University of Pittsburgh

The Stochastic Modeling of TiO_2 Memristor and Its Usage in Neuromorphic System Design

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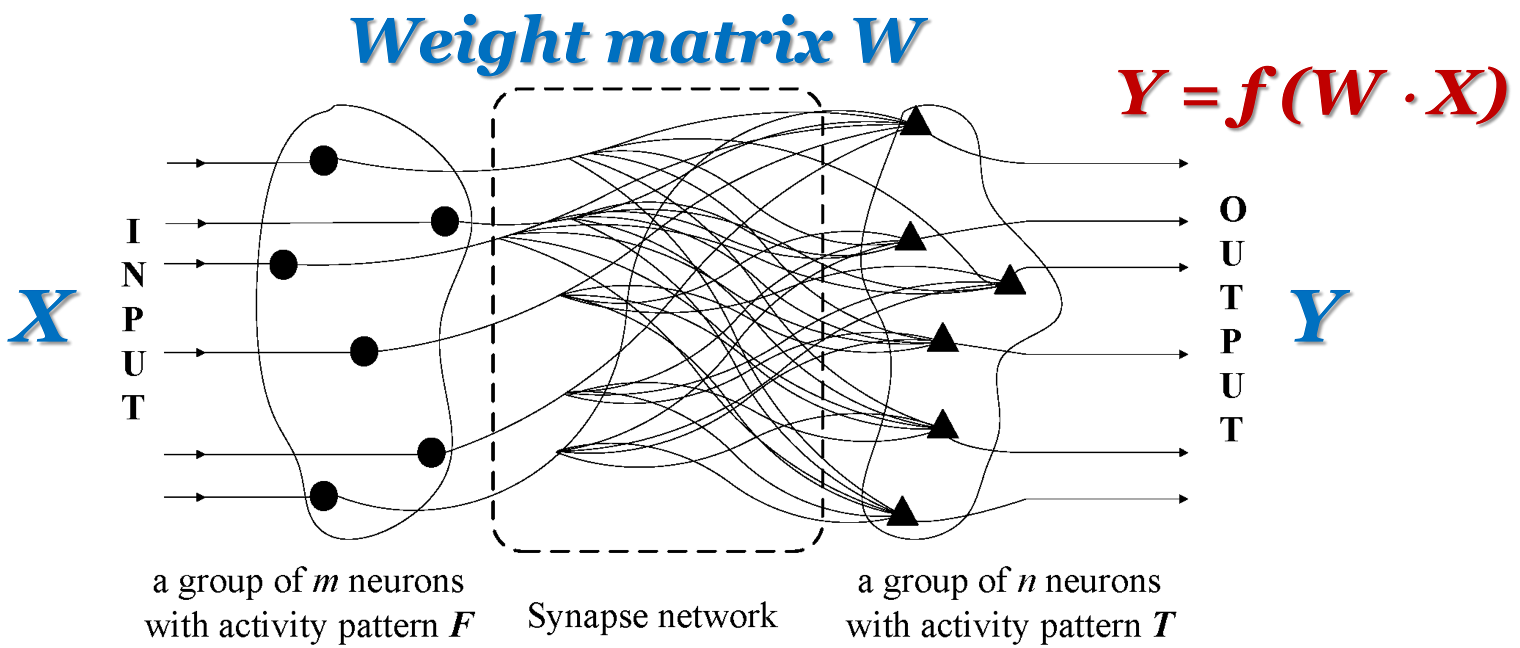
Outline

- Neural network and memristor
- Gaps between previous memristor models and real devices
  - Continuous and arbitrary states vs. binary or multi-level states
  - Deterministic vs. stochastic
- Stochastic modeling of TiO_2 memristor
  - ON and OFF static states
  - Dynamic switching process
- Neuromorphic applications
  - Weight storage unit
  - Stochastic neuron
- Conclusion and future work

Neural Network: Abstract of Bio Systems

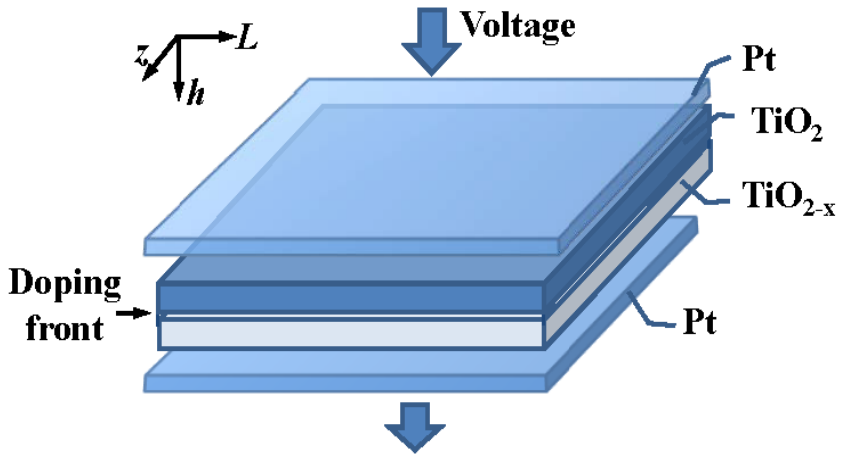


Neural network for pattern recognition:

- ***Training***: Learn from different prototype patterns.
- ***Recognize***: Output the most-likely prototype pattern for a given input.

