Qubit Placement to Minimize Communication Overhead in 2D Quantum Architectures

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Supported by the IARPA Quantum Computer Science Program
Outline

- Introduction
  - Quantum Computing Technologies
- Geometric Constraints
  - Nearest Neighbor Architectures
- Proposed Solution
  - MIP-based Qubit Placement
  - Force-directed Qubit Placement
- Results
- Conclusion
Quantum Computing

- **Motivation: Faster Algorithms**
  - Shor’s factoring algorithm (Superpolynomial)
  - Grover’s search algorithm (Polynomial)
  - Quantum walk on binary welded trees (Superpolynomial)
  - Pell's equation (Superpolynomial)
  - Formula evaluation (Polynomial)

- **Representation**

  Quantum Algorithm → Quantum Circuit → Physical Realization (PMD)

  PMD: Physical Machine Description

[Link to NIST Quantum Zoo](http://math.nist.gov/quantum/zoo/)
Quantum Circuits

- **Qubits**
  - Data is carried by quantum bits or qubits
  - Physical objects are ions, photons, etc.

- **Quantum Gates**
  - Single-qubit: H (Hadamard), X (NOT)
  - Two-qubit: CNOT (Controlled NOT), SWAP

- **Quantum Circuit**
Quantum PMDs

- **Move-based PMDs**
  - Explicit move instruction
    - There are routing channels for qubit routing
  - Examples: Ion-Trap, Photonics, Neutral Atoms

- **SWAP-based PMDs**
  - No move instruction
    - There are no routing channels
  - Qubit routing via SWAP gate insertion
  - Examples: Quantum Dot, Superconducting

- Focus of this presentation is on **SWAP-based PMDs**
Geometric Constraints

- **Limited Interaction Distance**
  - Adjacent qubits can be involved in a two-qubit gate
  - Nearest neighbor architectures

- **Route distant qubits to make them adjacent**
  - Move-based: MOVE instruction
  - SWAP-based: insert SWAP gates
SWAP-based PMDs

- **SWAP insertion**
  - **Objective**
    - Ensure that all two-qubit gates perform local operations (on adjacent qubits)
  - **Side effects**
    - More gates, and hence more area
    - Higher logic depth, and thus higher latency and higher error rate
  - Minimize the number of SWAP gates by placing frequently interacting qubits as close as possible on the fabric
    - This paper: MIP-based qubit placement
    - Future work: Force-directed qubit placement (a more scalable solution)

MIP: Mixed Integer Programming
Example on Quantum Dot

- Simple qubit placement: place qubits considering only their immediate interactions and ignoring their future interactions.

Two SWAP gates
Example on Quantum Dot (cont’d)

- Improved qubit placement: place qubits by considering their future interactions

No SWAP gate
Qubit Placement

- Assign each qubit to a location on the 2D grid such that frequently interacting qubits are placed close to one another

\( x_{iw} \): assignment of \( q_i \) to location \( w \)

\( x_{jv} \): assignment of \( q_j \) to location \( v \)

\( m_{ij} \): number of 2-qubit gates working on \( q_i \) and \( q_j \)

\( \text{dist}_{wv} \): Manhattan distance between locations \( w \) and \( v \)

\( c_{iwjv} = m_{ij} \times \text{dist}_{wv} \)

\[
\text{Min} \quad \sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{iw} x_{jv}
\]

subject to

\[
\sum_{w=1}^{n} x_{iw} = 1, \quad i = 1, \ldots, n, \\
\sum_{i=1}^{n} x_{iw} = 1, \quad w = 1, \ldots, n, \\
x_{iw} \in \{0, 1\}, \quad i, w = 1, \ldots, n.
\]
Kaufmann and Broeckx’s Linearization

\[ \alpha_{iw} = \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv}, \quad i, w = 1, \ldots, n \]

\[ z_{iw} = x_{iw} \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{jv}, \quad i, w = 1, \ldots, n \]

\[
\begin{align*}
\text{Min} & \quad \sum_{i=1}^{n} \sum_{w=1}^{n} z_{iw} \\
\text{subject to} & \quad \sum_{w=1}^{n} x_{iw} = 1, \quad i = 1, \ldots, n, \\
& \quad \sum_{i=1}^{n} x_{iw} = 1, \quad w = 1, \ldots, n, \\
& \quad \alpha_{iw} x_{iw} + \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{jv} - z_{iw} \leq \alpha_{iw}, \quad i, w = 1, \ldots, n, \\
& \quad x_{iw} \in \{0, 1\}, \quad i, w = 1, \ldots, n, \\
& \quad z_{iw} \geq 0, \quad i, w = 1, \ldots, n.
\end{align*}
\]

