

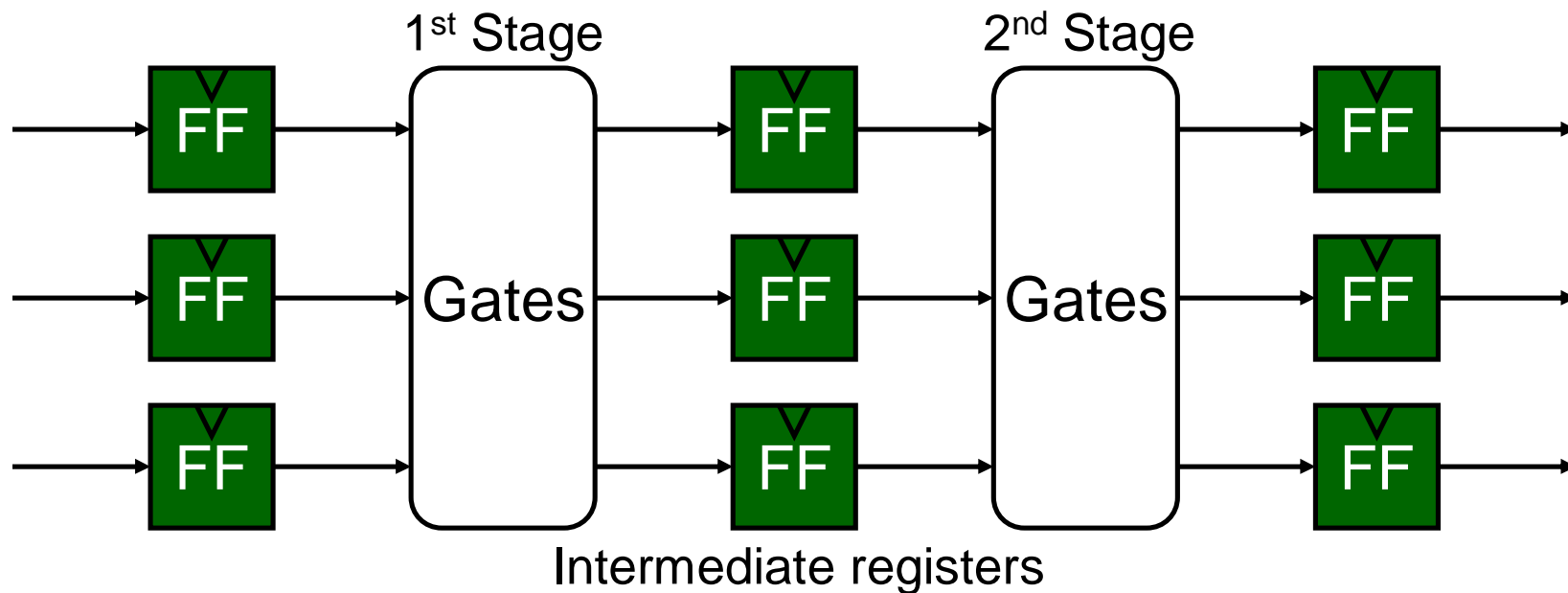


Low Area Pipelined Circuits by Multi-clock Cycle Paths and Clock Scheduling

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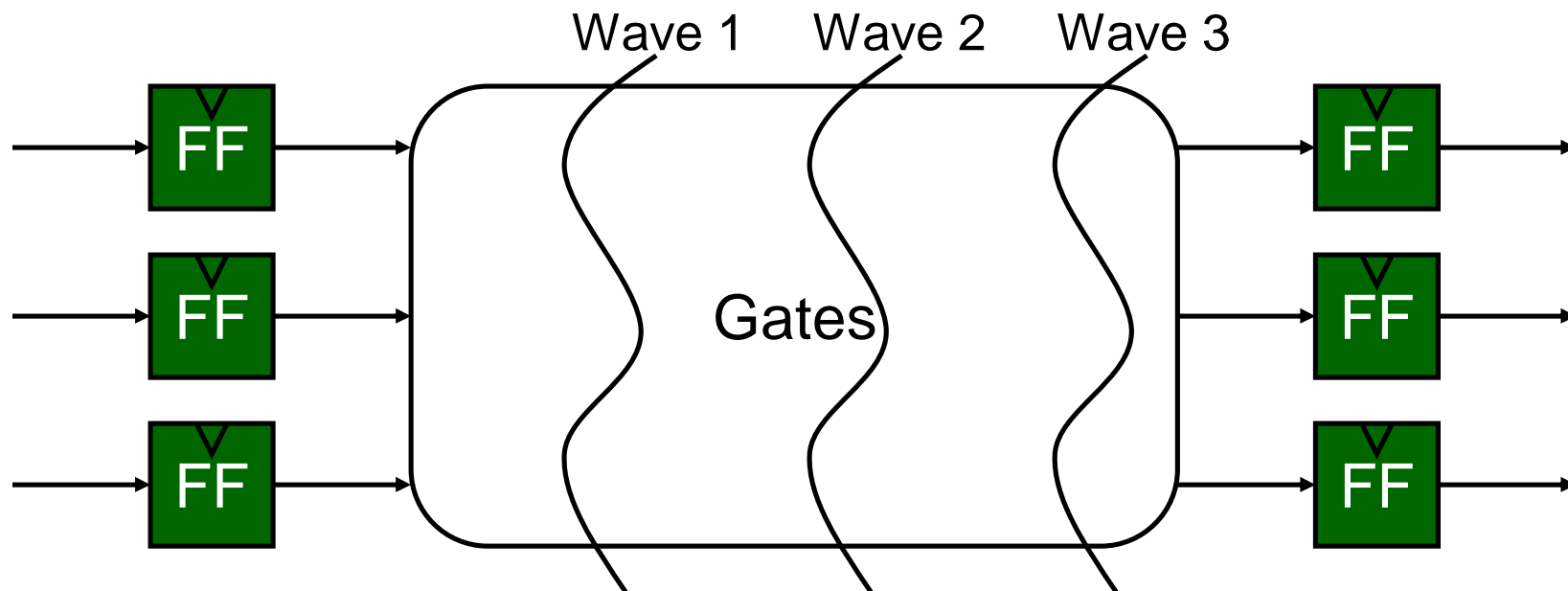
Pipelining

- A technique to shrink the clock period
- Circuit divided into number of stages.
- Intermediate registers inserted between stages.
- **Problem of current pipelining method:-**
 - Extra circuit area from the intermediate registers.
 - Increased the size of clock tree.



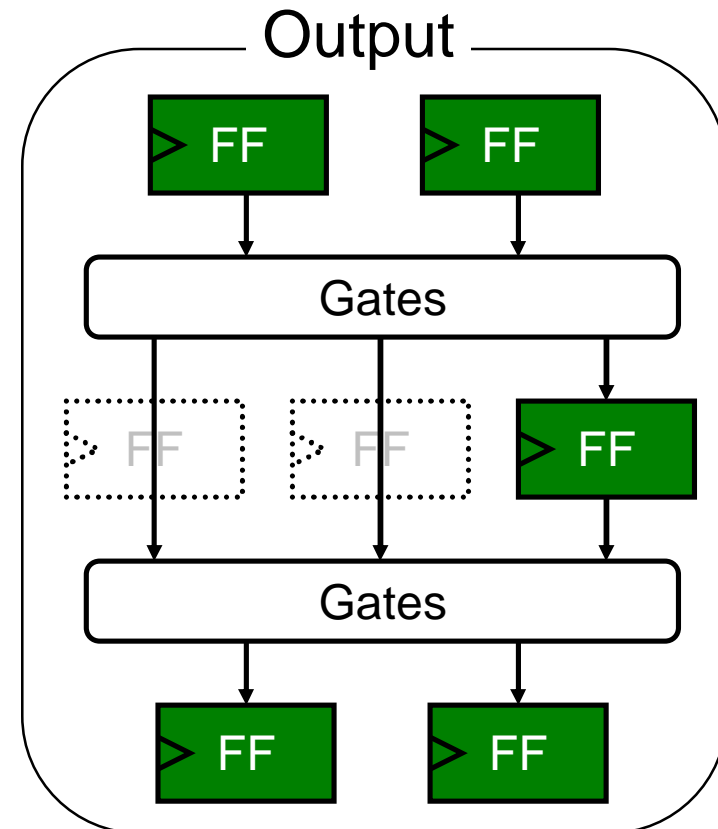
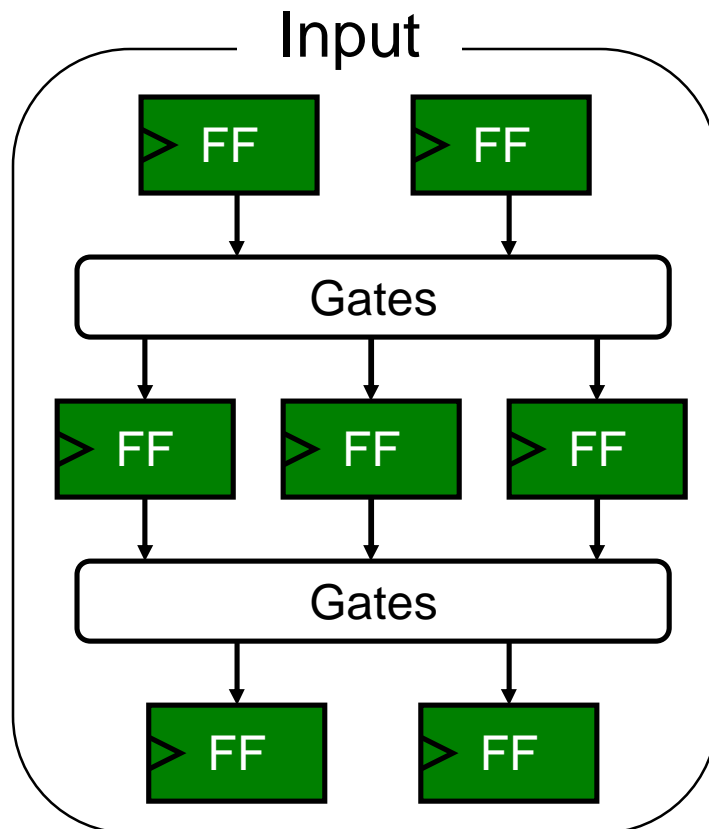
Wave Pipelining

