An Efficient Trajectory-based Algorithm for Model Order Reduction of Nonlinear Systems via Localized Projection and Global Interpolation

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Outline

- *Background*
- Proposed Approach
- Numerical Results
- Conclusion

Nonlinear Systems

- A nonlinear system can be described by

$$\frac{dg(x)}{dt} + f(x) = Bu(t)$$

$$x \in R^N$$

$g(\cdot): R^N \to R^N$ Nonlinear capacitance elements

$f(\cdot): R^N \to R^N$ Nonlinear resistance elements

$B \in R^{N \times p}$ Input incidence matrix

Trajectory Piecewise Linearization

- Linearize the nonlinear system around a series of points $\{x_1, x_2, \ldots, x_s\}$:

$$\sum_{i=1}^{s} w_i(x)\left(C_i \frac{dx}{dt} + G_i x + f(x_i) - G_i x_i \right) = Bu(t)$$

- Generate projection basis $\{V_1, V_2, \ldots, V_s\}$ for each piece of PWL system by Arnoldi/TBR method

$$C_i \frac{dx}{dt} + G_i x + f(x_i) - G_i x_i = Bu(t)$$

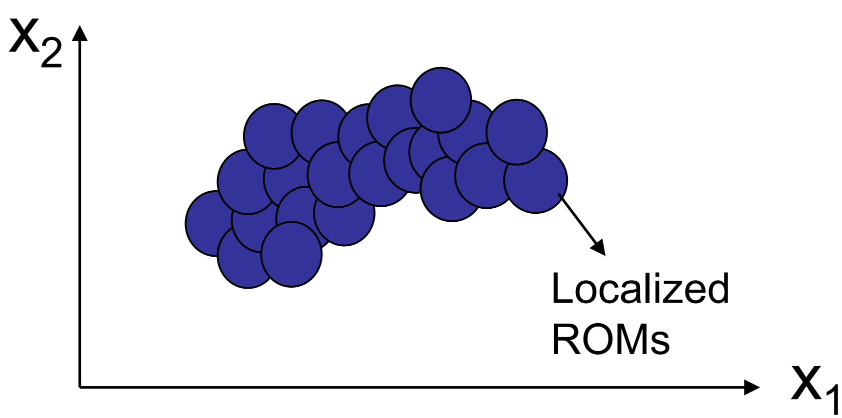
- Projection basis for nonlinear system:

$$V = \{V_1, V_2, \cdots, V_s\}$$

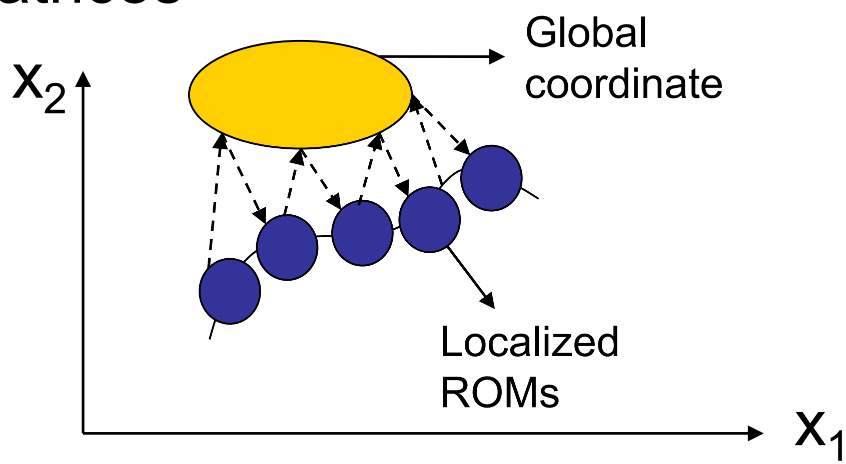
M. Rewienski et al, IEEE TCAD, 2003

Trajectory-based Algorithm via Localized Linear Projection

- Traditional trajectory-based algorithm depends on a global reduction matrix by combining subspaces at all points
- The dimension would be extremely large
- Localized linear projection: use localized projection matrices rather than global projection matrices



