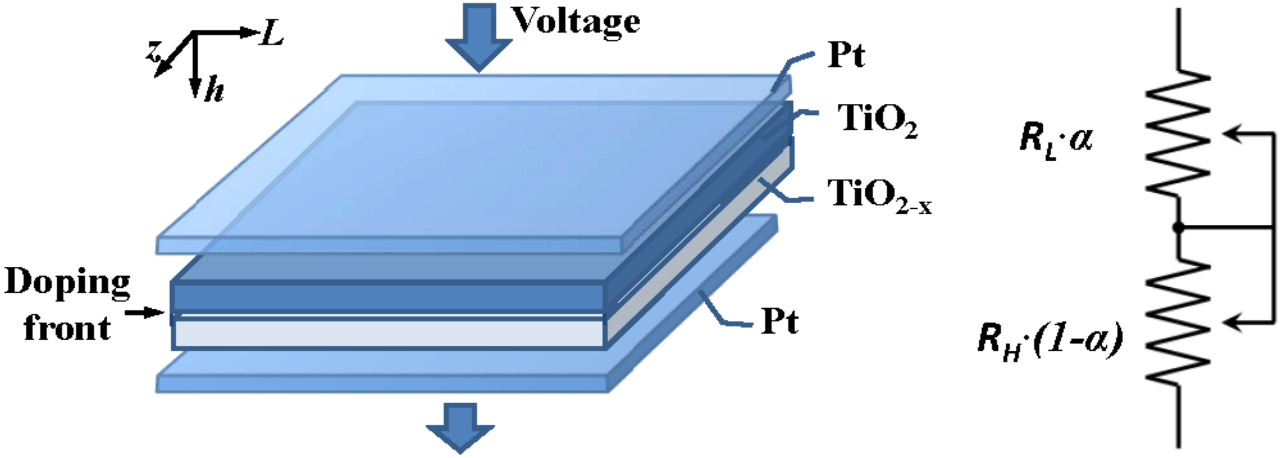
MEMRISTOR EXAMPLES

❑ TiO_2 thin-film memristor



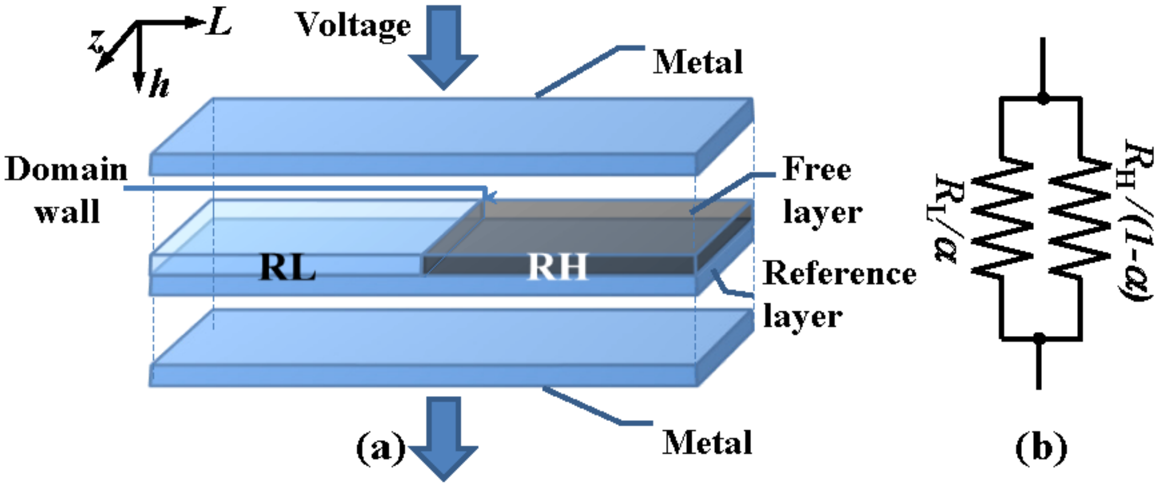
(a) (b)

$$M(a) = a \cdot R_L + (1-a) \cdot R_H$$

$$\frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)}$$

MEMRISTOR EXAMPLES

- ❑ TMR based spintronic memristor



$$M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1 - a)}$$

$$\frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} J(t) = \dfrac{V(t)}{M(a) \cdot l \cdot z}, & J(t) \geq J_{cr} \\ 0 & , J(t) < J_{cr} \end{cases}$$

DIFFERENCES

- Typical
- Many differences
  - Principles
  - Equivalent circuits
  - Sizes
- Make model a general solution

	Length (*L*)	**Width (*z*)**	**Thickness (*h*)**
TiO_2	50 nm	50 nm	10 nm
Spintronic	200 nm	10 nm	7 nm

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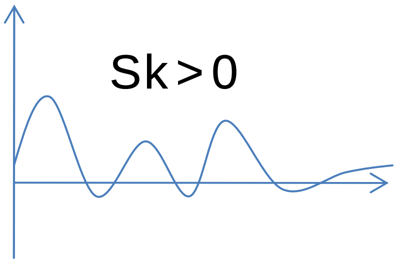
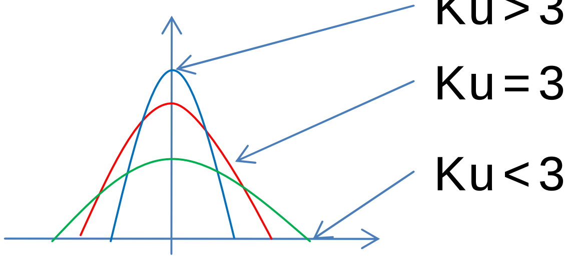
GEOMETRY VARIATIONS