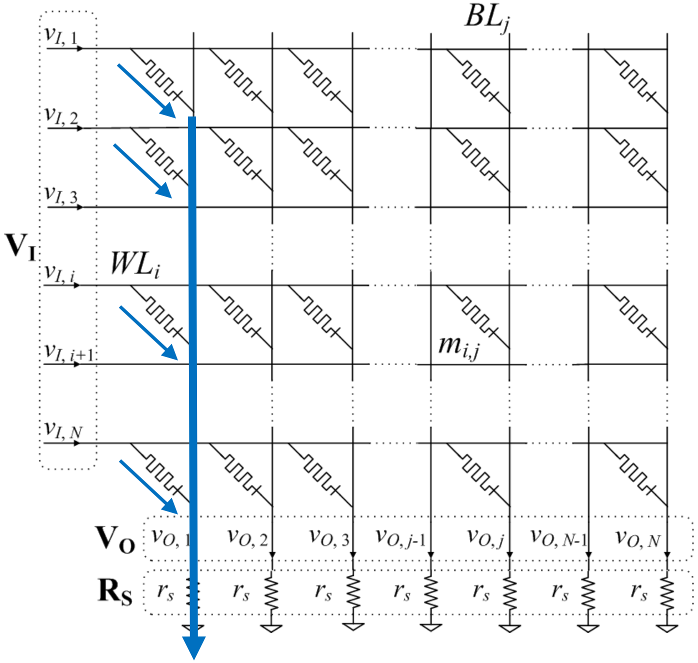
Memristor – Rebirth of Analog Approach

Memristor



$$M = R_L \cdot \alpha + R_H \cdot (1 - \alpha)$$

Memristor Crossbar



$$I = V_{M1}/M1 + V_{M2}/M_2 + \ldots + V_{Mn}/M_n$$

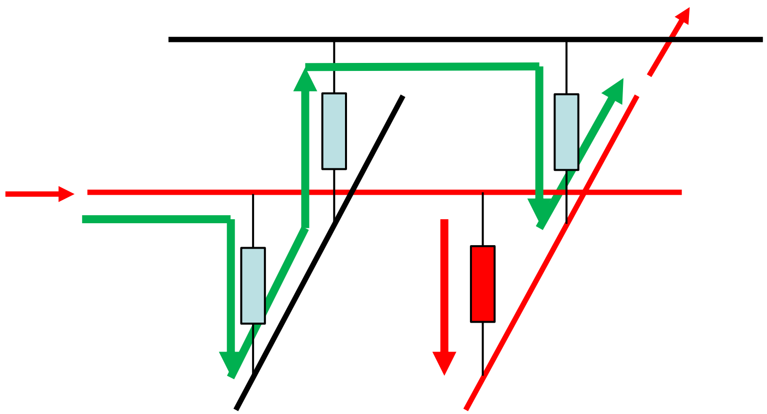
Natural weight carriers:

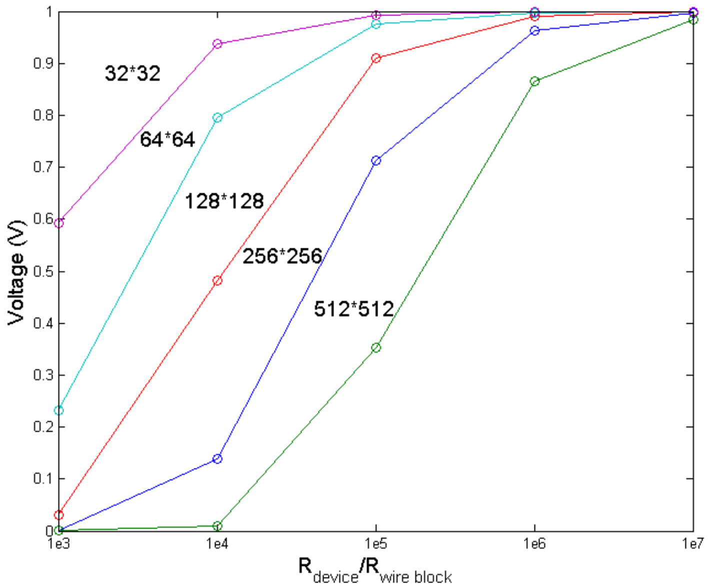
- Non-volatility, high density
- Analog resistance states
- Two terminal programming

- Natural weight summation
- MIMO ~ avoid reading sneak path
- Cost ~ O(N), not O(N^2)

Observation 1

It's difficult to *precisely* tune the state of every memristor in a large crossbar array.



