Piecewise-Polynomial Associated Transform Macromodeling Algorithm for Fast Nonlinear Circuit Simulation

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Outline

Background

- *Nonlinear model order reduction (NMOR)*
- *TPWL & PWP*
- *Associated transform*

PWPAT

- *Association of Laplace variables (s_1, s_2, s_3 etc.)*
- *Moment Matching by Associated Transform*
- *Piecewise-polynomial associated transform (PWPAT) NMOR*

Numerical examples

Conclusion

Nonlinear Model Order Reduction (NMOR)

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases}$$

$$\longrightarrow$$

$$\begin{cases} \dot{\hat{x}} = \hat{f}(\hat{x},u) \\ \hat{y} = \hat{h}(\hat{x},u) \end{cases}$$

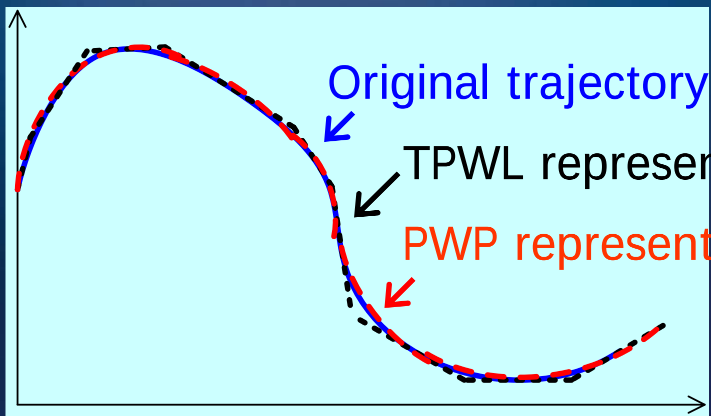
where $u(t), y(t) \in R^m$ are inputs and outputs of the *m-in-m-out* system, respectively. $x(t) \in R^N$ and $\hat{x}(t) \in R^q$ are the state variables of the original and reduced system, in which $q \ll N$.

Application:
System-level simulation, macromodel extraction ...

NMOR Methods:
Weakly nonlinear: NORM etc... Strongly nonlinear: TPWL, PWP etc...

TPWL & PWP

	TPWL (Trajectory Piecewise-Linear)	PWP (Piecewise-Polynomial)
Step 1	Divide the whole trajectory into segments	
Step 2	Approximate each segment by a linear ODE	Approximate each segment by a higher order ODE
Step 3	Reduce each linear ODE with a projection matrix V_i	Reduce each higher order ODE with a projection matrix V_i
Step 4	Aggregate all V_is together for whole projection matrix	



TPWL representation: Good at global accuracy

PWP representation: Good at global & local accuracies

Reduce Each Segment in PWP with a Higher Order ODE

NORM Algorithm (Li & Pileggi, *IEEE TCAD* 05)

$$H_1(s) = (sI - G_1)^{-1} b$$

$$H_2(s_1, s_2) = \frac{1}{2}\left((s_1 + s_2)I - G_1\right)^{-1} \left\{ G_2 \left[H_1(s_1) \otimes H_1(s_2) + H_1(s_2) \otimes H_1(s_1) \right] \right\}$$

$$H_3(s_1, s_2, s_3) = \frac{1}{3}\left((s_1 + s_2 + s_3)I - G_1\right)^{-1} \left\{ G_2 \left[H_1(s_1) \otimes H_2(s_2, s_3) + \cdots \right] \right\}$$

1. Use Taylor expansion to expand each transfer function (TF)
2. Match moments with Krylov subspaces

TF	Moments to be matched
$H_1(s)$	$1, s, s^2, s^3 \ldots$
$H_2(s_1,s_2)$	$1, s_1, s_2, s_1^2, s_1 s_2, s_2^2 \ldots$
$H_3(s_1,s_2,s_3)$	$1, s_1, s_2, s_3, s_1^2, s_1 s_2, s_1 s_3, s_2^2, s_2 s_3, s_3^2 \ldots$

