

Statistical Analysis of Process Variation Based on Indirect Measurements

Ivan Ukhov, Mattias Villani, Petru Eles, and Zebo Peng

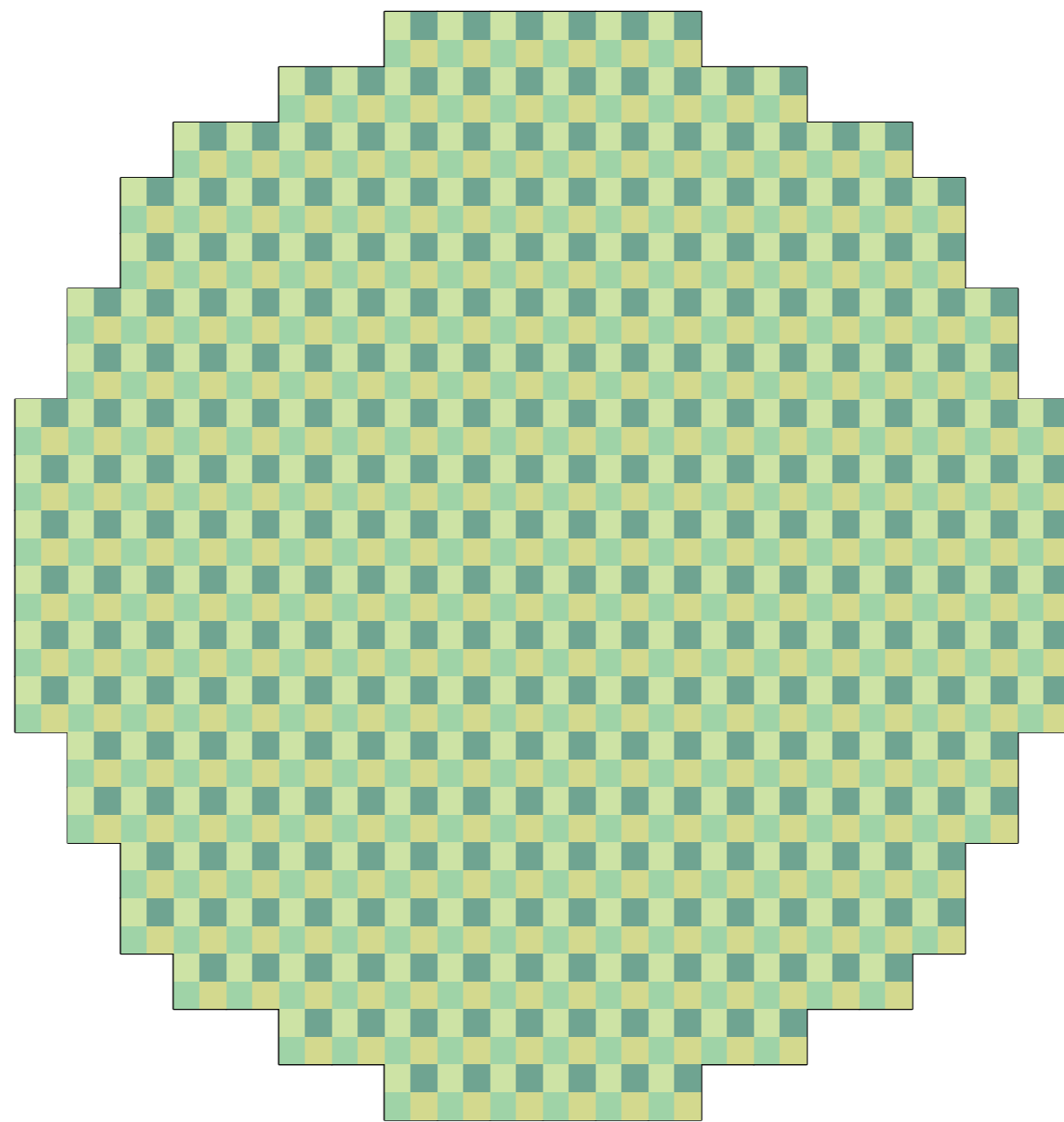
Linköping University

January 2014

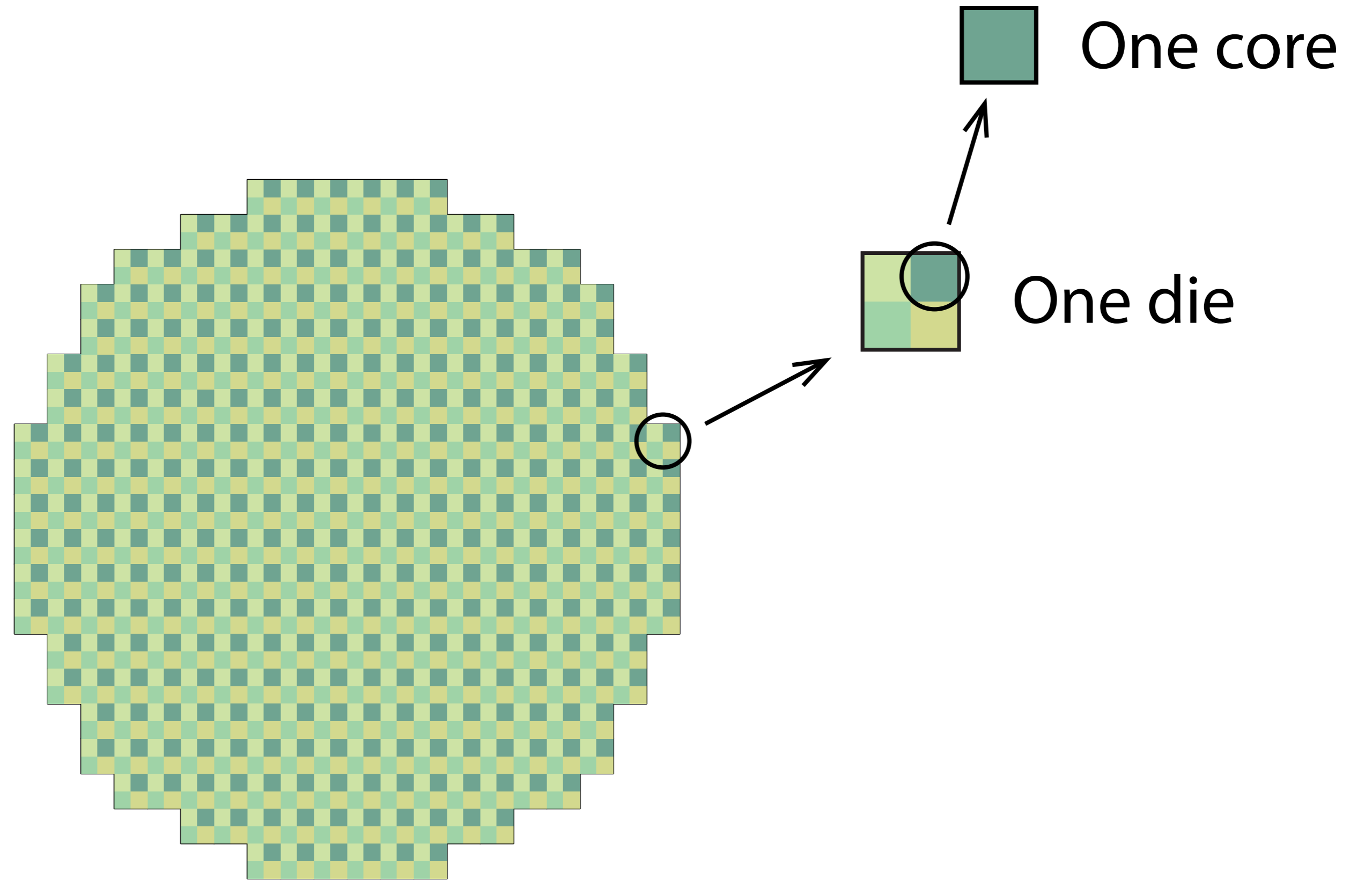
Outline

1. Introduction
2. Our goal and solution
3. Illustrative example
4. Technical details
5. Conclusion

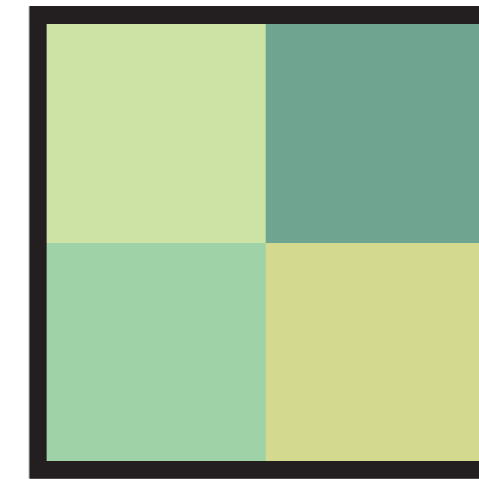
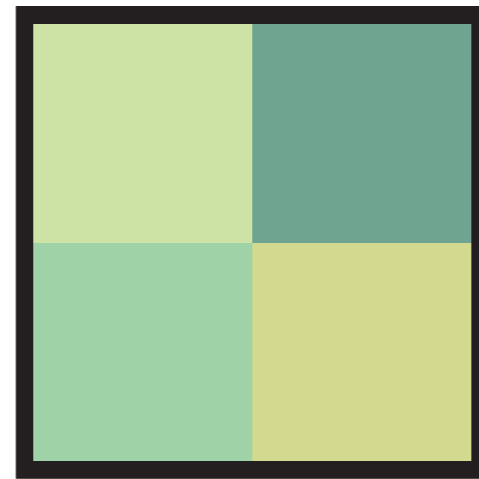
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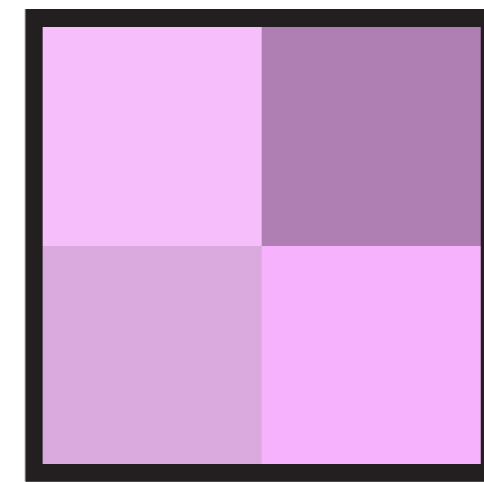
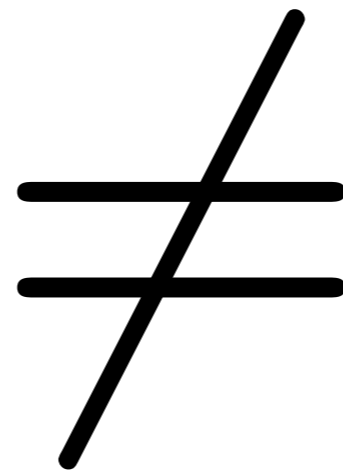
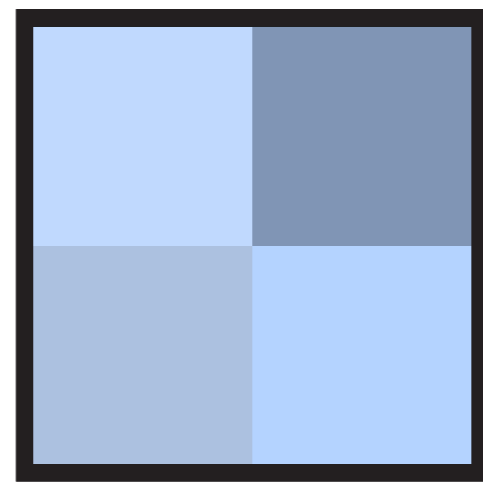
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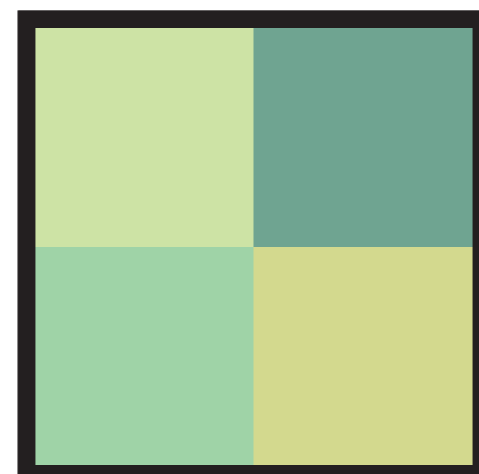
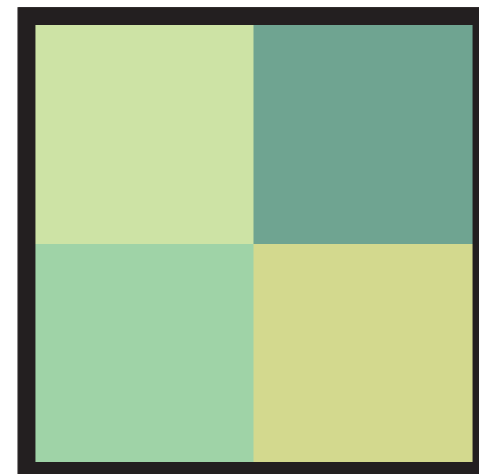
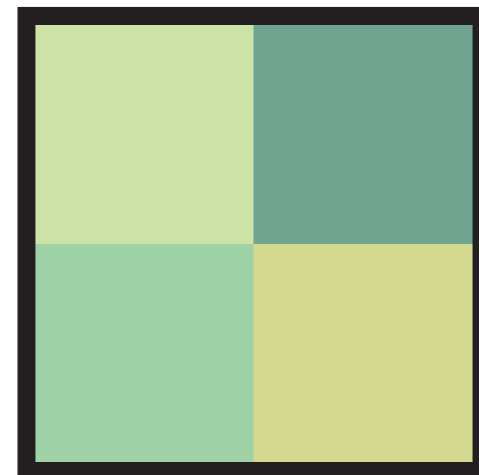
Process Variation