- Pipelining without the intermediate registers.
- Exist a number of waves of data in a stage.
- To avoid data collisions need delay balancing.
- **Problem of Wave Pipelining:-**
 - **Delay balancing may increases circuit area.**



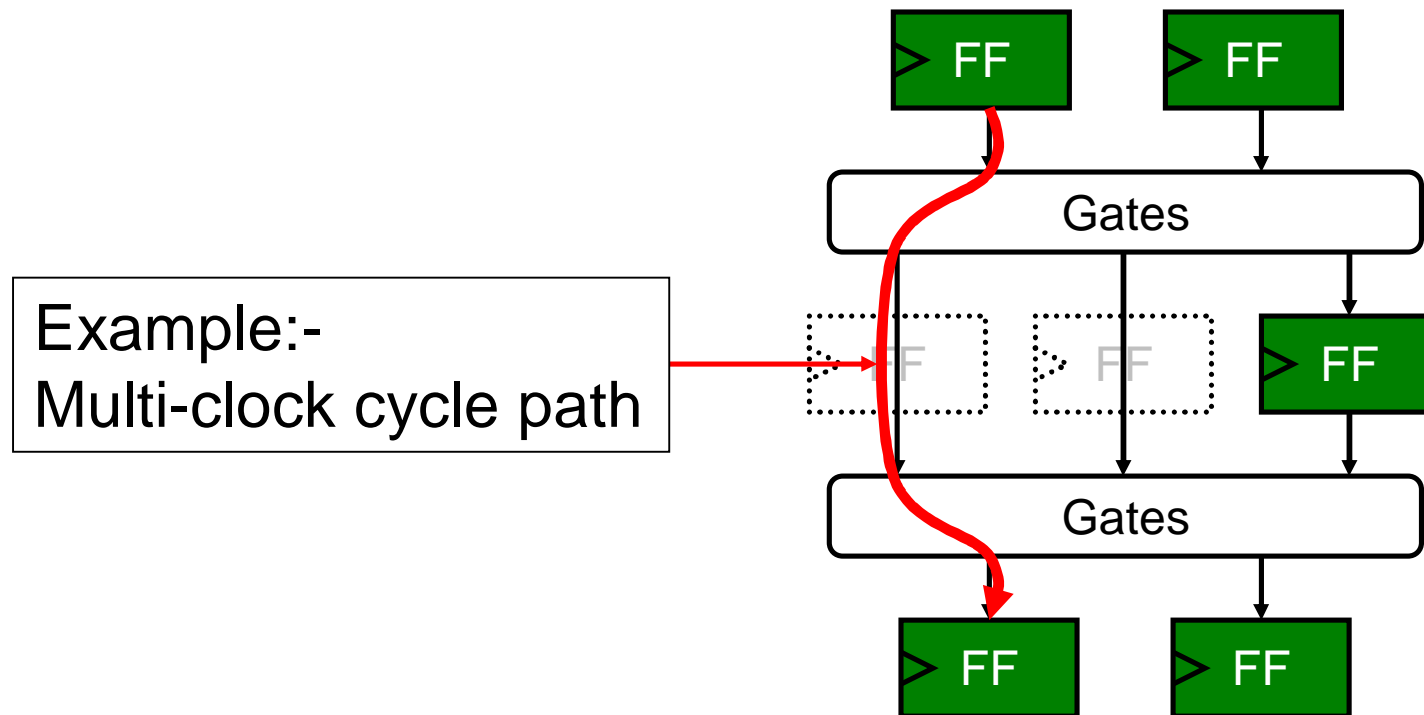
Purpose

- Circuit with smaller area and works at target clock period range.
- Input :- Pipelined Circuit, clock period range.
- Objective:- Reduce the number of intermediate registers.



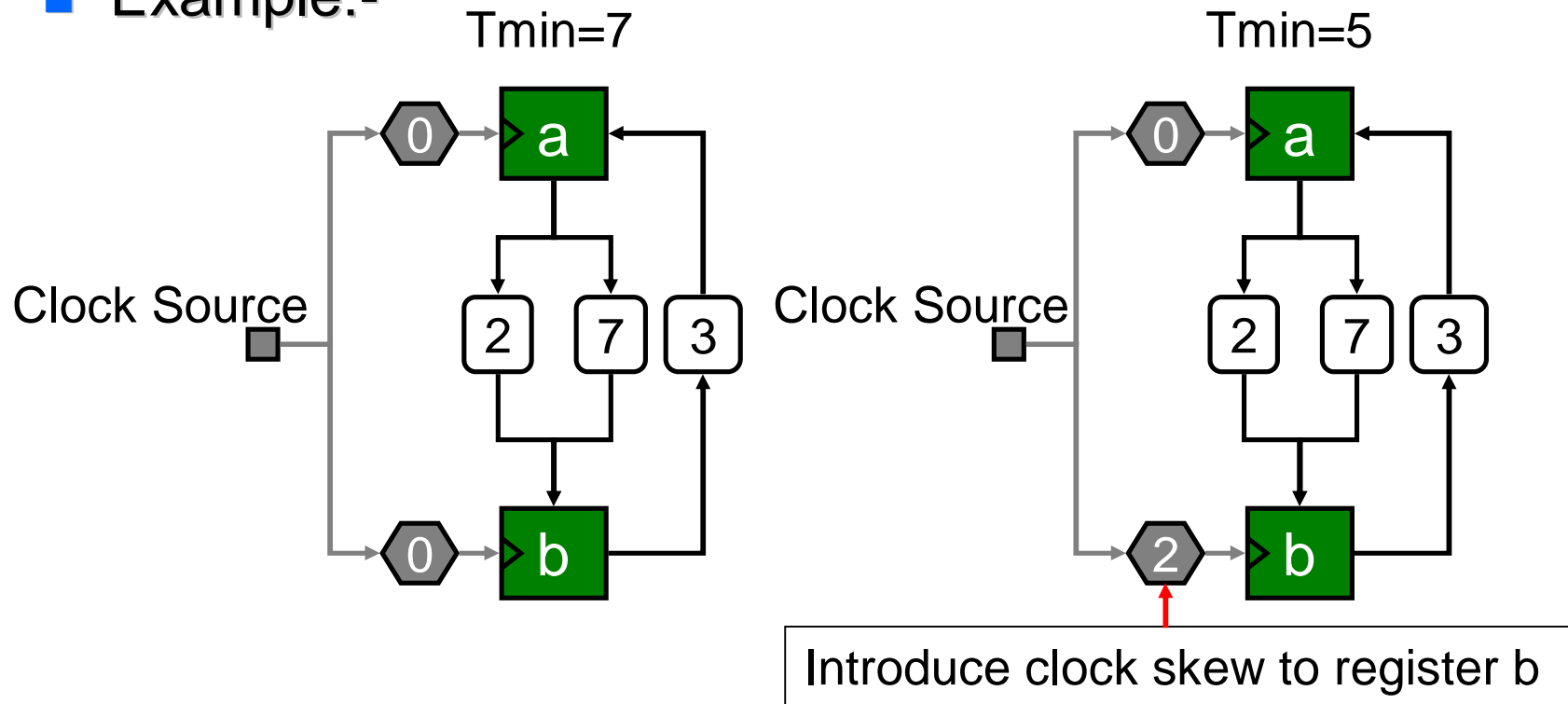
Method : Multi-clock Cycle Paths

- Data transmission more than one clock period.
- Allows intermediate registers to be removed.
- Timing constraints must be satisfied.
 - Need delay balancing.



