Exploiting Extended Krylov Subspace for the Reduction of Regular and Singular Circuit Models

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• Theoretical Background
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Introduction

• Efficient circuit simulation is among the most challenging problems facing the EDA industry today

• Power distribution network, multi-conductor interconnections, semiconductor substrate ...
  • The electrical models of the above subsystems are very large, consisting of hundreds of millions or billions of electrical elements

• Although their individual simulation is feasible, it is completely impossible to combine them and simulate the entire IC in many time-steps or frequencies.
Introduction

Step 1. PDE Field Solvers, SPICE Netlist, Measured Data

Large LTI descriptor systems (Thousands to Millions Eqns) very expensive to simulate...

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Model Order Reduction (MOR)

• Linear models typically need to be simulated multiple times during design, and/or in conjunction with external nonlinear circuitry
  • Inputs typically applied on a small subset $p \ll n$ of the circuit variables
  • Outputs typically constitute a small subset $q \ll n$ of the circuit variables
  • Very large linear model can be substituted for subsequent simulations by a model of reduced order, such that the input/output behavior is preserved
Most moment matching (MM) methods exploit the standard or the rational Krylov subspace in order to approximate the original model. 

- **expensive** parameter selection procedure, while the approximation quality is usually very sensitive to an inaccurate selection of these parameters.

Established moment matching methods construct the subspace only for positive directions, usually leading to a large approximated subspace to obtain a satisfactory error.
Contributions

• A MOR method which greatly decreases the error induced by MM methods by approximating both ends of the spectrum.
  • The proposed method exploits the superposition property in order to enable the simulation of many-port models

• A procedure for applying EKS-MM to large-scale regular and singular models, by implementing computationally efficient transformations in order to preserve the original form of the sparse input matrices

• The methodology is evaluated on industrial IBM power grids.
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Modified Nodal Analysis

- Modified Nodal Analysis (MNA) description of an $n$-node, $m$-branch (inductive), $p$-input, and $q$-output RLC circuit in the time domain:

$$\frac{dx(t)}{dt} = A \cdot x(t) + B \cdot u(t) + C \cdot y(t)$$

- Capacitance and inductance matrix
- Conductance matrix
- Vector of node voltages and currents
- Node-to-output connectivity matrix
- Input-to-node connectivity matrix

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Model Order Reduction

\[
\begin{align*}
\frac{dx(t)}{dt} &= A x(t) + B u(t) + n p y(t) = q C x(t) \\
\frac{d\tilde{x}(t)}{dt} &= \tilde{A} \tilde{x}(t) + \tilde{B} u(t) + r q \tilde{y}(t) = \tilde{C} \tilde{x}(t)
\end{align*}
\]

MOR

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Moment matching (MM) for MOR relies on the derivation of a reduced order model where some moments of the reduced-order transfer function match some moments of the original transfer function.

They are very efficient in circuit simulation problems and are formulated in a way that has a direct application to the linear model.

... but can be very sensitive to the selected expansion points.
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Extended Krylov Subspace Method

• The essence of MM methods is to iteratively compute a projection subspace, and then project the original system into this subspace in order to obtain the reduced-order model.

• The $k$-th dimensional extended Krylov subspace is

$$\mathcal{K}_k^E(A_E, B_E) = \mathcal{K}_k(A_E, B_E) + \mathcal{K}_k(A_E^{-1}, B_E) = \text{span}\{B_E, A_E^{-1}B_E, A_EB_E, A_E^{-2}B_E, A_E^2B_E, \ldots, A_E^{-(k-1)}B_E, A_E^{k-1}B_E\}$$
EKS-MM Algorithm

Algorithm 1: EKS computation by Arnoldi procedure

Input: $A_E = A^{-1}E, B_E = A^{-1}B$, desired order $r$, #ports $p$
Output: $V$

Function compute_EKS($A_E, B_E, r$):