Process Variation

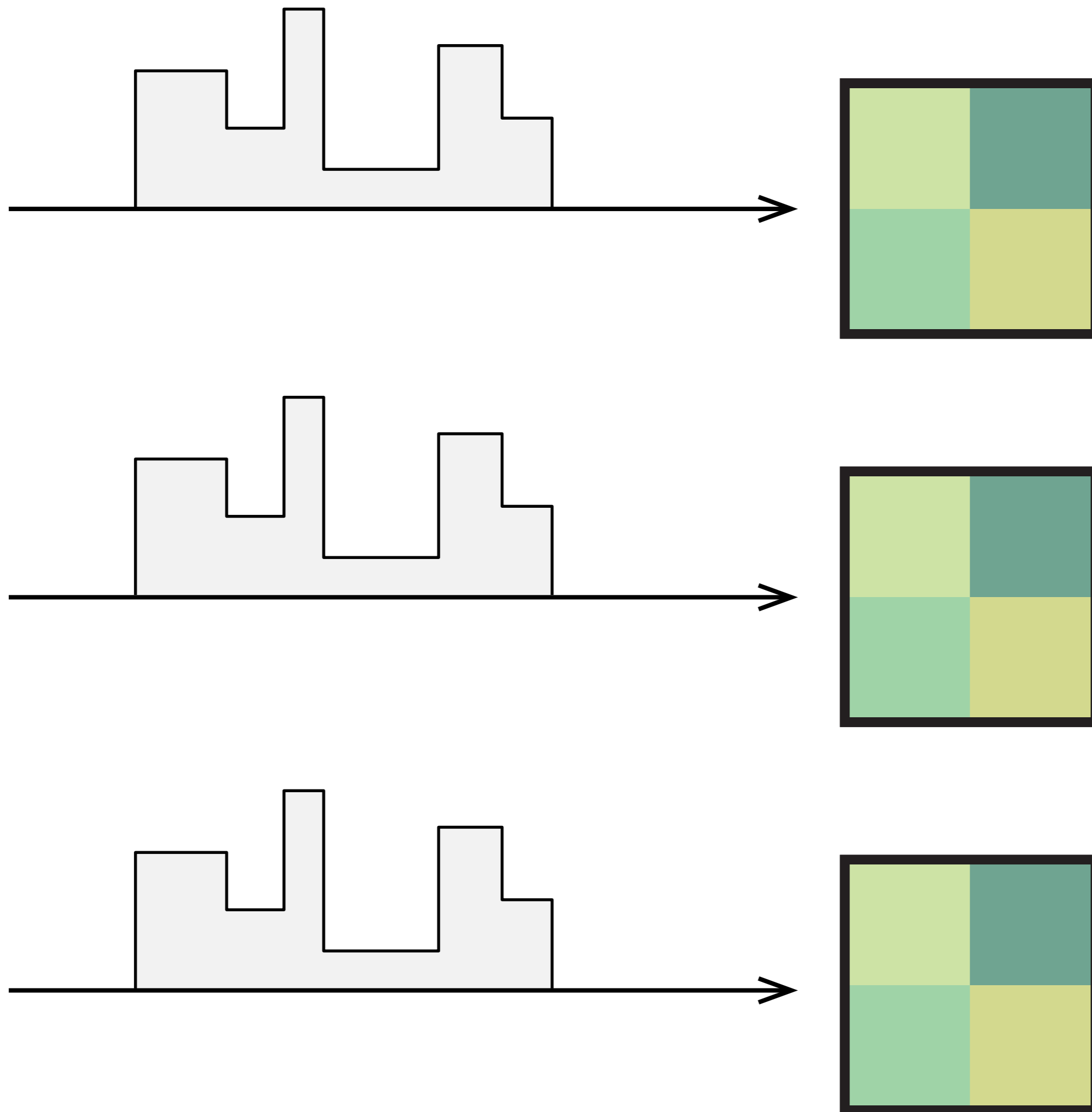


Process Variation



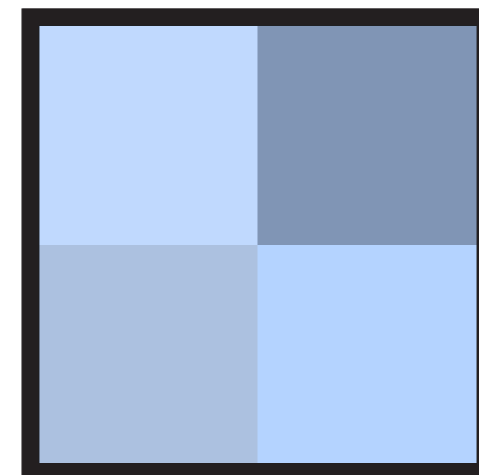
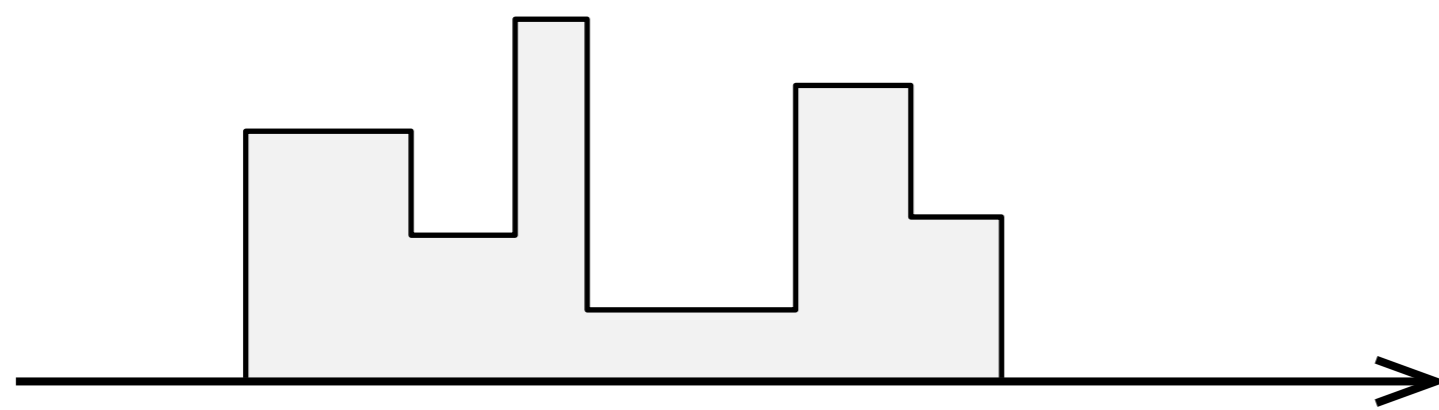
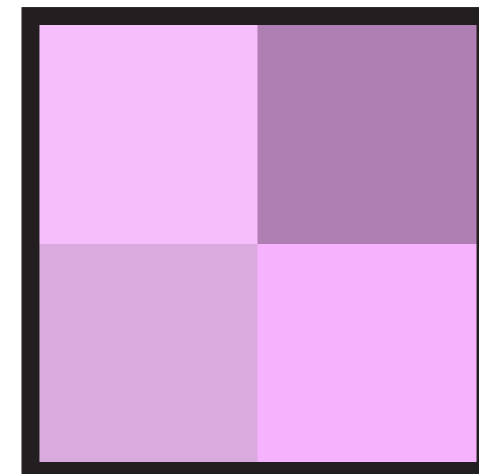
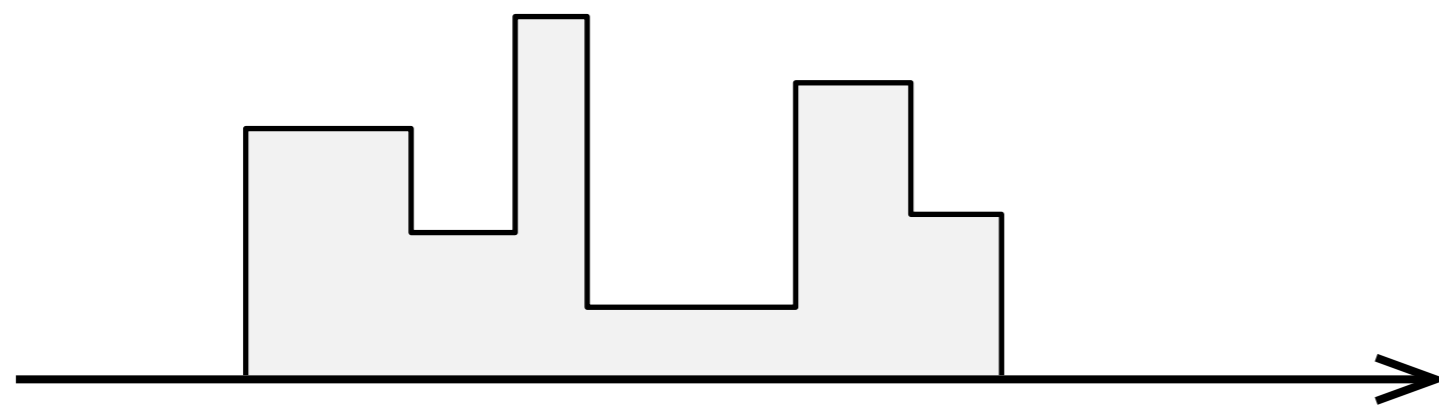
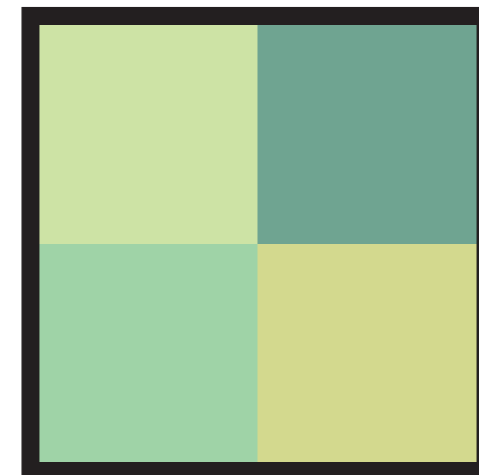
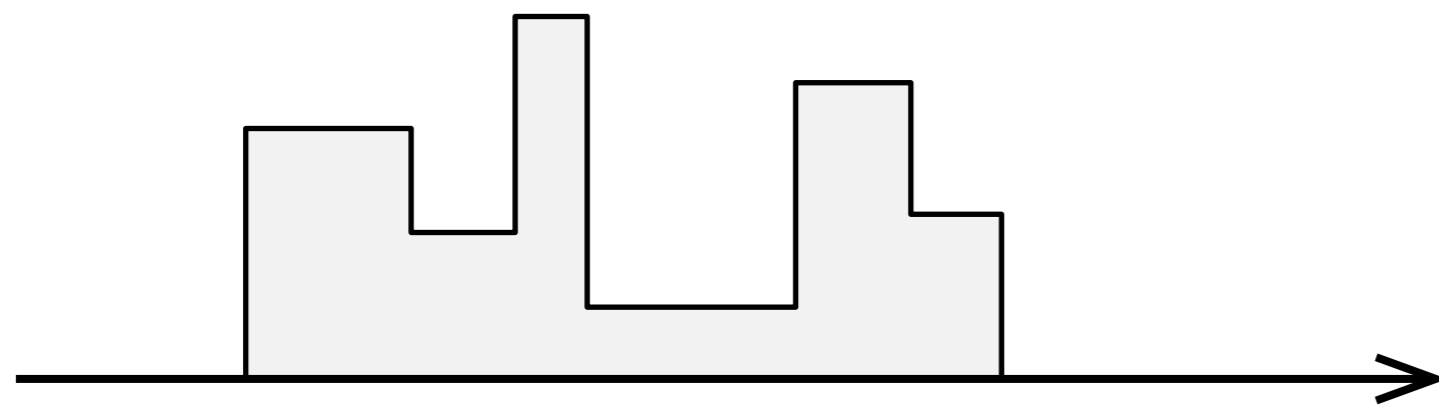
Process Variation

Workload

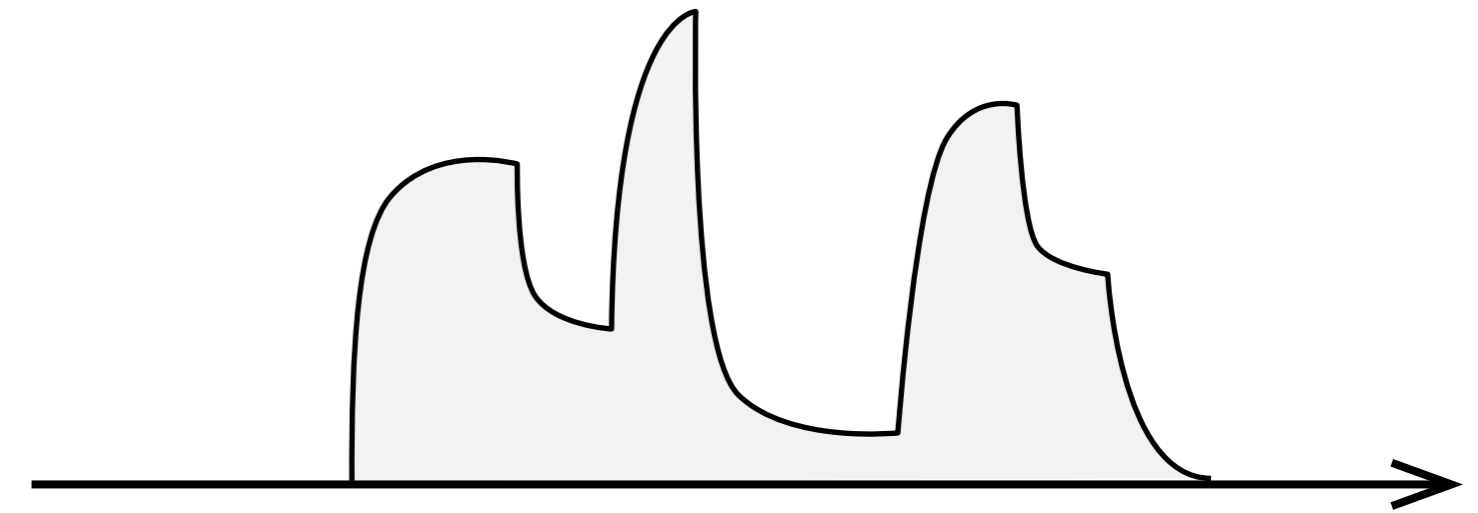
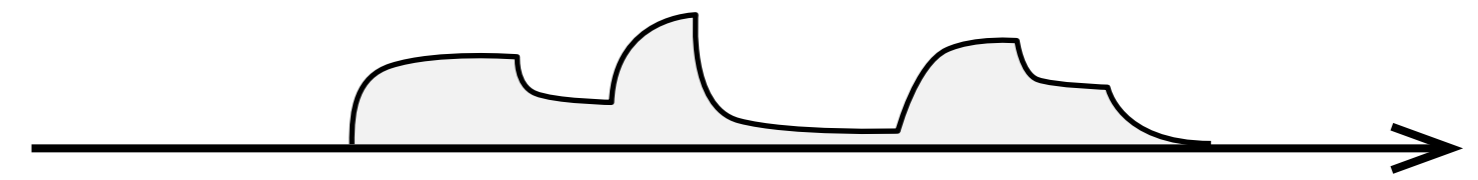
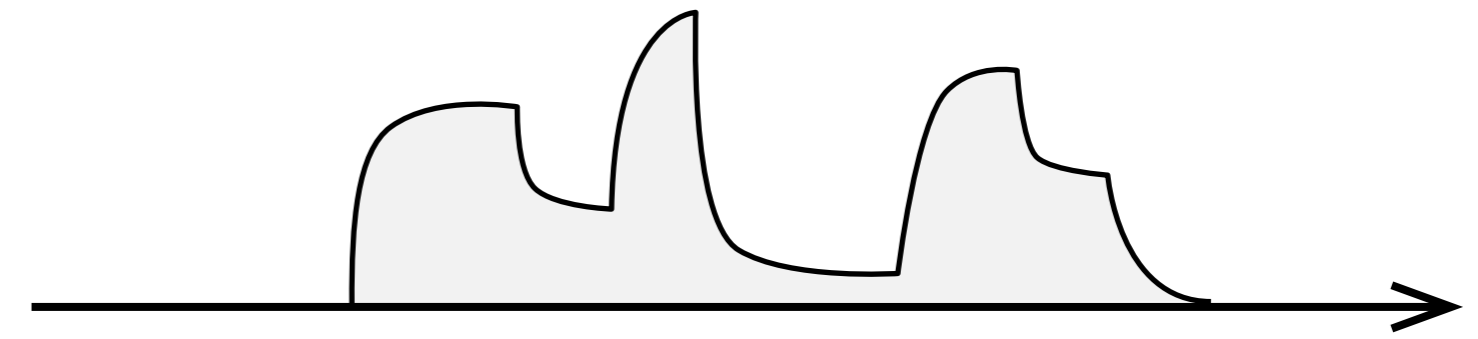