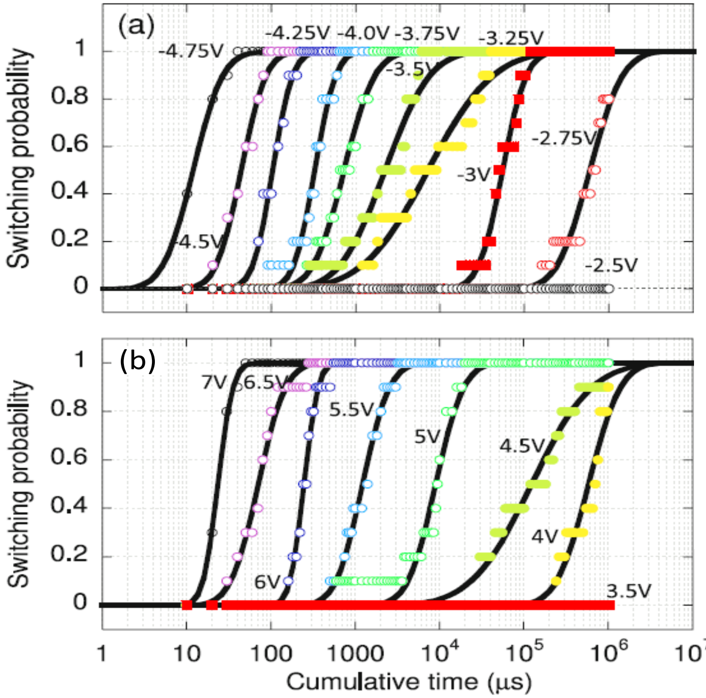


Sneak paths causes unexpected state changing on neighbor devices.

Wire resistance results in voltage degradation, especially on the device far from the driver.

Observation 2

- Metal oxide based memristors behave stochastically.
  - The stochastic feature is missing in existing physical models.
  - Modeling the stochastic feature which is heavily correlated with variations is very difficult.
  - Previous statistical analyses [1] consider only the binary switching, while ignoring memristor's continuous analog states.



A general model:

$$V = I \cdot M(w, V)$$

$$\frac{dw}{dt} = f(w, V)$$

[1] G. Medeiros-Ribeiro, et al., Nanotechnology, 2011.

Stochastic Modeling of TiO_2 Memristor

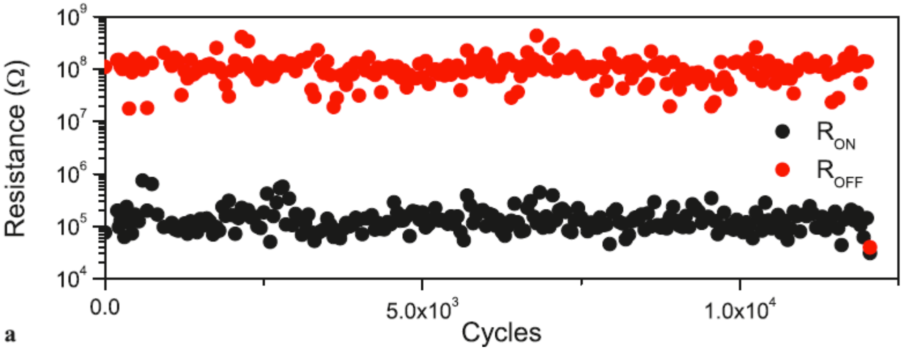
- A stochastic behavior model of TiO_2 memristor was firstly proposed in this work.

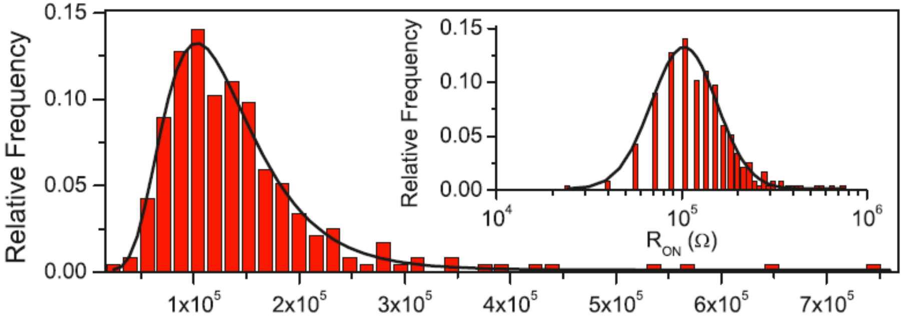
- The model bypasses material-related parameters by directly linking the device analog behavior to stochastic functions.
  - *Simpler configuration*: no need to decouple the impact of variations
  - *Simpler device model*: feasible to used in large-scale simulations.
  - *Better fitting the stochastic nature*: more statistically accurate.