Associated Transform

1st Link to h$_n$(t) : Multivariate Inverse Laplace

$$h_n(t_1,\cdots,t_n)=\mathcal{L}^{-1}\left(H_n\left(s_1,\cdots,s_n\right)\right)=\frac{1}{(2\pi j)^n}\int_{\sigma_n-j\infty}^{\sigma_n+j\infty}\cdots\int_{\sigma_1-j\infty}^{\sigma_1+j\infty}H_n\left(s_1,\cdots,s_n\right)e^{s_1t_1}\cdots e^{s_nt_n}ds_1\cdots ds_n$$

$$\begin{tikzcd}
H_n\left(s_1,s_2,\cdots,s_n\right) \arrow[r, "\text{Multidimensional } \mathcal{L}^{-1}"] \arrow[d, "\mathcal{A}_n"] & h_n\left(t_1,t_2,\cdots,t_n\right) \arrow[d, "t_1=t_2\cdots=t_n=t"] \\
H_n\left(s\right) \arrow[r, "\text{Single } \mathcal{L}^{-1}"] & h_n\left(t\right)
\end{tikzcd}$$

2nd Link to h$_n$(t): Association of variables

$$H_n(s)=\mathcal{A}_n\left(H_n\left(s_1,\cdots,s_n\right)\right)=\frac{1}{(2\pi j)^{n-1}}\int_{\sigma_n-j\infty}^{\sigma_n+j\infty}\cdots\int_{\sigma_2-j\infty}^{\sigma_2+j\infty}H_n\left(s-s_2-\cdots-s_n,s_2,\cdots,s_n\right)ds_2\cdots ds_n$$

Associated Transform – An illustrative Example

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\frac{1}{s_1+1}\left(\frac{1}{s_2+1}-\frac{1}{s_2+2}\right) \arrow[r, leftrightarrow, "\text{2-D Inverse Laplace}"] \arrow[d, "\text{Association of } s_i\text{'s}"] & e^{-t_1}\left(e^{-t_2}-e^{-2t_2}\right) \arrow[d, "\text{Unify all } t_i\text{'s}"] \\
\frac{1}{s+2}-\frac{1}{s+3} \arrow[r, leftrightarrow, "\text{1-D Inverse Laplace}"'] & e^{-2t}-e^{-3t}
\end{tikzcd}$$

Moment Matching by Associated Transform

Theorem 1

$$\mathcal{A}_2\left(\left(s_1 I_{n1} - A_1\right)^{-1} \otimes \left(s_2 I_{n2} - A_2\right)^{-1}\right) = \left(sI_{n1n2} - \left(A_1 \oplus A_2\right)\right)^{-1}$$

Theorem 2

$$\mathcal{A}_2\left(\left(s_1 I - A\right)^{-1} b\right) = b$$

$$H_1(s) = \left(sI - G_1\right)^{-1} b$$

$$H_2(s_1, s_2) = \frac{1}{2}\left(\left(s_1 + s_2\right) I - G_1\right)^{-1}\left\{G_2\left[H_1(s_1) \otimes H_1(s_2) + H_1(s_2) \otimes H_1(s_1)\right]\right\}$$

$$H_3(s_1, s_2, s_3) = \frac{1}{3}\left(\left(s_1 + s_2 + s_3\right) I - G_1\right)^{-1}\left\{G_2\left[H_1(s_1) \otimes H_2(s_2, s_3) + \cdots\right]\right\}$$

$$\downarrow$$

$$\mathcal{A}_2\left(H_2(s_1, s_2)\right) = \left(sI - G_1\right)^{-1}\left(G_2\left(sI - G_1 \oplus G_1\right)^{-1}\left(b \otimes b\right) + D_1 b\right)$$