Process Variation

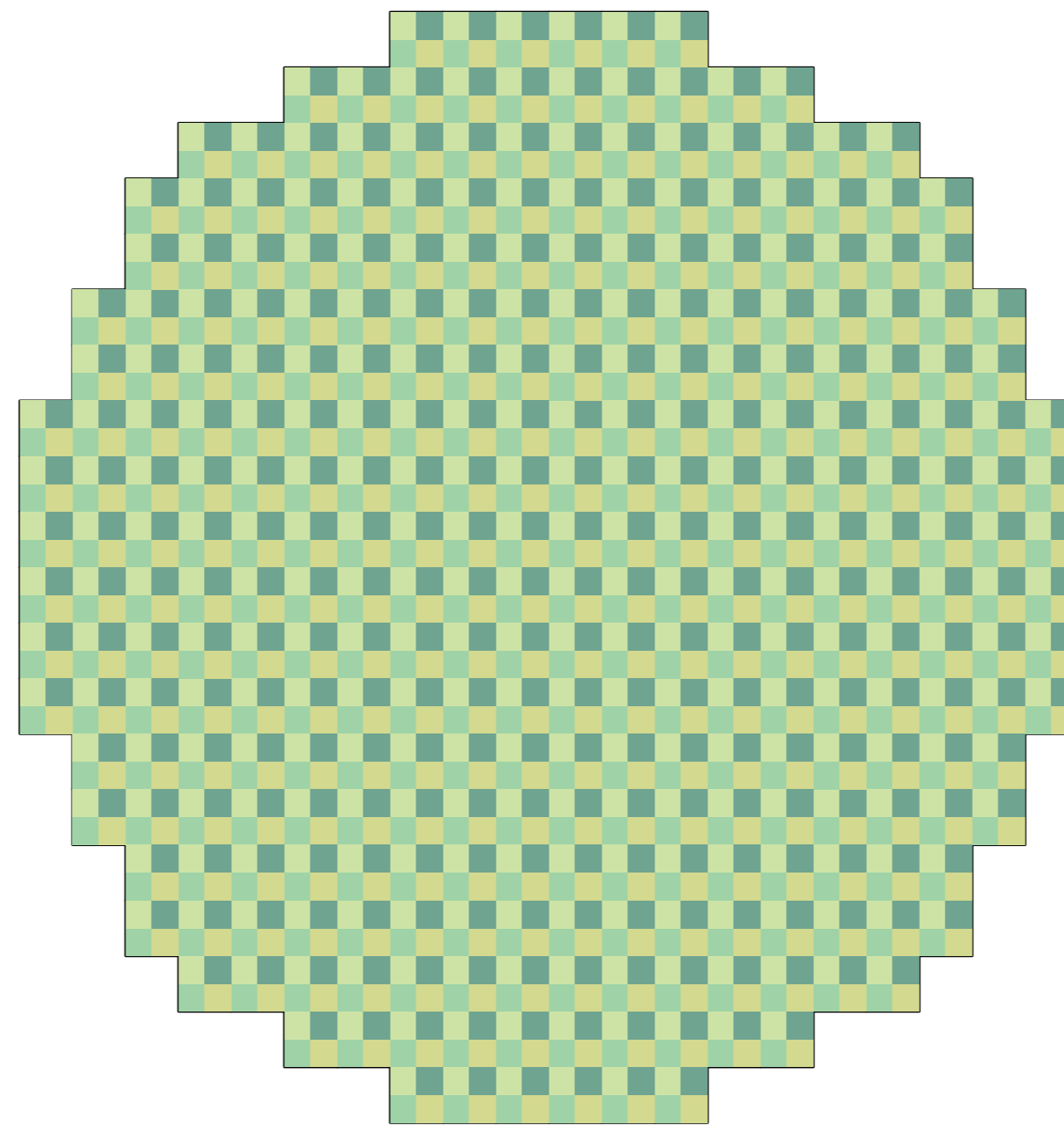
Workload



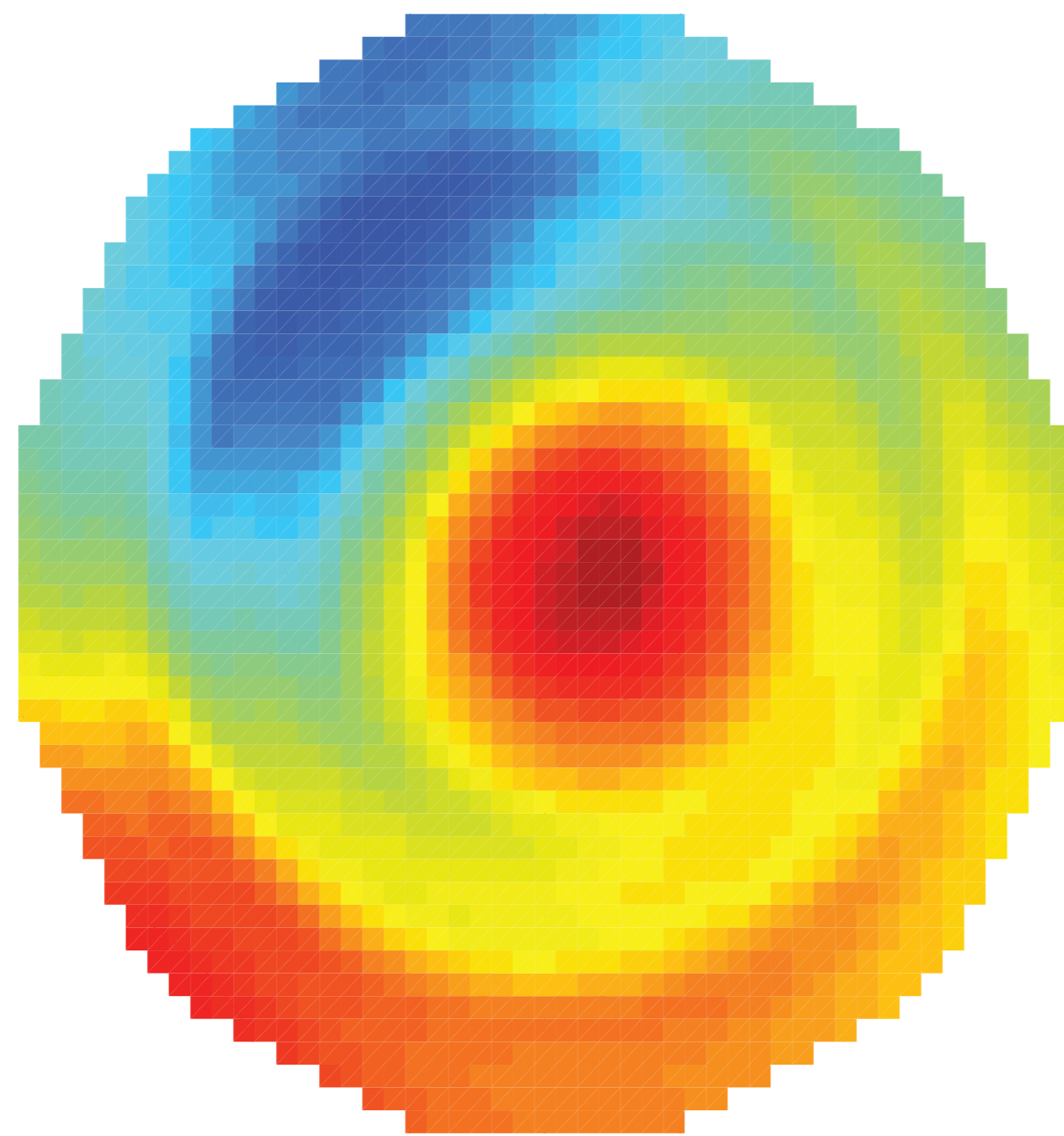
Temperature



Process Variation



Process Variation

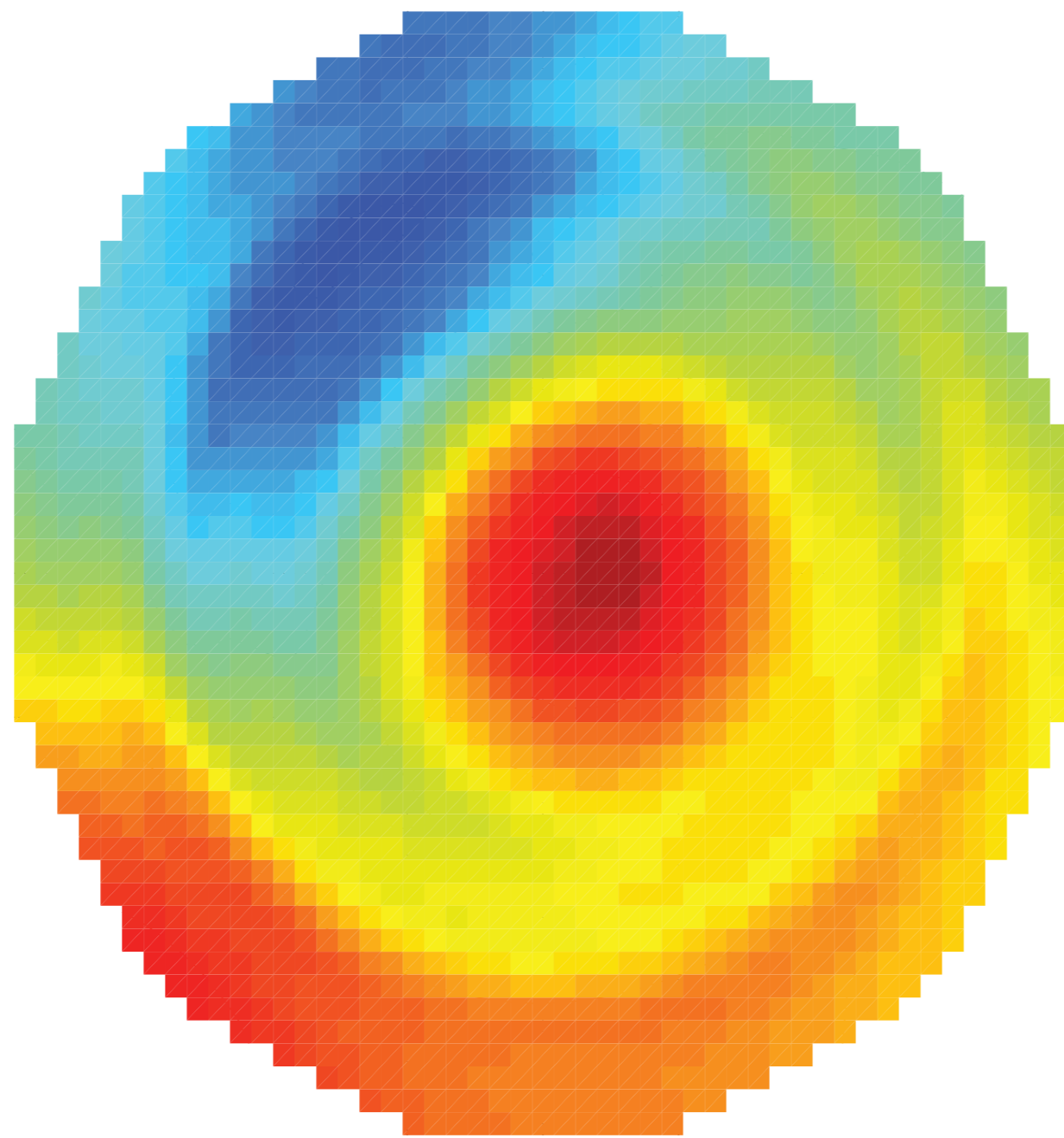


Longer



Shorter

Our Goal

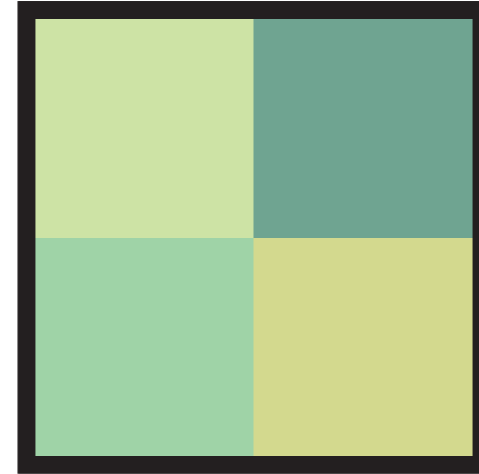


Our Goal

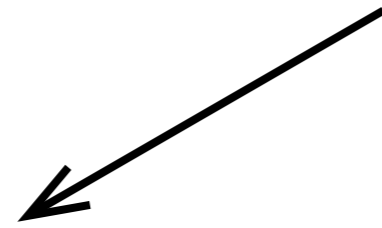
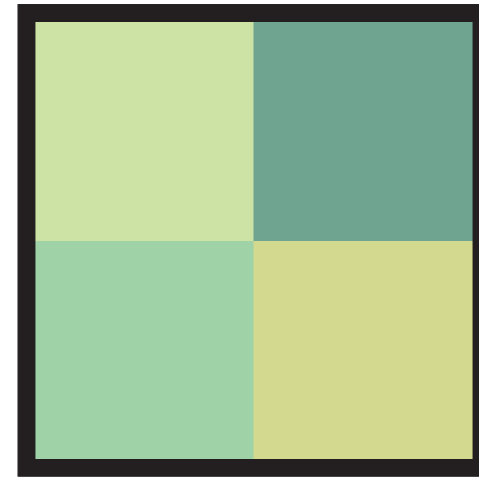


Quantity of interest

Alternatives

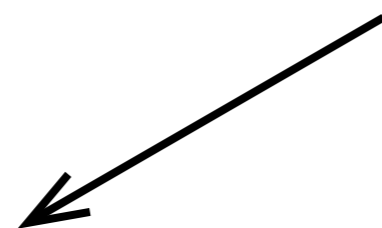
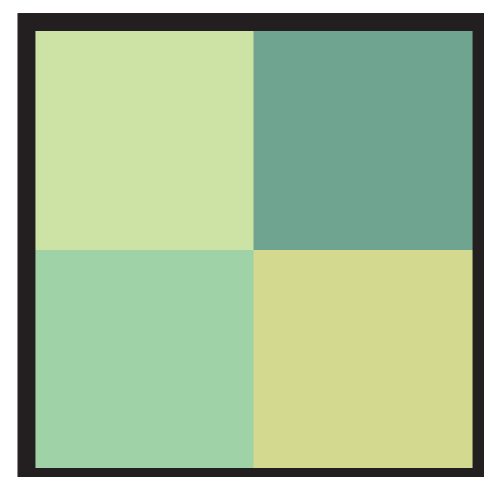


Alternatives

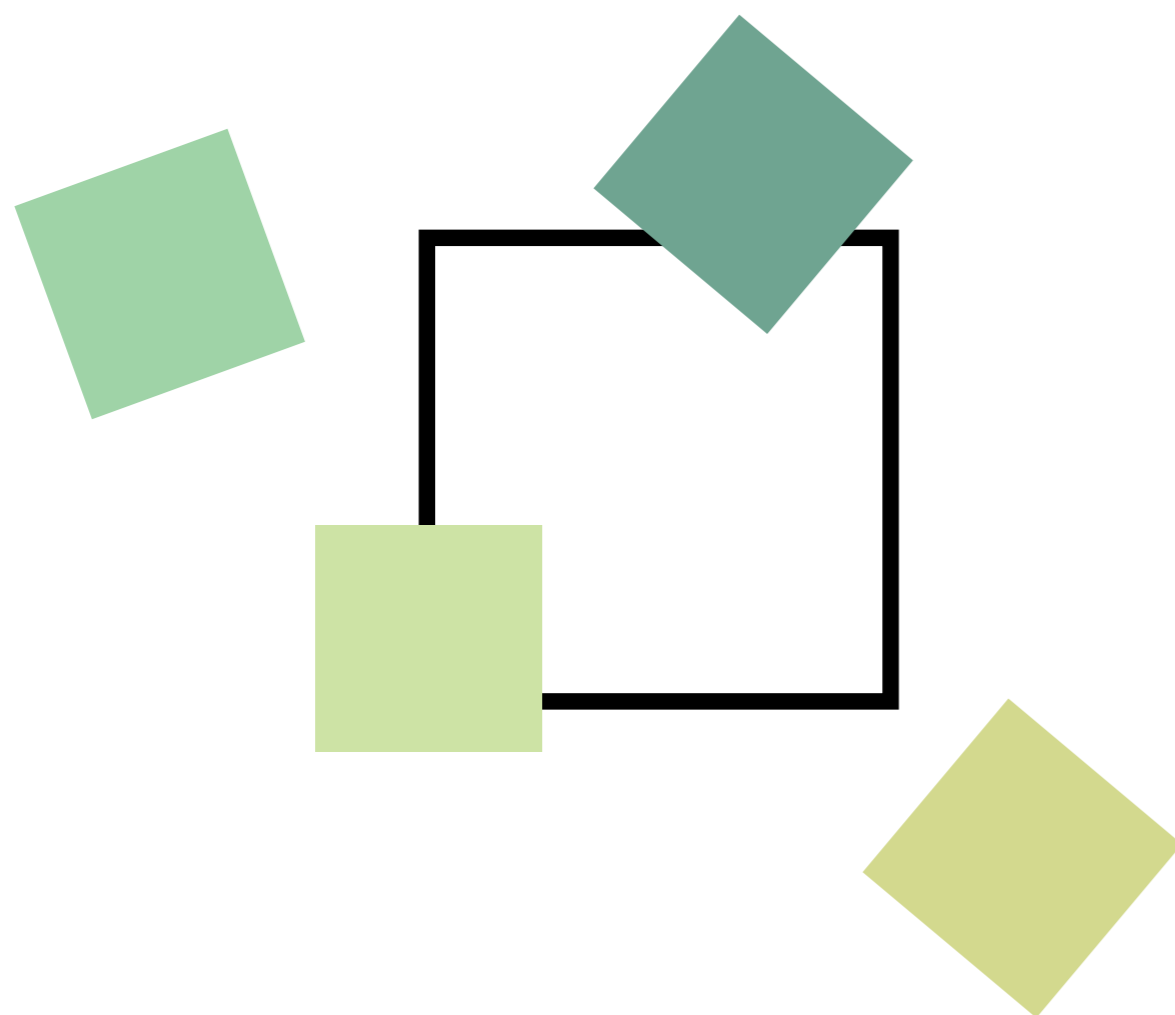


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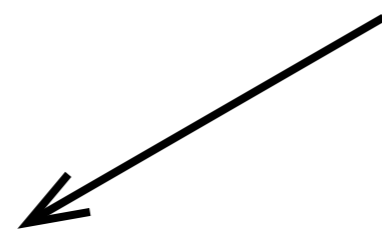
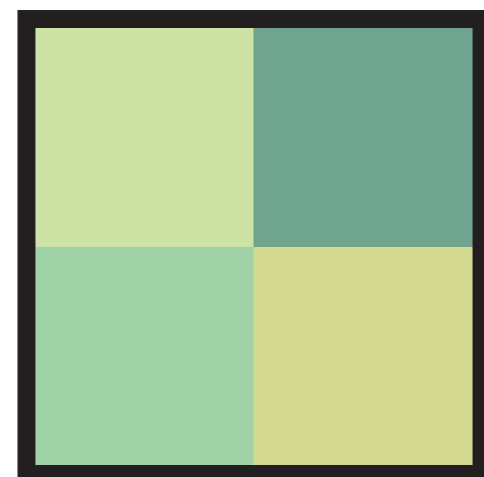
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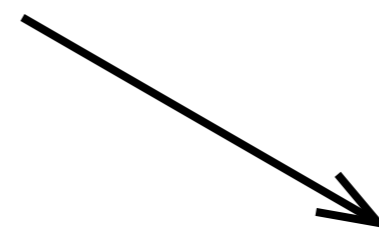
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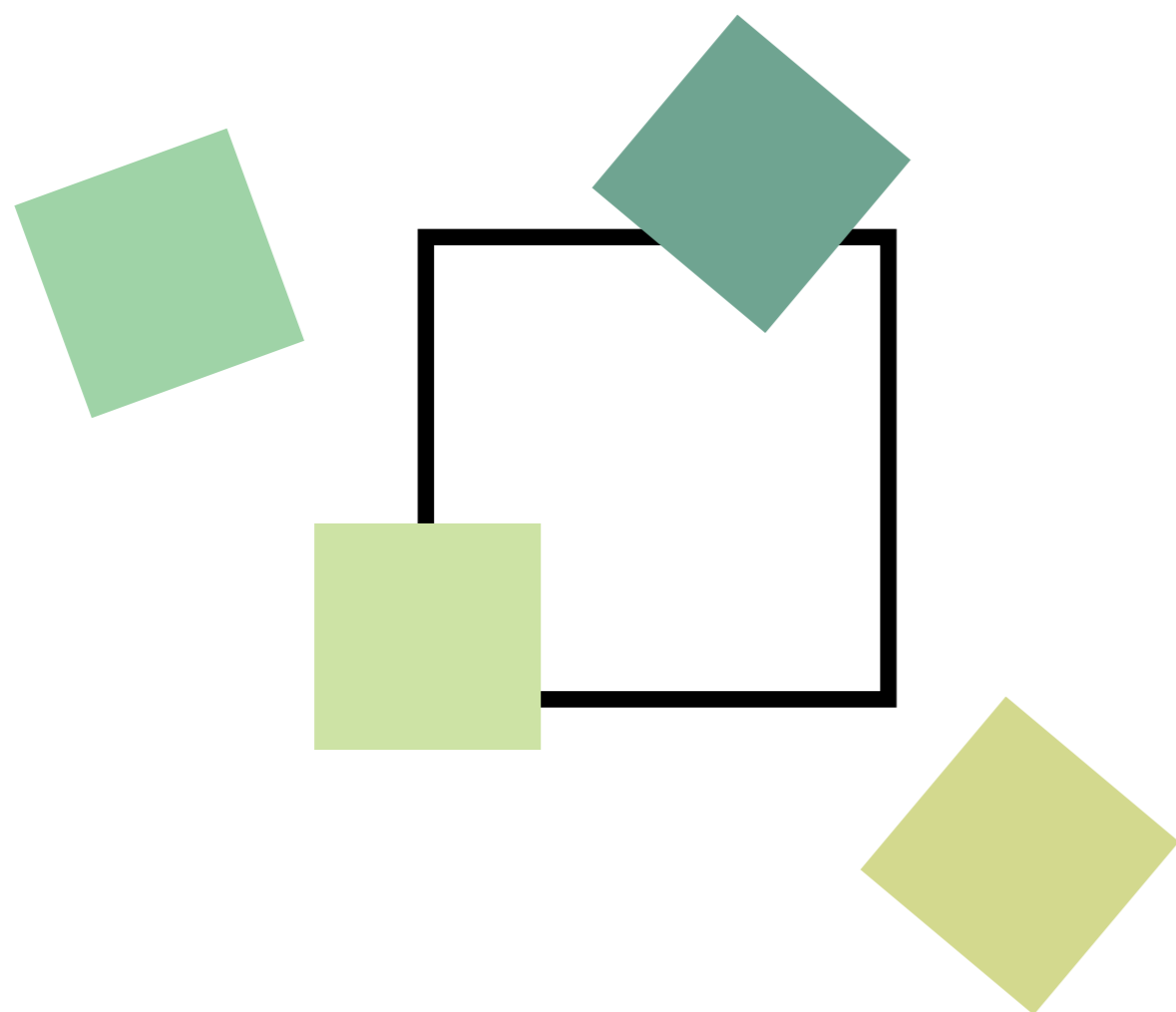
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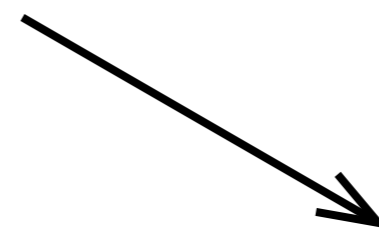
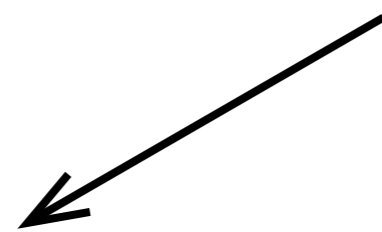
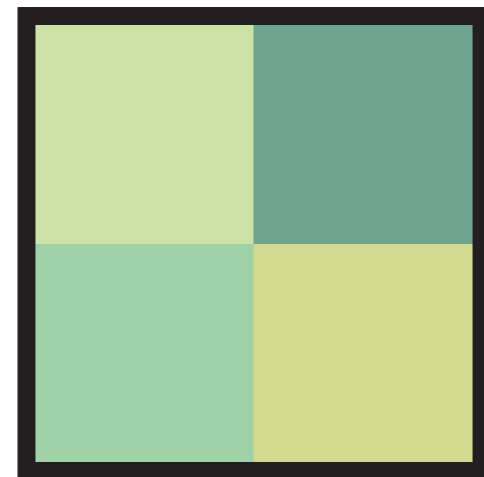
u