Method : Clock Scheduling

- Intentionally introduce clock skew of register
- Can relax timing constraints.
 - Can improve circuit performance.
- Example:-



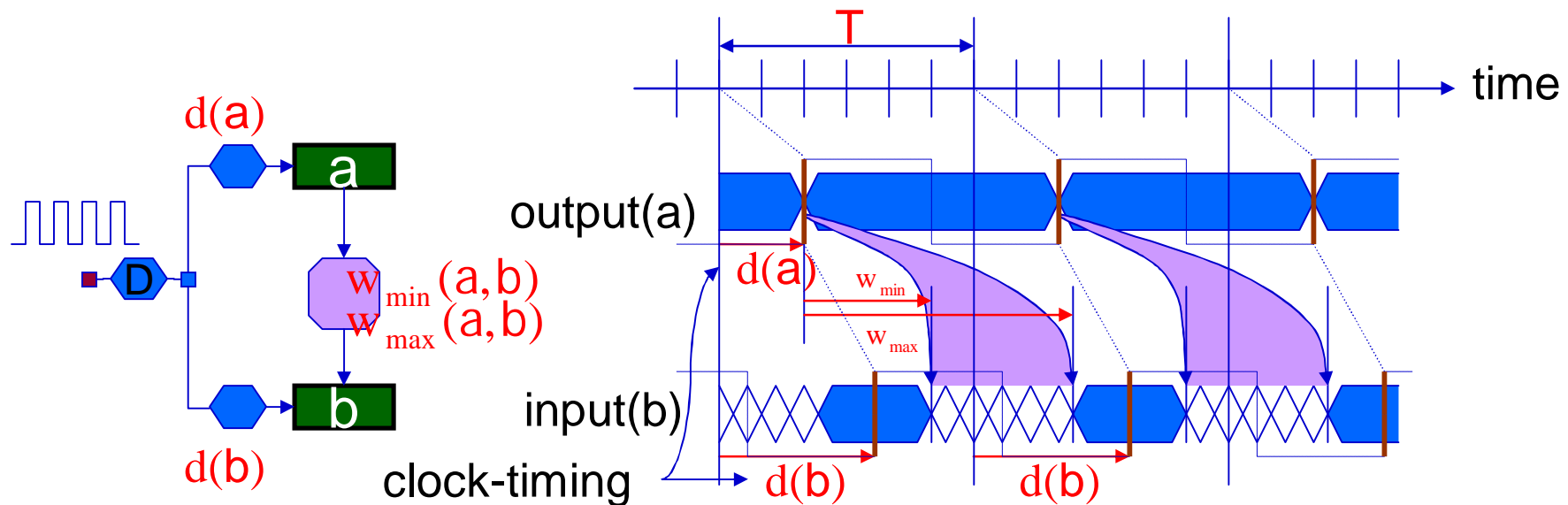
Clocking Constraints(1-clock-cycle)

- Setup Constraint (No zero-clocking)

$$d(a) + w_{\max}(a, b) \leq T + d(b)$$

- Hold Constraint (No double-clocking)

$$d(b) \leq d(a) + w_{\min}(a, b)$$



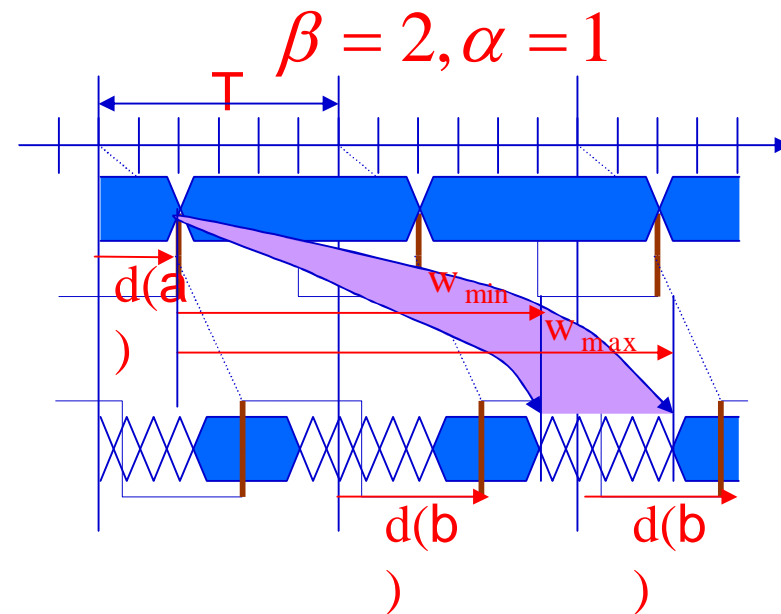
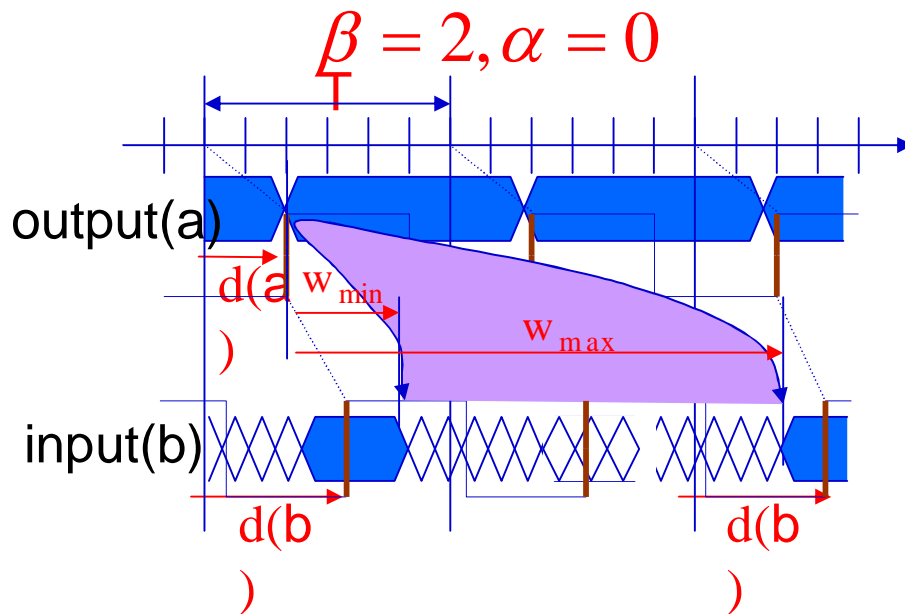
Clocking Constraints(Multi-clock-cycle)

- Setup Constraint (No zero-clocking)

$$d(a) + w_{\max}(a, b) \leq \beta_{a,b}T + d(b)$$

- Hold Constraint (No double-clocking)

$$d(b) + \alpha_{a,b}T \leq d(a) + w_{\min}(a, b) \quad \beta_{a,b} > \alpha_{a,b} \geq 0$$



Bound of feasible clock period

- Setup Constraint (Decide lower bound, T_{min})

$$d(a) + w_{\max}(a, b) \leq \beta_{a,b}T + d(b)$$

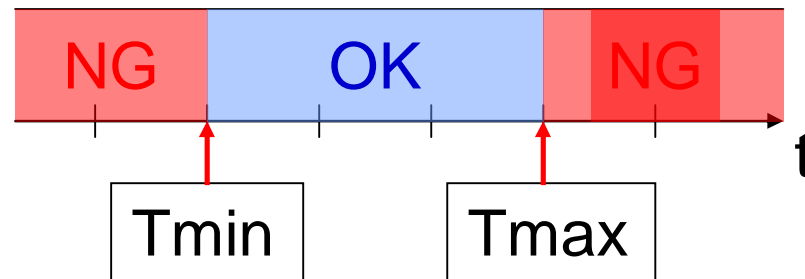
$$\beta_{a,b}T \geq d(a) - d(b) + w_{\max}(a, b)$$

- Hold Constraint (Decide upper bound, T_{max})

$$d(b) + \alpha_{a,b}T \leq d(a) + w_{\min}(a, b)$$

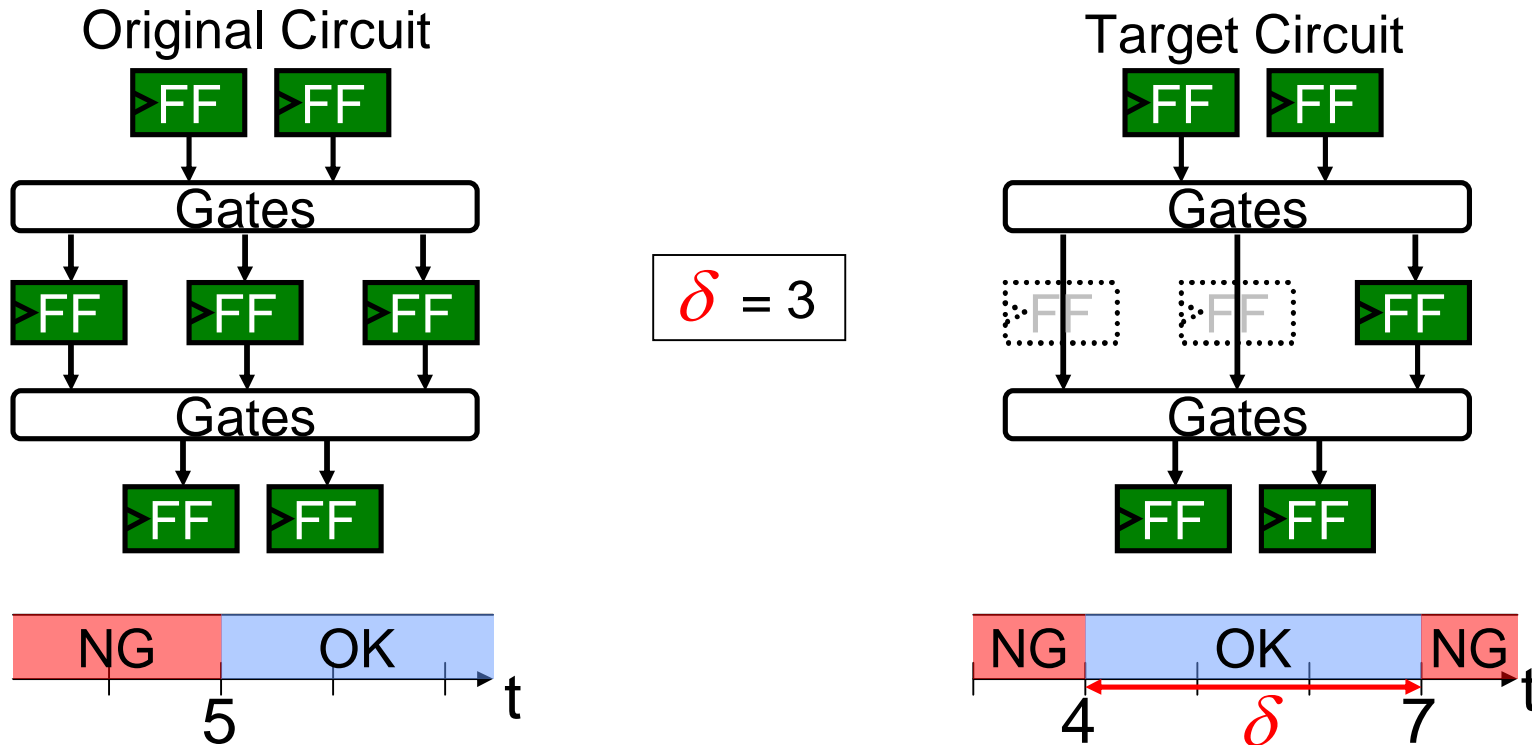
$$\alpha_{a,b}T \leq d(a) - d(b) + w_{\min}(a, b)$$

- If $\alpha_{a,b} \neq 0$ exist an upper bound of clock period.



