

# Compact Nonlinear Thermal Modeling of Packaged Microprocessors



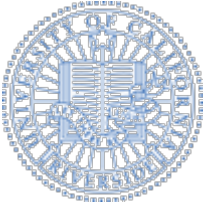
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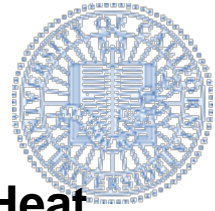
Intel Corporation  
Chandler, AZ

# Content



- Introduction to thermal modeling
- Problem of subspace-based thermal modeling
- Proposed method
- Experimental result
- Conclusion

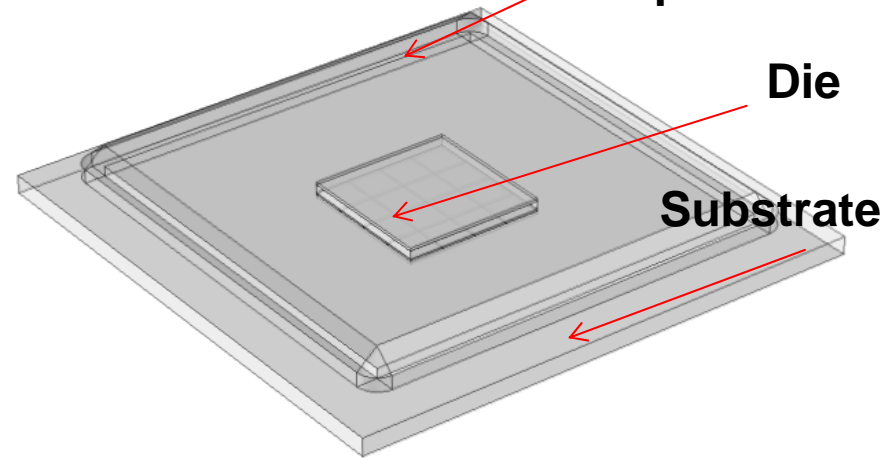
# Thermal modeling of packaged microprocessor



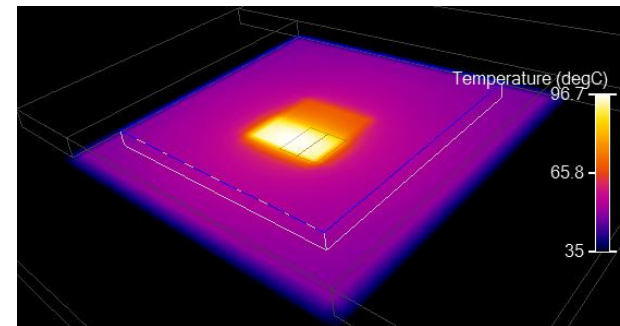
Heat Spreader

Die

Substrate

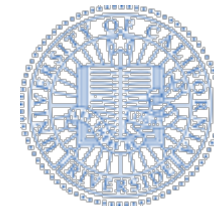


- Temperature has become a major concern for high performance microprocessors
- Even for severe for multi/many core and emerging 3D stacked systems
  - Longer thermal paths
  - Loaded dependent hotspots
  - Large thermal gradients and dynamic thermal effect related reliability issues.
- Compact thermal model at package level is vital for efficient thermal aware design and management.
  - Enable thermal-aware design flow
  - Enable accurate online thermal management and regulation



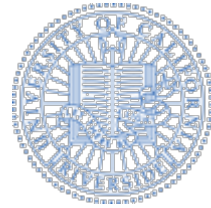
Simulated Temp distribution using Cu sink (390 W/m K)

# Bottom-up thermal modeling methods

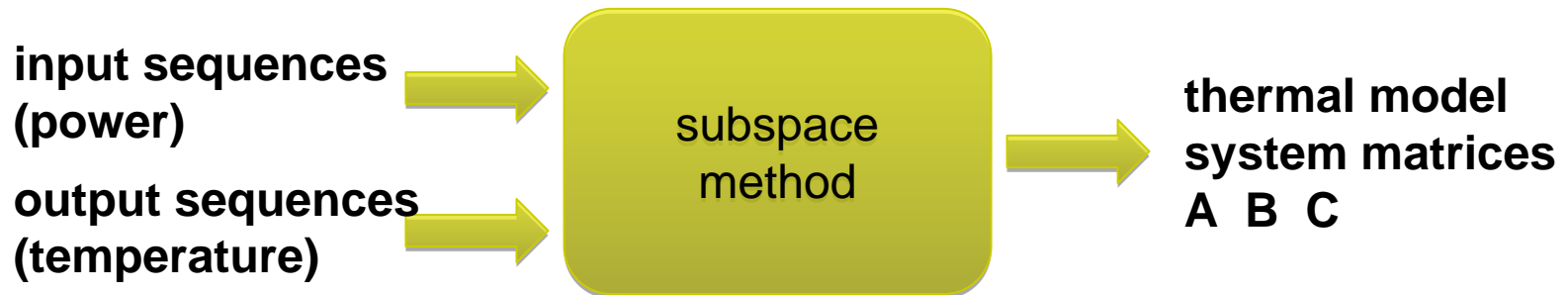


- FDM (finite difference)
- FEM (finite element) [Lasance:SEMITHERM'95, Christiaens:TCPMT'98]
  - **Limitation:**
    - Knowledge of detailed thermal structures is not easy to obtain
    - Impractical for large scale circuits
- HotSpot [Huang:DAC'04, Skadron:ISCA'03]
  - Mainly for architectural level design exploration
  - **Limitation:**
    - Accuracy losses in lumped model

# Behavioral thermal modeling method

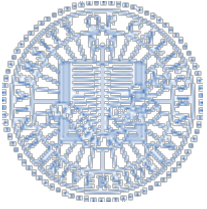


- Matrix pencil or subspace methods [Overschee:book'06] [Eguia and Tan: TVLSI'10]
  - Obtains thermal model through input/output information
  - No need for detailed thermal structures



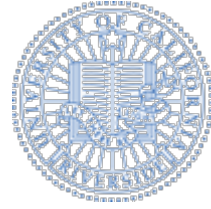
- Compact model suffers from accuracy losses due to non-linearity of the practical thermal system

# Content

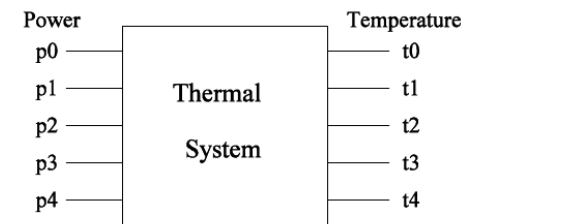
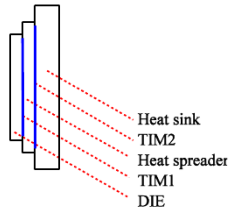
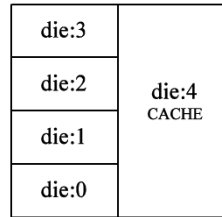


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# Thermal modeling for given power density under correlated power inputs