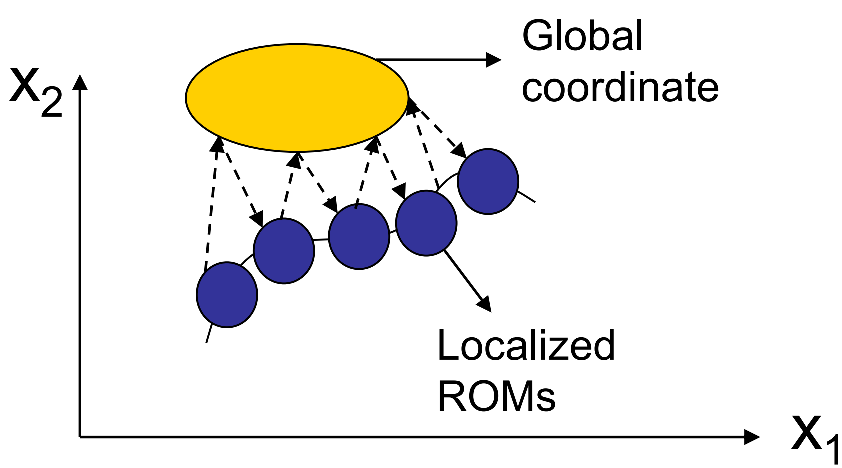
Reduction: build small overlapped ROMs with localized projection matrices



Simulation: jump from one local coordinate to another one

Limitations of Localized Linear Projection

- The localized ROMs should be overlapped to guarantee accuracy. The coordinates of neighboring localized ROMs are different, they cannot be interpolated globally.
- A global projection matrix is needed to provide a global view. The transformation between local and global subspaces would consume considerable amount of time.



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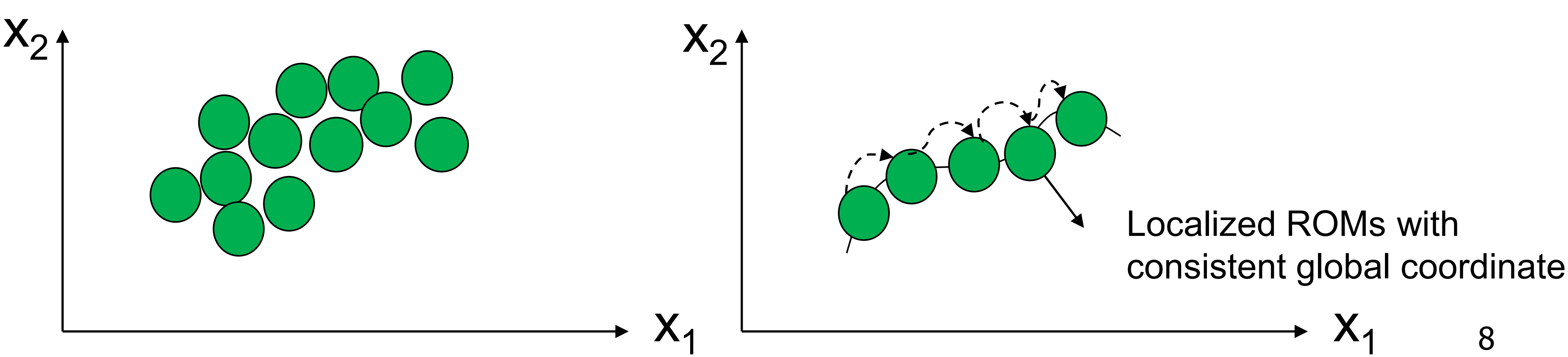
Key Idea of the Proposed Approach

- Transform the coordinates of localized ROMs into consistent global coordinate
- Benefits:
  - Enable global interpolation: overlapped ROMs are no longer necessary
  - Smooth transformation between localized ROMs: no coordinate transformation is needed