\( n^2 \) binary variables \((x_{iw})\), \( n^2 \) real variables \((z_{iw})\), and \( n^2 + 2n \) constraints

MIP Optimization Framework

- GUROBI Optimizer 5.5 ([http://www.gurobi.com](http://www.gurobi.com))
  - Commercial solver with parallel algorithms for large-scale linear, quadratic, and mixed-integer programs (free for academic use)
  - Uses linear-programming relaxation techniques along with other heuristics in order to quickly solve large-scale MIP problems

- Qubit placement (the MIP formulation) does not guarantee that all two-qubit gates become localized; Instead, it ensures the placement of qubits such that the frequently interact qubits are as close as possible to one another
  - SWAP insertion
SWAP Insertion

CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
CNOT 2, 4
CNOT 6, 8
CNOT 1, 3
CNOT 2, 6

CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
SWAP 2, 7
SWAP 2, 3
CNOT 2, 4
CNOT 6, 8
CNOT 1, 3
CNOT 2, 6

CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
SWAP 2, 7
SWAP 2, 3
CNOT 2, 4
CNOT 6, 8
SWAP 1, 2
CNOT 1, 3
CNOT 2, 6
Solution Improvement (1)

- Two qubits may interact with one another at different times
  - Not satisfactorily captured by a global qubit placer
  - Solution: Partition the circuit into $k$ sub-circuits ($S_1, \ldots, S_k$)

(1) The placement tool finds initial qubit placements ($P^i$).

(2) A SWAP insertion block generates final qubit placements ($P^f$) by inserting *intra-set* SWAP gates.

(3) A swapping network inserts *inter-set* SWAP gates to change the final placement of $S_j$ to the initial placement of $S_{j+1}$ as generated by the qubit placer.
Solution Improvement (2)

- In the previous solution, $P_j^f$ is obtained without considering $P_{j+1}^i$, for $j \geq 2$
- Large swapping networks
- Objective function of (1) only minimizes the intra-set communication distances
- Solution: Add a new term to the objective function in order to capture inter-set communication distances

$q_{i,s}$: qubit $i$ in sub-circuit $s$

$x_{i,w}^s$: assignment of $q_{i,s}$ to location $w$

$x_{j,v}^s$: assignment of $q_{j,s}$ to location $v$

$m_{ij}^s$: number of 2-qubit gates working on $q_{i,s}$ and $q_{j,s}$
Improved Qubit Placement

Intra-set communication distance

\[
\begin{align*}
\text{Min} & \quad \sum_{s=1}^{k} \sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{n} m_{ij}^{s} \text{dist}_{wv} x_{iw}^{s} x_{jv}^{s} + \\
& \quad \sum_{s=1}^{k} \sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{v=1}^{n} \text{dist}_{wv} x_{iw}^{s} x_{jv}^{s+1}
\end{align*}
\]

subject to

\[
\begin{align*}
\sum_{w=1}^{n} x_{iw} &= 1, \quad i = 1, \ldots, n, \\
\sum_{i=1}^{n} x_{iw} &= 1, \quad w = 1, \ldots, n, \\
x_{iw} &\in \{0, 1\}, \quad i, w = 1, \ldots, n.
\end{align*}
\]

Inter-set communication distance

Intra-set communication distance

Inter-set communication distance
Force-directed Qubit Placement

- **Attractive forces**
  - A force proportional to $m_{ij}^s$ between $q_{i,s}$ and $q_{j,s}$.
  - A (unit) force between between $q_{i,s}$ and $q_{i,s+1}$.
- Can be solved by quadratic programming
### Results (1)

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## Results (2)

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On average 27
Results (3)

Improvement over best 1D solution

[Bar chart showing improvement over best 1D solution for various tasks, with some tasks showing significant improvement (e.g., 27%)]
Conclusion

- Qubit placement methods for 2D quantum architectures
  - Directly applicable to Quantum Dot PMD
- 27% improvement over best 1D results

- Future work: force-directed qubit placement
  - Better results by considering both intra- and inter-set SWAP gates in the optimization problem
References


Thank you!