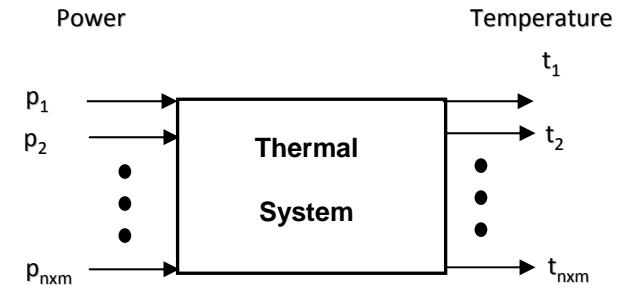
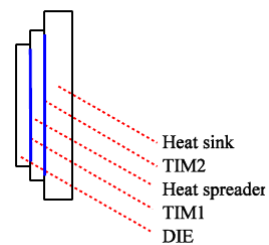
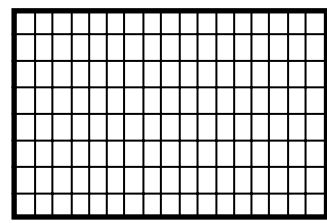


## Current:

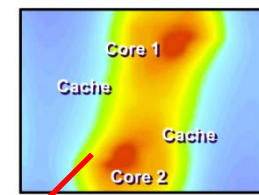
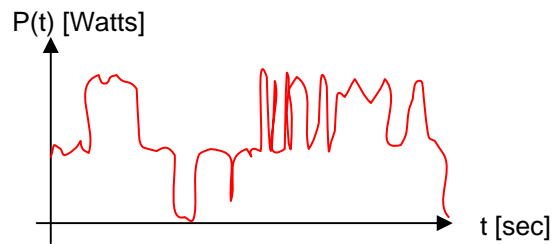


## Proposed:

$n \times m$  sections

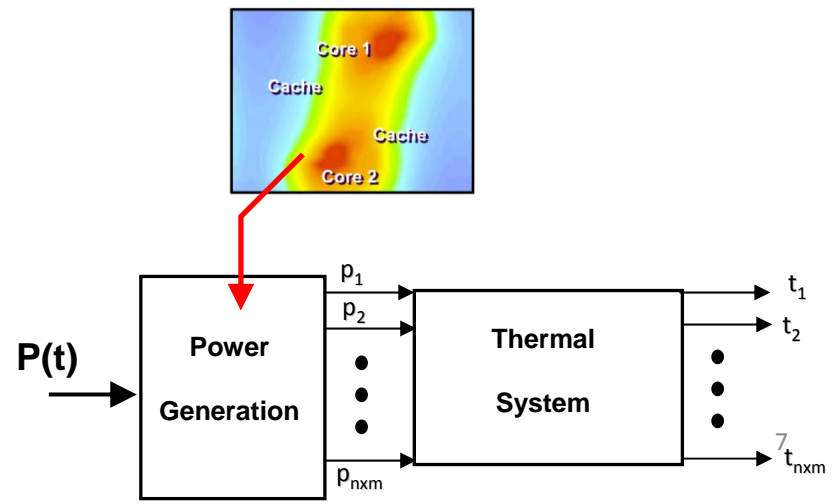


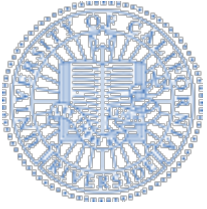
$p_1, p_2, \dots, p_{n \times m}$  generated from a power map and are highly correlated & overall processor power  $P(t)$



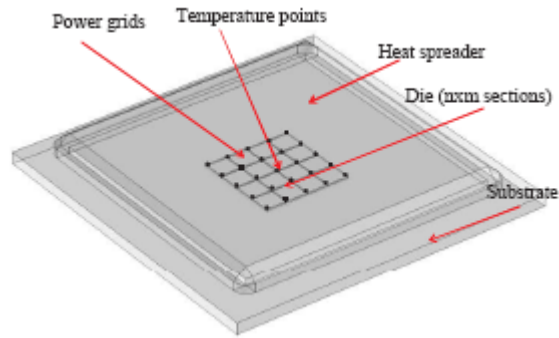
– Desired : Stand Alone Application

- Inputs:  $P(t)$ , Power Map, Identification method (POF or Subspace ID),  $t_1, t_2, \dots, t_{n \times m}$
- Output:  $H(s) \rightarrow n \times m$  transfer function matrix

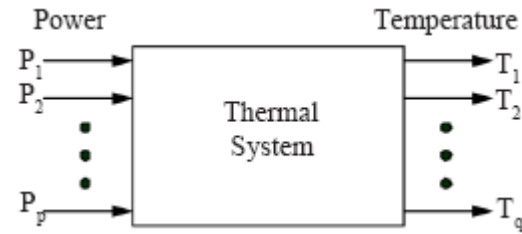
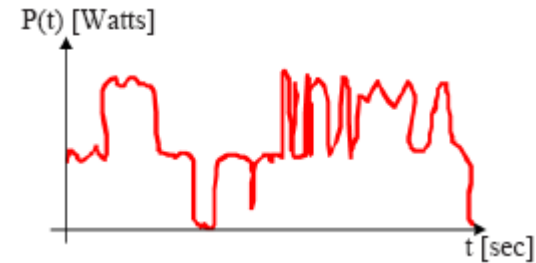




# State space model of thermal system



chip partition



The abstracted model with power as input

**The linear model** of the thermal system can be described by state space equation:

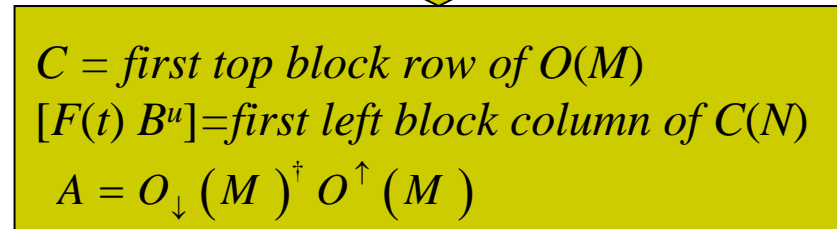
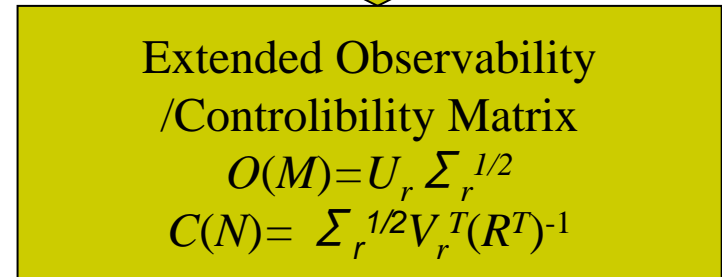
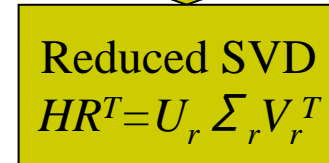
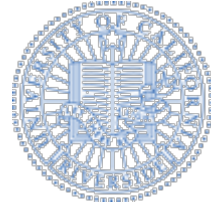
$$x(t+1) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

Given input  $u(t)$  and output  $y(t)$ , the state matrices  $A$ ,  $B$ ,  $C$  can be identified by subspace method.