Target type of Circuit

- Reduce number of intermediate registers.
- Works at target clock period range δ .



- To reduce number of intermediate registers:-
 - Need to analyze a circuit with multi-clock cycle paths.
 - Need to know minimum feasible clock period of the circuit.

Clock Period Range δ

- Setup Constraint (Decide lower bound, T_{\min})

$$\beta_{a,b} T_{\min} \geq d(a) - d(b) + w_{\max}(a,b)$$

- Hold Constraint (Decide upper bound, T_{\max})

$$\alpha_{a,b} T_{\max} \leq d(a) - d(b) + w_{\min}(a,b)$$


$$\delta = T_{\max} - T_{\min}$$

$$\alpha_{a,b} (T_{\min} + \delta) \leq d(a) - d(b) + w_{\min}(a,b)$$

- If clock period range given, by using above constraints:-
 - Can get circuit works correctly between T_{\min} and $(T_{\min} + \delta)$.

Minimum Clock Period with clock period range

■ Problem formulation

- Input: $w_{\max}(a, b)$, $w_{\min}(a, b)$, clock period range δ .
- Output: minimum clock period T .

- Subject to: for each path between registers

- $d(a) - d(b) \leq \beta T - w_{\max}(a, b)$

- $d(b) - d(a) \leq w_{\min}(a, b) - \alpha\delta - \alpha T$

β, α : Given constants

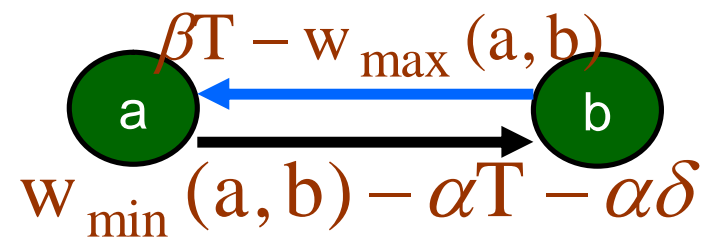
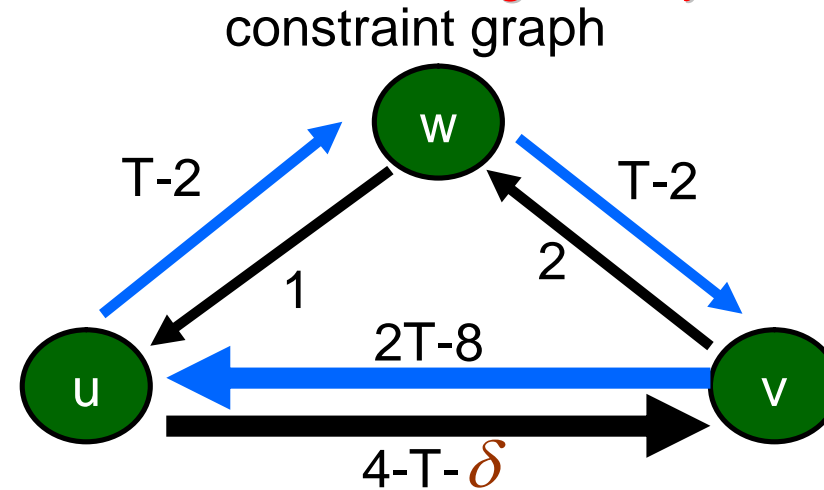
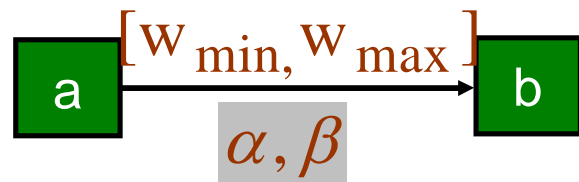
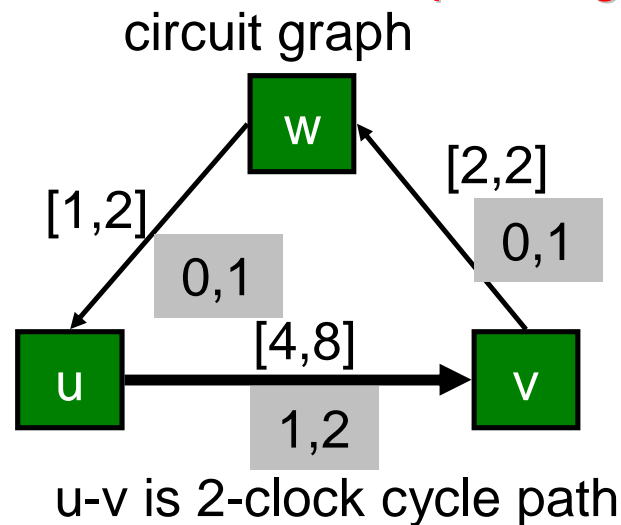
$d(a)$: Clock input timing (variable)

- Assumption

- Arbitrary clock input timing is possible.
- Clock-timing realization is independent of circuit.
(given signal propagation delays are fixed)

Feasibility Check by Constraint Graph

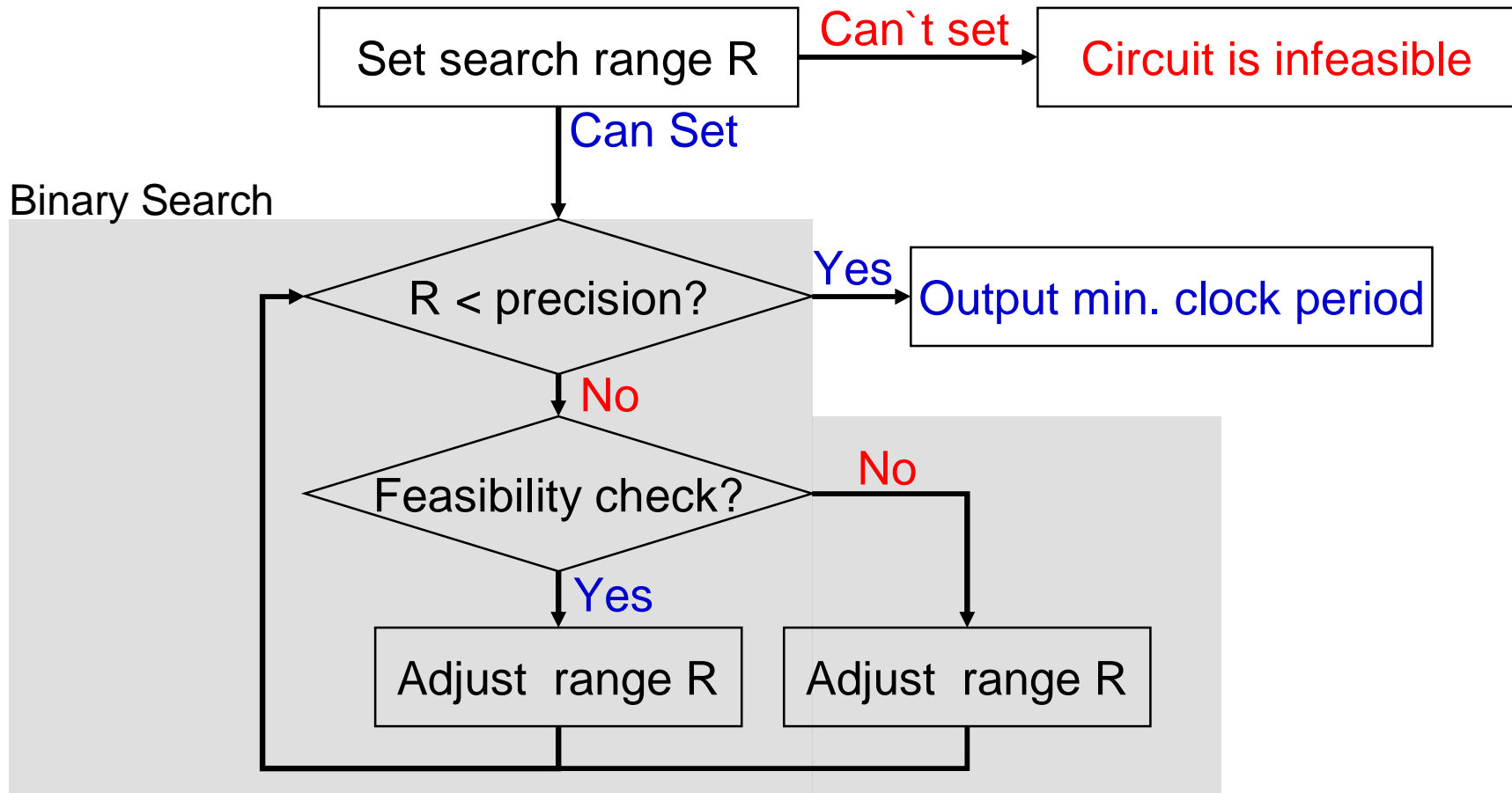
- Clock period T is **feasible** if and only if constraint graph has **no negative weight cycle**.
- Use **shortest path algorithm** to check there are negative cycle or not.