ROMs of Locally Linearized Systems

- Consider the localized linearized systems along the expansion points $\{x_1,x_2,\ldots,x_s\}$ on the trajectories:

$$C_i \frac{dx}{dt} + G_i x + I_i = Bu(t) \qquad \text{i=1,…s}$$

- With a set of projection matrices $\{V_1,V_2,\ldots,V_s\}$ of sizes $N \times p$, the ROMs can be expressed as

$$\tilde{C}_i \frac{d\tilde{x}}{dt} + \tilde{G}_i \tilde{x} + \tilde{I}_i = \tilde{B}_i u(t) \qquad \text{i=1,…s}$$

where $\tilde{C}_i = V_i^T C_i V_i$, $\tilde{G}_i = V_i^T G_i V_i$, $\tilde{I}_i = V_i^T I_i$, $\tilde{B}_i = V_i^T B_i$.

Transforms the ROMs to Ones with Consistent Coordinate

- A subspace can be described using different projection bases.
- For V_i, a new equivalent projection basis V_i' can be obtained by using an orthogonal matrix Q_i

$$V_i' = V_i Q_i, i = 1, 2, \cdots, s$$

- V_i' and V_i span the same subspace. The ROMs obtained by V_i' and V_i are equivalent but with different coordinates.

- Key idea: transform the ROMs into equivalent ROMs with consistent coordinates

Transforms the ROMs to Ones with Consistent Coordinate

- Consider two projection bases V_i and V_j, we aims to find a transformation matrix Q_{ij} such that the coordinates of the ROMs obtained by V_i and V_j are consistent

$$\min_{Q_{i,j} \in R^{p \times p}} \left\| V_j Q_{i,j} - V_i \right\|_F$$

which is equivalent to minimize the difference of the projection matrices V_i and V_j.

- The analytical solution of this minimization problem is

$$Q_{i,j} = U_{i,j} Z_{i,j}^T$$

where $U_{i,j} S Z_{i,j}^T$ is SVD of $V_i^T V_j$.

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D. Amsallem et al, SIAM J. SCI. COMPUT., 2011

Algorithm for transforming localized ROMs to Ones with consistent coordinate

- For a set of localized ROMs, randomly choose a reference, and transform the other ROMs according to the reference.

Algorithm 1 Algorithm for transforming localized ROMs to ROMs with consistent coordinate

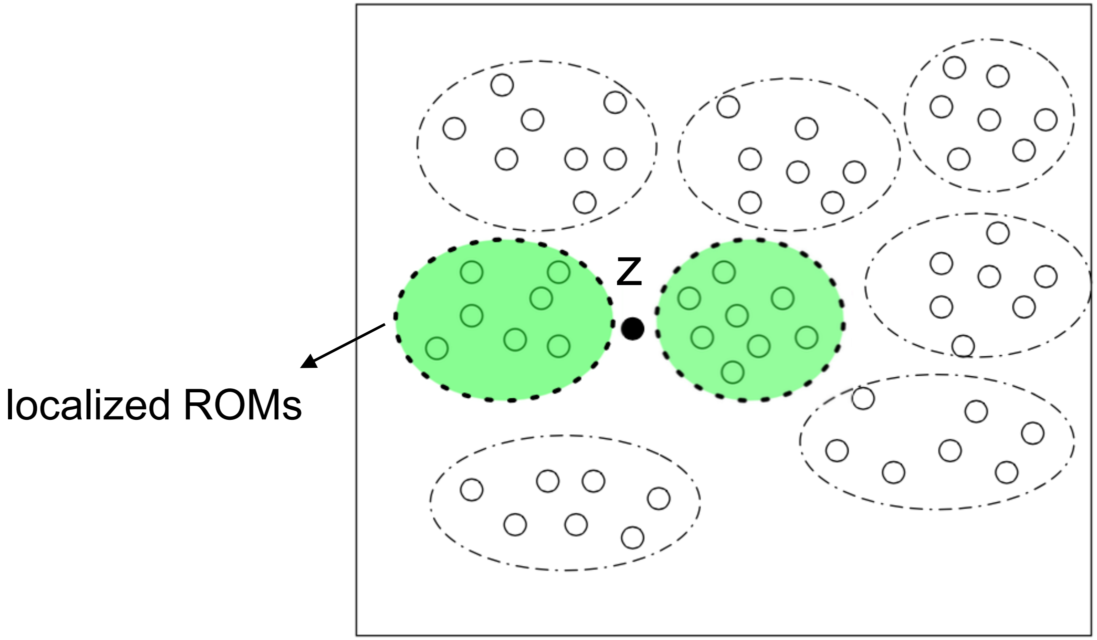
Let $V_{local j}$ be the local reduction matrix of ROM j, $j = \{1, 2..., m\}$
Randomly choose $r \in j$ as reference
for all j except r **do**
    $P = V_{local j}^T V_{local r}$
    $[U, S, Z] = \text{svd(P)}$
    $Q = UZ^T$
    $V'_{local j} = V_{local j} Q$
    $\tilde{G}'_j = V'^T_{local j} G_j V'_{local j}$
    $\tilde{C}'_j = V'^T_{local j} C_j V'_{local j}$
    $\tilde{I}'_j = V'^T_{local j} I_j$
    $B'_j = V'^T_{local j} B$
    $\tilde{L}'_j = V'^T_{local j} L_j$
    $\tilde{x}'_j = V'^T_{local j} x_j$
end for