$$q = h(u)$$

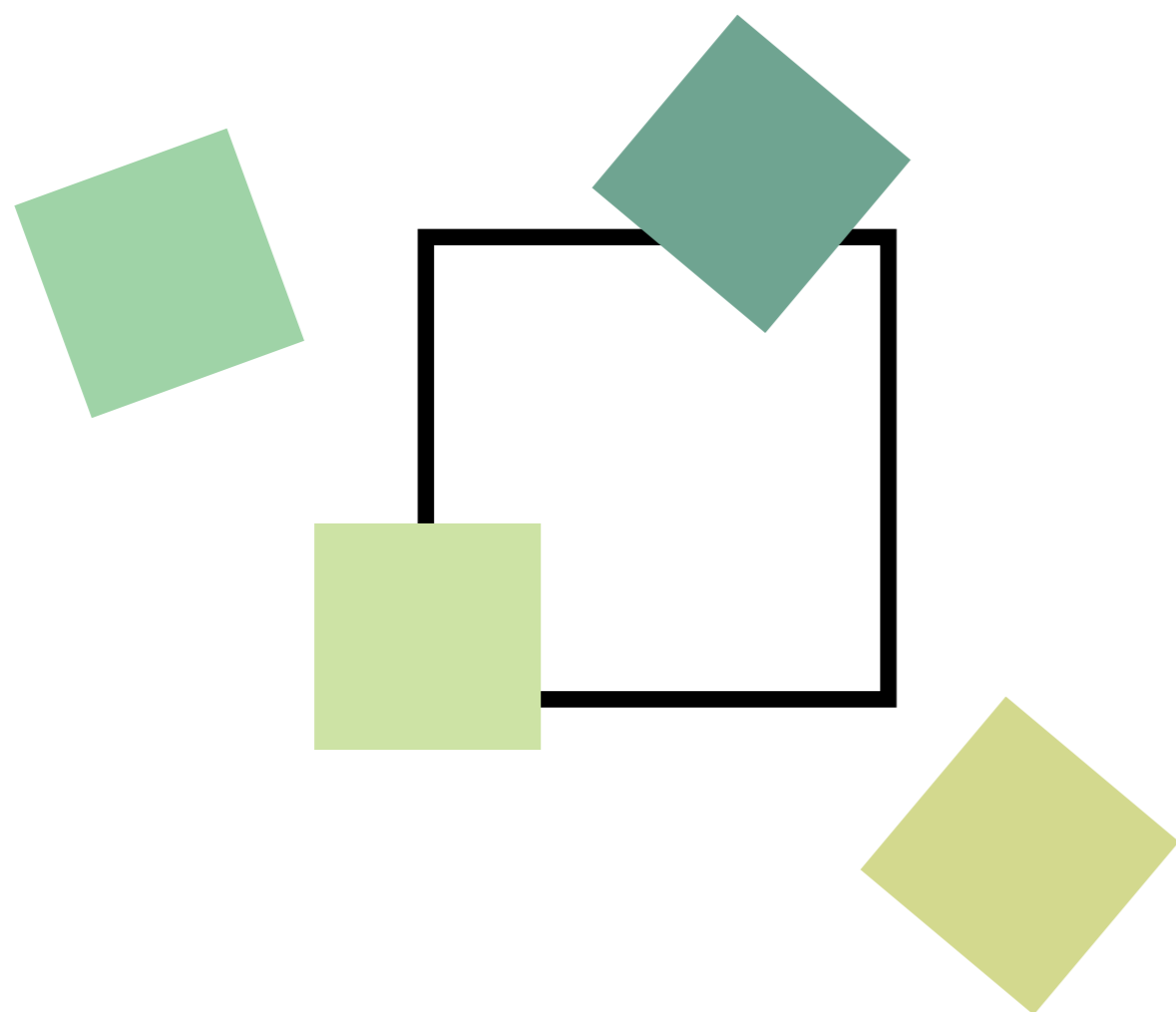


Alternatives



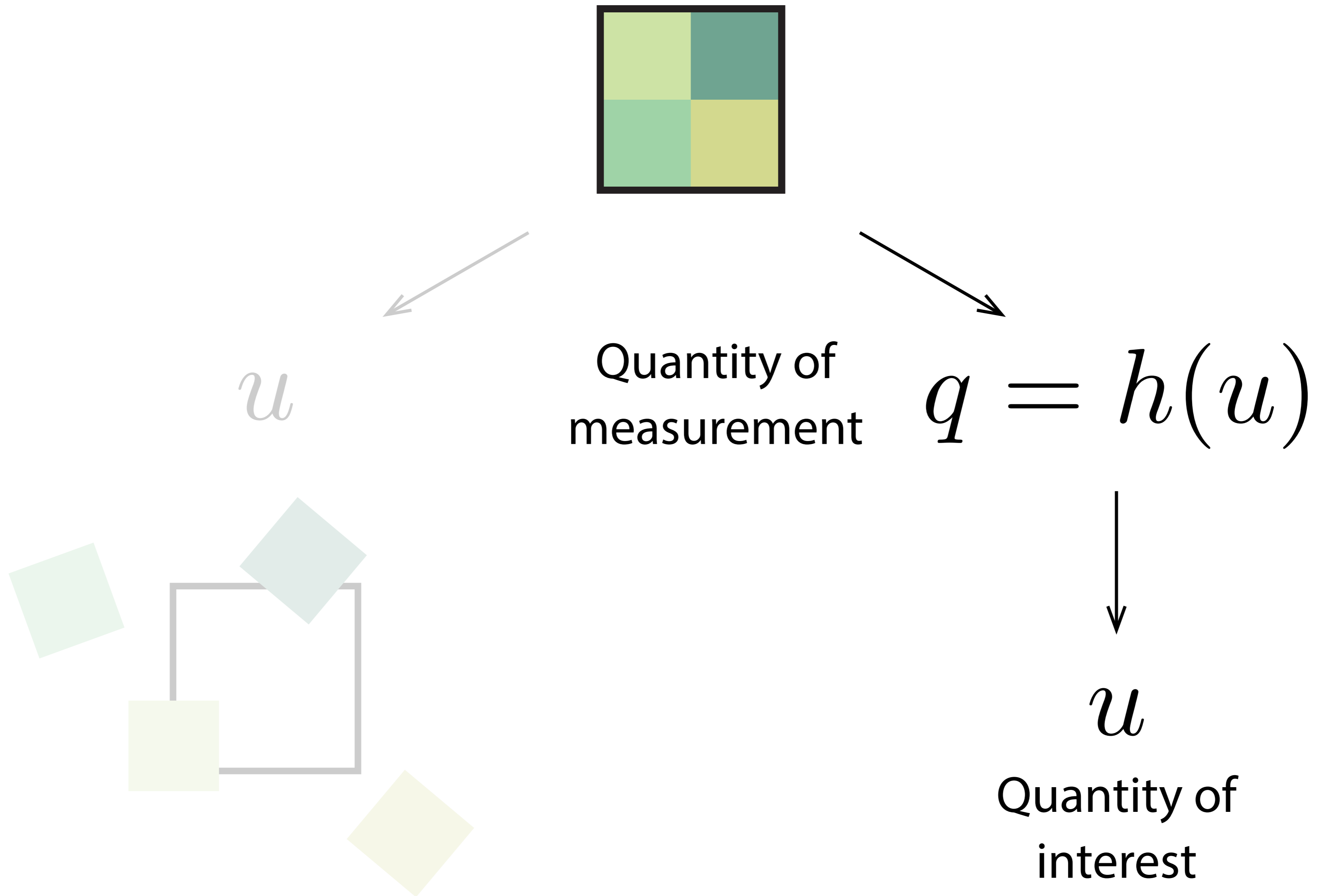
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$$q = h(u)$$

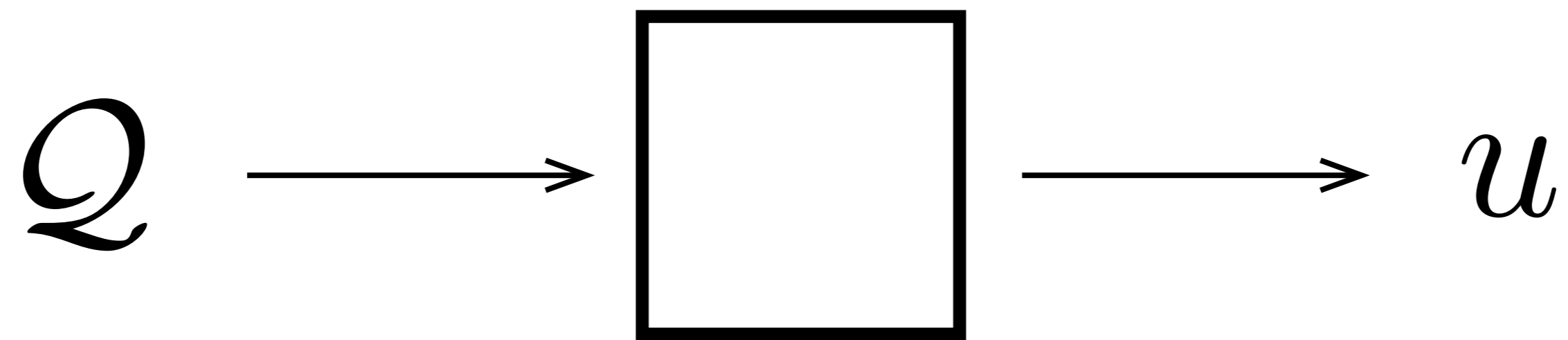


u ?