$d(a) - d(b) \leq \beta T - w_{\max}(a, b) : \text{Z-Edge (Setup)}$
$d(b) - d(a) \leq w_{\min}(a, b) - \alpha T - \alpha \delta : \text{D-Edge (Hold)}$

Original Algorithm [1] – Overview

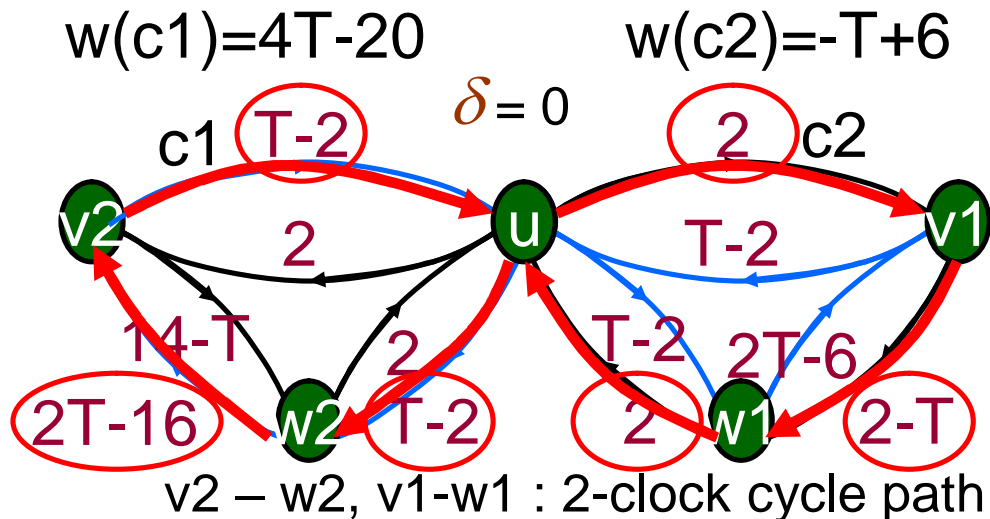
- Target circuit : Contains **1-clock cycle** path only.



[1] : Practical Fast clock schedule design (Takahashi, 2005 18th karuizawa workshop)

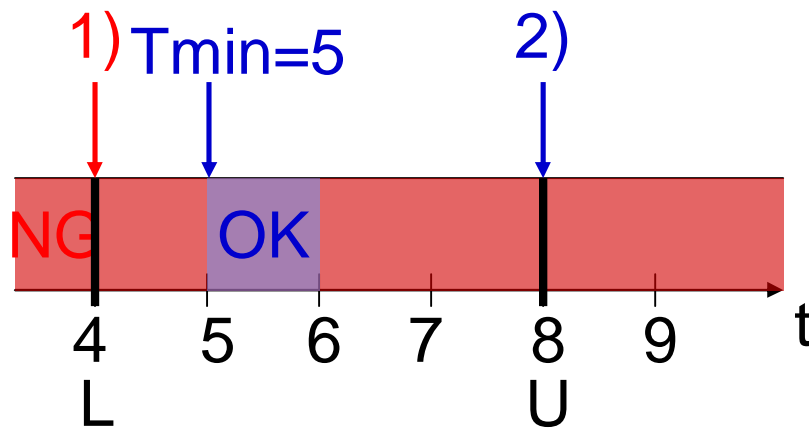
Problem of Original Algorithm

- Apply original algorithm to circuit contains **multi-clock** cycle path.



Searching Range ($L=4, U=8$)

- ($L=4, U=8$) Check L :-
 - $T=4, w(c1) = 4T - 20 = -4$
c1 is negative
- ($L=4, U=8$) Check U :-
 - $T=8, w(c2) = -T + 6 = -2$
c2 is negative

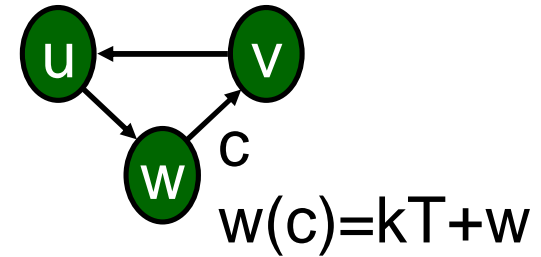


Algorithm assumes that the circuit is infeasible at any clock period.

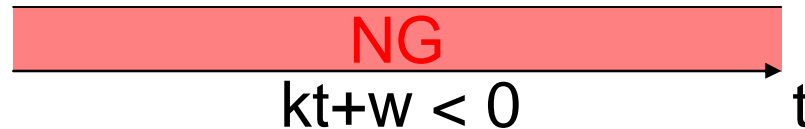
Z-Edge (Setup) D-Edge (Hold)

Definition for type of cycle

- Cycle weight of C, $w(C) = kT + w$.
 k, w : constants
 T : clock period

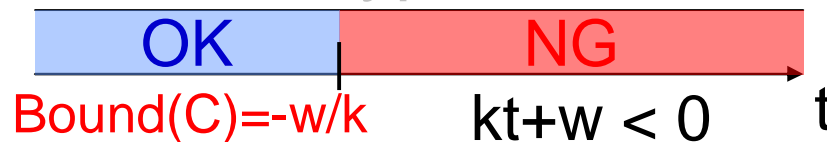


- $k=0$, C is 0-type.
 - If $w < 0$, circuit is infeasible for any T.

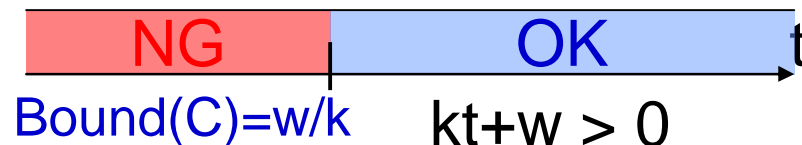


- If $w \geq 0$, no problem.

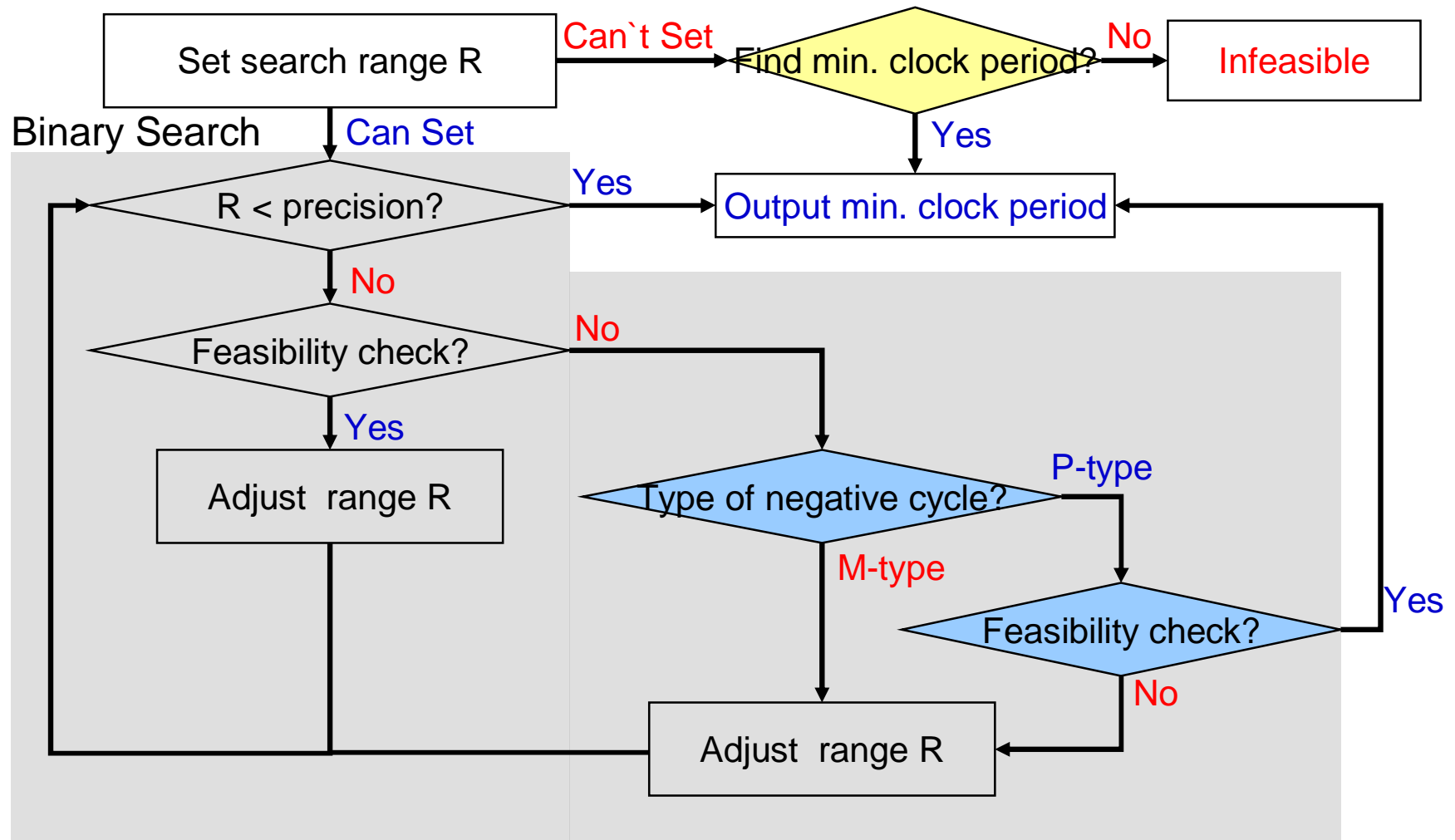
- $k < 0$, C is M-type.