Model Construction – Static States

- The *Lognormal* fitting is used for ON and OFF states of TiO_2 memristor [2].
- The internal variable w follows a normal distribution.
- The device resistance has an exponential relation with w.

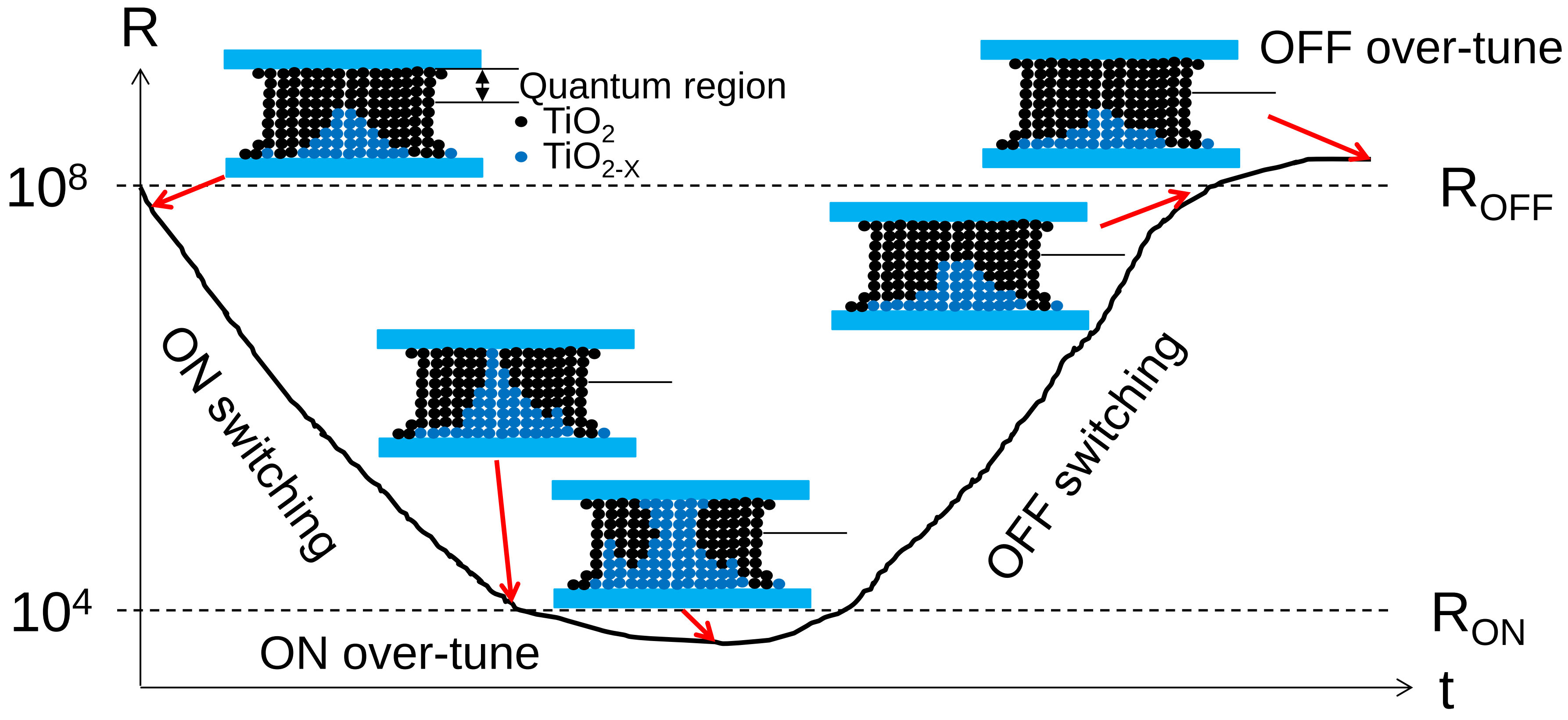
[2] W. Yi, et al., *Appl. Phys. A*, 2011.





Model Construction – Dynamic Switching

The stochastic switching process is illustrated as below.



Analog Switching Process

- The time dependency of switching probability can be approximated by the *cumulative probability function* (CDF) of lognormal distribution [1].

$$P(\text{Success switch}) = F(t_{\text{switch}}; \mu_t, \sigma_t) = \frac{1}{2}\text{erfc}\left[-\frac{(\ln t_{\text{switch}}/\mu_t)^2}{\sqrt{2}\sigma_t{}^2}\right]$$

[1] G. Medeiros-Ribeiro, et al., Nanotechnology, 2011.