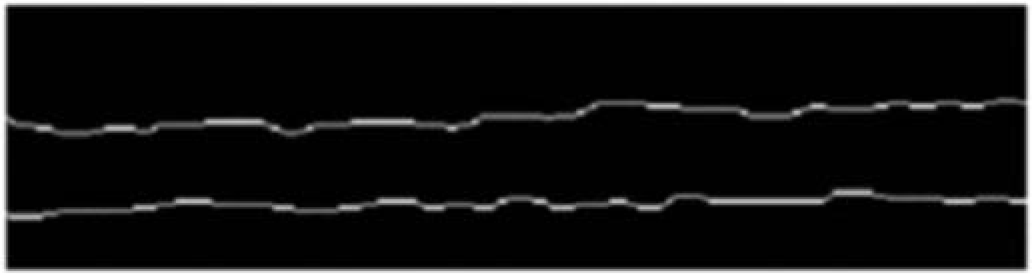
- ☐ Line Edge Roughness (LER)
  - Beyond the capability of analytic model
  - The most difficult part
- ☐ Thickness fluctuation (TF)
  - Follows Gaussian distribution
- ☐ Random doping (RDD)
  - Not significant, not considered

LER CHARACTERIZATION

- ☐ LER's characteristics*:
  - Root Mean Square (RMS)
  - Skewness (Sk)
  - Kurtosis (Ku)
  - Power spectral density (PSD)
  - Auto-correlation function (ACF)

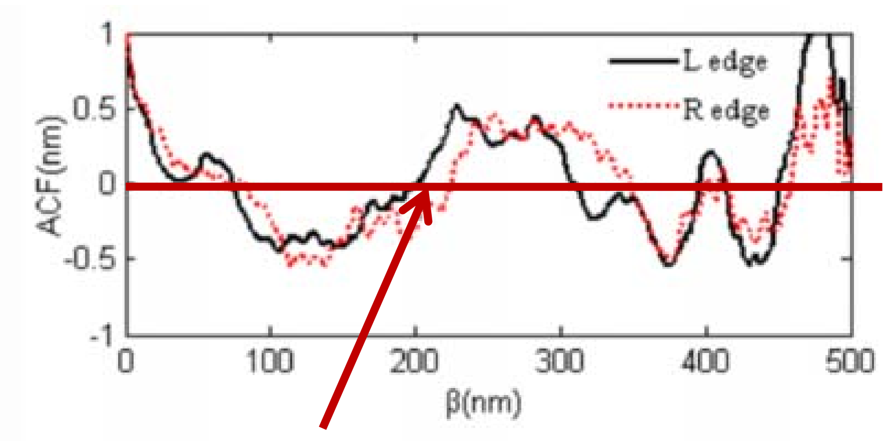




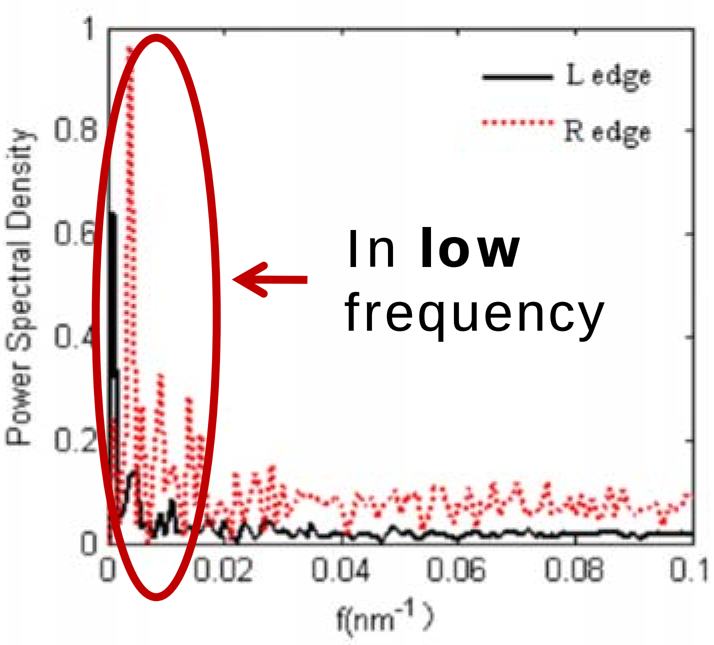


*Z. Jiang, "Characterization of Line Edge Roughness and Line Width Roughness of Nano-scale Typical Structures," 2009.