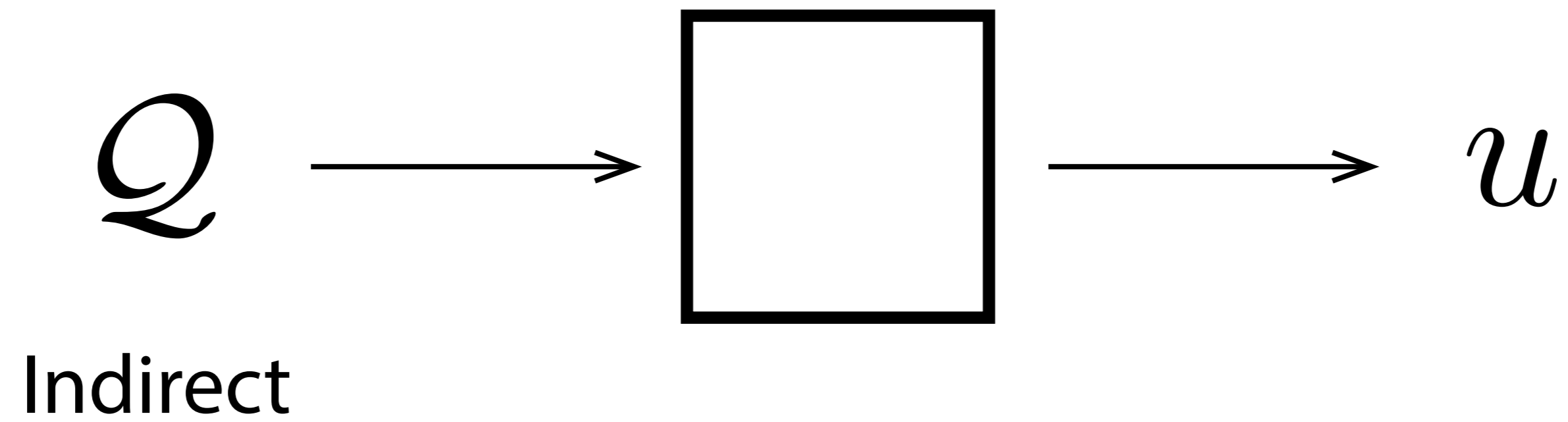
Proposed Framework



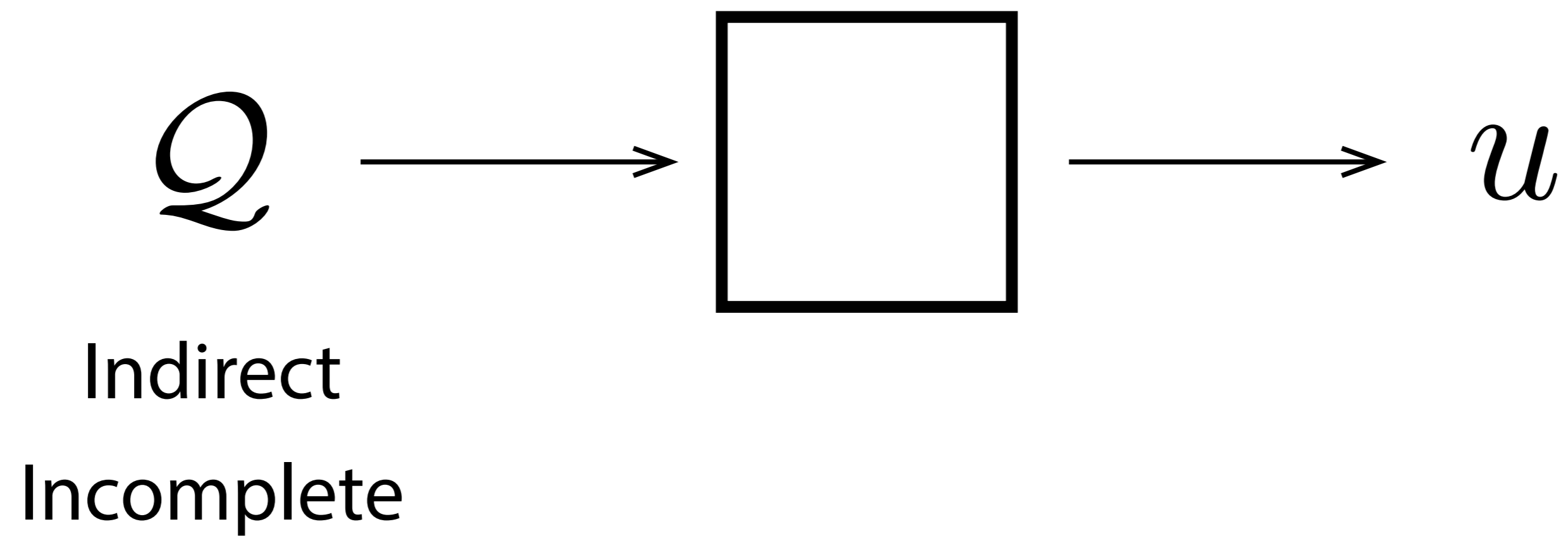
Proposed Framework



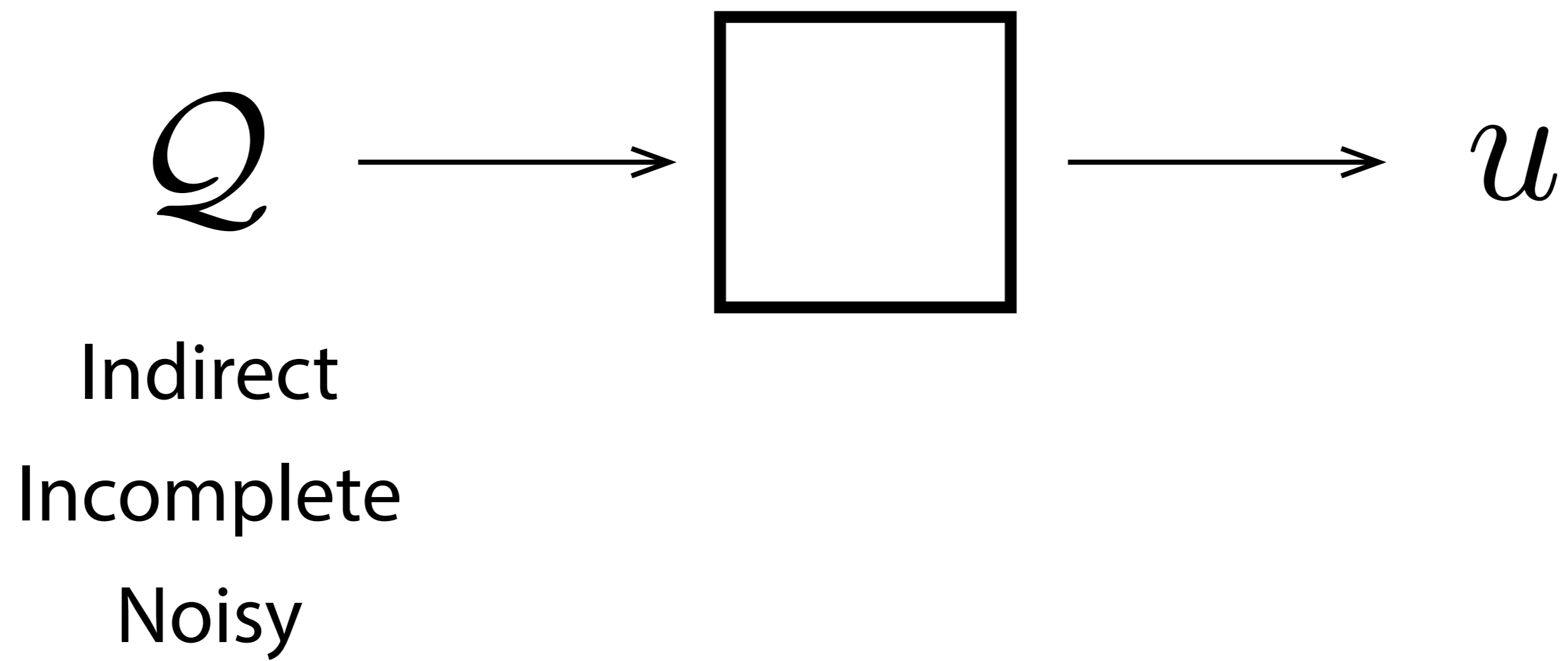
Proposed Framework



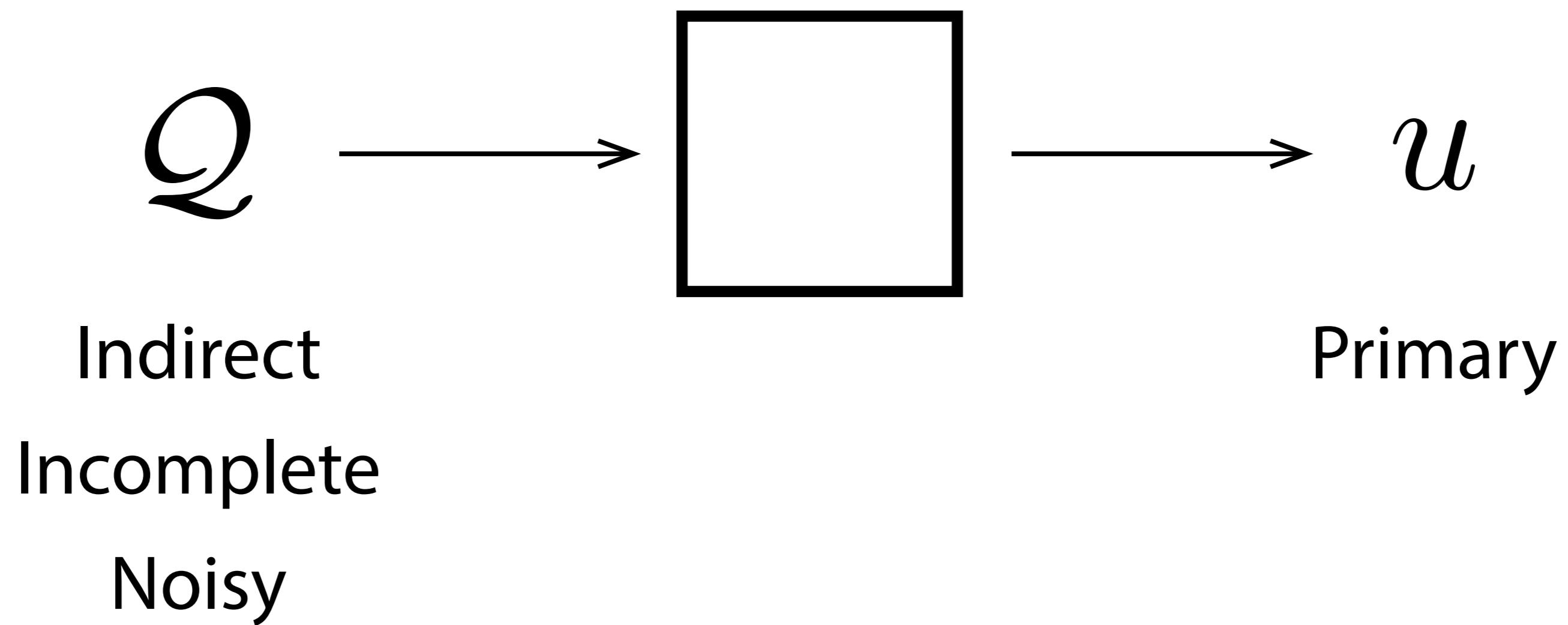
Proposed Framework



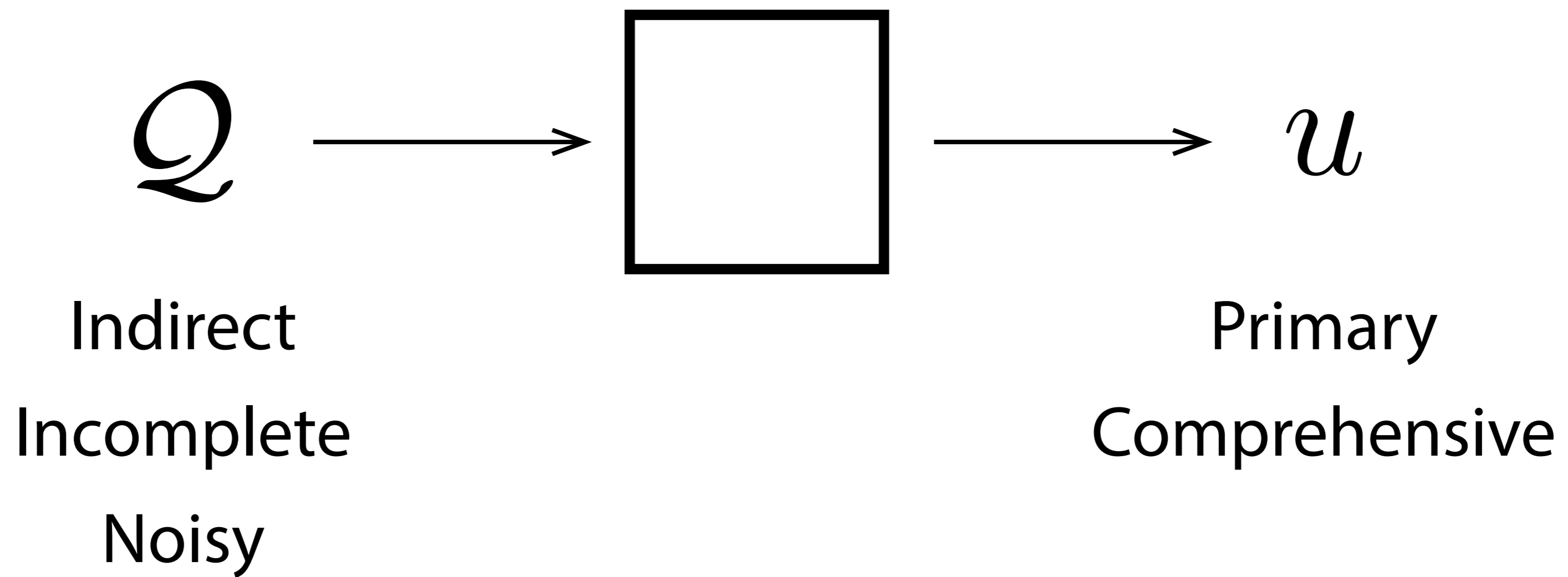
Proposed Framework



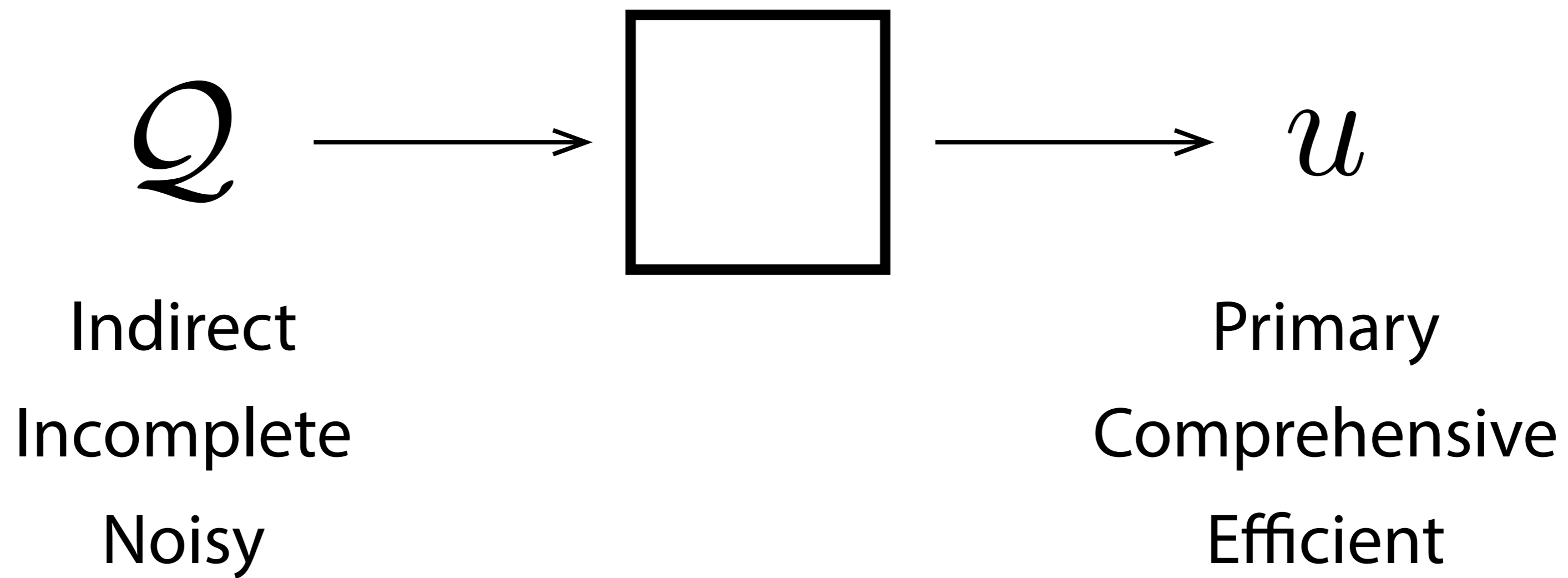
Proposed Framework



Proposed Framework



Proposed Framework



Illustrative Example

u Effective channel length

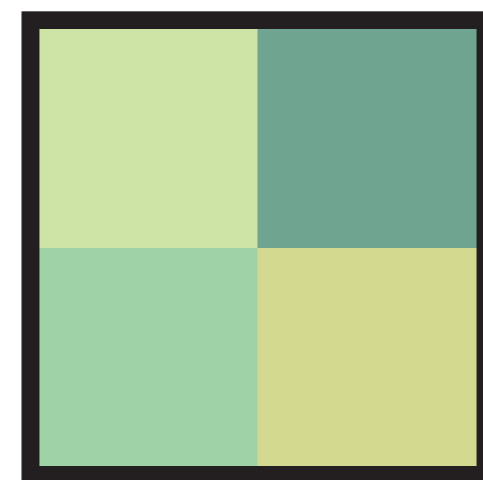
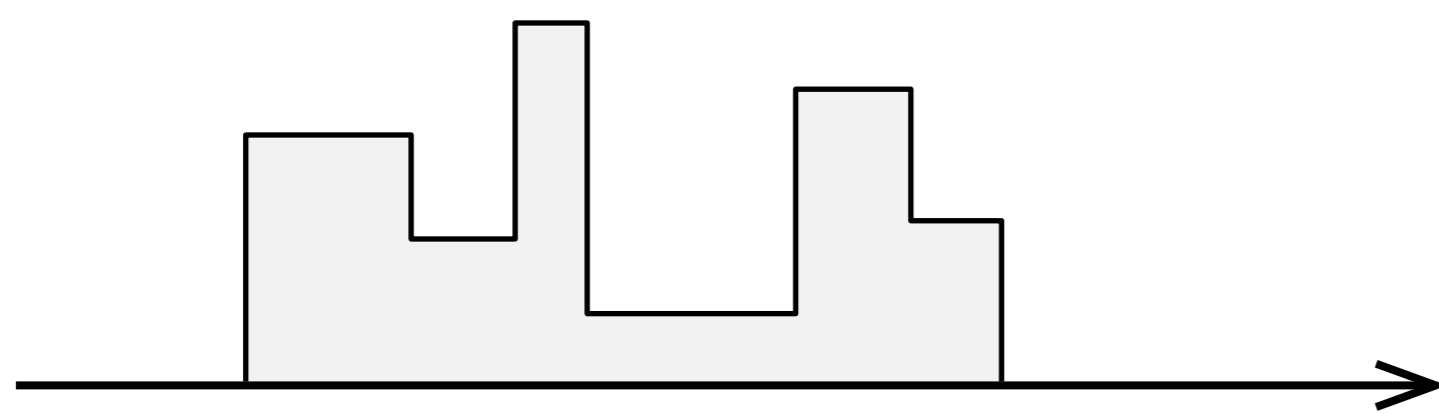
q Temperature