- We further expend it to fit analog process:

$$\frac{dP(\text{Success switch})}{dt_{\text{switch}}} = f_{t_{\text{switch}}}(t_{\text{switch}}; \mu_t, \sigma_t)$$

$$\frac{dR}{dt} = (R_{\text{off}} - R_{\text{on}}) \cdot f_{t_{\text{switch}}}(t_{\text{switch}}; \mu_t, \sigma_t)$$

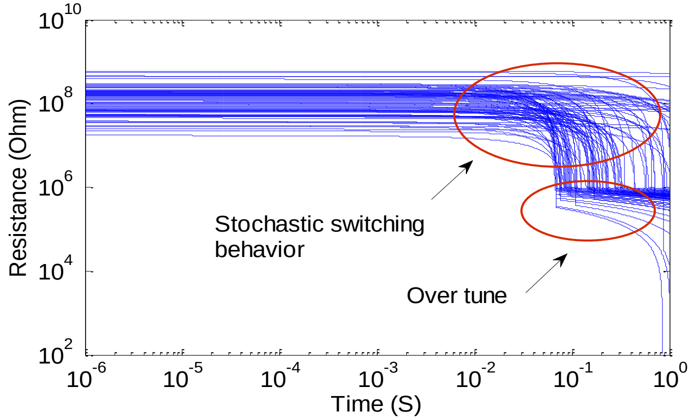
Over-Tune

- The device mechanism is different in over-tune situation.
- Considering the slow and relatively small resistance changing in over-tune situation, a linear approximation is adopted in which *e* is a fitting parameter:

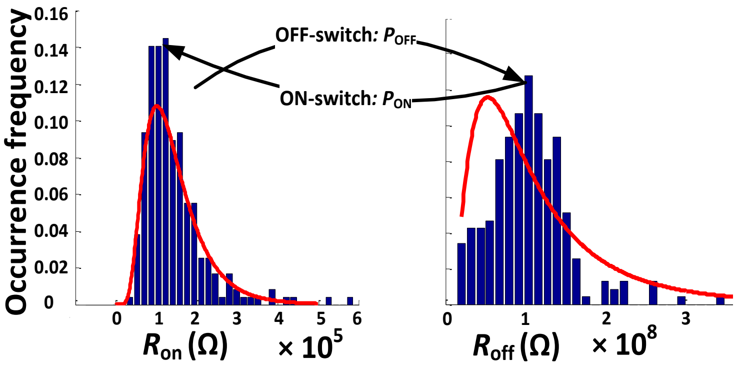
$$\mu_{\text{shift}} = e \cdot q = e \cdot \left(\frac{V}{M}\right) \cdot t$$

