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A Quasi-Newton Preconditioned Newton-Krylov Method for Robust and Efficient Time-Domain Integrated Circuit Simulation

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Outline

- Motivation
- Review of previous work
- Preconditioned Newton-Krylov Method
- Conclusions

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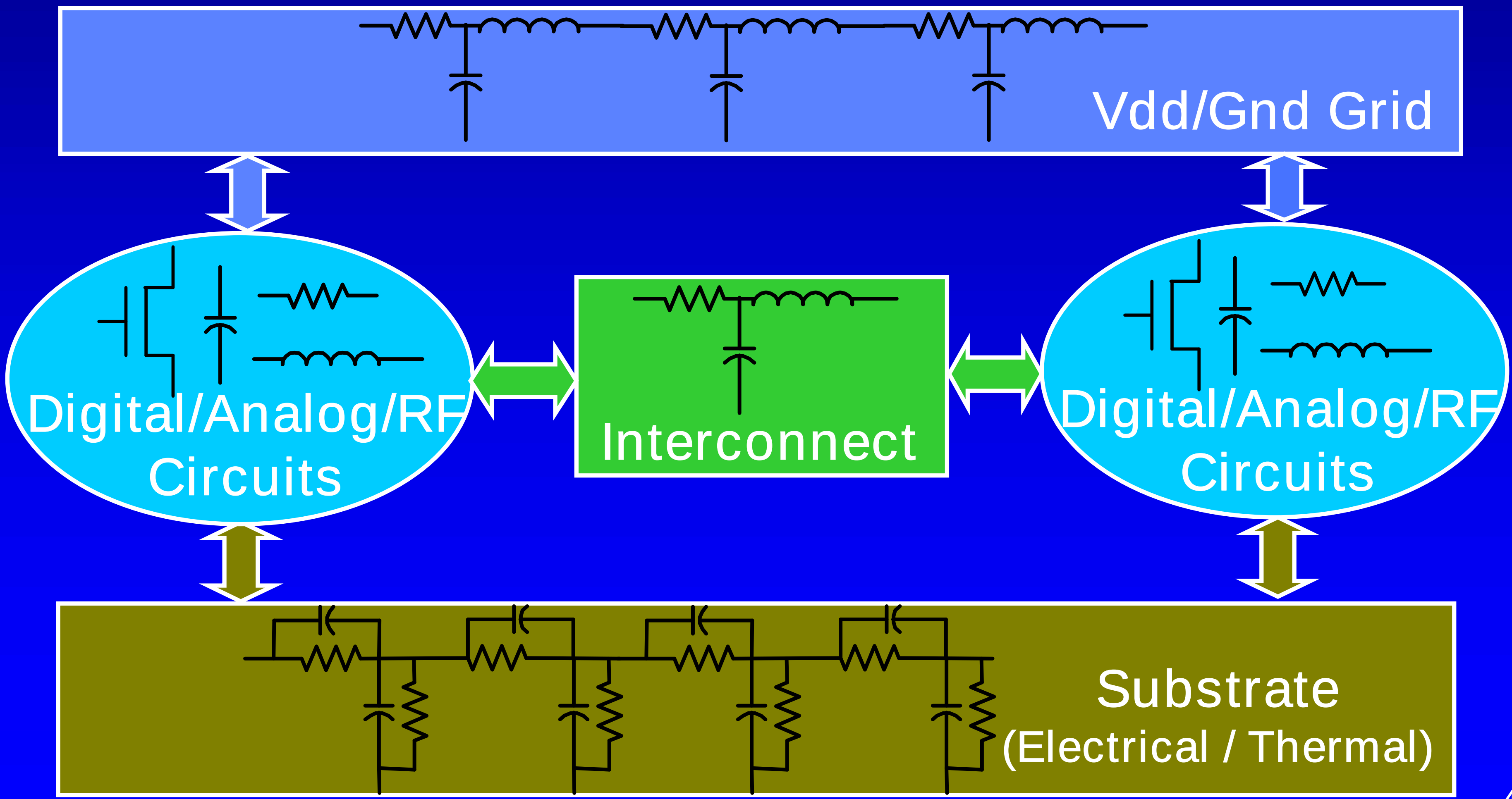
Deep-Submicron VLSI Circuit Simulation

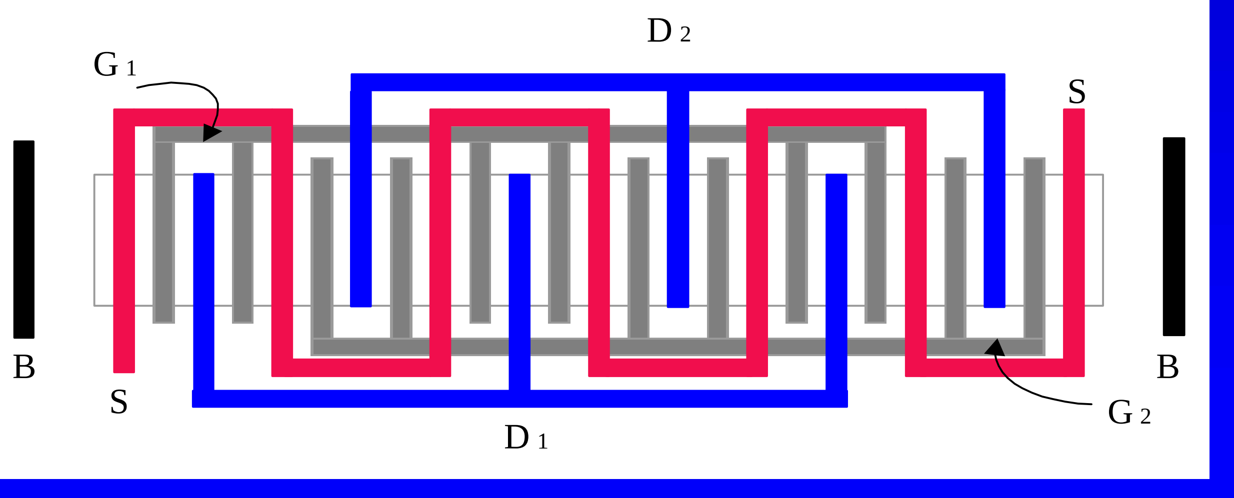
- Couplings with surrounding parasitic environments