- $k > 0$, C is P-type.



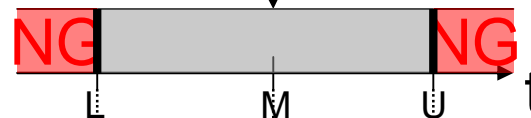
Proposed Algorithm – Overview



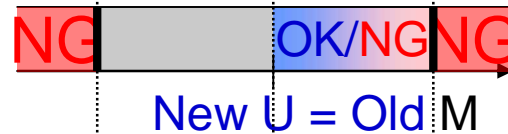
Proposed Algorithm: Binary Search

- Feasibility check

Check $M=(L+U)/2$

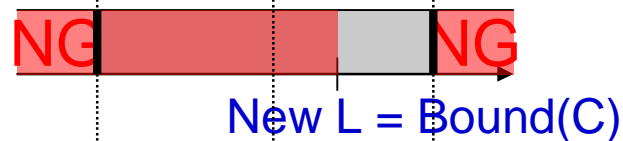


- Feasible:-

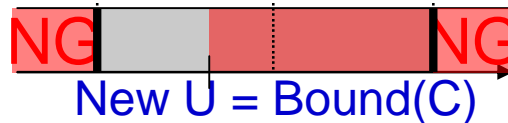


- Infeasible, a negative cycle C is found:-

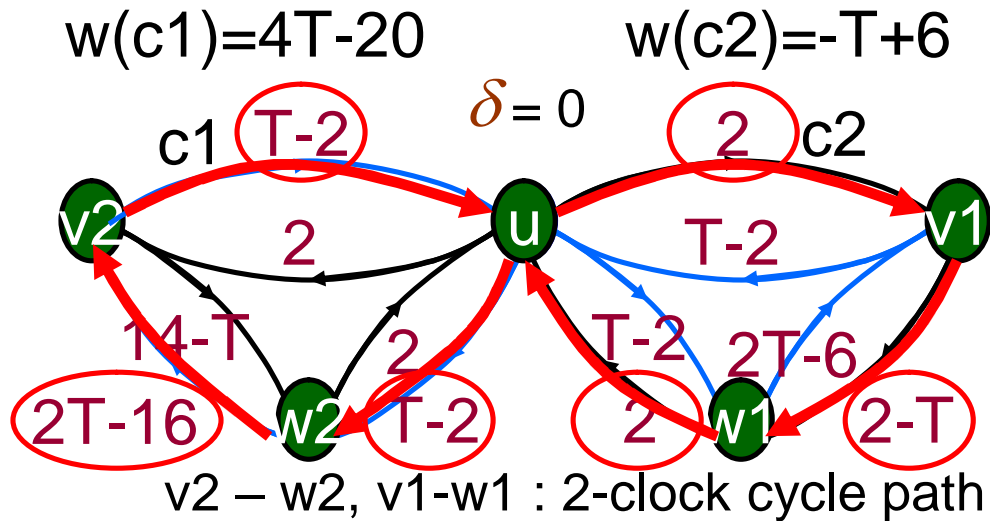
- C is P-type



- C is M-type



Proposed Algorithm: Example



Searching Range ($L=4, U=8$)

1) ($L=4, U=8$) Check L :-

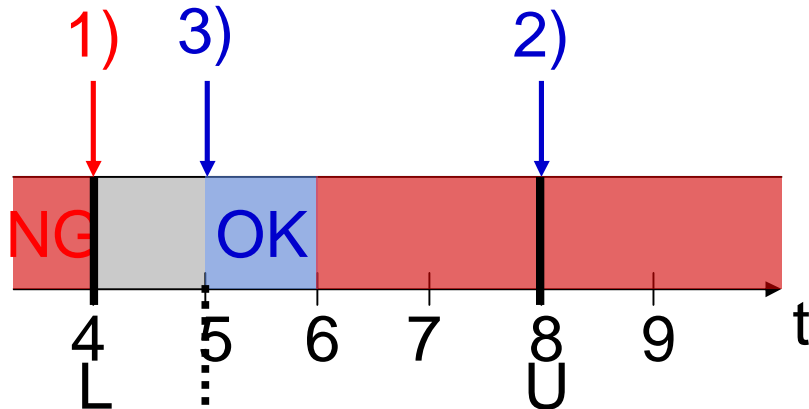
- $T=4, w(c1)=4T-20=-4$
c1 is negative, P-type

2) ($L=4, U=8$) Check U :-

- $T=8, w(c2)=-T+6=-2$
c2 is negative, M-type
Bound(c2)=6 = new U

3) ($L=4, U=6$)
Check $M=(4+6)/2=5$:-

- $T=5$, no negative cycle



$T_{min} = 5, Precision=1$

Z-Edge (Setup) D-Edge (Hold)

Experiment Results

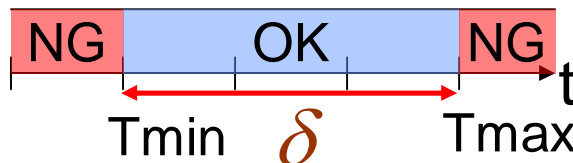
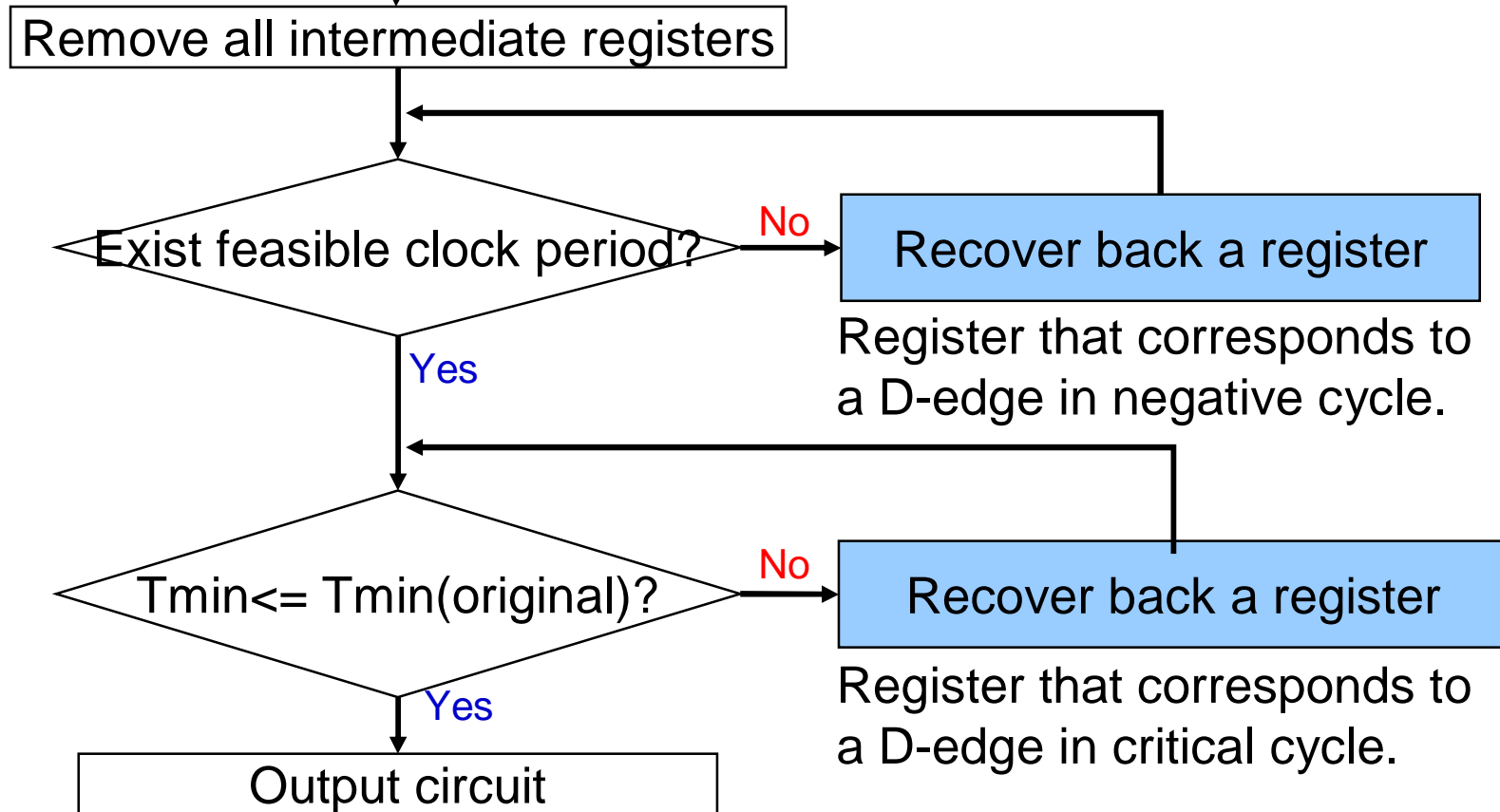
		Proposed		Original	
* δ [ps]	circuit	Tmin [ps]	Tmax [ps]	Tmin [ps]	Tmax [ps]
0	Add1	2407	2407	NA	NA
150	Add1	2560	2710	NA	NA
0	Add2	2231	2231	2231	2231
250	Add2	2560	2810	NA	NA