Transforms the ROMs to Ones with Consistent Coordinate

- It would be difficult to make all the ROMs of the linearized systems share the same consistent coordinates.
- We propose a hybrid strategy
  - Group the state points employing some data mining methods.
  - For each group, derive a global projection matrix by aggregating the localized projection matrices
  - Use Algorithm 1 to make global projection matrices from different groups share the consistent coordinate

Global Interpolation of Localized ROMs with Consistent Coordinate

- During simulation, because the coordinates of the localized ROMs are consistent
  - For a new state point *z*, we find k nearest localized ROMS around *z*
  - Use these localized ROMs to interpolate the final ROMs



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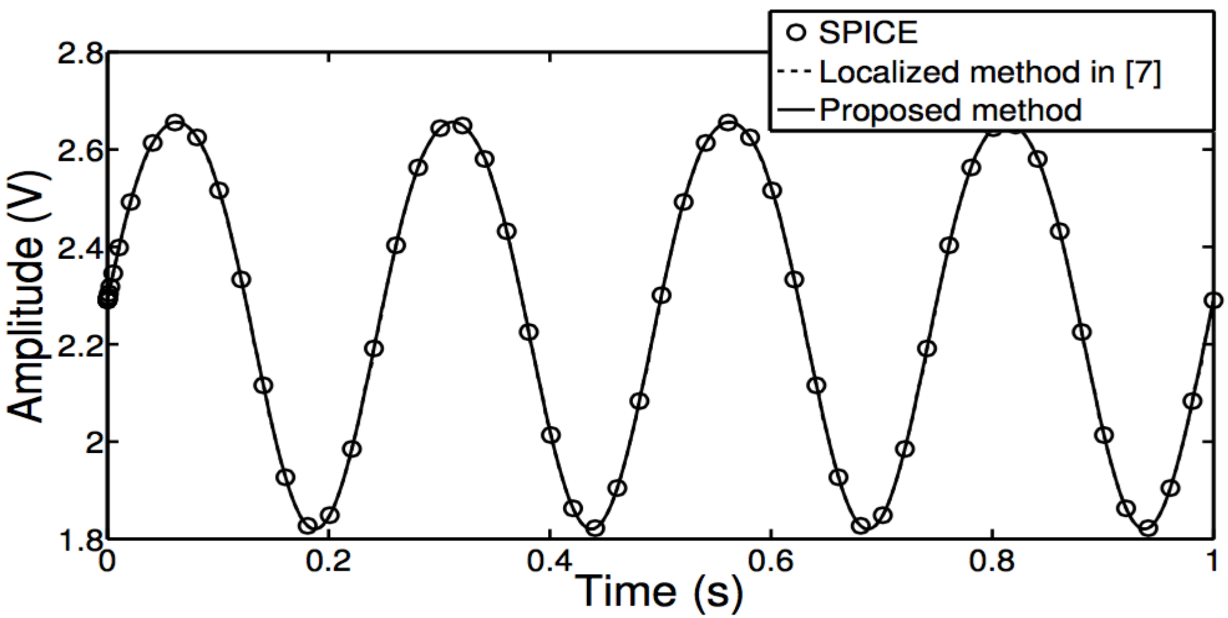
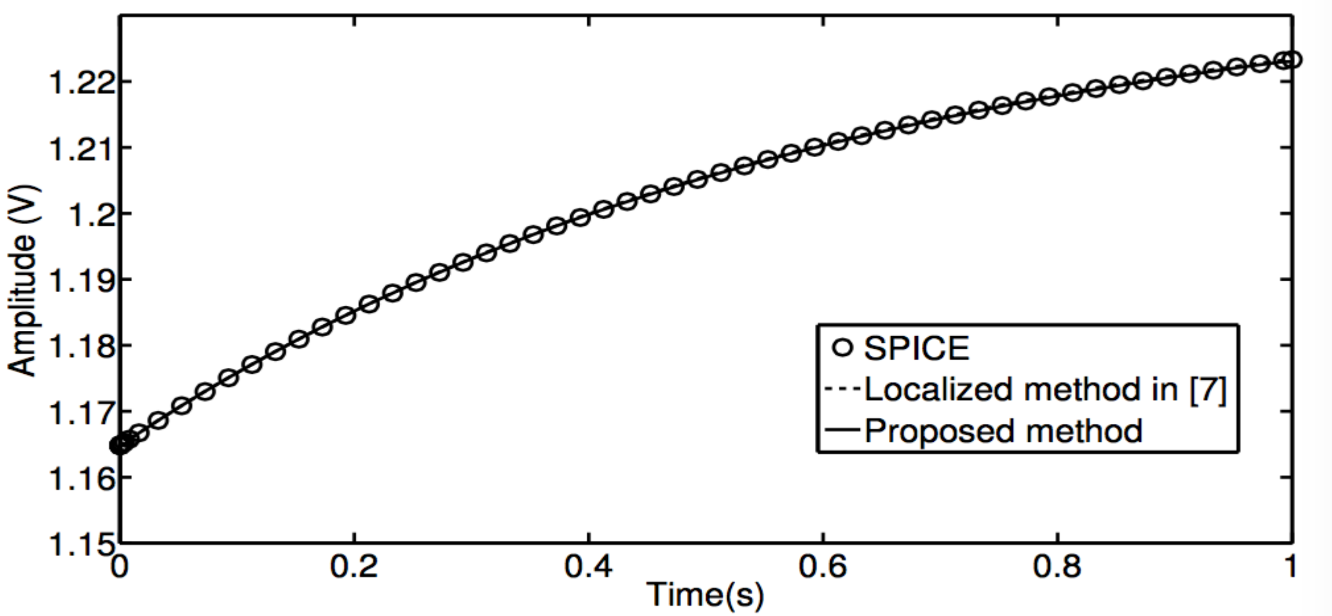
Case 1: Telescopic Operational Amplifier

- Order of the original system: 83
- 72 state points are extracted. A test input v = 1.5+0.01sin(8π*t) is applied to the model.
- The simulation results of the traditional local projection [7] and our proposed method match the SPICE simulation results well.