# Subspace based modeling flow



$$W = \begin{bmatrix} U_{future} & W_{past} & Y_{future} \end{bmatrix}$$

$$= \begin{bmatrix} Q_1 & Q_2 & Q_3 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \\ 0 & 0 & R_{33} \end{bmatrix}$$

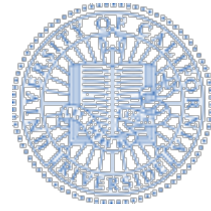
$R_{23}^T$  is the weighted Hankel matrix  $HR^T$   
 $R_{22}$  is the weight matrix  $R$

$$H(N, M) = \begin{bmatrix} h_1(N, 1) & h_1(N, 2) & \dots & h_1(N, N) \\ h_2(N, 1) & h_2(N, 2) & \dots & h_2(N, N) \\ \vdots & \vdots & \ddots & \vdots \\ h_M(N, 1) & h_M(N, 2) & \dots & h_M(N, N) \end{bmatrix}$$

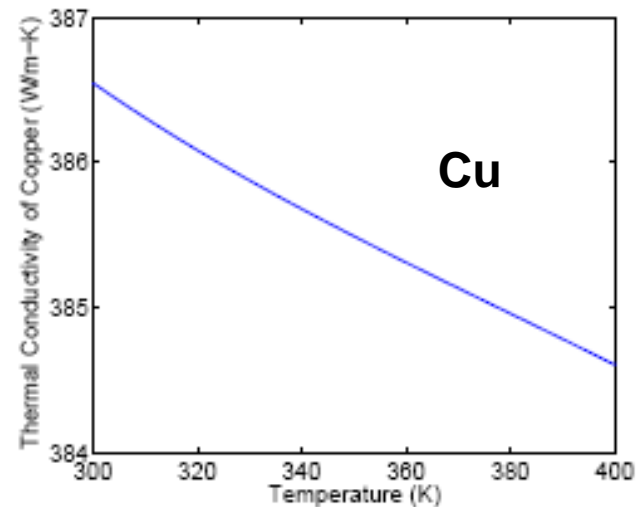
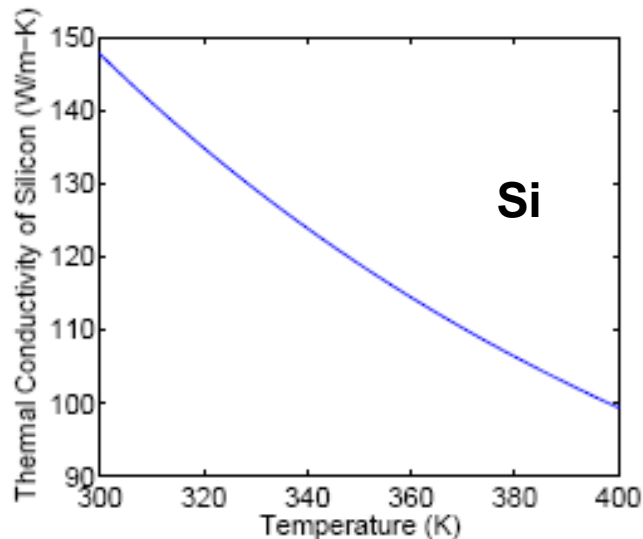
$$= \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{M-1} \end{bmatrix} \begin{bmatrix} B & SB & \dots & S^{N-1}B \end{bmatrix} = O(M)C(N)$$

$$O^{\uparrow}(M) = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{M-1} \end{bmatrix}, \quad O_{\downarrow}(M) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{M-2} \end{bmatrix}$$

# Thermal systems are actual nonlinear!

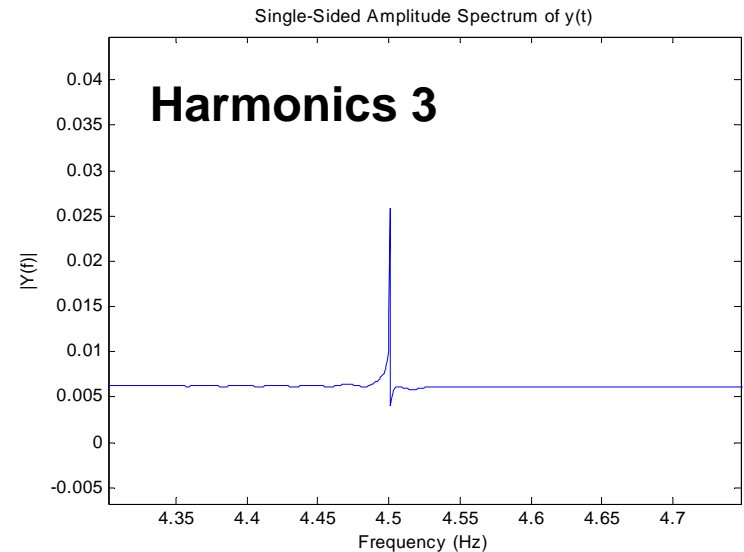
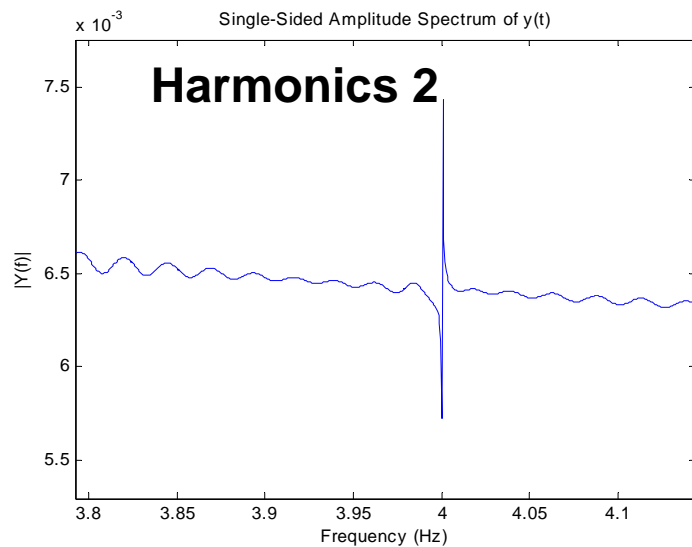
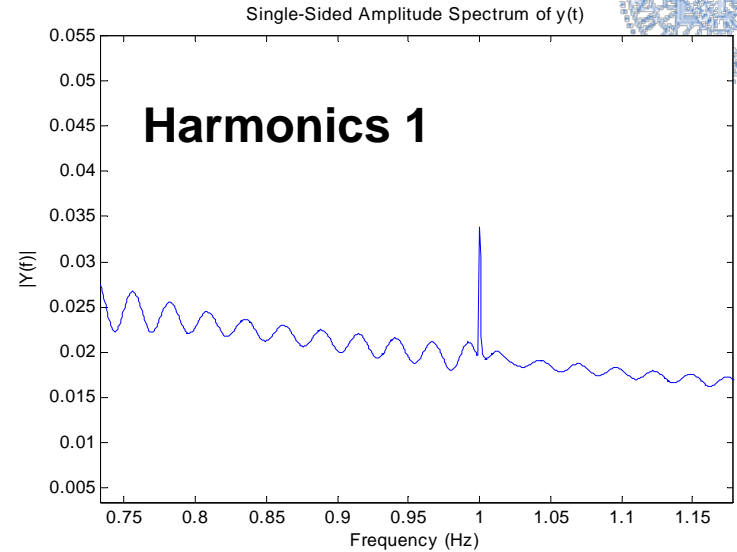
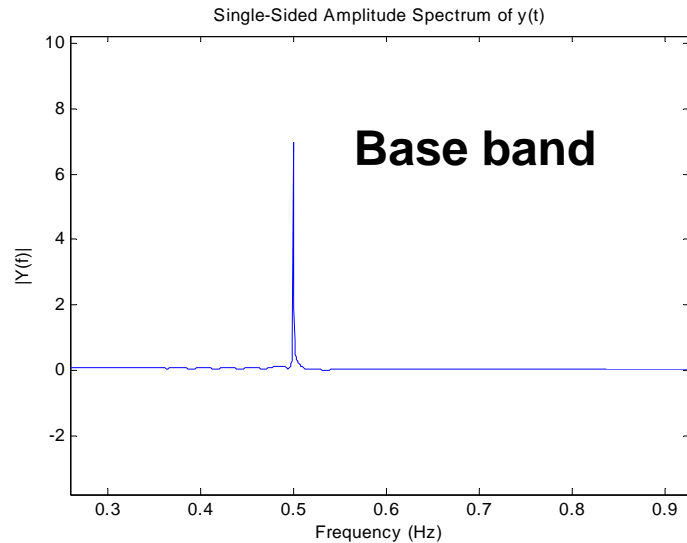
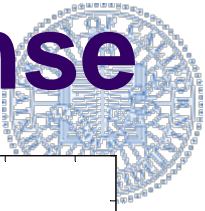