1. $j = 1$
2. $V(j) = qr([B_E, A_E^{-1}B_E])$
3. $k = \frac{r}{p}$
4. while ($j < k$) do
5.   $k_1 = 2p(j - 1), k_2 = k_1 + p; k_3 = 2pj$
6.   $V_1 = [A_E V(j)(:, k_1 + 1 : k_2), A_E^{-1} V(j)(:, k_2 + 1 : k_3)]$
7.   $V_2 = \text{orth}_\text{wrt}(V_1, V(j), p)$
8.   $V_3 = qr(V_2)$
9.   $V(j+1) = [V(j), V_3]$
10. $j = j + 1$
11. end
12. $V = V(:, 1 : 2r)$
13. return $V$

Can be implemented as sparse linear solves

The projection space is constructed iteratively

Matrices can remain sparse even for manipulating singular systems
EKS Method using Superposition Property

• Output response of the initial multi-input multi-output (MIMO) model can be computed as the sum of the output responses of single-input multi-output (SIMO)

• This property can be employed for the parallel computation of the reduced-order model

\[
\mathcal{K}_k^E(A_E, B_{iE}) = \mathcal{K}_k(A_E, B_{iE}) + \mathcal{K}_k(A_{E}^{-1}, B_{iE}) = \\
\text{span}\{b_{iE}, A_{E}^{-1}B_{iE}, A_E B_{iE}, A_{E}^{-2}B_{iE}, A_{E}^{2}B_{iE}, \ldots, A_{E}^{-(k-1)}B_{iE}, A_{E}^{-(k-1)}B_{iE}\}
\]
Outline

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Experimental Setup for Circuit Models

• For the experimental evaluation of the proposed methodology we have used the available IBM power grid benchmarks

• The reduced-order models (ROMs) were evaluated in the frequency range of \([\omega_1, \omega_2] = [10^0, 10^{12}]\)
  • purely for testing purposes

• All experiments were executed on a Linux workstation Intel Core i7 processor with 8 cores running at 3.6GHz and 32GB memory
# Experimental Results for Circuit Models

<table>
<thead>
<tr>
<th>Ckt</th>
<th>Dimension</th>
<th>#ports</th>
<th>ROM Order</th>
<th>#moments</th>
<th>Max Error</th>
<th>Runtime(s)</th>
<th>#moments</th>
<th>Max Error</th>
<th>Error Reduction percentage</th>
<th>Runtime(s)</th>
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<tbody>
<tr>
<td>ibmpg1</td>
<td>44946</td>
<td>600</td>
<td>1200</td>
<td>2</td>
<td>0.037</td>
<td>0.146</td>
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<td>62.16%</td>
<td>0.146</td>
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<td>500</td>
<td>2000</td>
<td>4</td>
<td>0.233</td>
<td>1.206</td>
<td>2</td>
<td>0.131</td>
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<td></td>
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<td>800</td>
<td>1600</td>
<td>2</td>
<td>0.253</td>
<td>11.029</td>
<td>1</td>
<td>0.146</td>
<td></td>
<td></td>
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<td>600</td>
<td>2400</td>
<td>4</td>
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<td>1200</td>
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<td>0.242</td>
<td>10.228</td>
<td>1</td>
<td>0.063</td>
<td>73.97%</td>
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<td>1000</td>
<td>6000</td>
<td>6</td>
<td>0.161</td>
<td>19.155</td>
<td>3</td>
<td>0.130</td>
<td>19.25%</td>
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<td>54265</td>
<td>400</td>
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<td>4.767</td>
<td>0.259</td>
<td>1</td>
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<td>0.273</td>
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<td>3600</td>
<td>4</td>
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<td>0.250</td>
<td>2</td>
<td>0.411</td>
<td>47.64%</td>
<td>0.268</td>
</tr>
</tbody>
</table>

Negligibly larger runtime
Comparison of transfer functions of ROMs from standard MM and EKS MM in ibmpg2t and ibmpg1 benchmarks at ports (4,4) and (9,9) respectively.

- Response of EKS-MM ROM is performing very close to the original model, while the response of MM ROM exhibits a clear deviation.
- Responses of ROMs produced by MM do not capture effectively the dips and overshoots that arise in some frequencies.
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Conclusions

• An efficient use of EKS to enhance the accuracy of moment matching methods for descriptor circuit models was presented

• Our method provides clear improvements in reduced-order model accuracy compared to a standard Krylov subspace moment matching technique

• Proposed methodology remains computationally efficient, introducing only a small overhead in the reduction process.
QUESTIONS?