LER CHARACTERIZATION



oscillate around **zero**



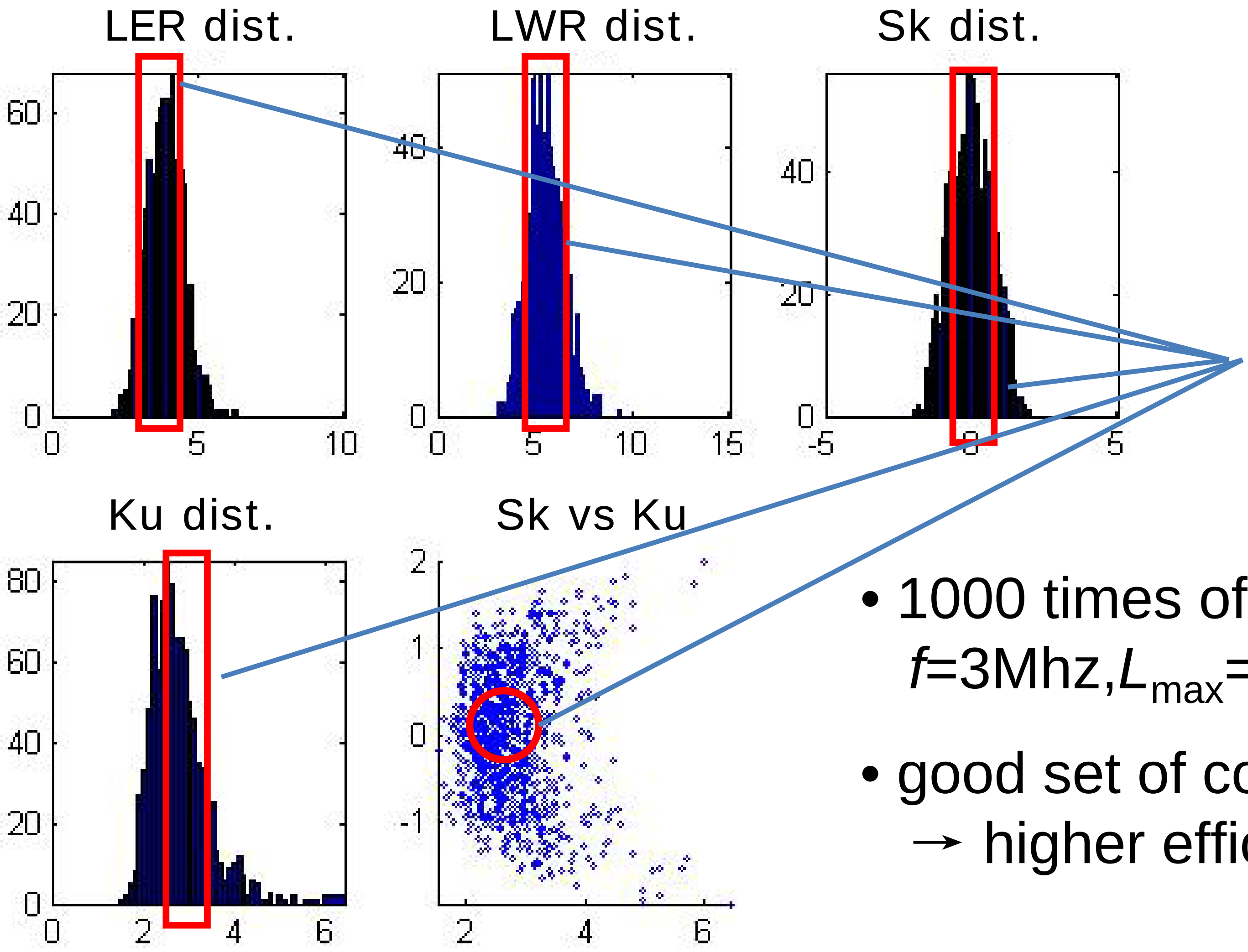
In **low** frequency

$$\Delta L = L_{\max} \cdot \sin(f \cdot x) + L_{normal} \cdot p$$

LER Noise ***Low Frequency Noise*** ***Gaussian Noise***

*from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV

LER SIMULATION RESULTS



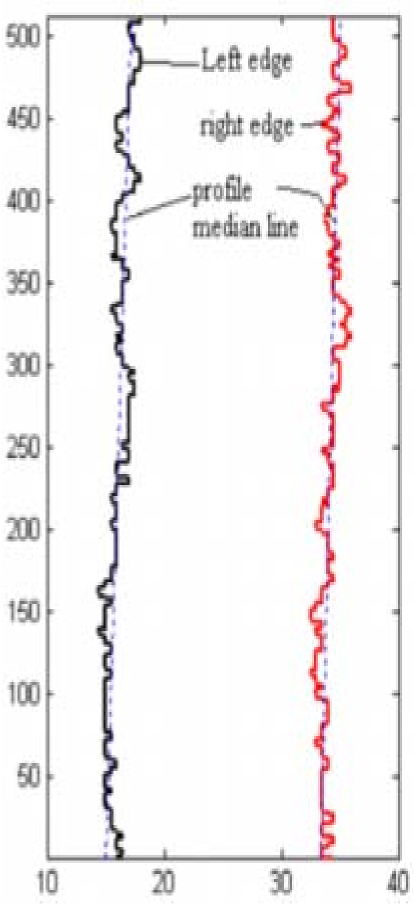
Acceptable Margins

- 1000 times of simulation with f=3Mhz,L_{max}=1,L_{normal}=10
- good set of conditions → higher efficiency

LER CHARACTERIZATION

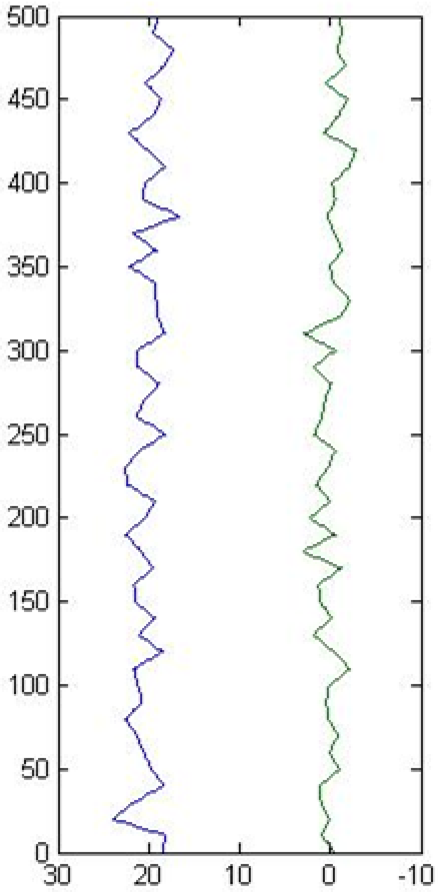
Left line:
$3\sigma_{LER}$:3.52
Sk:0.1703
Ku:2.9458