- Nonlinearity is caused by the temperature dependent properties of the package materials.



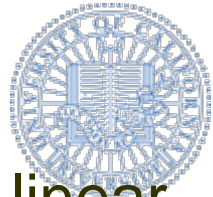
**Example: Temperature dependence of thermal conductivity**

# Harmonics of the thermal response

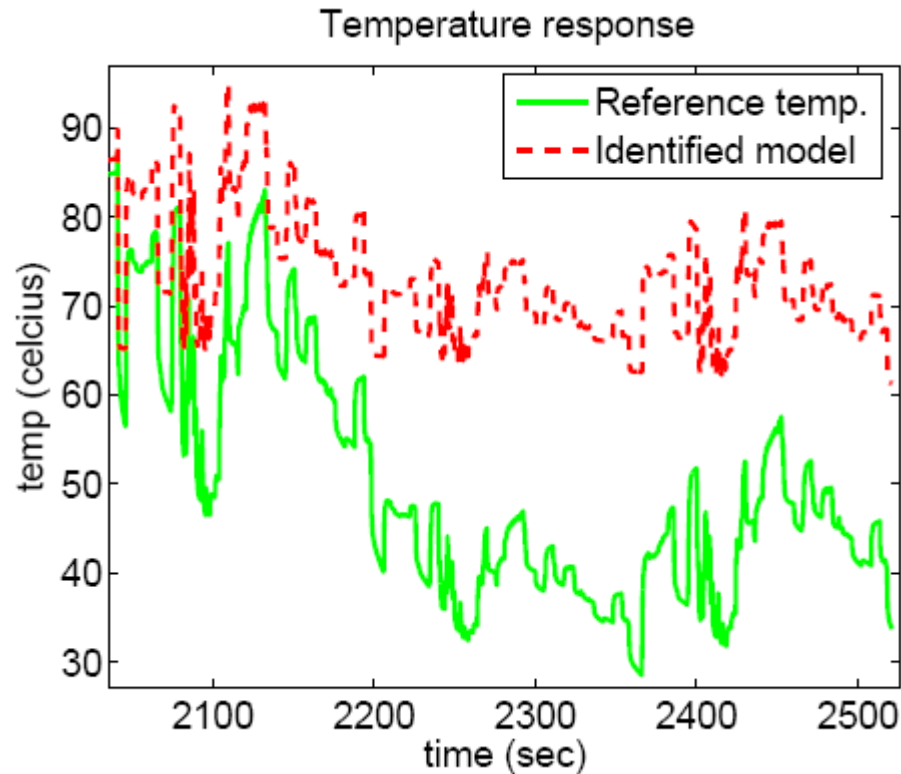


- The output frequencies (The input signal is  $P_{in} = P_{sin}(\pi t)$ ,  $t = 0:0.1:1599.9$ )

# Accuracy loss of linear model

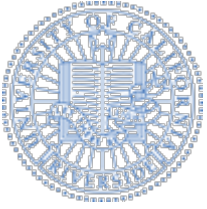


- Nonlinearity results in accuracy losses of the compact linear state space model (order=4).

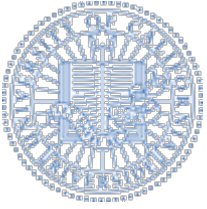


**Temperature output of the identified model (linear)**

# Content

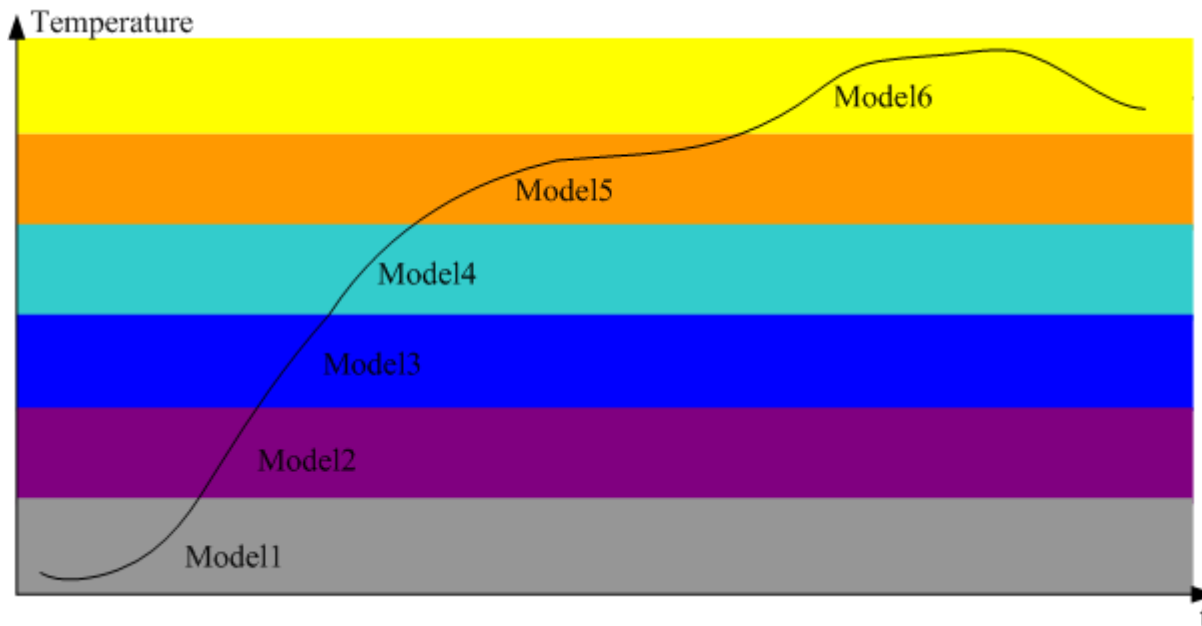


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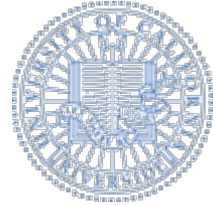