$$= \left[\begin{array}{c|c} G_1 & I_n \\ \hline I_n & 0 \end{array}\right] \cdot \left[\begin{array}{c|c} G_1 \oplus G_1 & b \otimes b \\ \hline G_2 & D_1 b \end{array}\right] = \left[\begin{array}{cc|c} G_1 & G_2 & D_1 b \\ 0 & G_1 \oplus G_1 & b \otimes b \\ \hline I_n & 0 & 0 \end{array}\right] = \left[\begin{array}{c|c} \tilde{G}_2 & \tilde{b}_2 \\ \hline \tilde{c}_2 & 0 \end{array}\right]$$

$$\mathcal{A}_3\left(H_3(s_1, s_2, s_3)\right) = \left(sI - G_1\right)^{-1}\left(G_2 \tilde{H}_3(s) + D_1^2 b\right)$$

where: $\tilde{H}_3(s) = \left(I_n \otimes \tilde{c}_2\right)\left(sI - G_1 \oplus \tilde{G}_2\right)^{-1}\left(b \otimes \tilde{b}_2\right) + \left(\tilde{c}_2 \otimes I_n\right)\left(sI - \tilde{G}_2 \oplus G_1\right)^{-1}\left(\tilde{b}_2 \otimes b\right)$

Moment Matching by Associated Transform (Cont'd)

Higher-order TFs into LTIs:

$H_1(s) = (sI - G_1)b$ where G_1 is $n \times n$, and b is $n \times 1$

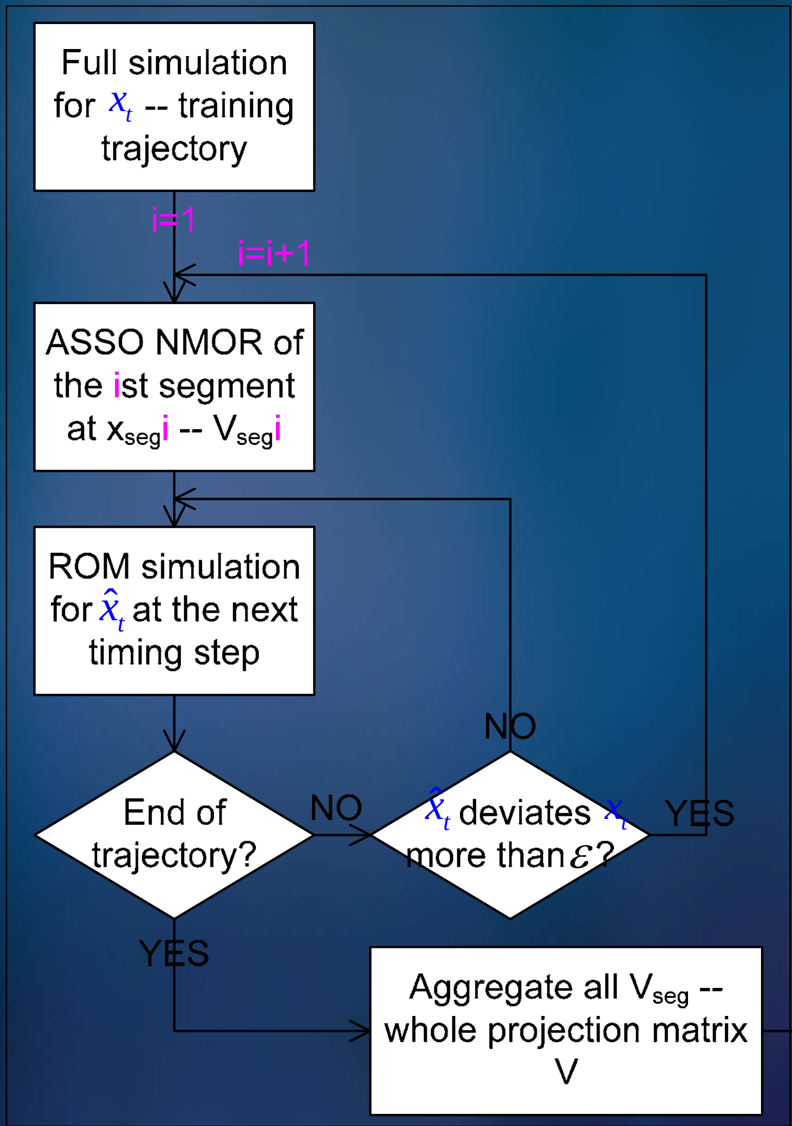
$H_2(s) = N_2(sI - M_2)F_2$ where N_2 is $n \times (n^2 + n)$, M_2 is $(n^2+n)\times(n^2+n)$, and F_2 is $(n^2+n)\times 1$