Real Data →



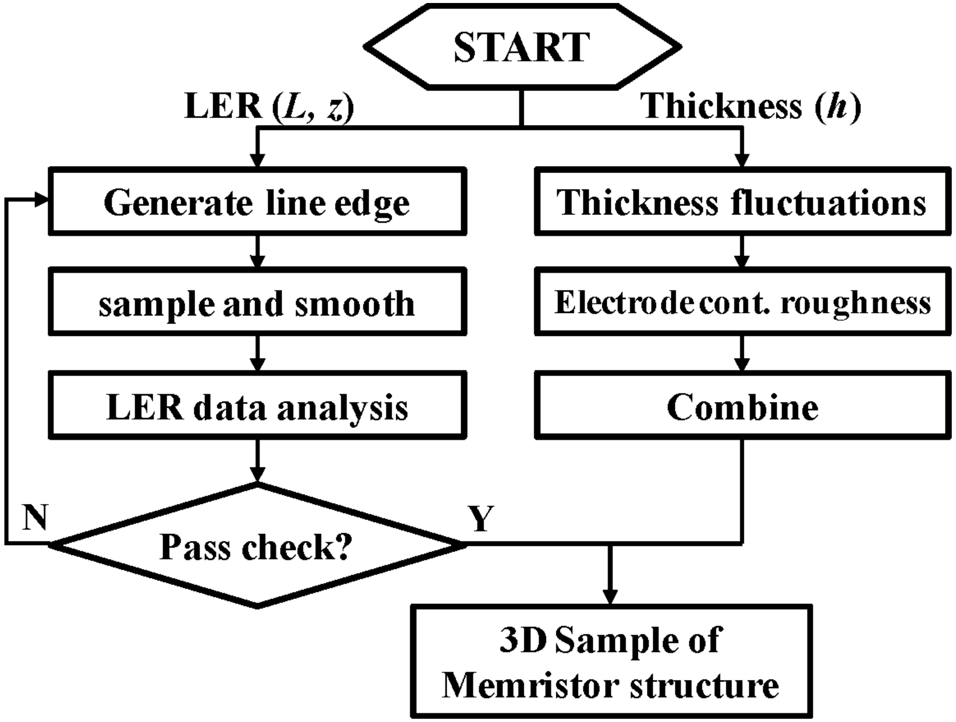
Right line:
$3\sigma_{LER}$:2.65
Sk:0.1127
Ku:3.0212

Simulation Result →



*from nano-scale resist pattern fabricated with CABL-9000C EBL system of Crestec. Co. of Japan with the accelerating voltage 30kV

GEOMETRY VARIATIONS GENERATION FLOW


START

LER (*L*, *z*)

Thickness (*h*)

Generate line edge

sample and smooth

LER data analysis

Pass check?