# Proposed PWL modeling method

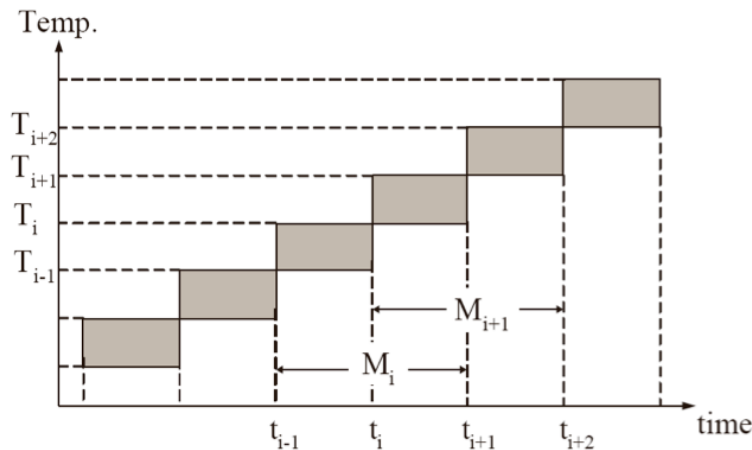
- Use piecewise linear model to approximate the thermal response of the chip for different temperature ranges



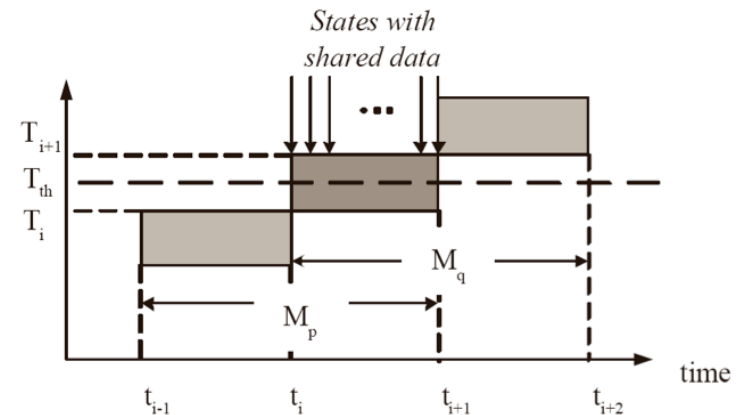
# Outline of the proposed method



- Preparing training data sets for model identification in different temperature ranges
- Improved subspace method is used to identify the sub-models for each temperature range
- Linear transformation is used to build the piecewise linear model

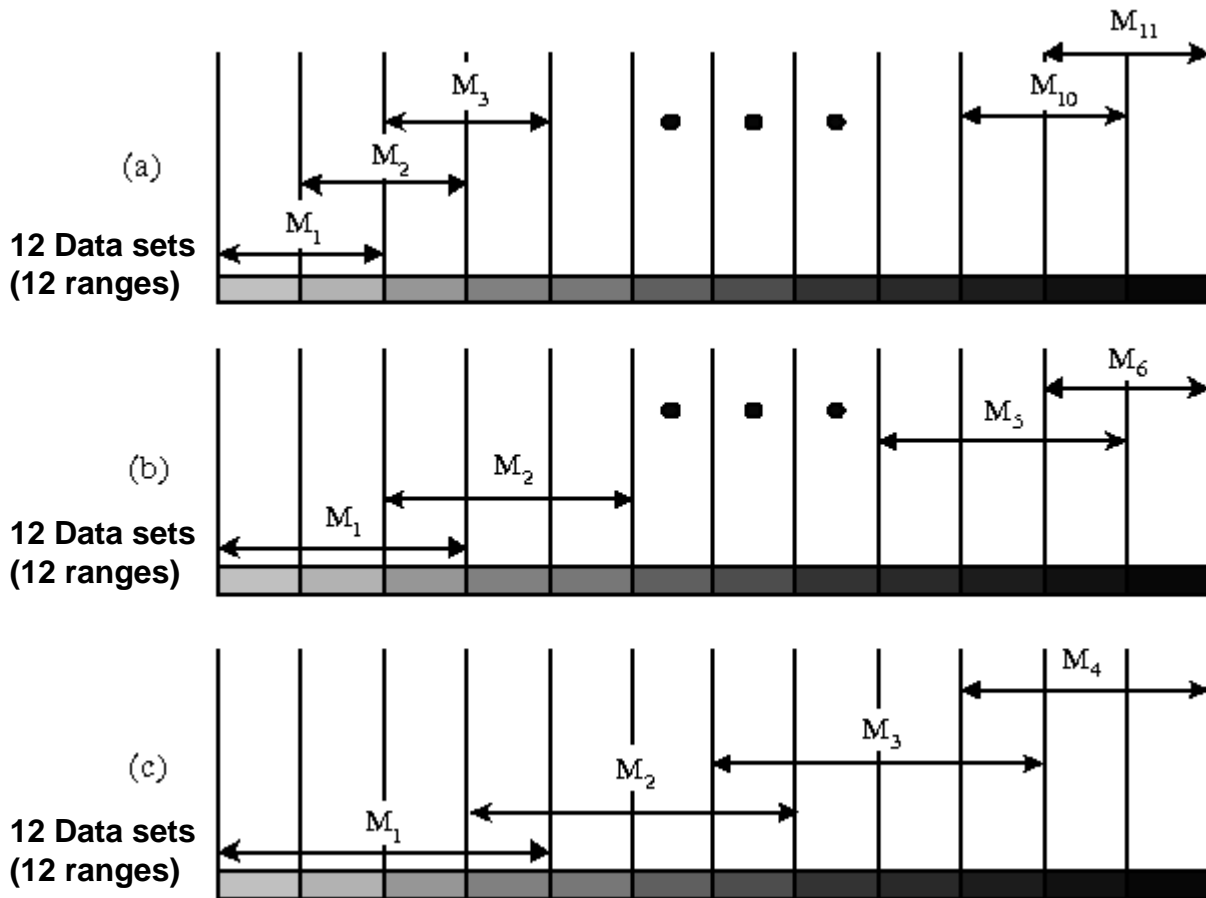
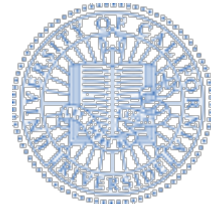


Identification of different models at different temp ranges



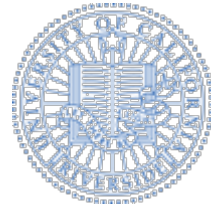
Modeling transition from  $M_p$  to  $M_q$

# Data partition for model identification



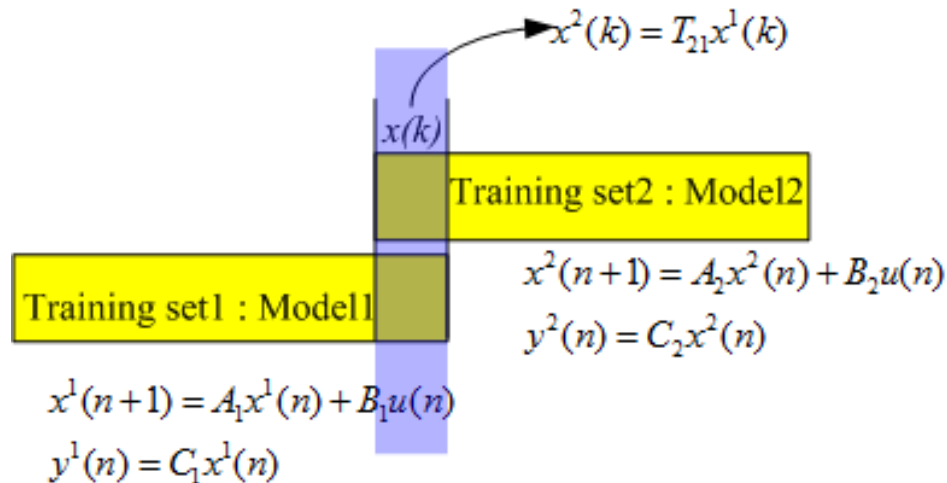
**Data partition scheme to build piecewise linear model (a) use 11 sub-models (b) use 6 sub-models (c) use 4 sub-models**