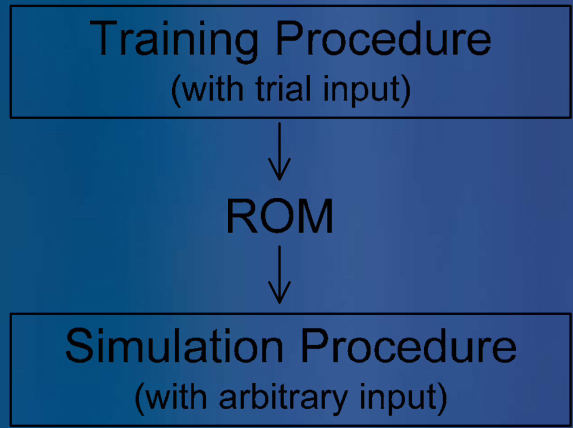
$H_3(s) = N_3(sI - M_3)F_3$ where N_3 is $n \times (2n^3 + 2n^2 + n)$, M_3 is $(2n^3+2n^2+n)\times(2n^3+2n^2+n)$,
and F_3 is $(2n^3+2n^2+n)\times 1$

TF	Moments to be matched
$H_1(s)$	$1, s, s^2, s^3 \ldots$
$H_2(s)$	$1, s, s^2, s^3 \ldots$
$H_3(s)$	$1, s, s^2, s^3 \ldots$

ROM is much more compact!

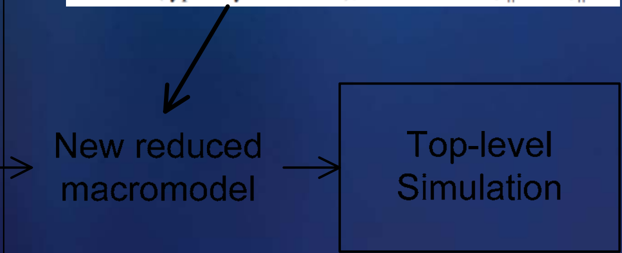
PWPAT NMOR





$$\hat{E}\dot{z} = \sum_{i=1}^{k} (\omega_i(z))(\hat{A}_i(z-z_i) + \hat{W}_i(z-z_i) \otimes (z-z_i)$$
$$+\hat{K}_i + \hat{B}(u))$$
$$y = C\left[\sum_{i=1}^{k} (\omega_i(z))(x_i + V(z-z_i))\right]$$

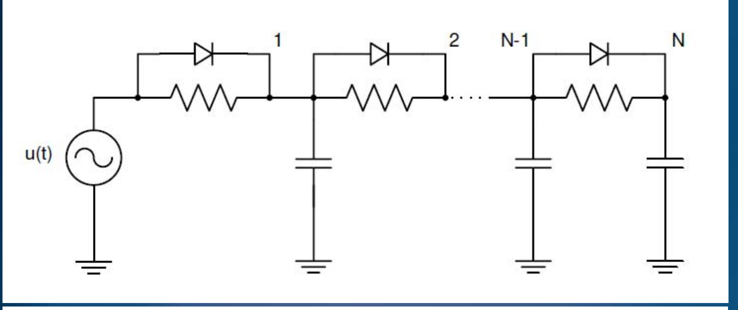
in which $w_i(z) = \frac{\exp\left(\frac{-\beta\|z-z_i\|}{m}\right)}{\sum_{i=1}^{k}\exp\left(\frac{-\beta\|z-z_i\|}{m}\right)}$ where β is some constant (typically we use 25), and $m = \min_i \|z-z_i\|$.