N

Y

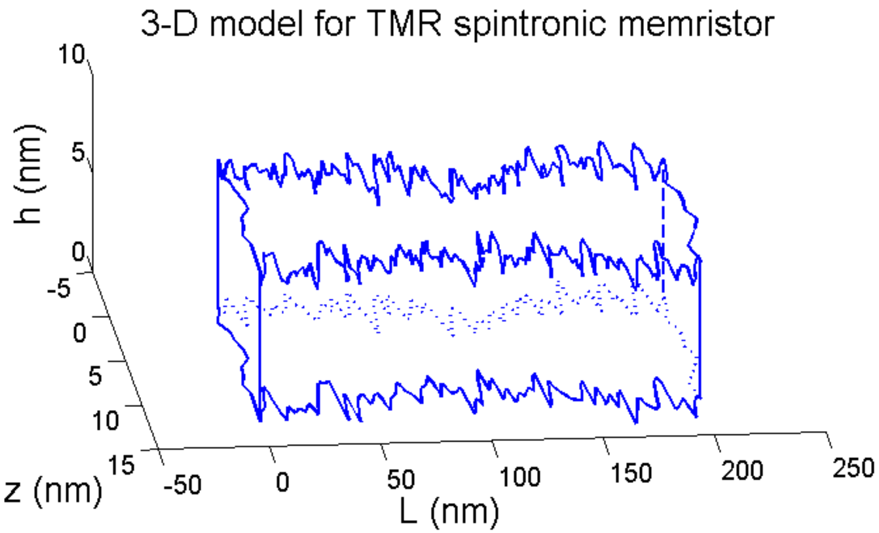
Thickness fluctuations

Electrode cont. roughness

Combine

3D Sample of Memristor structure

Generation Flow



An Example of Spintronic Memristor

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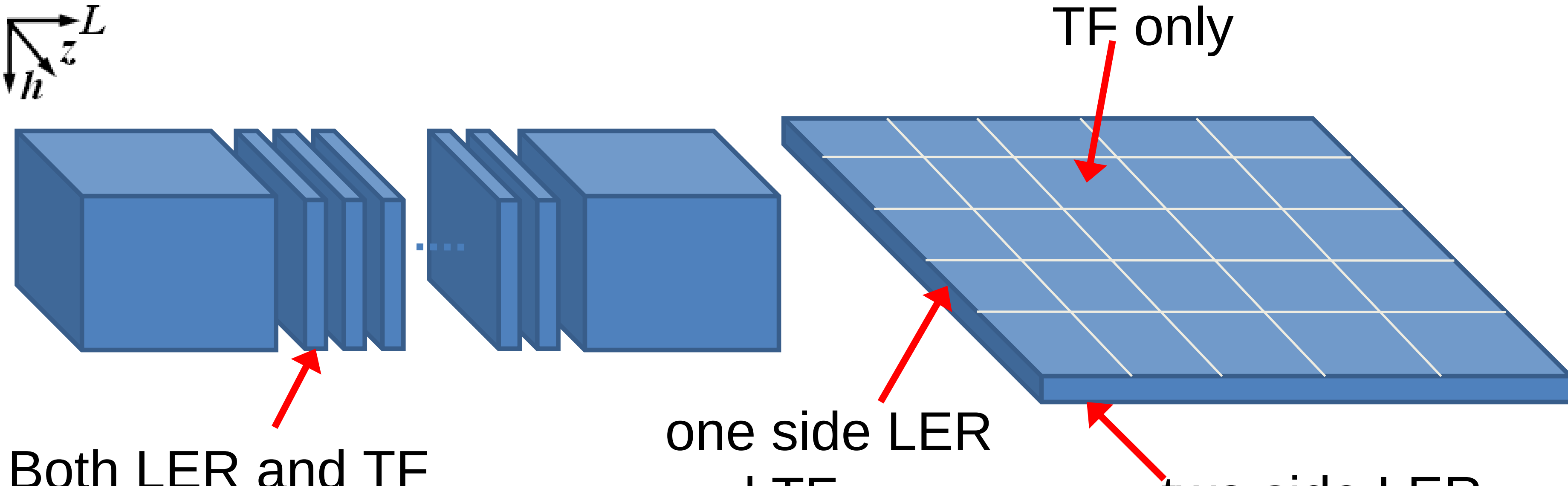
STATISTICAL ANALYSIS

Spintronic memristor

TiO_2 memristor



TF only

Both LER and TF

one side LER and TF

two side LER and TF

- ***Compute state of each filament***
- ***Combine them to achieve the overall performance***

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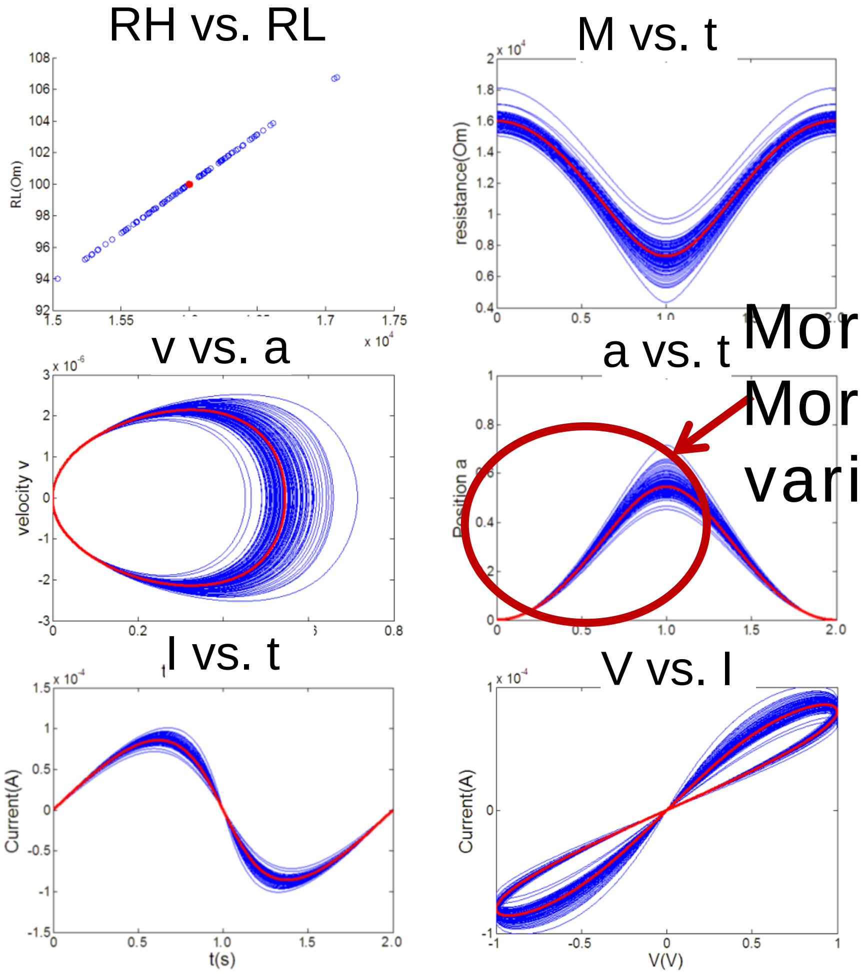
PERFORMANCE ANALYSIS