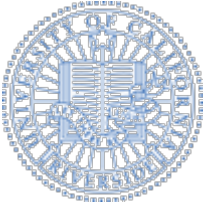


# Build PWL model from sub-models

- To build PWL model, it is necessary to convert the state vectors of different sub-models to the same basis
- The identified sub-models are not on the same basis.
- At the transition region, the states in Model1 and Model2 differ by a linear transformation  $T_{21}$ :  $x^2(k_1 : k_N) = T_{21}x^1(k_1 : k_N)$
- $T_{21}$  could be determined using least square method.
- $x^2(k) = T_{21}x^1(k)$  transforms Model2 to the basis of Model1

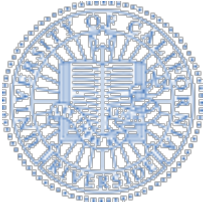


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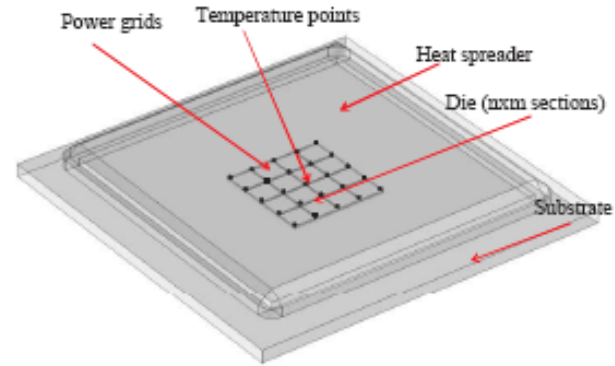


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# Experimental setup

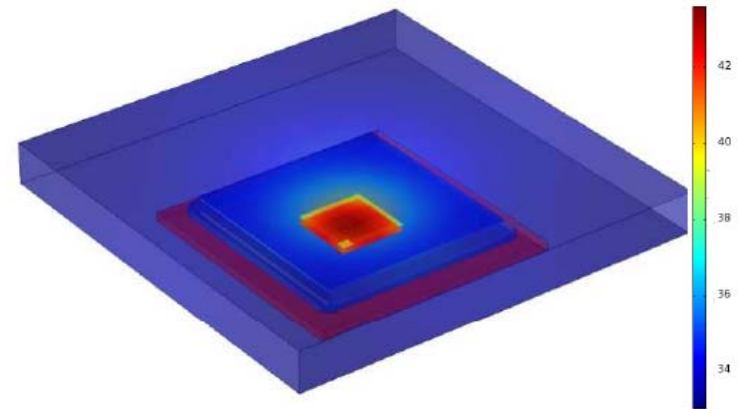


- Treat the meshed thermal chip package as a 16-input (power) and 25-output (temperature) system.



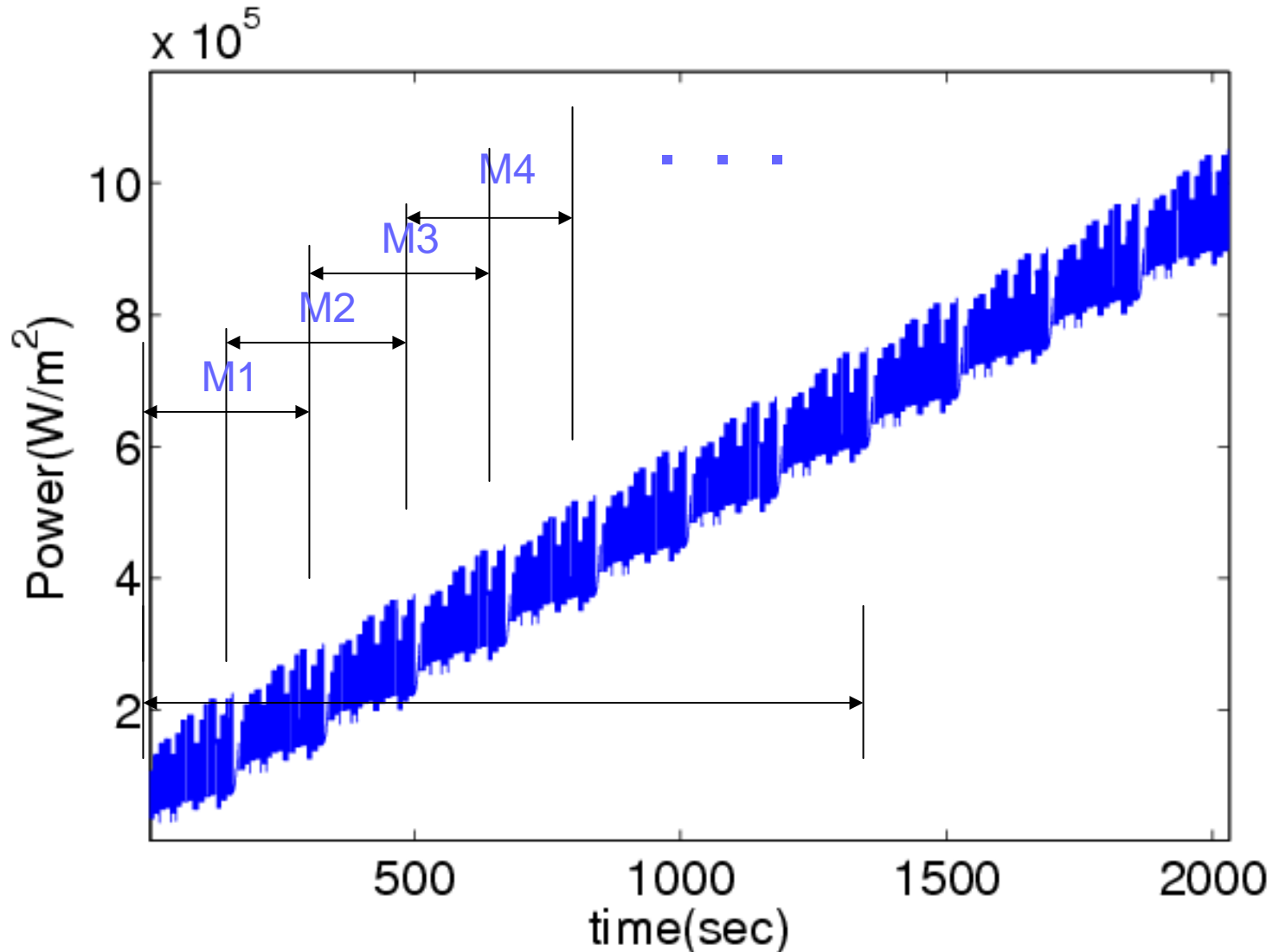
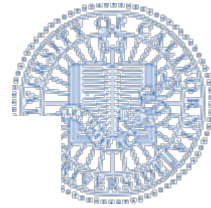
**Chip partition**