Illustrative Example

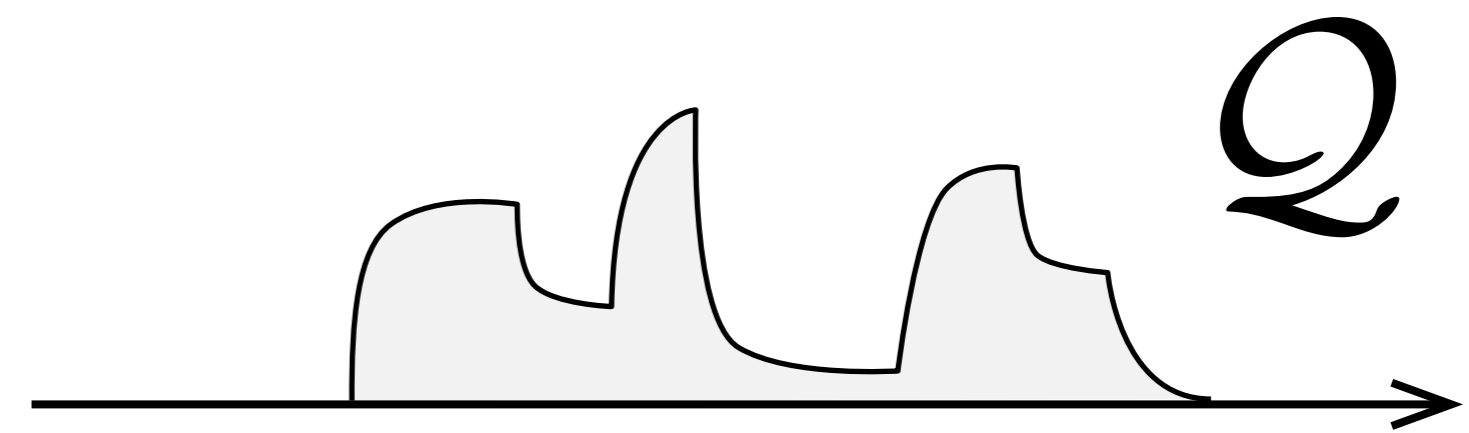
u Effective channel length

q Temperature

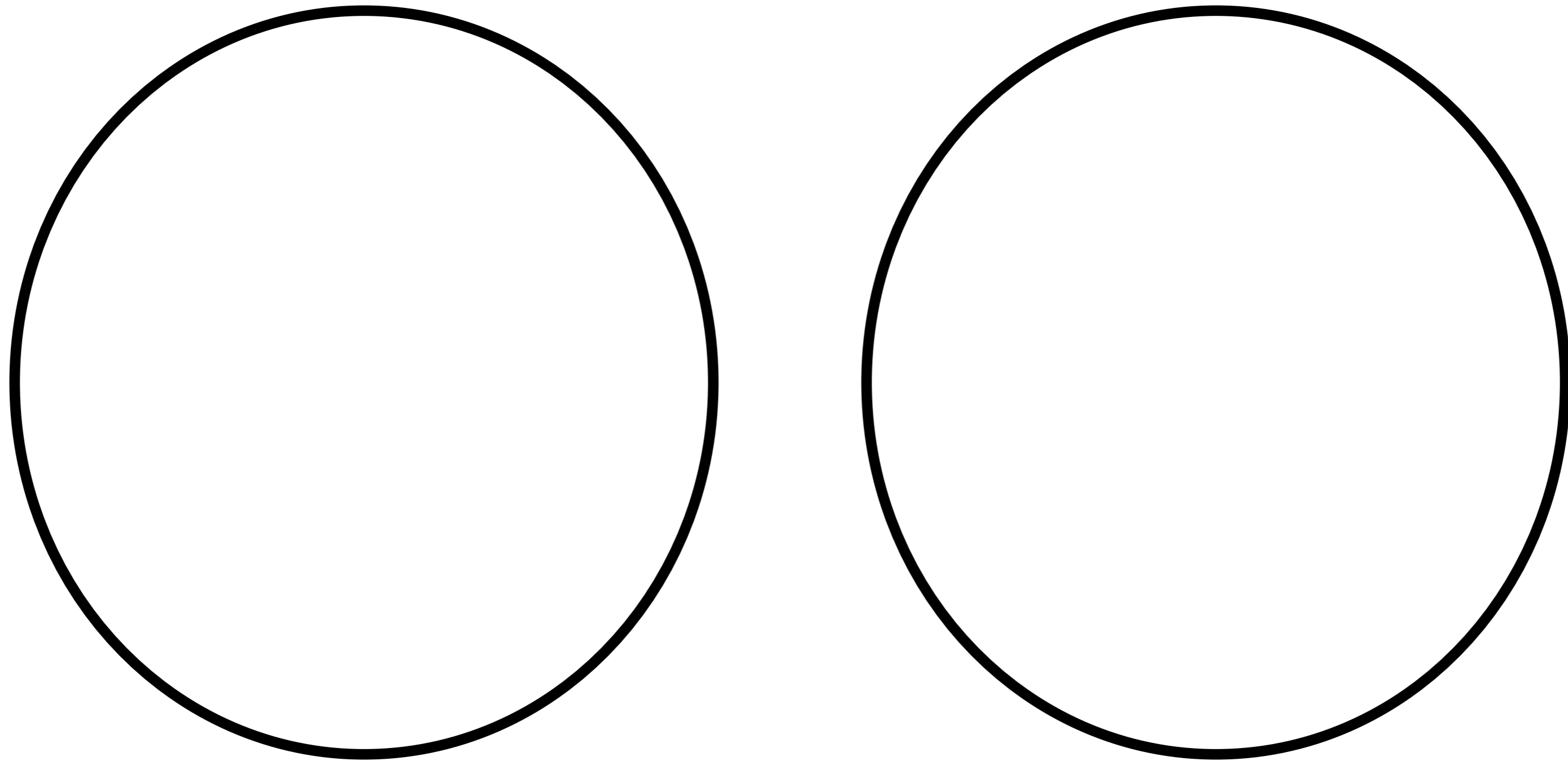
Workload



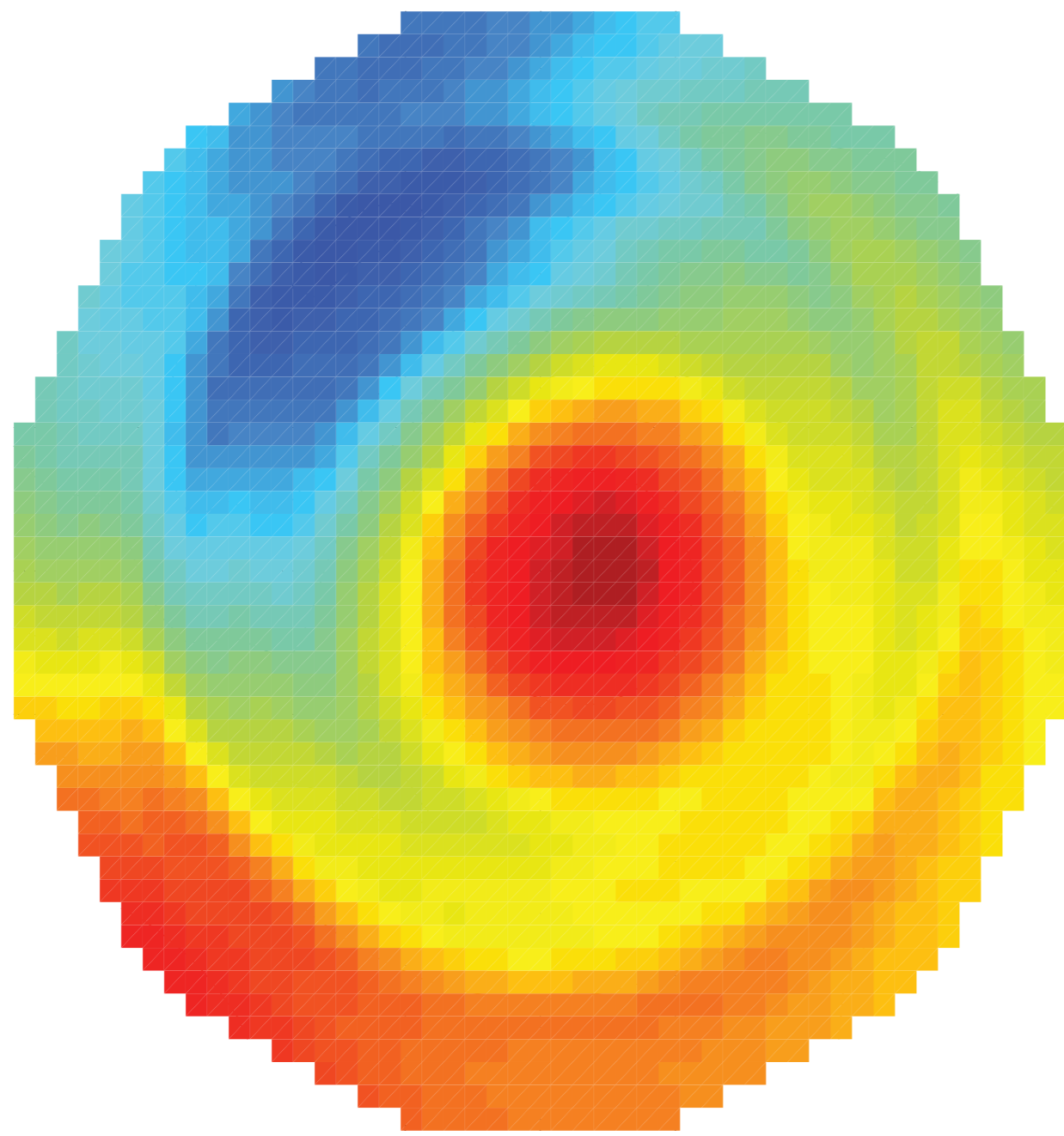
Temperature



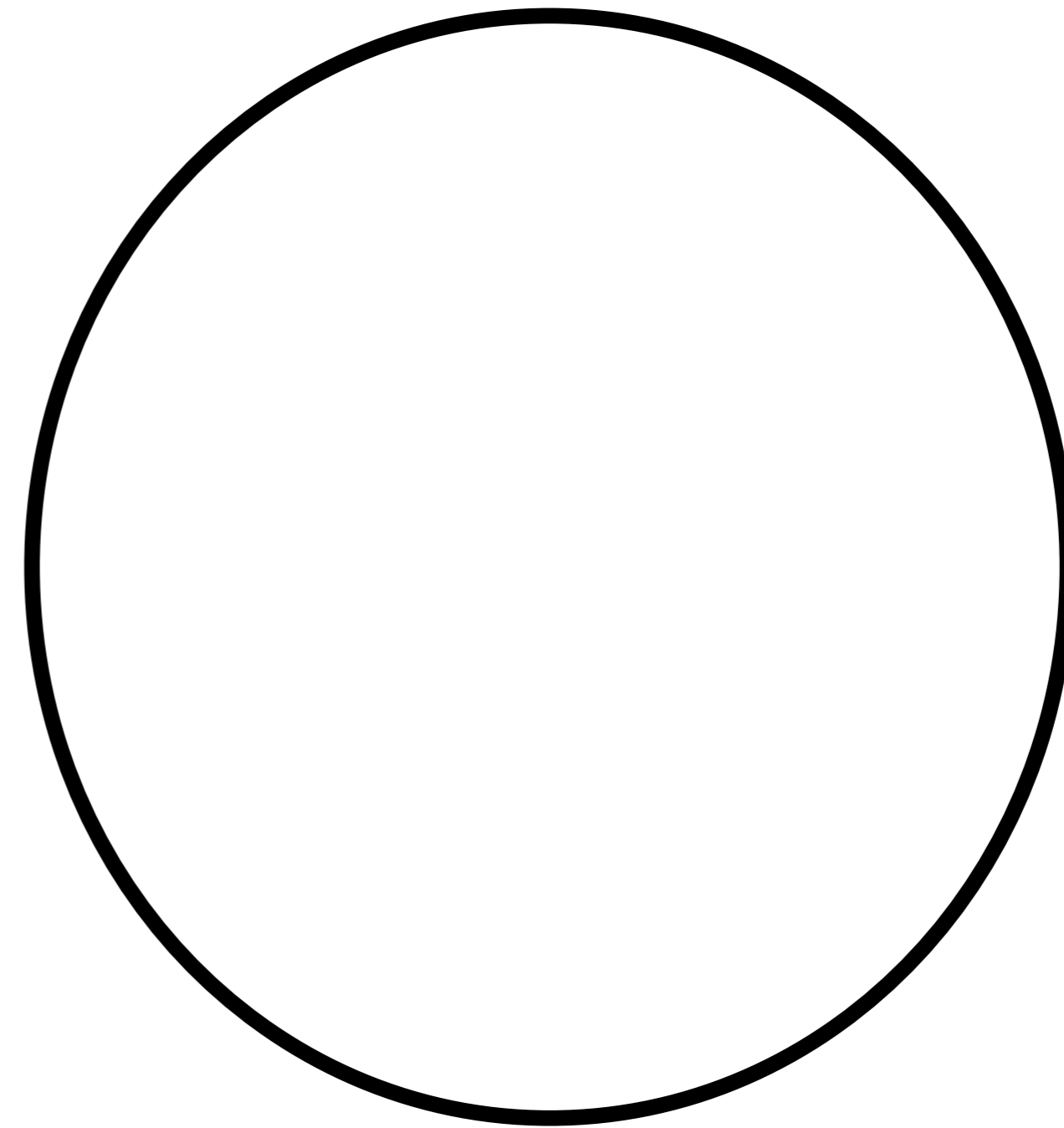
Illustrative Example



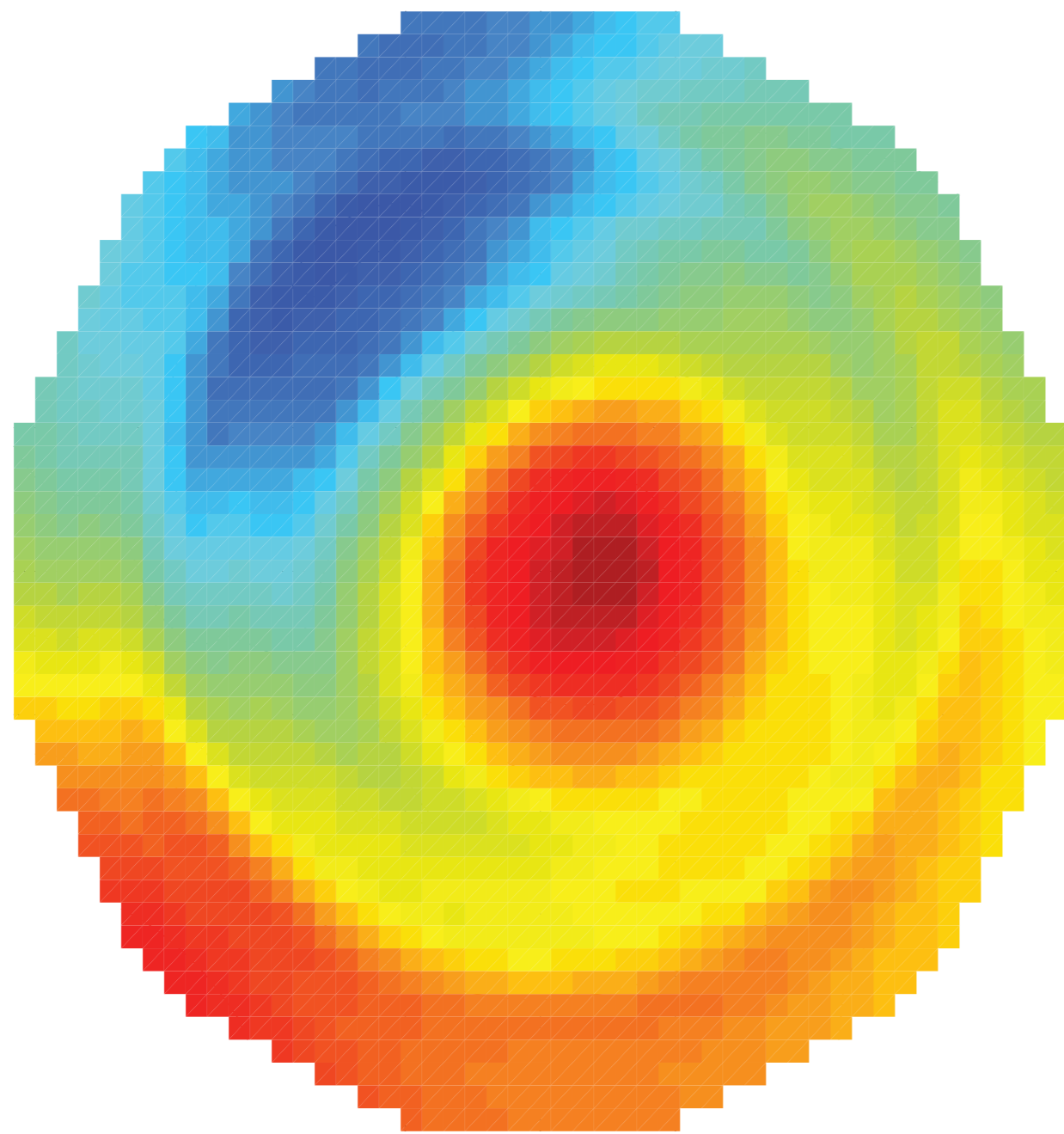
Illustrative Example



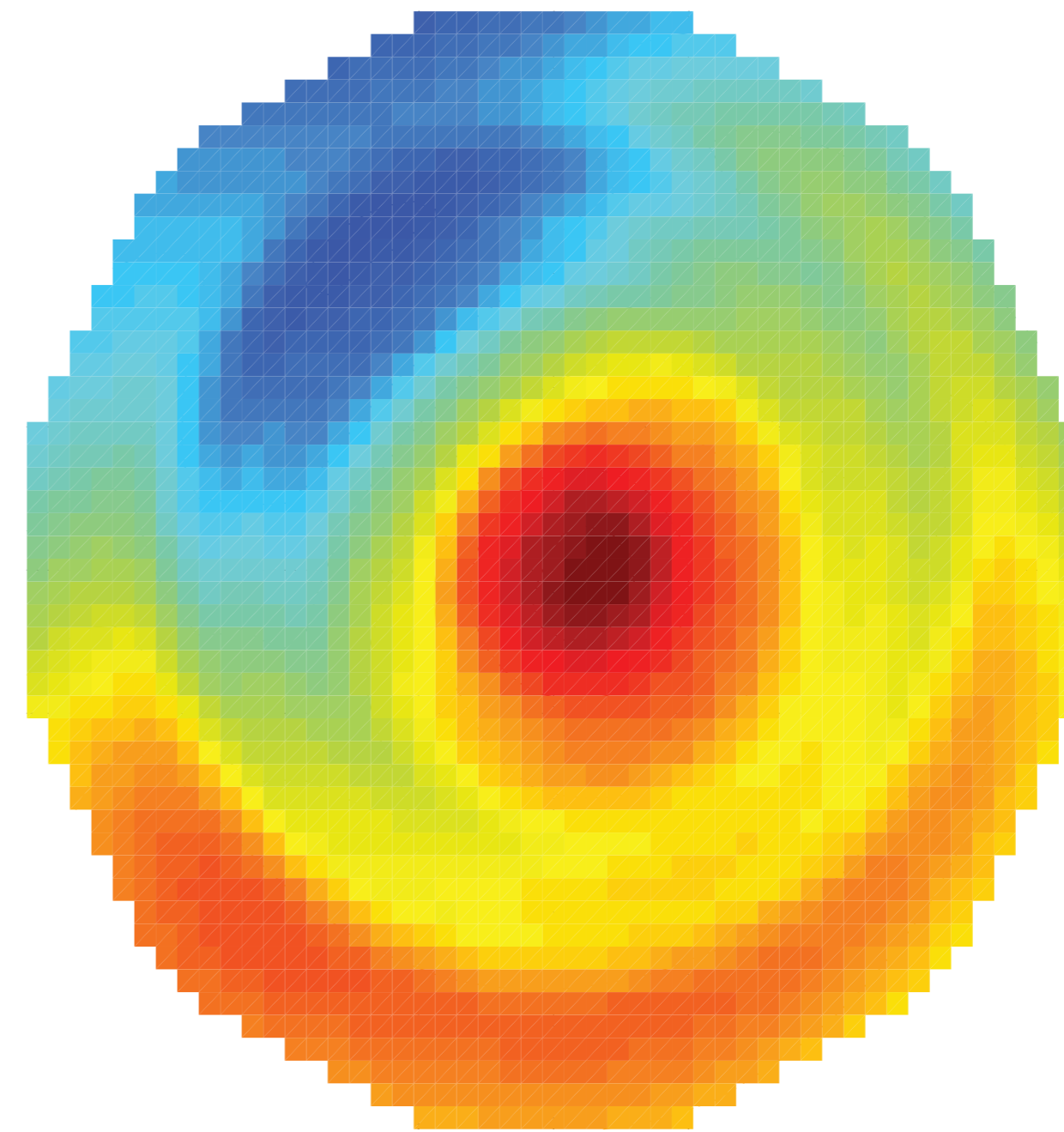
True



Illustrative Example

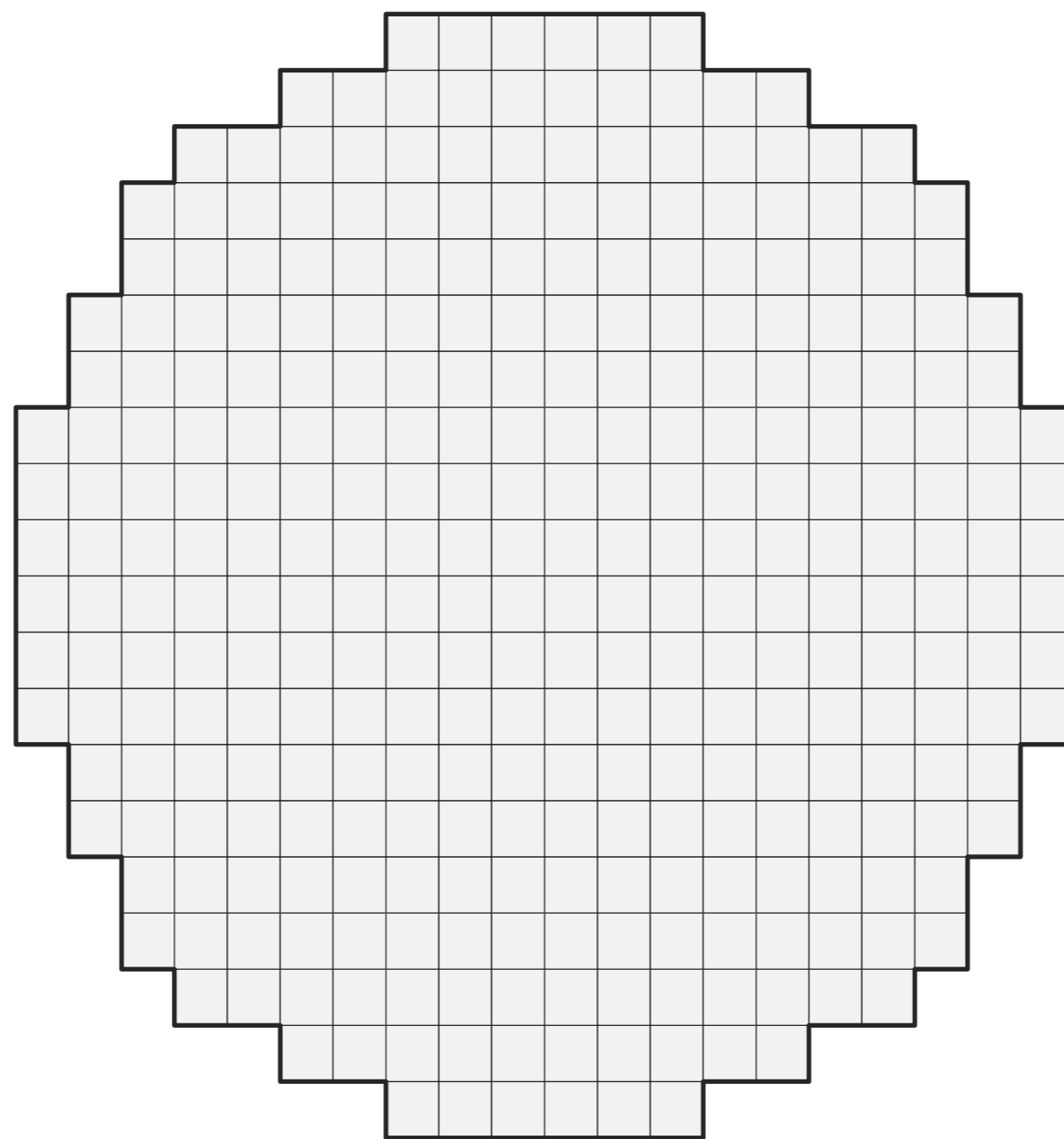


True



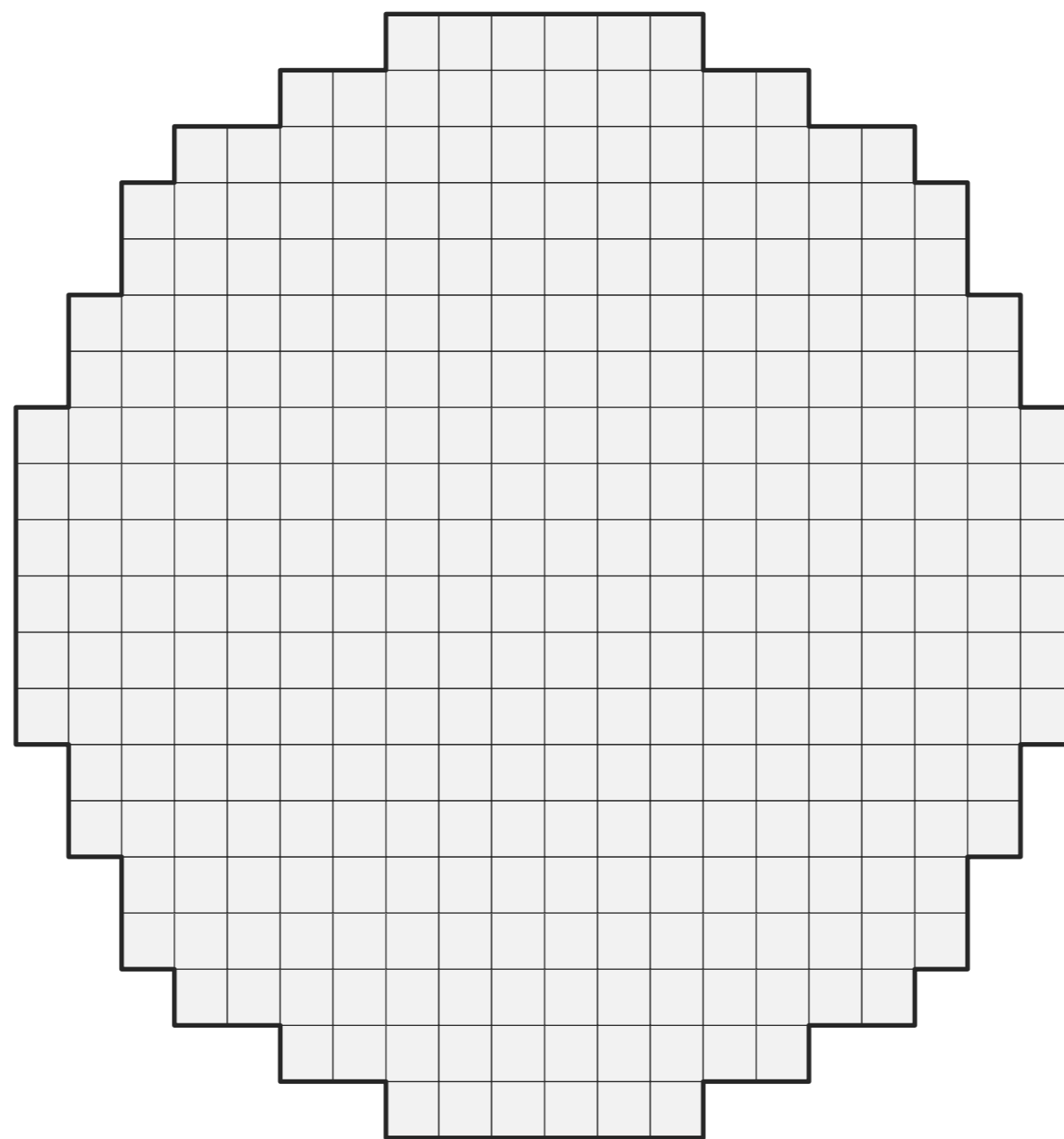
Inferred

Decision Making



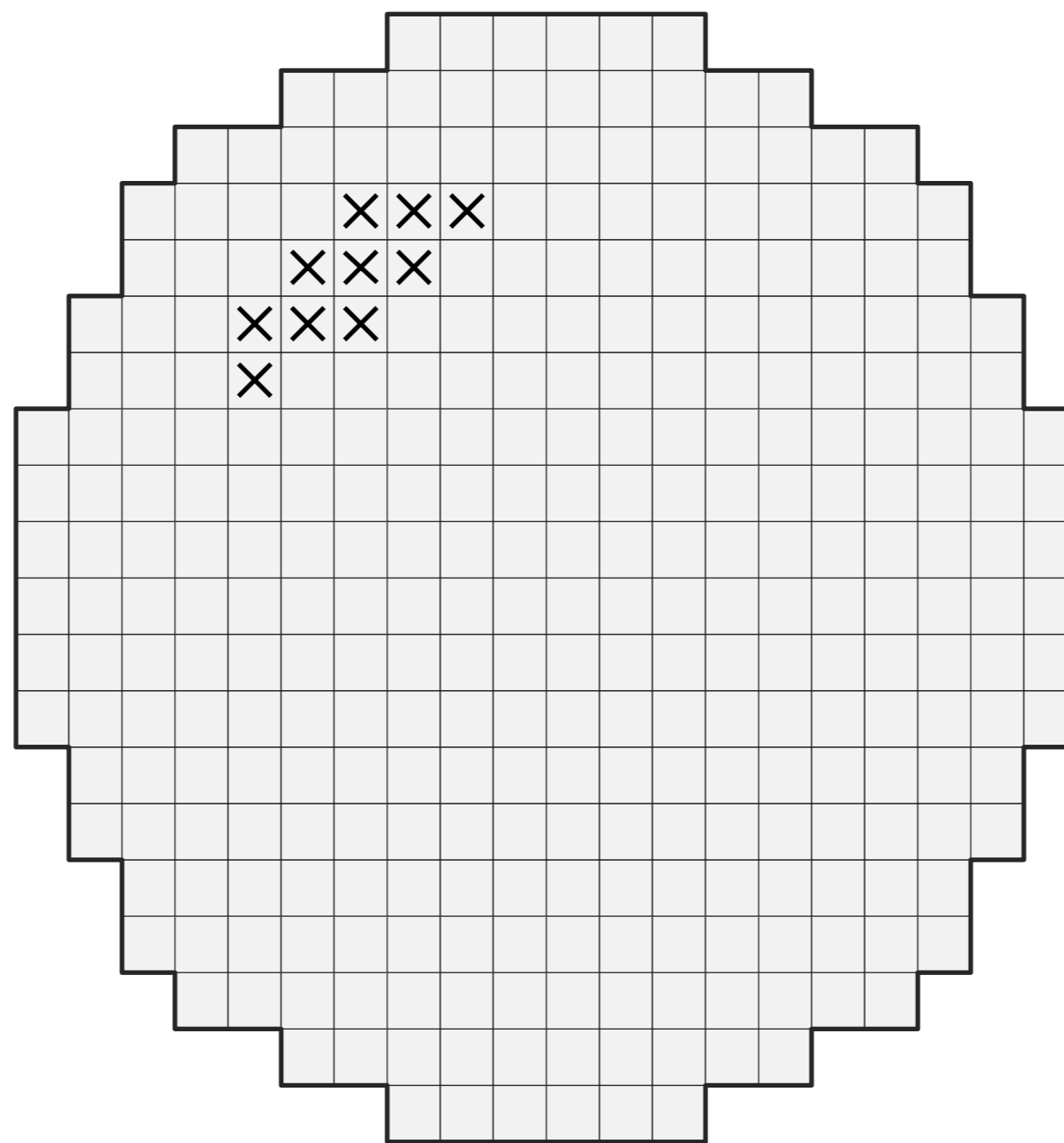
Decision Making

$$u > u_*$$



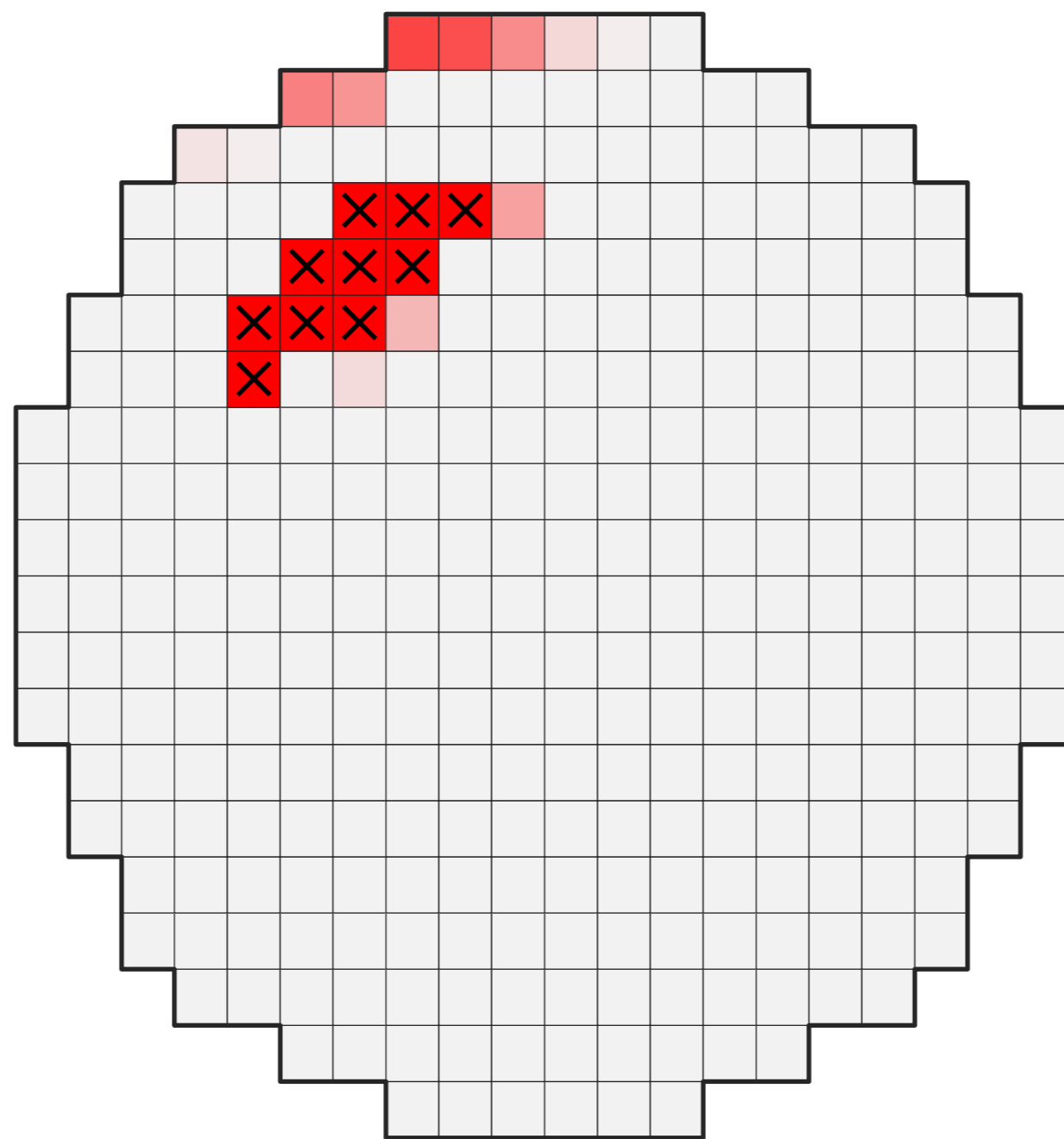