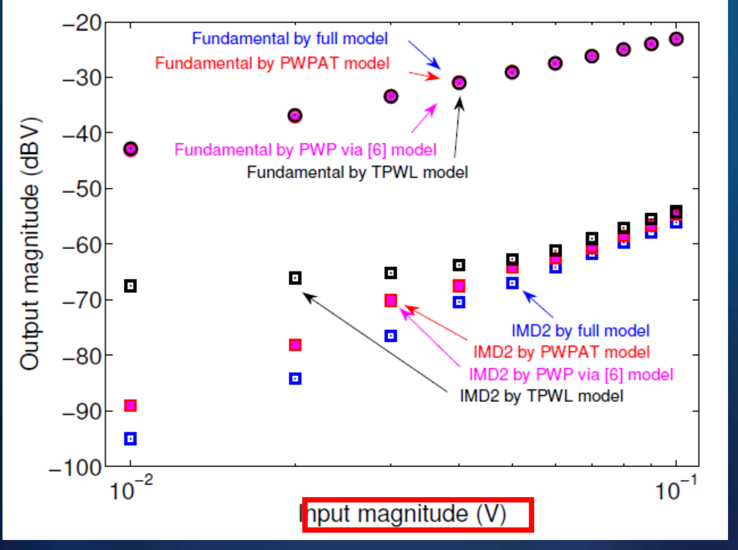


Numerical Examples

Nonlinear trasnmission line

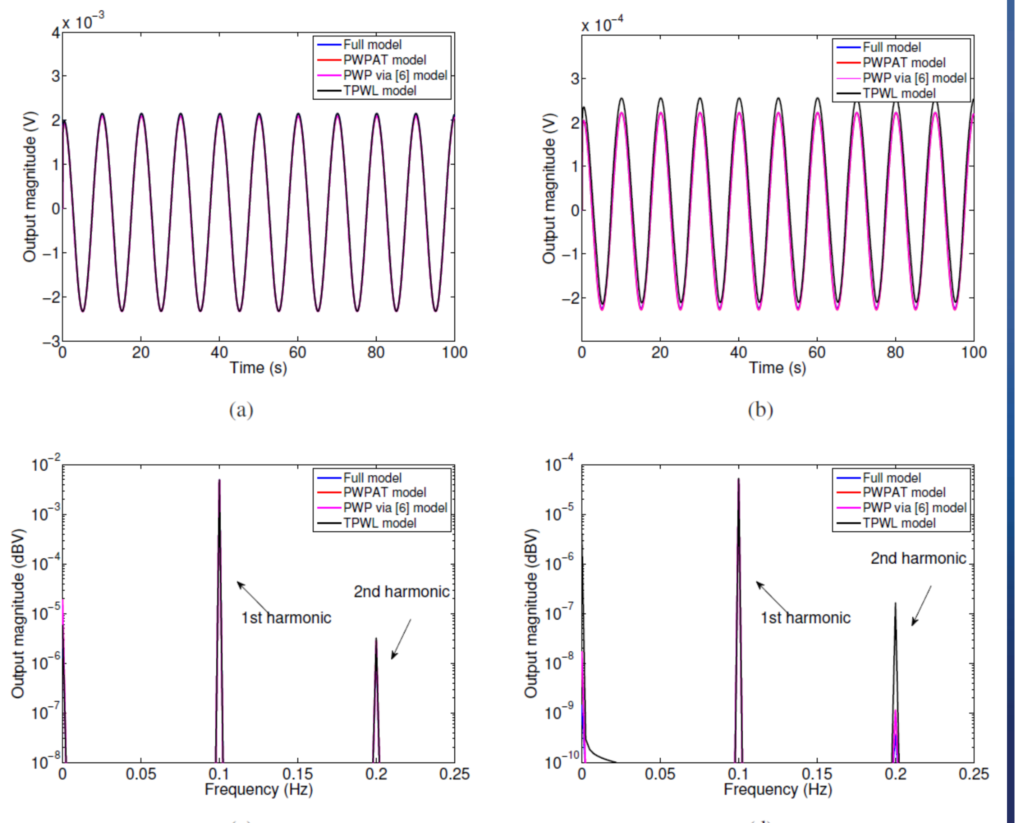
Schematic





Large input

Small input



(a)

