A Time Domain Behavioral Model for Oscillators Considering Flicker Noise

ASP-DAC 2017

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Chiba, Japan, Jan.18, 2017
Outline

- Introduction
- Relationship between jitter and phase noise
  - The link for thermal noise
  - Discuss and derive the link for flicker noise in detail.
- Model Implementation
- Theory and model verification
  - Theory verification
  - Model verification
    - Comparison of the phase noise
    - Comparison of the period jitter’s PSD
- Conclusions
Time Domain Behavioral Model Needed

- Large-signal time domain model is the only suitable model for the circuit without steady-state solution.
  - Fractional-N PLL
  - Bang-bang PLL
  - ........
- Design space exploration can be done efficiently by the behavioral model.
Phase Noise in Oscillators

- The -20dB/dec and -30dB/dec regions are up-converted by the thermal and flicker noise respectively.
State-of-the-Art

Predicting the Phase Noise and Jitter of PLL-Based Frequency Synthesizers

Ken Kundert  
Designer’s Guide Consulting, Inc.

Citation: 169 (based on Google Scholar)

- Most cited paper in modeling PLL and the oscillator behaviorally
- Jitter-based time domain and phase domain model
- To be improved: “This excludes flicker noise.”
State-of-the-Art

Event-Driven Simulation and Modeling of Phase Noise of an RF Oscillator
R.B. Staszewski et al., TCAS-I, 2005
Citation: 101

- Second most cited paper in modeling oscillator behaviorally
- Jitter-based time domain model
- To be improved: “A further correction has to be made” when model the 30dB/dec rolling off region of the phase noise.
Both of the two top cited models are jitter-based.

- **Efficient**: noise is represented only on the timing of the transitions (in the form of jitter)
- **Available**: jitter extraction methodology is based on the commercially available simulator such as SpectreRF
Link between Jitter and Phase Noise Considering Only Thermal Noise

- The relationship between the period jitter variance and the phase noise with only the thermal noise is

\[ \sigma_{thermal}^2 = L(\Delta f) \frac{\Delta f^2}{f_0^3}. \]

- The jitter extracted from this formula is proved to be accurate in modeling the -20dB/dec of the phase noise.
Extract the Jitter due to Flicker Noise

Hajimiri (JSSC99), McNeil (ISCAS04) et al.:

\[ \sigma^2(\tau) = \frac{2}{(\pi f_0)^2} \int_{0}^{+\infty} L_\phi(\Delta f) \sin^2 (\pi \Delta f \tau) d\Delta f \]

- The formula is not closed-form.
- Flicker noise is nonstationary.

R.B. Staszewski (TCAS-I, 2005):

\[ \sigma_{\Delta T, \frac{1}{f}} = \frac{\Delta f_{c,1}}{f_0} \cdot \sqrt{T_0} \cdot \sqrt{2 \mathcal{L}\{\Delta \omega_{c,1}\}}. \]

- Not rigorous
- Further correction to be made in modeling the -30dB/dec region
The flicker noise is ‘postulated’ as an stationary stochastic process by introducing a cut-off frequency.

Solving the integral analytically will establish a link.
We relate the variance of the period jitter with the phase noise for flicker noise as

\[
\sigma_{1/f}^2(t) = 2 \left[ \ln \left( \frac{t}{T_0} \right) - 0.9151 \right] \frac{\Delta f^3}{f_0^4} L(\Delta f)
\]

This expression is CLOSED-FORM and COMPACT.

- \( t \): the observation time
- \( \Delta f \): the offset frequency
- \( L(\Delta f) \): the single-sided spectral noise density
- \( f_0 \): the nominal frequency
Time Domain Model of Oscillators Including the White and Flicker Noise

**Model Schematic**

**Perturbed Frequency**

**Frequency to Phase**

**Phase to Clock**

**Clock_Edge_Analyzer**

**Noise_Source**
The model with only flicker noise is used.

Jitter’s variance grows along \[ \ln \left( \frac{t}{T_0} \right) - 0.9151 \].

It is predicted by our theory and formula.
The variance of the period jitter is fixed.

The phase noise is predicted by our formula by

\[ L(\Delta f) = \frac{\sigma_{1/f}(T_0)}{2\left[\ln\left(\frac{f}{T_0}\right)-0.9151\right]} f_0^4 \Delta f^3. \]

The simulation results conforms the prediction.
Model Verification with Real Oscillator Circuits

- Parameter Extraction
  - 1.422GHz LC oscillator circuit with about 100KHz noise corner frequency
  - Thermal noise jitter: $\sigma_{thermal}^2 = L(\Delta f) \frac{\Delta f^2}{f_0^3}$
  - Flicker noise jitter: $\sigma_{1/f(t)}^2 = 2 \left[ \ln \left( \frac{t}{T_0} \right) - 0.9151 \right] \frac{\Delta f^3}{f_0^4} L(\Delta f)$
Model Verification with Real Oscillator Circuits

Parameter Extraction (15.911GHz ring oscillator circuit with about 60MHz noise corner frequency)

- Thermal noise jitter: \( \sigma_{\text{thermal}}^2 = L(\Delta f) \frac{\Delta f^2}{f_0^3} \)

- Flicker noise jitter: \( \sigma_{1/f}(t) = 2 \left[ \ln \left( \frac{t}{T_0} \right) - 0.9151 \right] \frac{\Delta f^3}{f_0^4} L(\Delta f) \)
Further Model Verification with Real Oscillator Circuits

- Extract the period jitter by transient noise analysis
  - Setup: noisemin is 10K, noisefmax is 500G
  - Runtime: 10 days 😞 to complete 1ms simulation (server with E5 processor and 16G memory)
Further Model Verification with Real Oscillator Circuits

- Comparison of the period jitter spectrum between
  - Our model (extracted by the link between the phase noise and the jitter)
  - That extracted directly in time domain by the transient noise analysis
**Conclusions**

- We have detailly discussed and derived the link between jitter and phase noise for the flicker noise.
  - A closed-form analytical expression is given without any approximation.
  - Demonstrate the link between period jitter and phase noise by simulation for the first time.

- Present a time domain behavioral model for oscillators considering the flicker noise.
  - The first work to model the up-converted flicker noise region of the phase noise accurately in time domain
  - Universal and accurate for either LC or ring oscillators

- Two different ways are used to verify the model, both observe excellent agreements.
Acknowledgements: This research is supported by NSFC (61471011) and R&D projects of Shenzhen city (JCYJ20150331102721193, JCYJ20160229094148396).