* δ is target clock period range (Input)

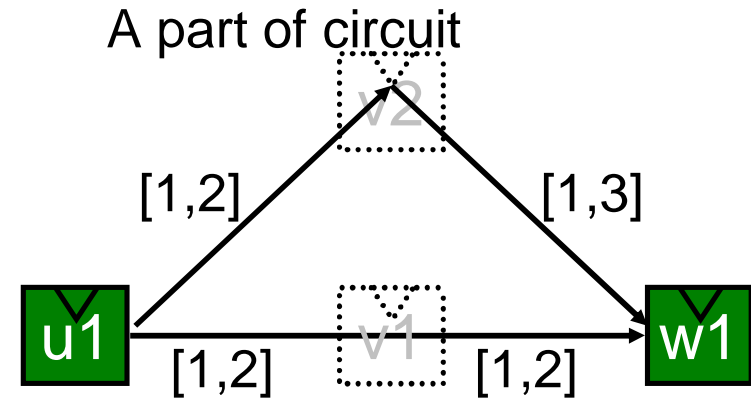
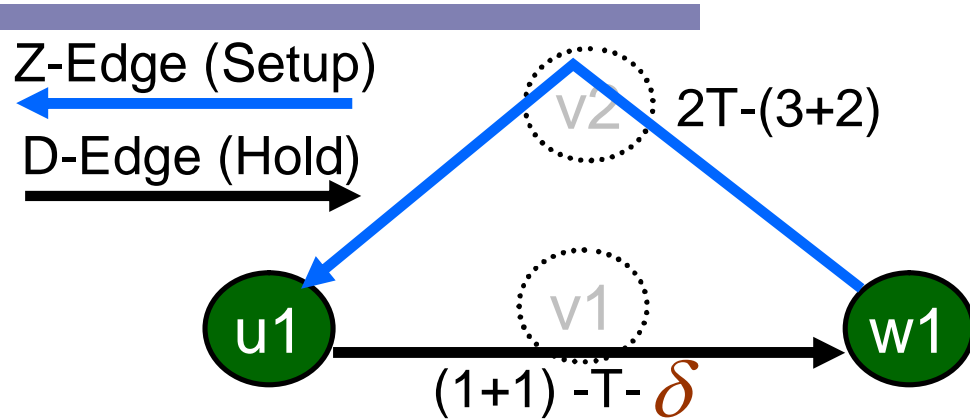
- Proposed algorithm can find minimum feasible clock period of a circuit with multi-clock cycle paths.

Reduction on the number of registers

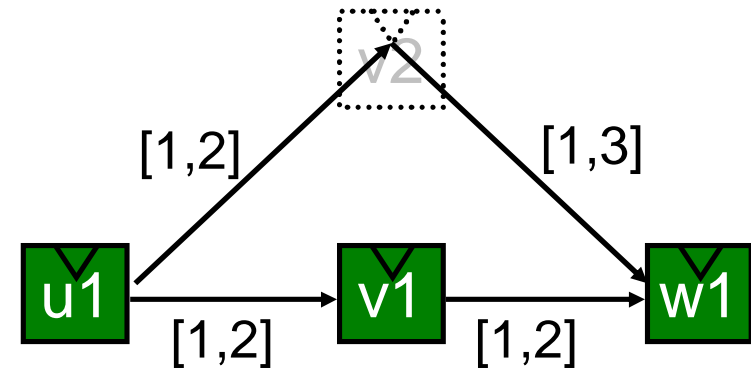
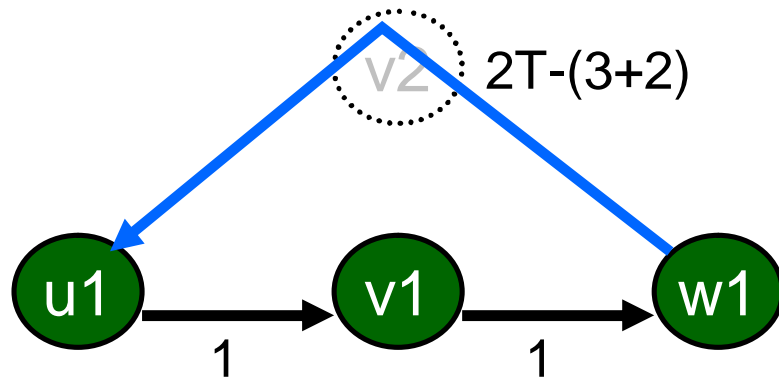
Input : Pipelined circuit ($T_{\min}(\text{original})$) and clock period range δ



Why recover back a register corresponds to D-Edge?



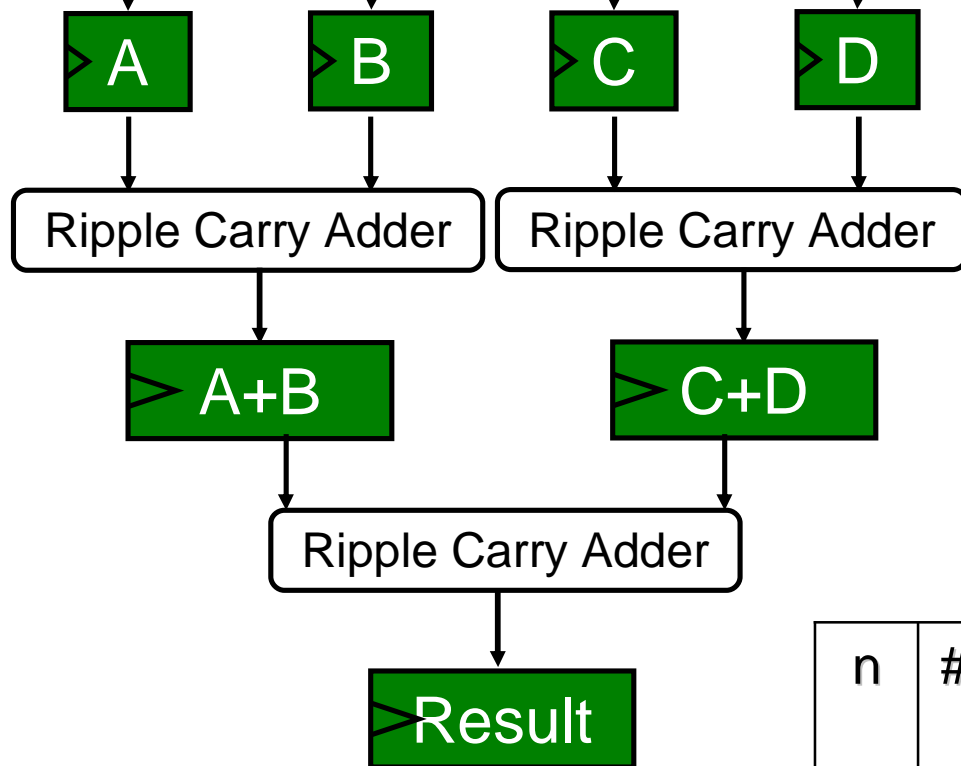
$T=2, \delta=1$ $w(u1, w1, u1) = 2T-(3+2) + (1+1)-T-\delta = -2$: negative cycle due to minimum delay path $u1, v1, w1$.



$T=2, \delta=1$ Recover back register $v1$
 $w(u1, v1, w1, u1) = 2T-(3+2) + 1+1 = 1$

Experiments : Input Circuit (Pipelined Adder)

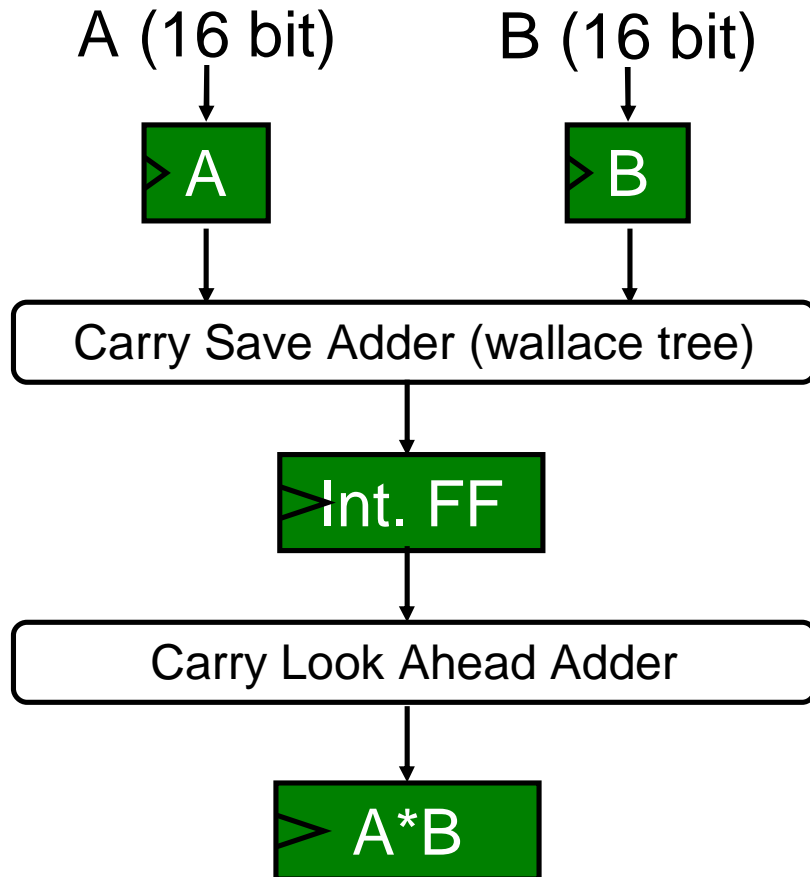
A (n bit) B (n bit) C (n bit) D (n bit)