(b)

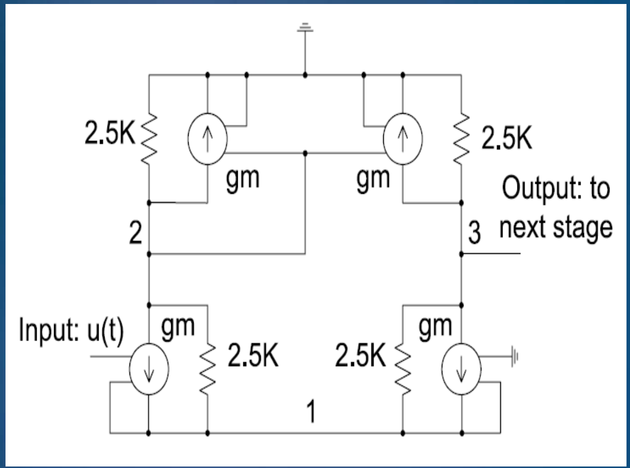
(c)

(d)

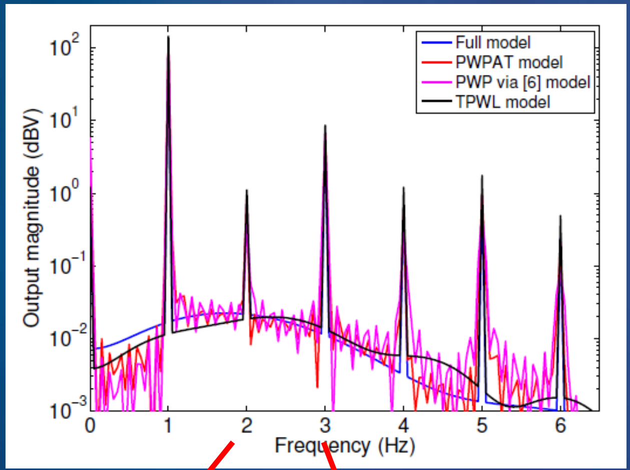
Numerical Examples (Cont'd)

Analog buffer circuit

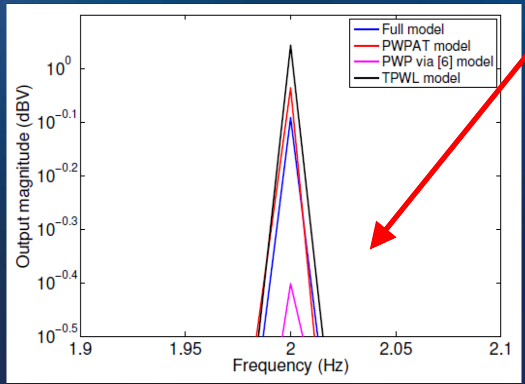
Schematic

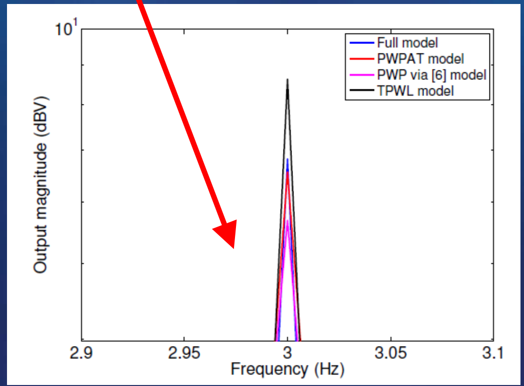


Output spectrum



Under similar ROM dimensions, PWPAT has better accuracy than TPWL, especially at higher order harmonics.





Conclusions

Piecewise-Polynomial + Associated Transform NMOR

Pros:

- Better accuracy in higher order characteristics
- Much smaller ROM with comparable accuracy
- LTI techniques reusable

Cons:

- Higher MOR cost but parallelization possible

Thank You