TiO$_2$ Thin-film



RH vs. RL

M vs. t

v vs. a

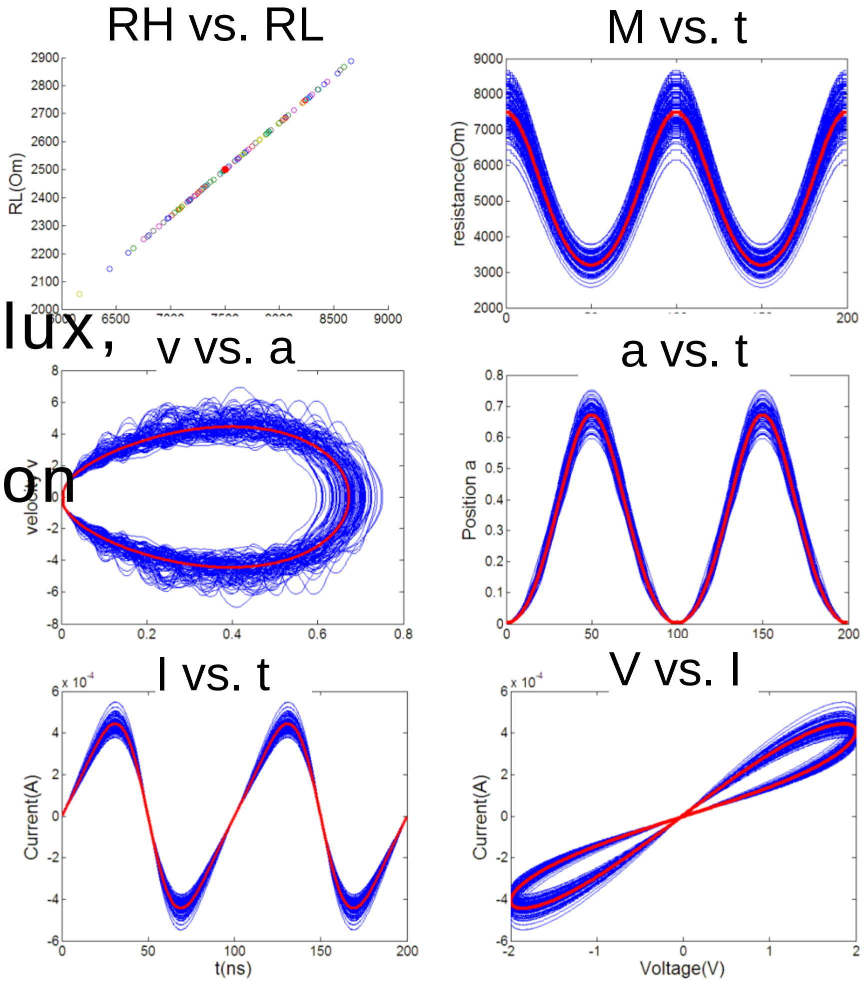
a vs. t

I vs. t

V vs. I

More flux, More variation

Spintronic



RH vs. RL

M vs. t

v vs. a

a vs. t

I vs. t

V vs. I

PERFORMANCE ANALYSIS

- ☐ Main source of process variation
  - TiO_2 memristor: TF
  - Spintronic memristor: LER
- ☐ Variation estimation of M:
  - TiO_2 memristor: -36.5% to 24.1%
  - Spintronic memristor: -16.3% to 21.1%
- ☐ Signal type does not affects the variation

  Flux does

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- ☐Motivation
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- ☐Memristor model with geometry variations
- ☐Model simplification
- ☐Statistical analysis
- ☐Summary

SUMMARY

- We evaluate the impact of geometry variations quantitatively:
  - TiO2 thin-film and spintronic memristors;
  - Electrical properties of memristors; and
  - Static and memristive parameters.
- A simple LER sample generation algorithm is proposed to speed up the related Monte-Carlo simulations.
- This device modeling methodology can be expended to other materials.
- The process-variation analysis will benefit memristor-based design.
- Model download:

  http://eeweb.poly.edu/hli/index_files/memristors.htm

Q & A?

Supplementary slides

MODEL EQUATIONS

- ☐ Thin-film memristor

$$M(a) = a \cdot R_L + (1 - a) \cdot R_H$$

$$\frac{da(t)}{dt} = \mu_v \cdot \frac{R_L}{h^2} \cdot \frac{V(t)}{M(a)}$$

- ☐ For spintronic memristor

$$M(a) = \frac{R_H \cdot R_L}{R_H \cdot a + R_L \cdot (1 - a)}$$

$$\frac{da(t)}{dt} = \frac{\Gamma_v}{l} \cdot J_{eff}(t), J_{eff} = \begin{cases} J(t) = \dfrac{V(t)}{M(a) \cdot l \cdot z}, & J(t) \geq J_{cr} \\ 0, & J(t) < J_{cr} \end{cases}$$