Model Verification

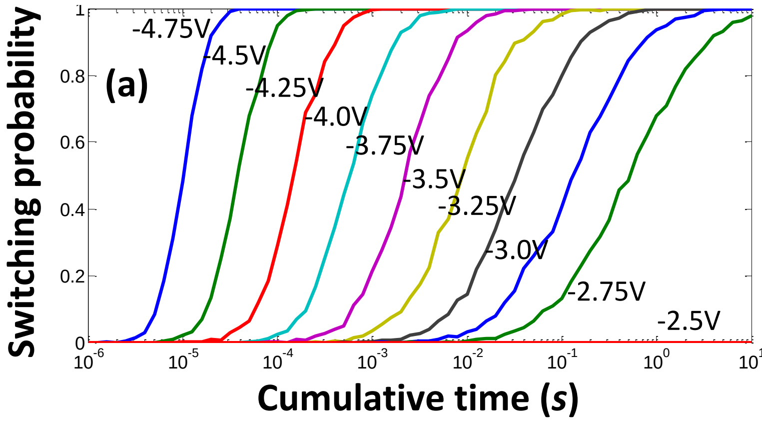
Example of 100 cycles



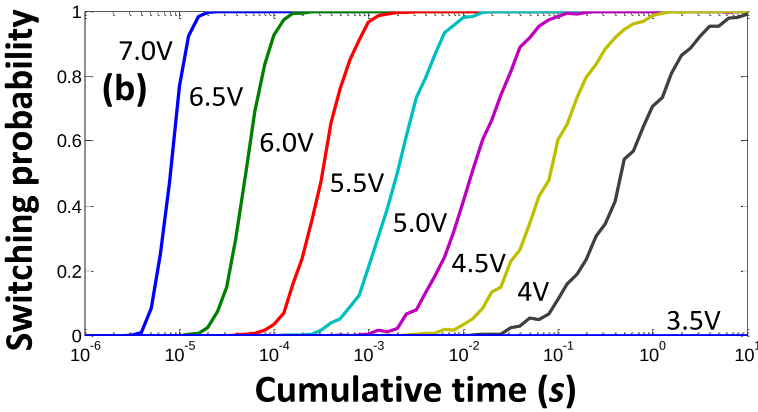
Static states fitting



ON switching fitting



OFF switching fitting

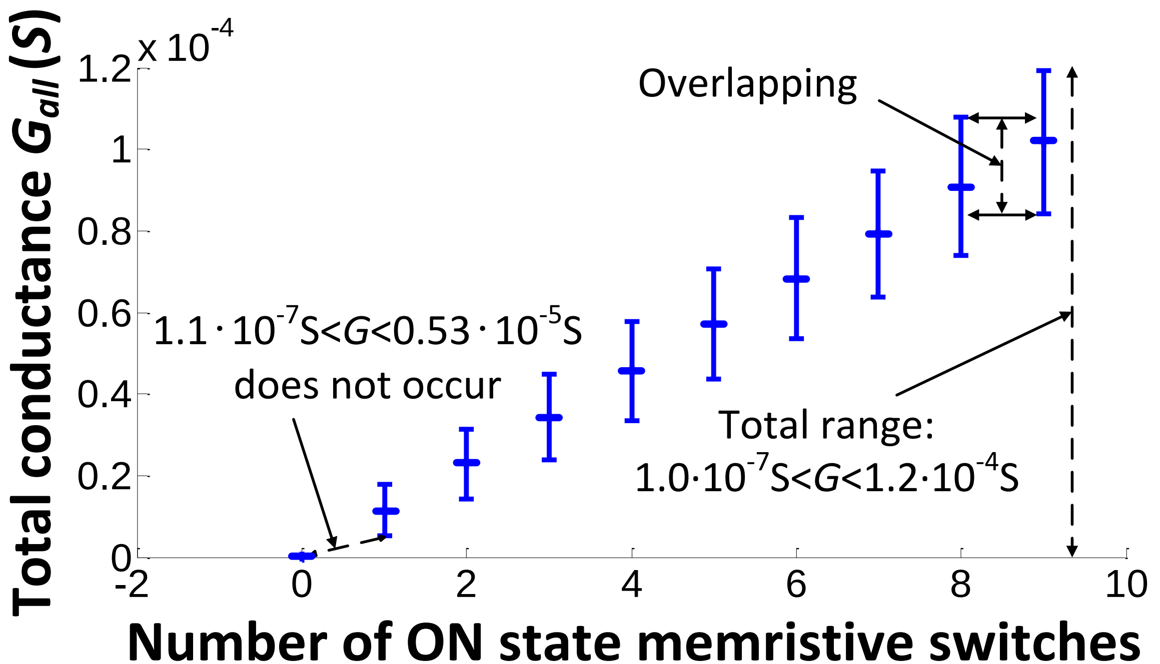


Neuromorphic Applications

- Our primary interest is to effectively utilize memristive switches and provide feasible designs for NN hardware.
  - Continuous weight storage unit
    - Alleviating the impact of stochastic
    - Using binary states of memristor to represent continuous value
  - Stochastic neuron
    - Making use of stochastic feature
    - Replacing pseudo-random number generators in traditional NN hardware

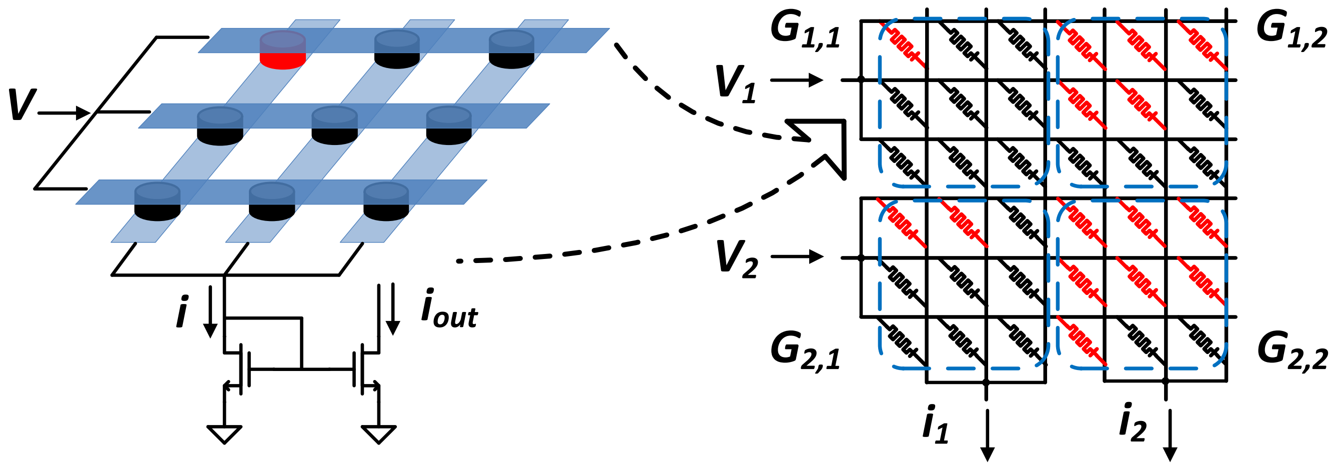
Continuous Weight Storage Unit

- Distribution of parallel connected memristors
- An example consisting of 9 parallel connected memristors



Continuous Weight Storage Unit

- A macro cell
  - Containing multiple memristive switches in crossbar structure.
  - A larger memristive switch crossbar can be partitioned into many macro cells for continuous weight storage.