Decision Making

$$u < u_*$$

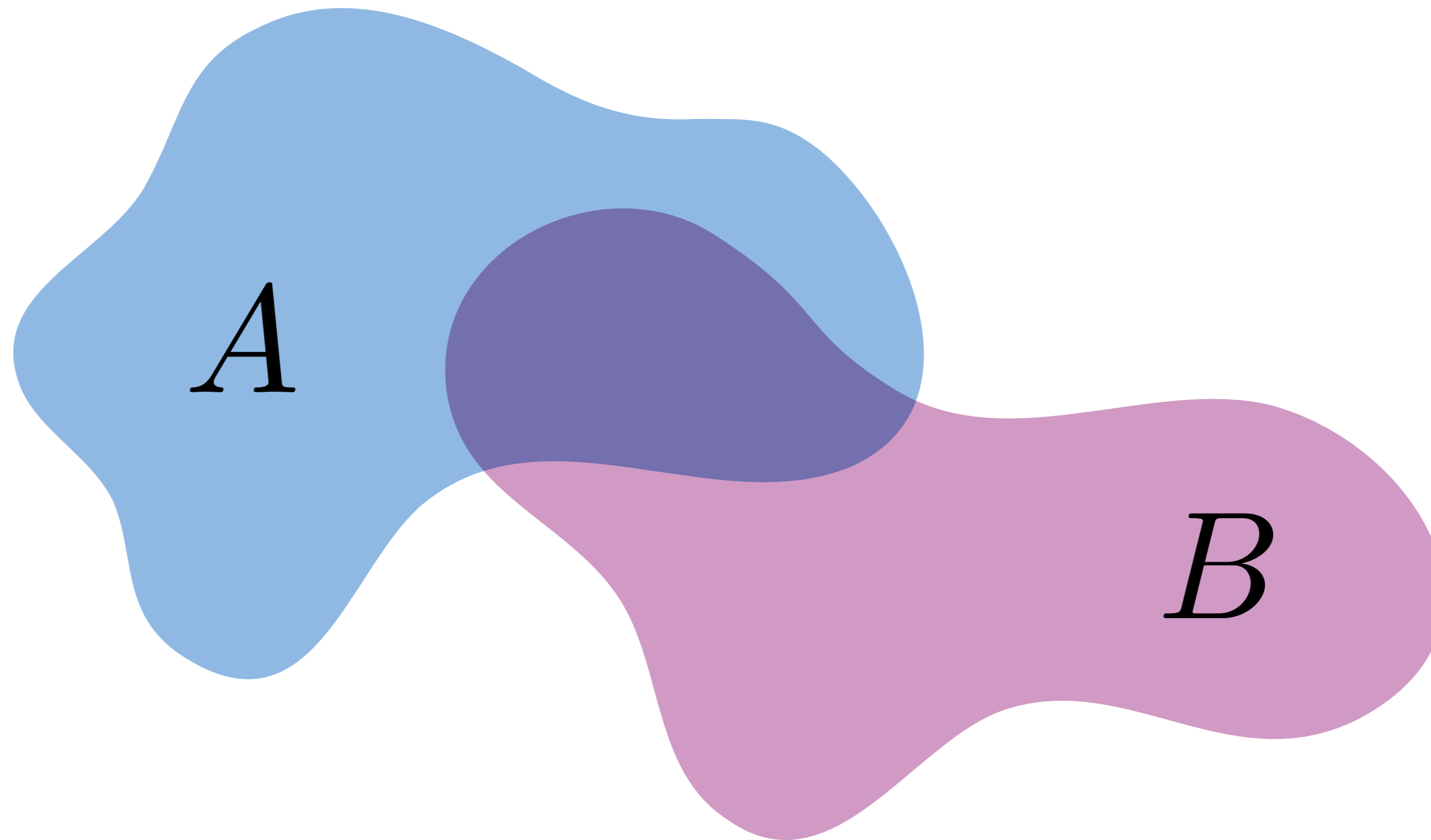


Decision Making

$$P(u < u_*)$$

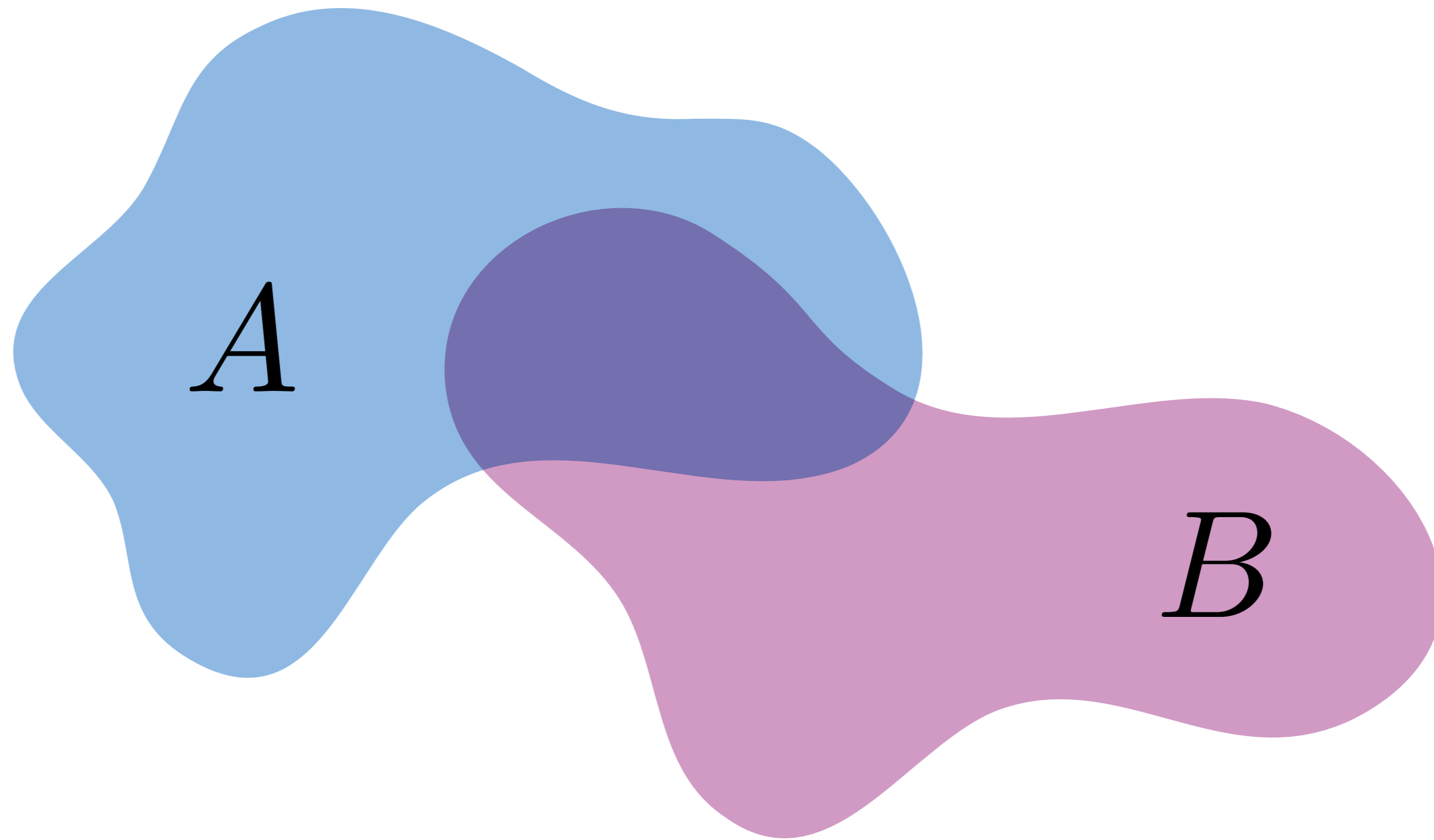


Bayesian Inference



Bayesian Inference

$$p(A|B) = ?$$



Bayesian Inference

$$p(A|B) \propto p(B|A) \times p(A)$$

Bayesian Inference

Posterior

Likelihood

Prior

$$p(A|B) \propto p(B|A) \times p(A)$$

Bayesian Inference

Posterior

Likelihood

Prior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$

Prior

$$p(u)$$

Prior

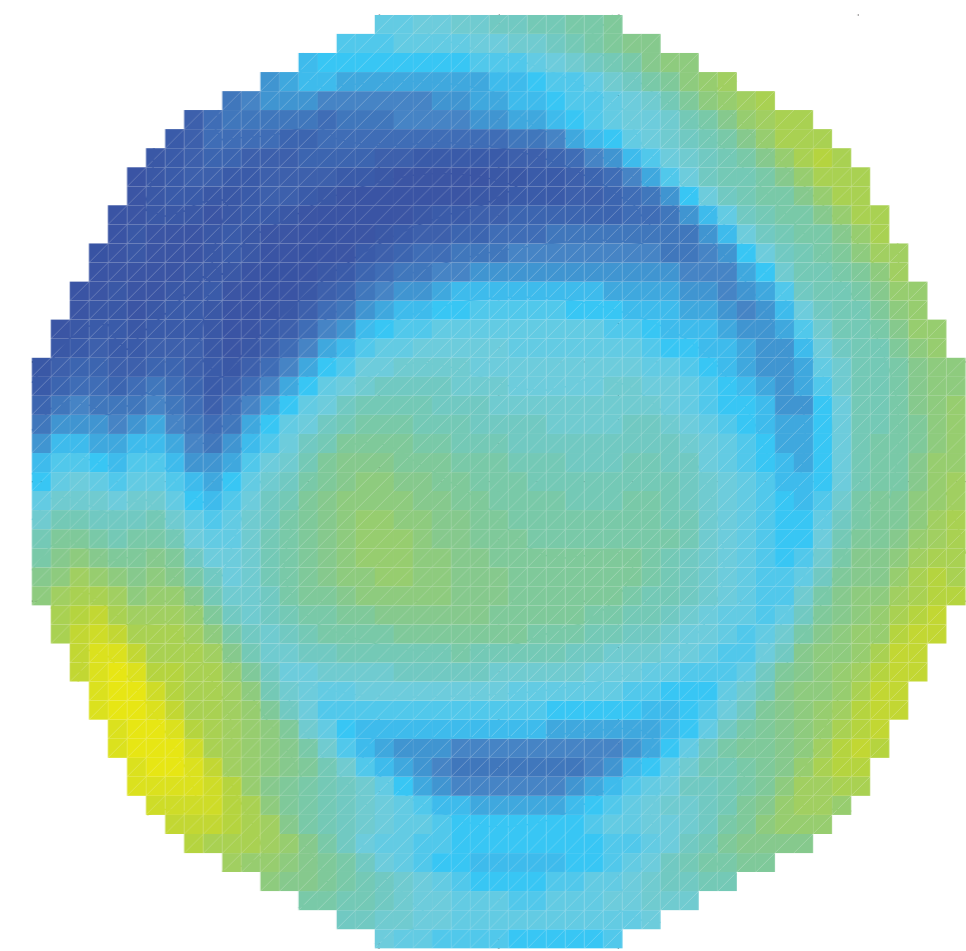
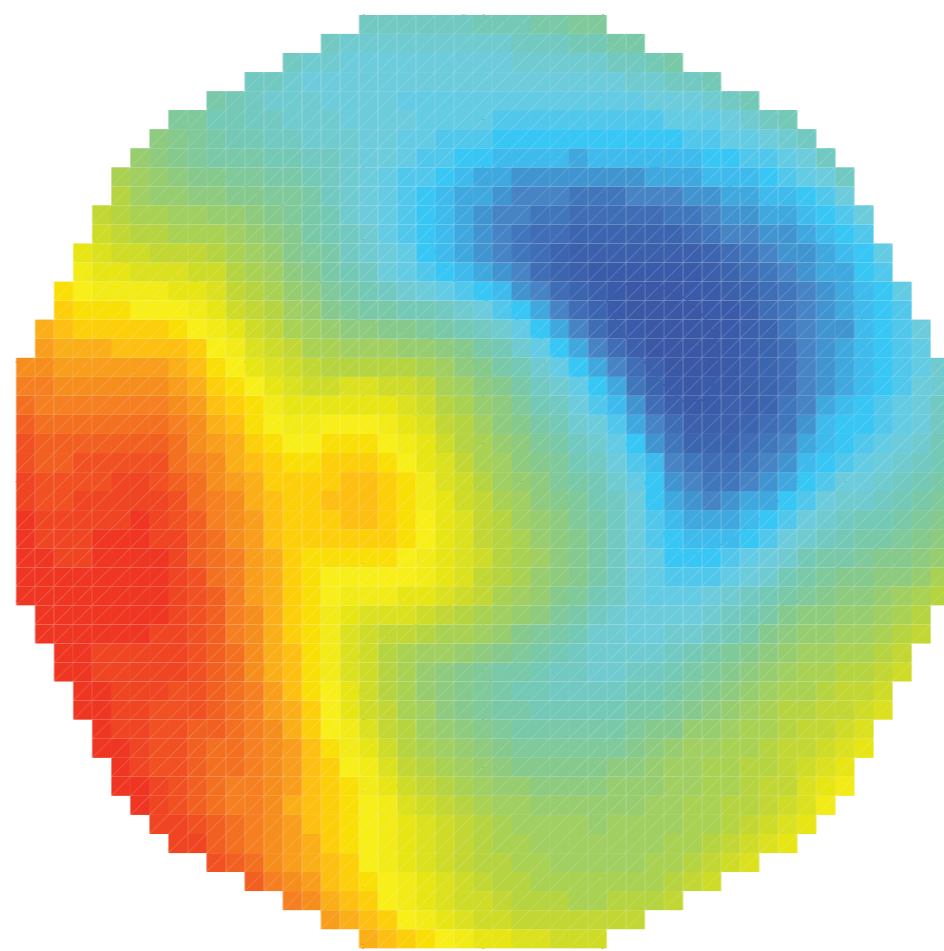
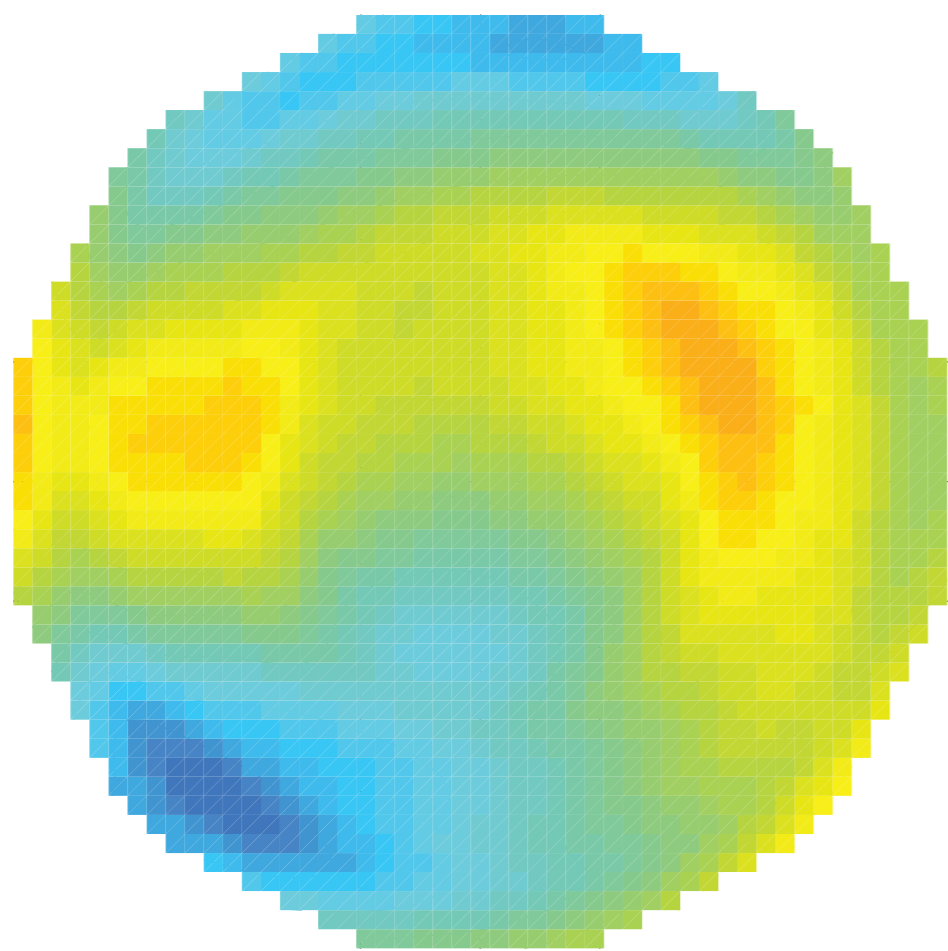
$$p(u)$$