RESULTS

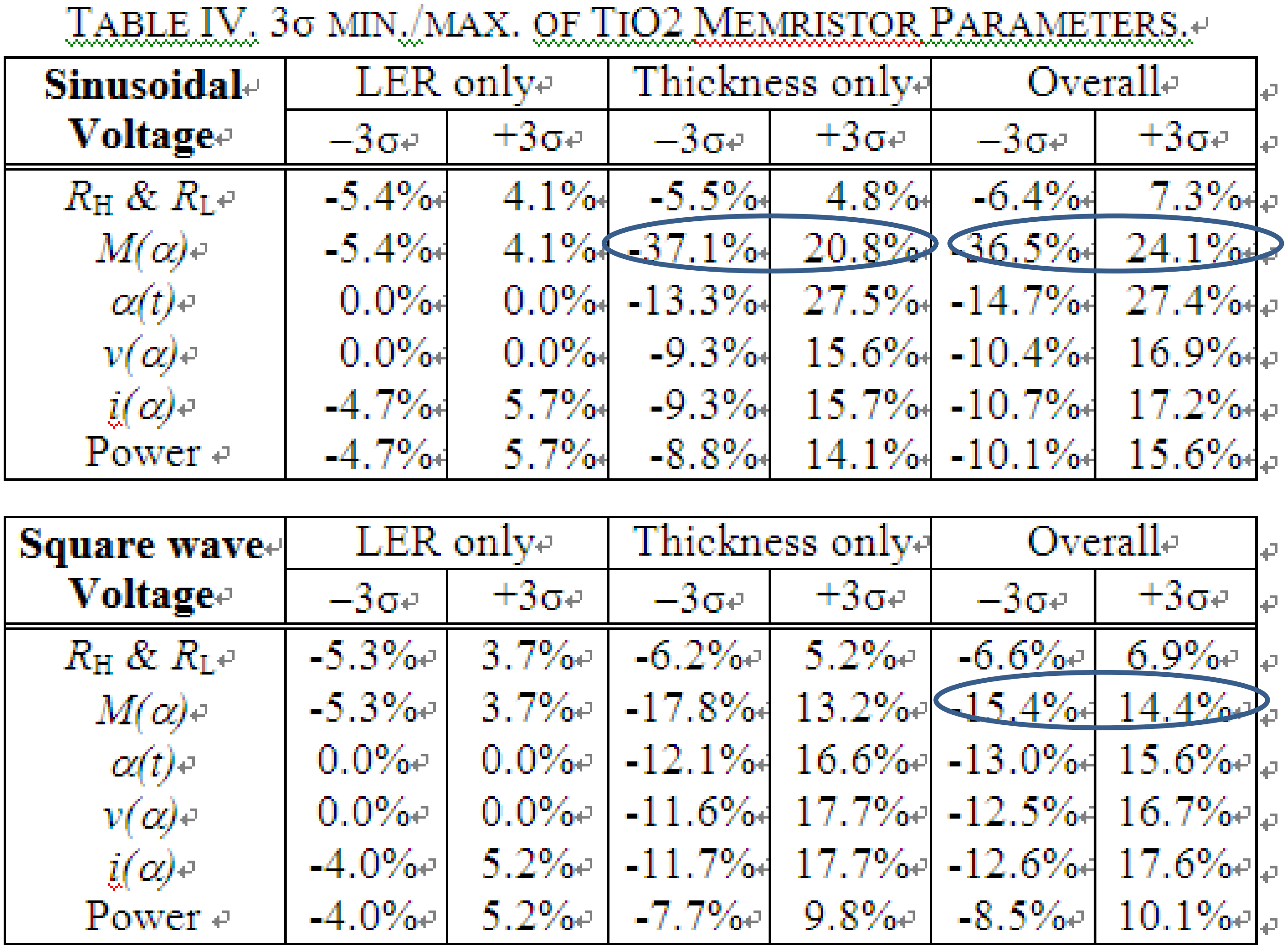

TABLE IV. 3σ MIN./MAX. OF TIO2 MEMRISTOR PARAMETERS.

Sinusoidal Voltage	LER only		Thickness only		Overall	
	–3σ	+3σ	–3σ	+3σ	–3σ	+3σ
R_H & R_L	-5.4%	4.1%	-5.5%	4.8%	-6.4%	7.3%
$M(\alpha)$	-5.4%	4.1%	-37.1%	20.8%	-36.5%	24.1%
$\alpha(t)$	0.0%	0.0%	-13.3%	27.5%	-14.7%	27.4%
$v(\alpha)$	0.0%	0.0%	-9.3%	15.6%	-10.4%	16.9%
$i(\alpha)$	-4.7%	5.7%	-9.3%	15.7%	-10.7%	17.2%
Power	-4.7%	5.7%	-8.8%	14.1%	-10.1%	15.6%

Square wave Voltage	LER only		Thickness only		Overall	
	–3σ	+3σ	–3σ	+3σ	–3σ	+3σ
R_H & R_L	-5.3%	3.7%	-6.2%	5.2%	-6.6%	6.9%
$M(\alpha)$	-5.3%	3.7%	-17.8%	13.2%	-15.4%	14.4%
$\alpha(t)$	0.0%	0.0%	-12.1%	16.6%	-13.0%	15.6%
$v(\alpha)$	0.0%	0.0%	-11.6%	17.7%	-12.5%	16.7%
$i(\alpha)$	-4.0%	5.2%	-11.7%	17.7%	-12.6%	17.6%
Power	-4.0%	5.2%	-7.7%	9.8%	-8.5%	10.1%