- Use COMSOL to simulate its transient temperature response to obtain the temperature data for piecewise linear system identification
- Full simulation time steps: 20412



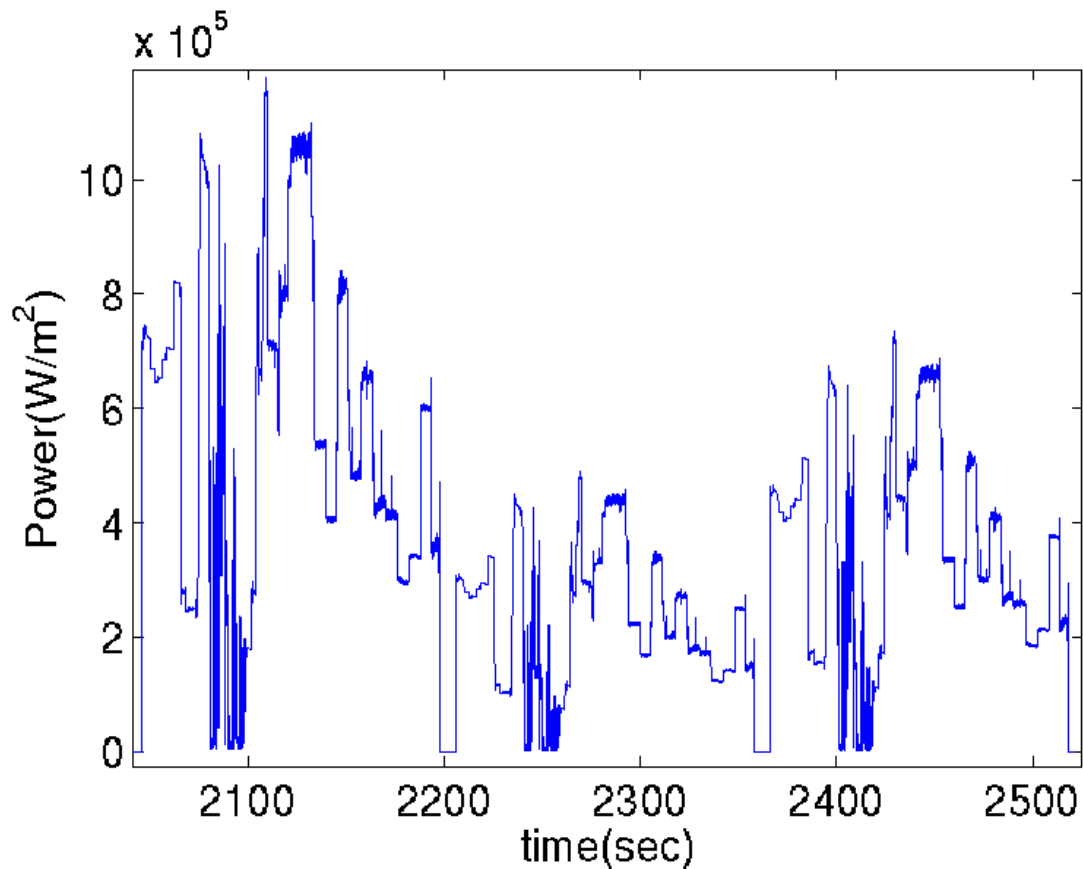
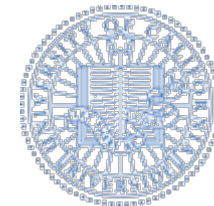
**Steady state temperature distribution**

# Input power waveform (I)



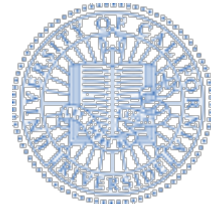
Input waveform used for model identification  
(PRBS signals with stairs-like envelop)