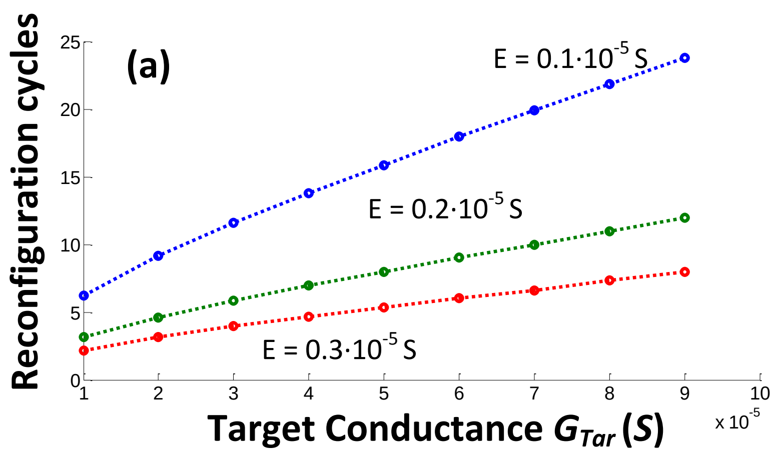


(a) (b)

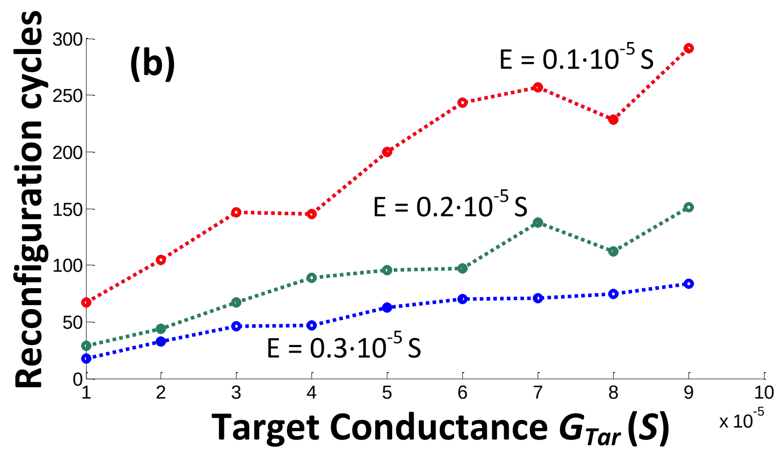
Feedback Switching Scheme

- *Step 1*: Decide the number of ON state memristors in a macro cell.
- *Step 2*: Switch memristors and detect conductance of macro cell.
- *Step 3*: Repeat *Step 2* until conductance falls in acceptable range.