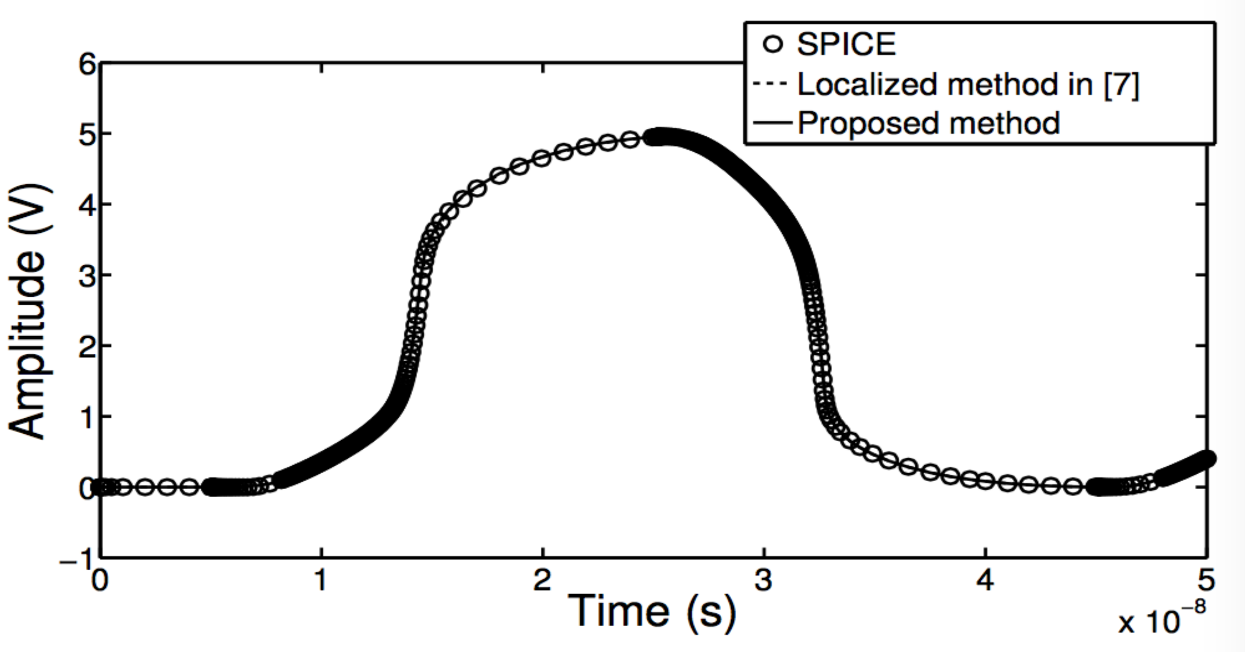
Case 2: Current Mirror OTA

- Order of the original system: 70
- 66 state points are extracted. A test input $v = 0.9+2*10^{-6}(1-e^{-t/0.6})$ is applied to the model.
- The simulation results of the traditional local projection [7] and our proposed method match the SPICE simulation results well.



Case 3: Clock Driver Circuit

- Order of the original system: 5642
- 439 state points are extracted. Another pulse input with different frequency is applied to the model.
- The simulation results of the traditional local projection [7] and our proposed method match the SPICE simulation results well.



Comparison of the Time/Memory Consumption

Test cases	Our proposed method		Method in [7]		SPICE
	# ROMs	sim. time(s)	# ROMs	sim. time(s)	sim. time(s)
OPA	15	0.0124	68	0.0169	0.068
OTA	13	0.0218	60	0.0266	2.25
CDR	55	0.318	412	0.445	290.86

- The proposed method can significantly reduce the number of localized ROMs and hence the memory consumption
- For the CDR case, the memory requirement for [7] is 1815MB, while our proposed method needs only 219MB
- The simulation time is also slightly reduced

Conclusion

- We transform the coordinates of localized ROMs into consistent global coordinate
- Global interpolation is possible and the overlapped ROMs is no longer necessary, which significantly reduce the number of localized ROMs
- Smooth transformation between localized ROMs. No coordinate transformation is needed
- The proposed method can achieve the same accuracy, while significantly reduce the number of localized ROMs and slightly reduce the simulation time

Thanks for your attention!

Q&A