# Input power waveform (II)

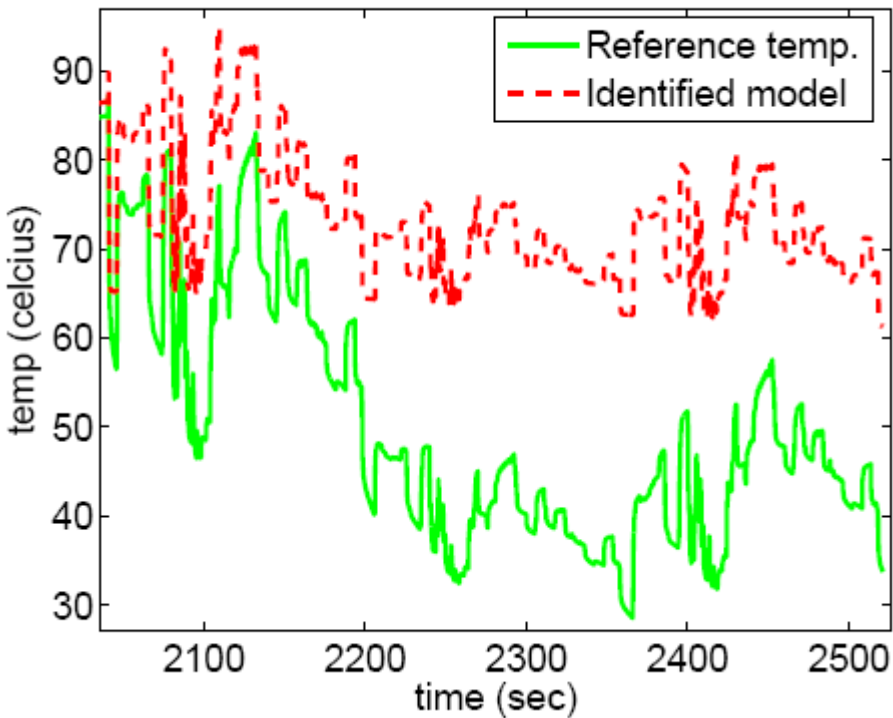


**Input waveform used for model validation (Intel's signal)**

# Transient response of 16-input and 25 output system at section(1,1)

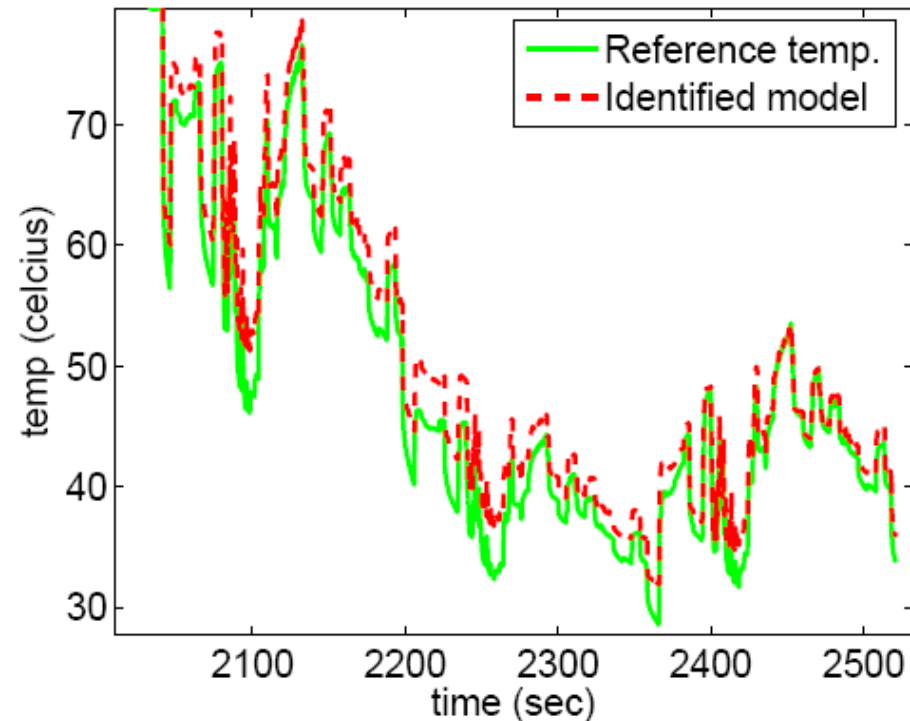


Temperature response

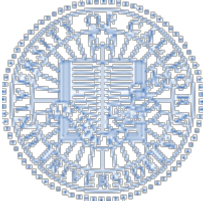


**Use 1 linear model:  
Order = 4**

Temperature response

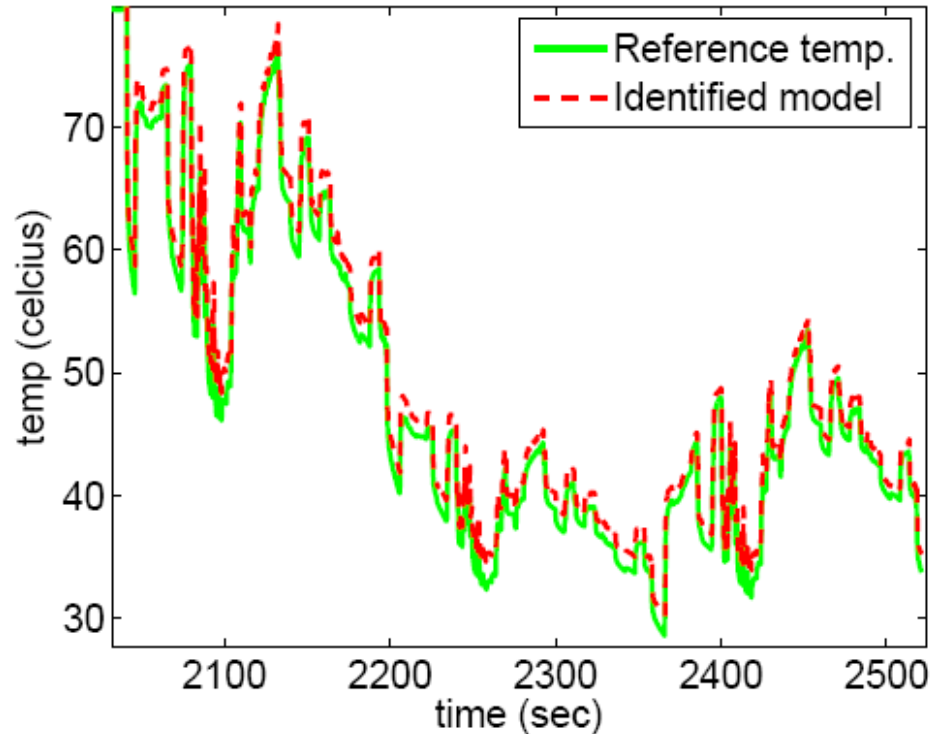


**Use piecewise linear model:  
Number of sub-model used: 4  
Order =4**



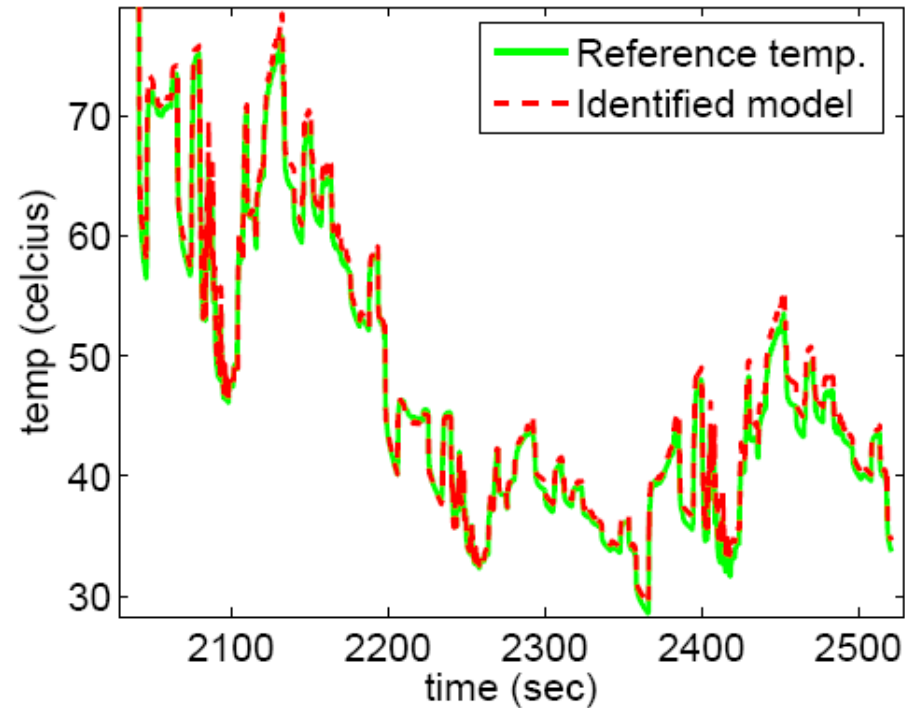
# Transient response of 16-input and 25 output system at section(1,1)

Temperated response



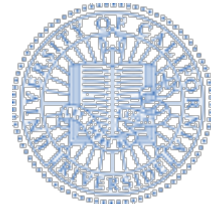
**Number of sub-model used: 6**  
**Order = 4**

Temperature response



**Use piecewise linear model:**  
**Number of sub-model used: 11**  
**Order =4**

# Summary of errors of 16-temperature output of the piecewise linear models

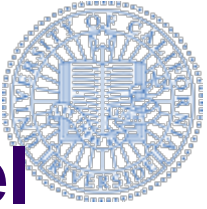


Error with PWL models (order:4)

Num. of linear models in use	11	6	4
Maximum of mean errors	2.1%	3.9%	5.9%

**Mean errors are calculated during the entire transient simulation**





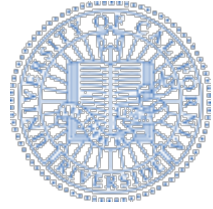
# Performance comparison with linear model

Comparison items	Error	Identification time	Simulation time
PLM(order:4)	2.1%	63.8 sec	7.88 sec
LM (order:15)	2.3%	627.1 sec	22.2 sec

**PLM – Piecewise linear model**

**LM – Linear model**

# Conclusion



- Piecewise linear model scheme has been proposed to consider nonlinear effects in thermal systems.
- Linear sub-models are identified for different temperature ranges using subspace identification method.
- A linear transformation method has been proposed to build piecewise linear model.
- Our experiment results show that Piecewise linear model is more efficient for fast thermal modeling and simulation of packaged microprocessor.