Average case

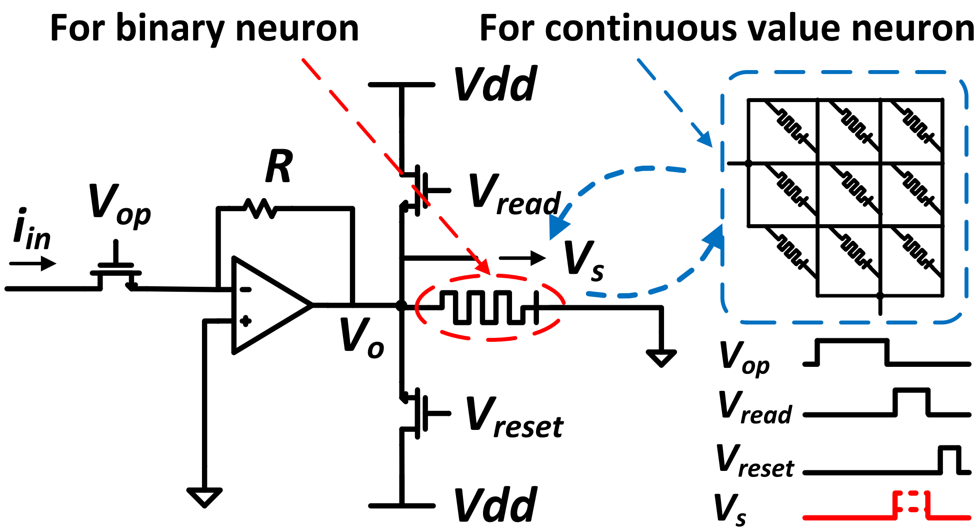


Worst case



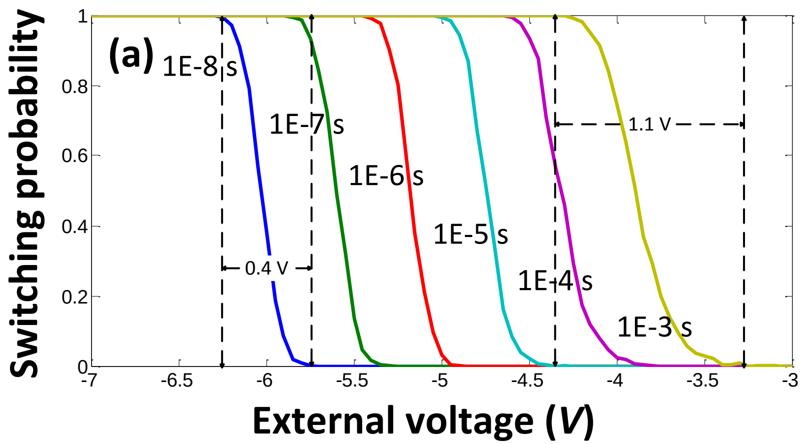
Stochastic Neuron

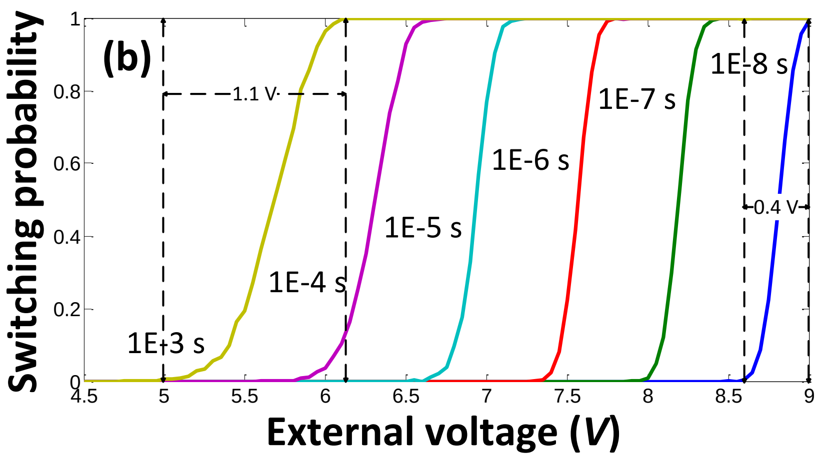
- Unlike weight storage, stochastic neuron employs the stochastic feature of memristor:
  - The probability of output states depends on the input voltage.



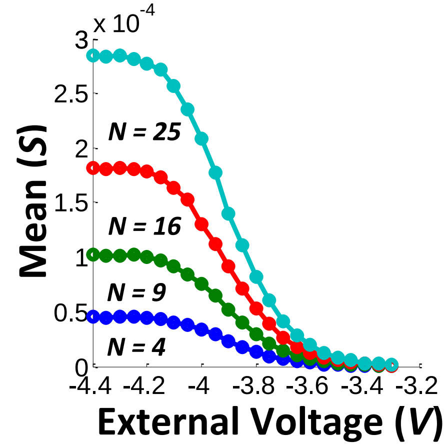
Stochastic neuron

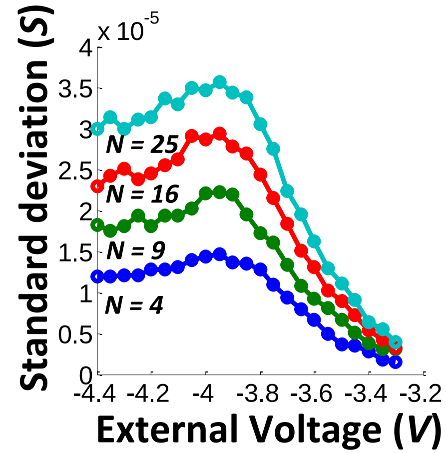
Binary neuron





Continuous neuron





Conclusion

- A simple and statistically accurate stochastic memristor model is firstly proposed.
- Two fundamental NN components are designed and analyzed by leveraging the proposed model.
- The weight storage unit:
  - Uses multiple devices to obtain the continuous analog state while bypassing complex tuning scheme.
  - On average, an analog value can be obtained within 25 attempts.
- Stochastic neuron:
  - Simple design structure by leverage the stochastic feature of memristors.
  - The neuron's stochastic function is determined by the device characteristics.

Thank you for attending my presentation!

Questions are welcome!