- 2-Stage.
- Add four n-bit numbers. (n=4,8,16).
- 1st stage : A+B and C+D.
- 2nd stage : A+B+C+D.
- Each adder : Ripple Carry Type.
- Process Library ROHM 0.35um.
- Scheduled I/O pins and registers.
- Circuit Statistics:-

n	#FF	1 st Stage (ps)		2 nd Stage (ps)	
4	32	588	2454	588	2840
8	60	598	4079	598	4474
16	116	598	7239	598	7634

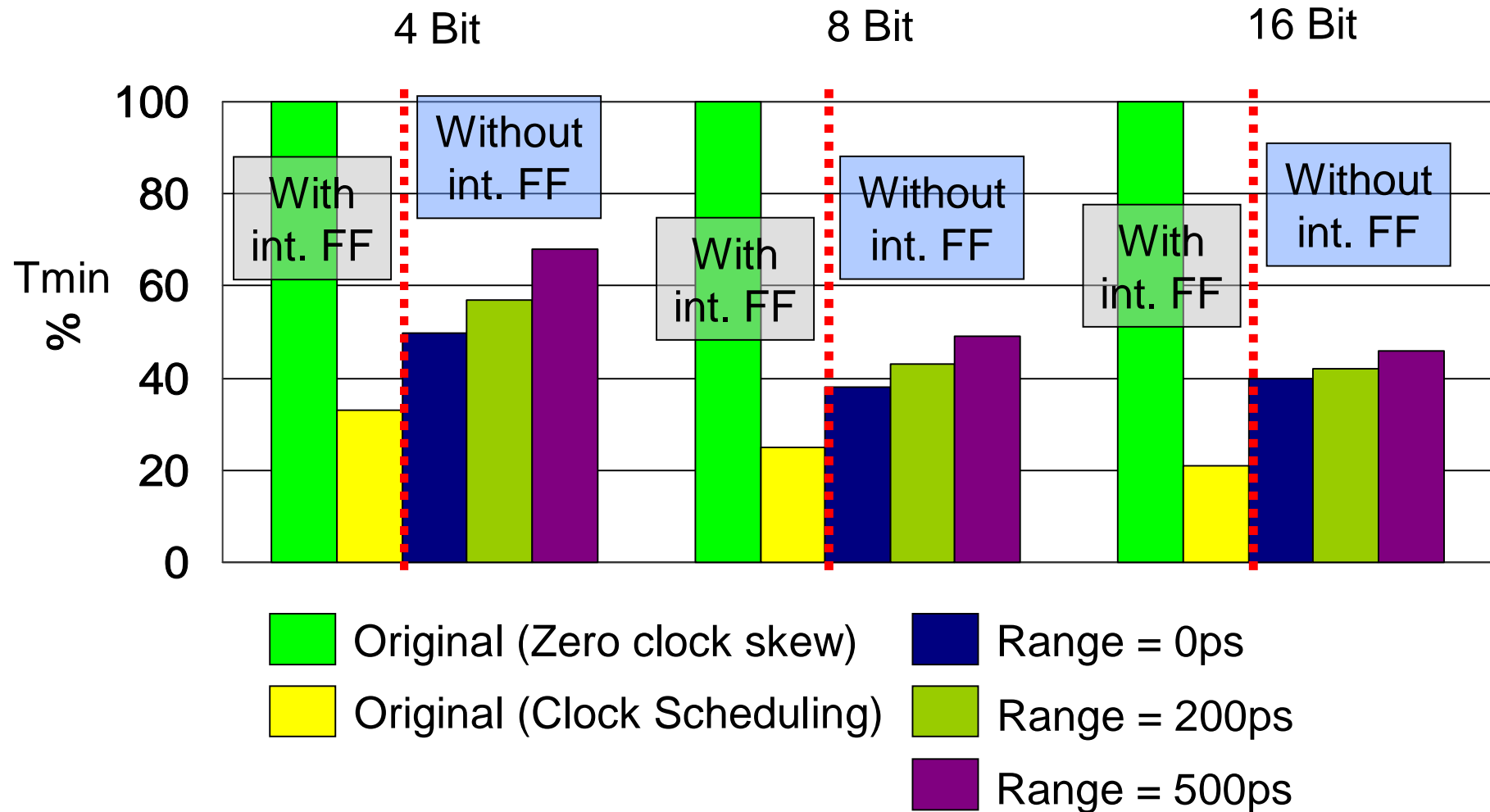
Experiments : Input Circuit (Pipelined Multiplier)



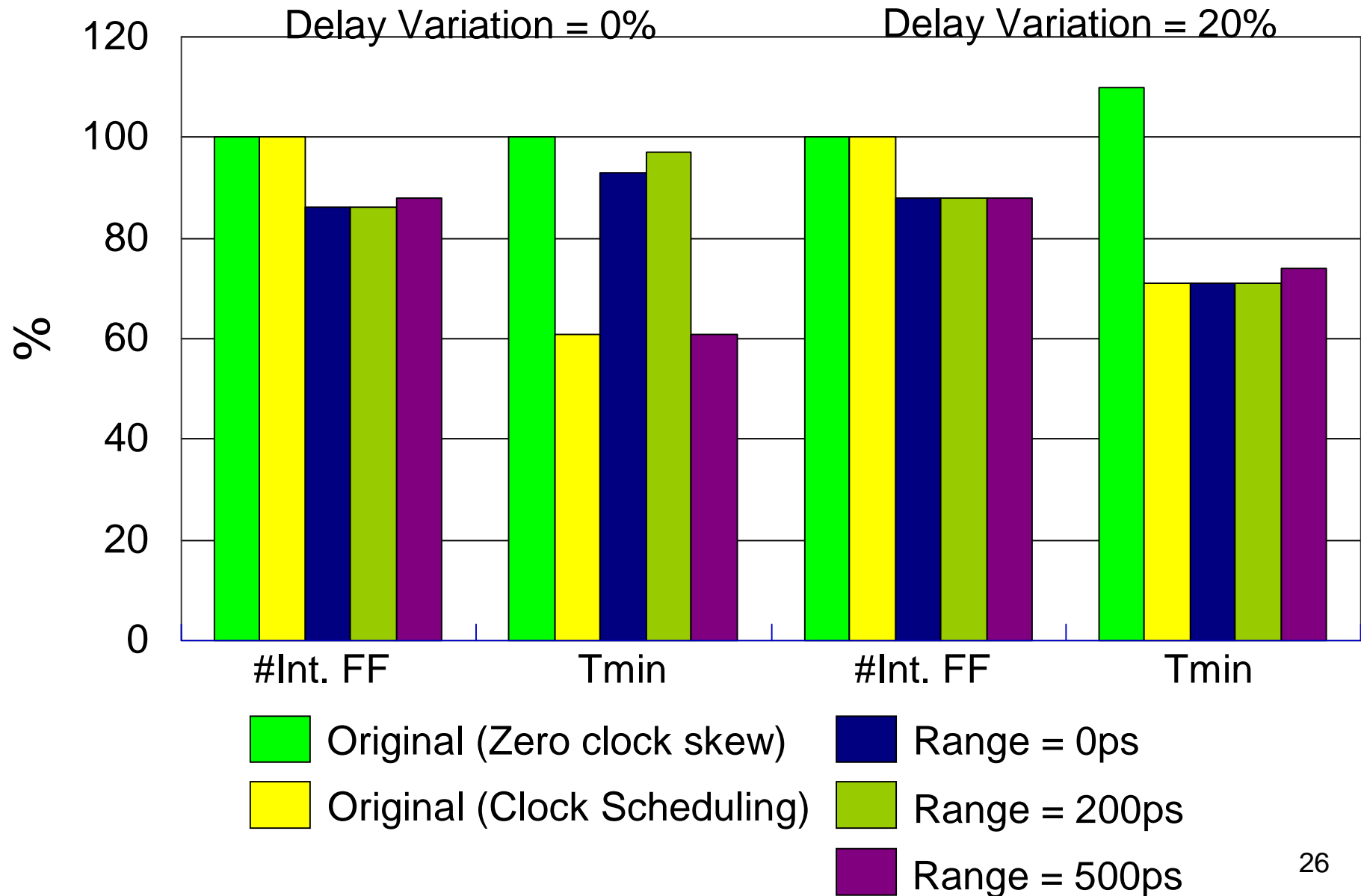
- 2-Stage.
- Multiply two 16-bit numbers.
- 1st stage : Carry Save adder with wallace tree structure.
- 2nd stage : Carry look ahead adder.
- Process Library ROHM 0.35um.
- Scheduled I/O pins and registers.
- Circuit Statistic:-

#FF	1 st Stage (ps)		2 nd Stage (ps)	
	120	757	5075	373

Experiments Results : Pipelined Adder



Experiments Results : Pipelined Multiplier



Conclusions

- Conclusion:-

- Number of intermediate registers can be reduced by multi-clock cycle paths and clock scheduling.

- Future Works:-

- Proposed algorithm only insert intermediate registers to satisfy timing constraints.
- Consider delay balancing together with intermediate register insertion.