RESULTS

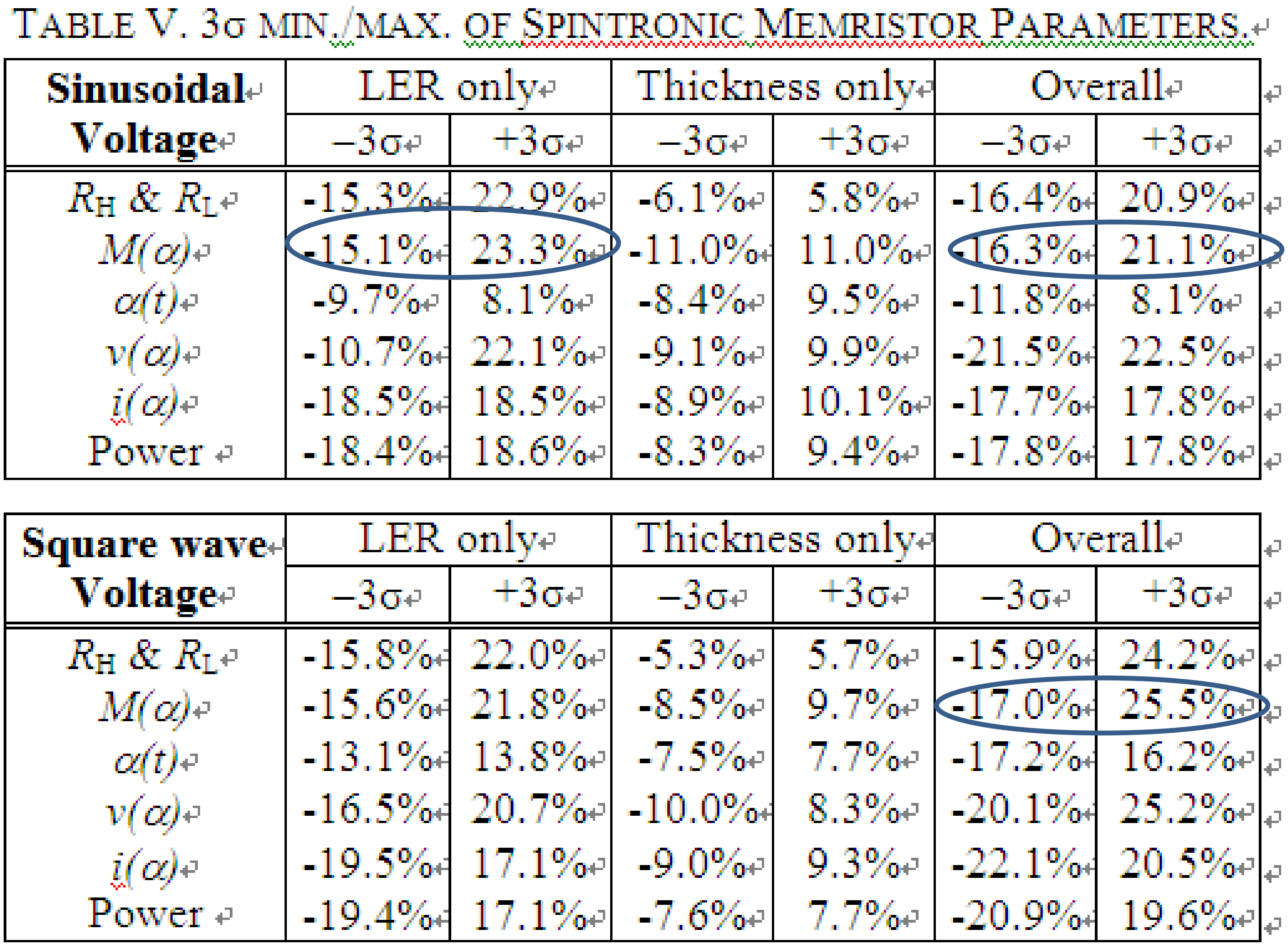

TABLE V. 3σ MIN./MAX. OF SPINTRONIC MEMRISTOR PARAMETERS.

Sinusoidal Voltage	LER only		Thickness only		Overall	
	−3σ	+3σ	−3σ	+3σ	−3σ	+3σ
R_H & R_L	-15.3%	22.9%	-6.1%	5.8%	-16.4%	20.9%
$M(\alpha)$	-15.1%	23.3%	-11.0%	11.0%	-16.3%	21.1%
$\alpha(t)$	-9.7%	8.1%	-8.4%	9.5%	-11.8%	8.1%
$v(\alpha)$	-10.7%	22.1%	-9.1%	9.9%	-21.5%	22.5%
$i(\alpha)$	-18.5%	18.5%	-8.9%	10.1%	-17.7%	17.8%
Power	-18.4%	18.6%	-8.3%	9.4%	-17.8%	17.8%

Square wave Voltage	LER only		Thickness only		Overall	
	−3σ	+3σ	−3σ	+3σ	−3σ	+3σ
R_H & R_L	-15.8%	22.0%	-5.3%	5.7%	-15.9%	24.2%
$M(\alpha)$	-15.6%	21.8%	-8.5%	9.7%	-17.0%	25.5%
$\alpha(t)$	-13.1%	13.8%	-7.5%	7.7%	-17.2%	16.2%
$v(\alpha)$	-16.5%	20.7%	-10.0%	8.3%	-20.1%	25.2%
$i(\alpha)$	-19.5%	17.1%	-9.0%	9.3%	-22.1%	20.5%
Power	-19.4%	17.1%	-7.6%	7.7%	-20.9%	19.6%

MODEL SIMPLIFICATION

- ☐ Compute state of each filament
  Combine them to get the overall performance
  - Spintronic memristor:
    - each filament is in either R_{iL} or R_{iH} states (under geometric variation)
    - the whole device can be regarded as serial connection of all the filaments

  - TiO_2 thin-film memristor:
    - each filament is a smaller memristor
    - TiO_2 memristor is the parallel connection of them

LER SIMULATION ALGORITHM

- We use this main function to mimic LER :

$$\Delta L = L_{\max} \cdot \sin(f \cdot x) + L_{normal} \cdot p$$

ΔL: LER noise per nm

L_{max}: weight of low frequency noise

L_{normal}: weight of normal distribution

f: Frequency for low frequency noise

x: random number of uniform distribution (0.5-1.5)

p: random number of standard normal distribution