$$u \sim \mathcal{GP}(\mu, k)$$

Prior

$$p(u)$$

$$u \sim \mathcal{GP}(\mu, k)$$



Likelihood

$$p(Q|u)$$

Likelihood

$$p(\mathcal{Q}|u)$$

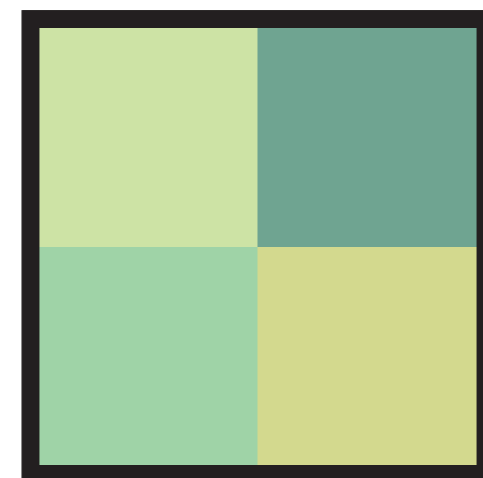
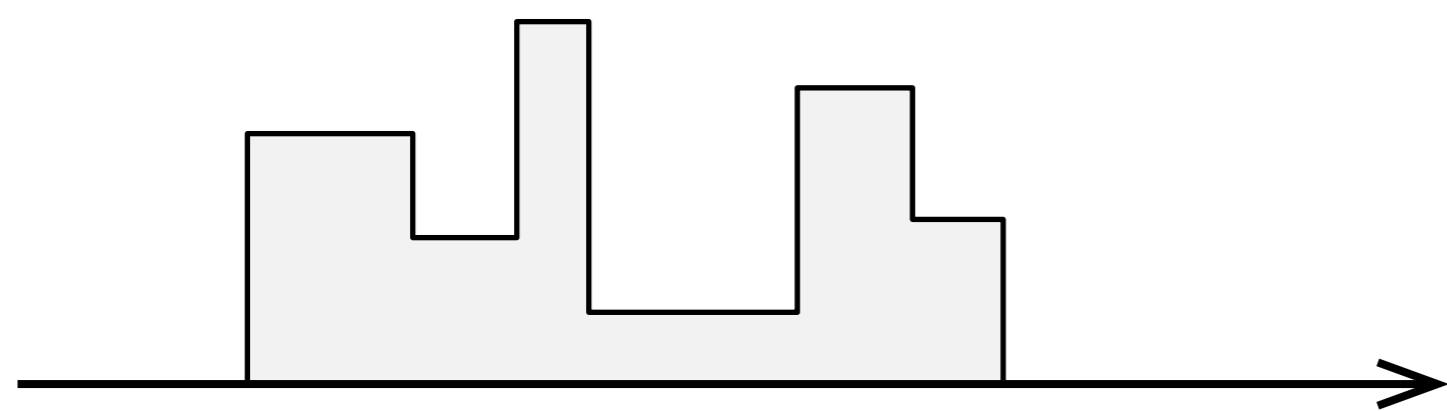
$$q = f(u)$$

Likelihood

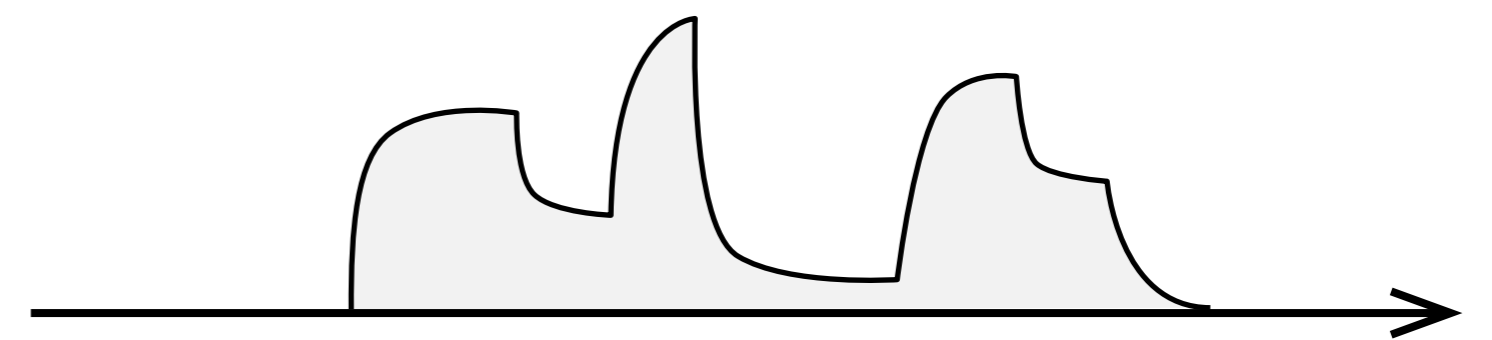
$$p(Q|u)$$

$$q = f(u)$$

Power



Temperature

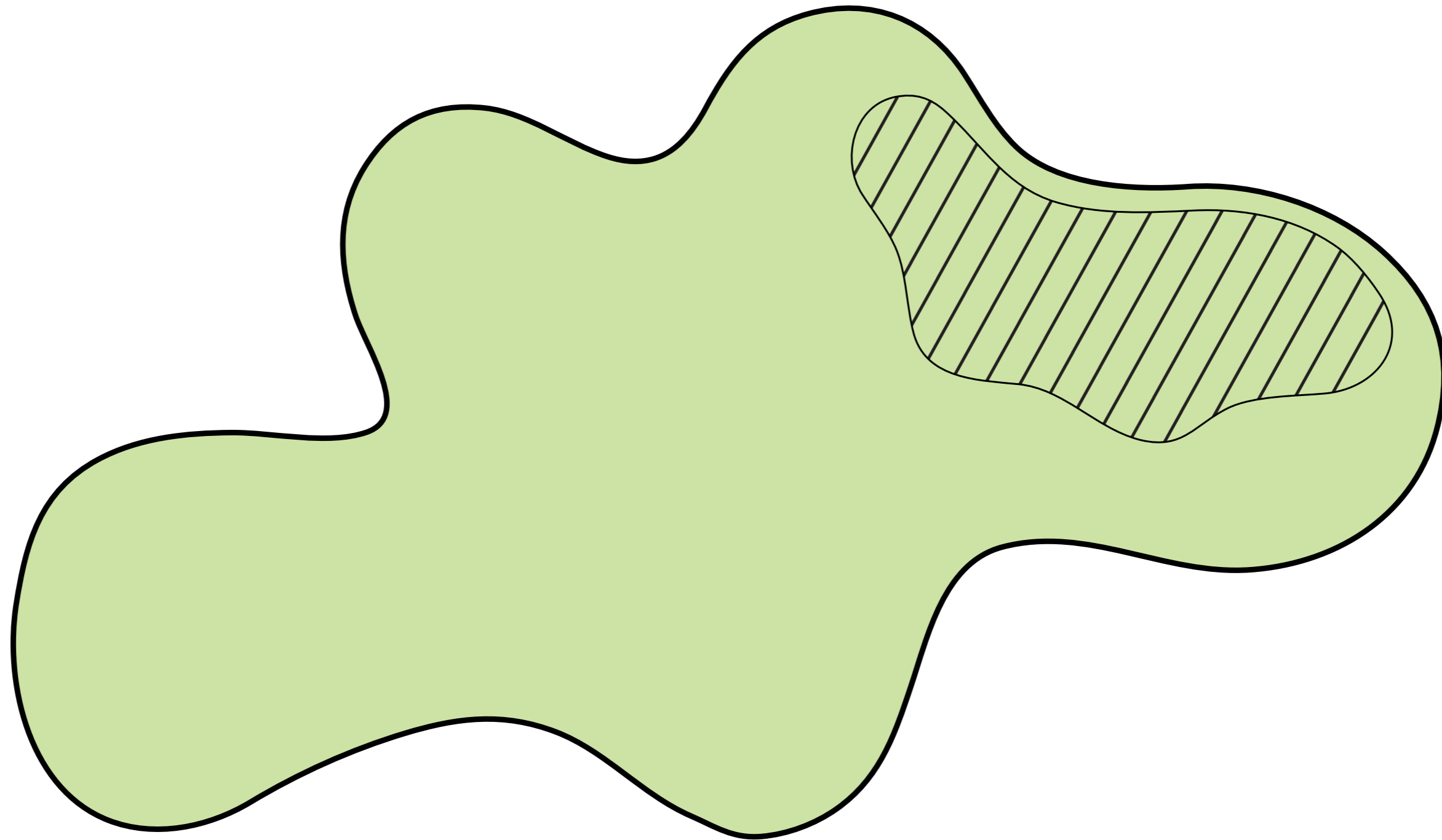


Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$

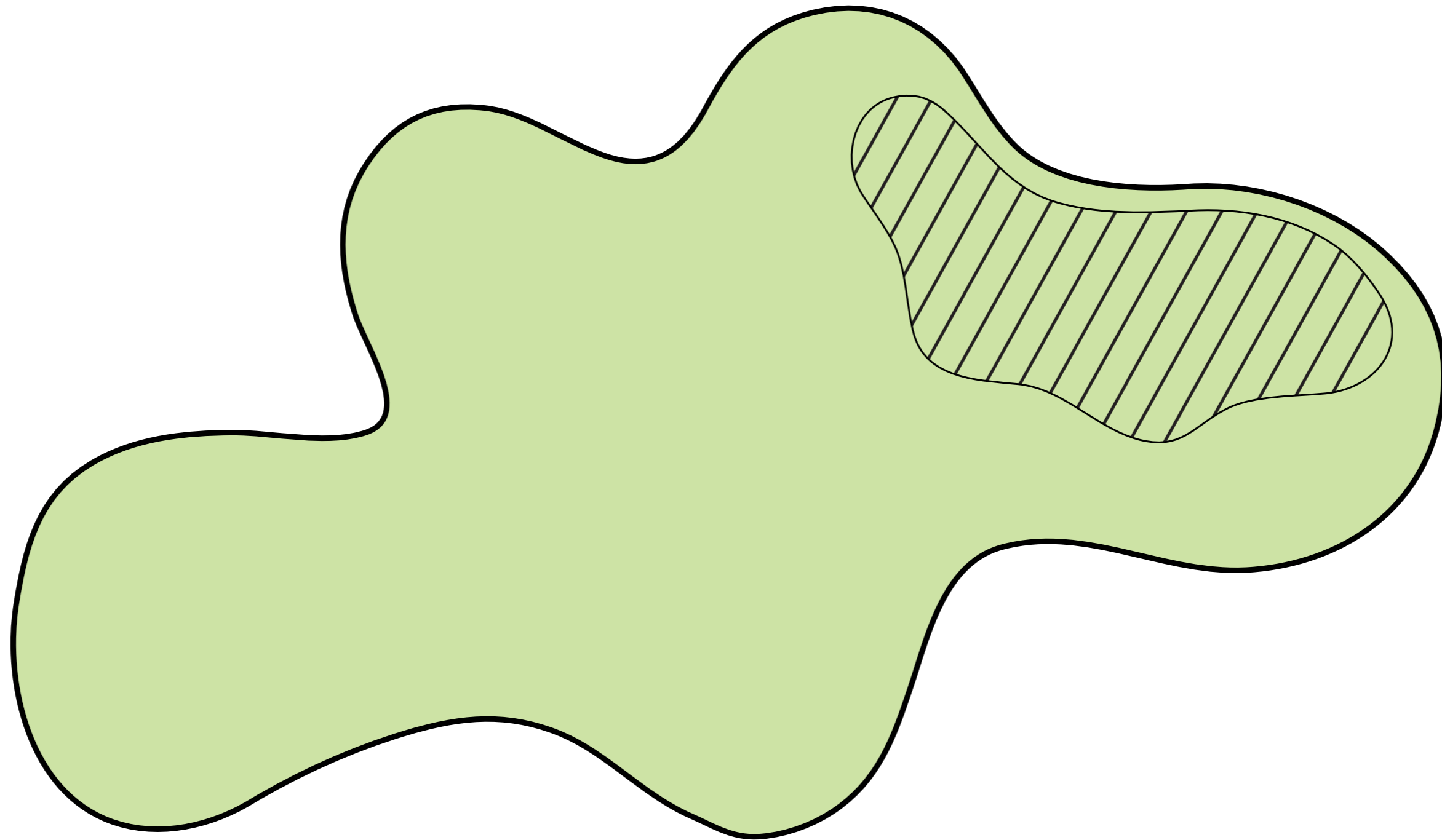
Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Posterior

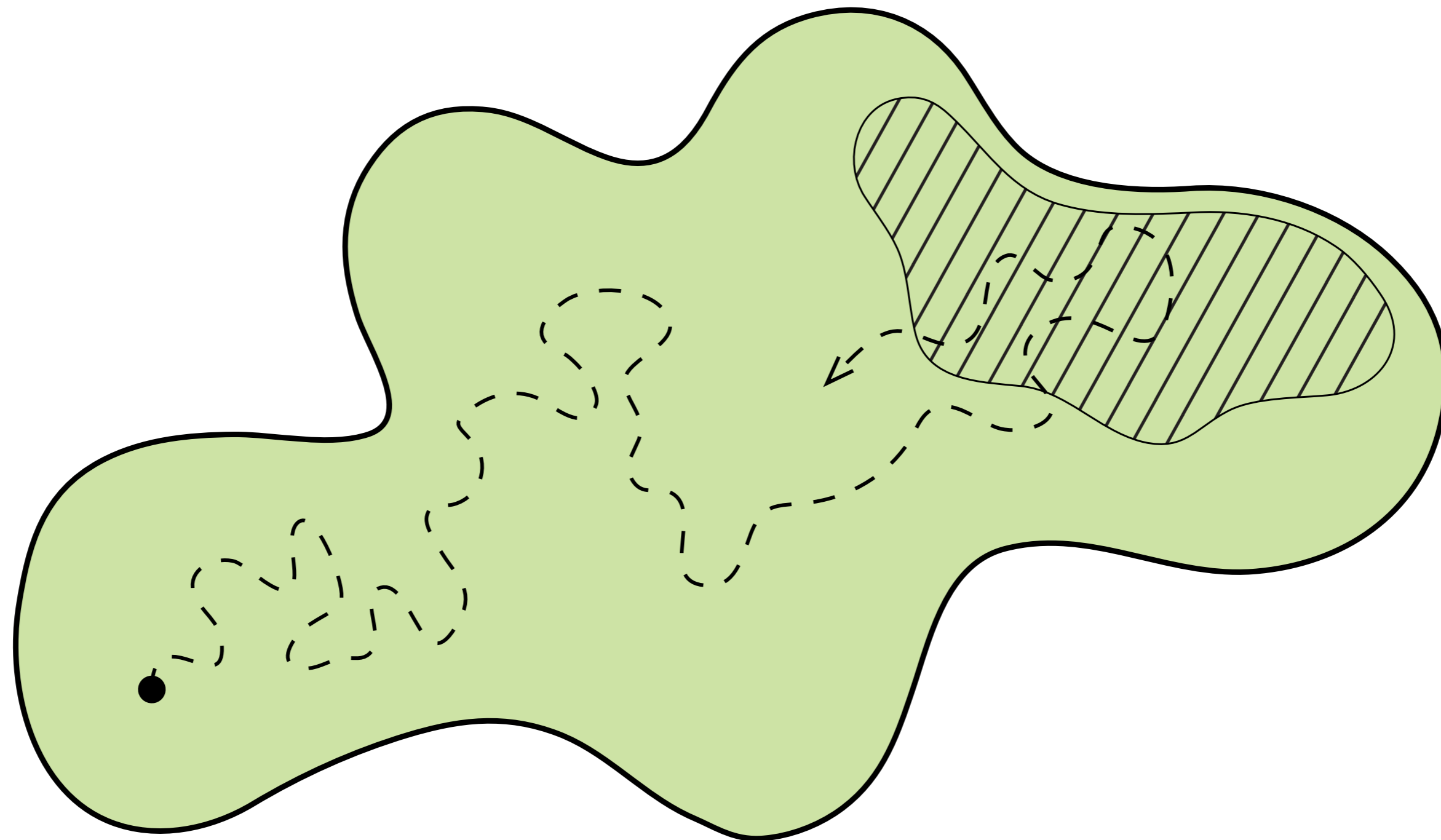
$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Monte Carlo

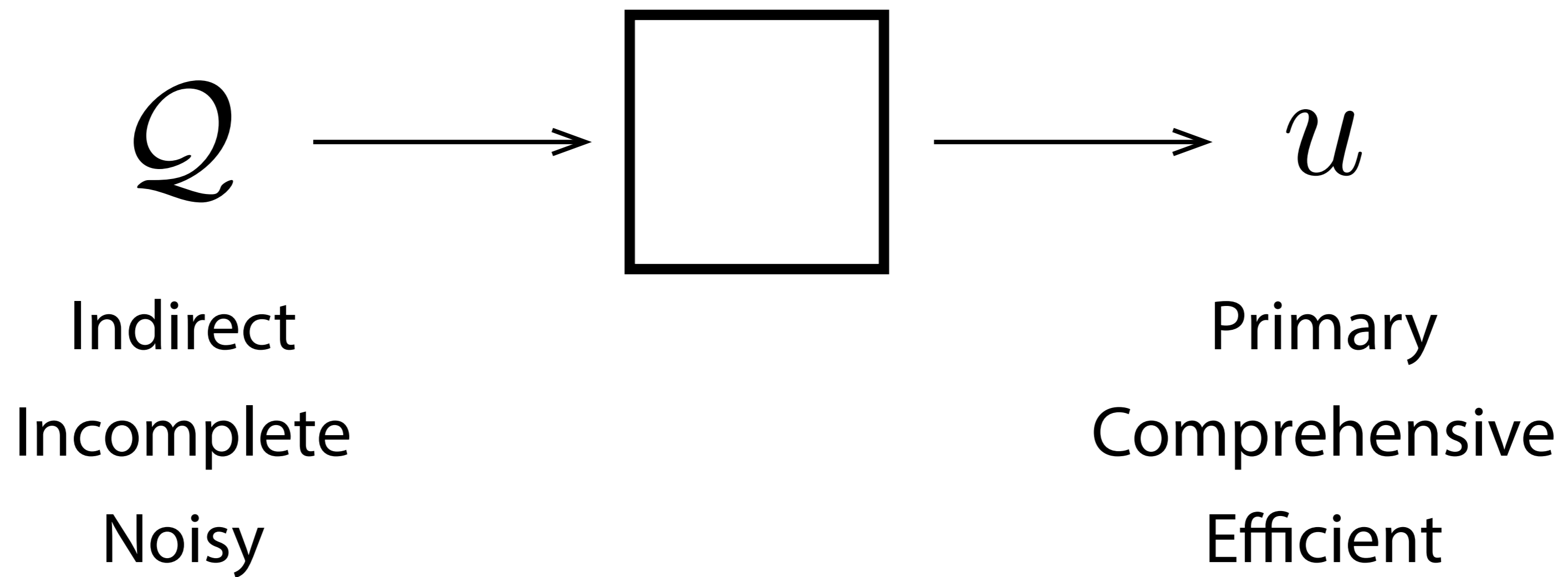
Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Markov chain Monte Carlo

Conclusion



Thank you! Questions?



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