High Frequency Analog/RF Circuit Simulation

- Coupled circuit-EM simulation
  - Sensitive structures modeled as large-scale strongly coupled linear networks, such as inductors in a LNA modeled with the PEEC method

- Extracted parasitics due to transistor layouts
  - Complicated layouts for transistors in an analog/RF circuit



Challenges for SPICE Time-Domain Simulation

- Big Linear Small Nonlinear
  - The number of linear elements modeling parasitic effects is much larger than the number of nonlinear devices
- Parasitic couplings make circuit structures denser and strongly coupled
  - Circuit matrices become denser and hard to partition
- Per-iteration cost is dominated by LU factorization

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Previous Work

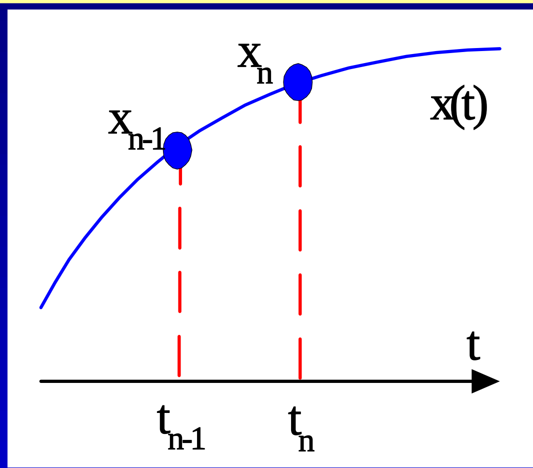
- Keep circuit matrices constant
  - Fixed leading coefficient numerical integration
  - Quasi-Newton methods for nonlinear iteration
- Make circuit matrices sparse (Partition)
  - Relaxation based iterative methods (Gauss-Seidel, SOR)
  - Semi-implicit methods (Phillips, ICCAD 2001)
  - Alternating-Direction-Implicit methods (Lee, ICCAD 2001)
- Use krylov subspace iterative methods
  - Conjugate gradient methods preconditioned with incomplete Cholesky decomposition (Chen, DAC 2001)
  - Multi-grid methods (Nassif, DAC 2000)

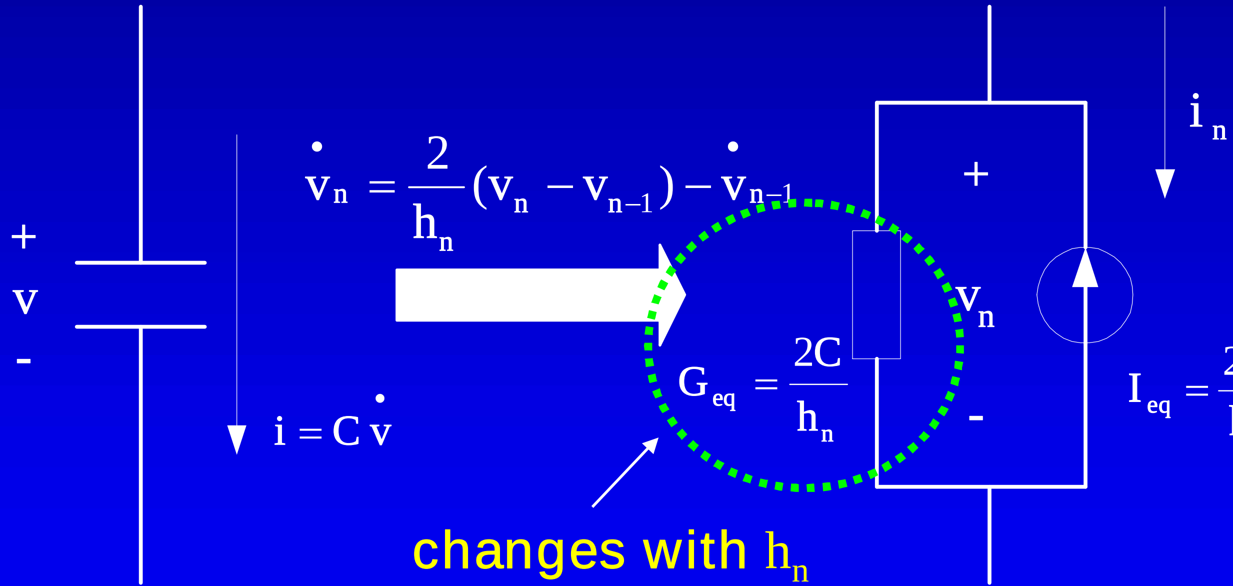
Our Viewpoints

- LU factorization based direct methods in SPICE are robust and accurate
  - Simulate nonlinear and linear circuits in a circuit matrix
  - Memory requirement is generally a minor factor
- To achieve efficiency, circuit matrices should be kept constant to reduce the number of LU factorizations

Matrix Change due to Numerical Integration (Trapezoid Formula)

$$\dot{x}_n \approx \frac{2}{h_n}(x_n - x_{n-1}) - \dot{x}_{n-1} \qquad (h_n = t_n - t_{n-1})$$





$$\dot{v}_n = \frac{2}{h_n}(v_n - v_{n-1}) - \dot{v}_{n-1}$$

$$i = C\,\dot{v}$$

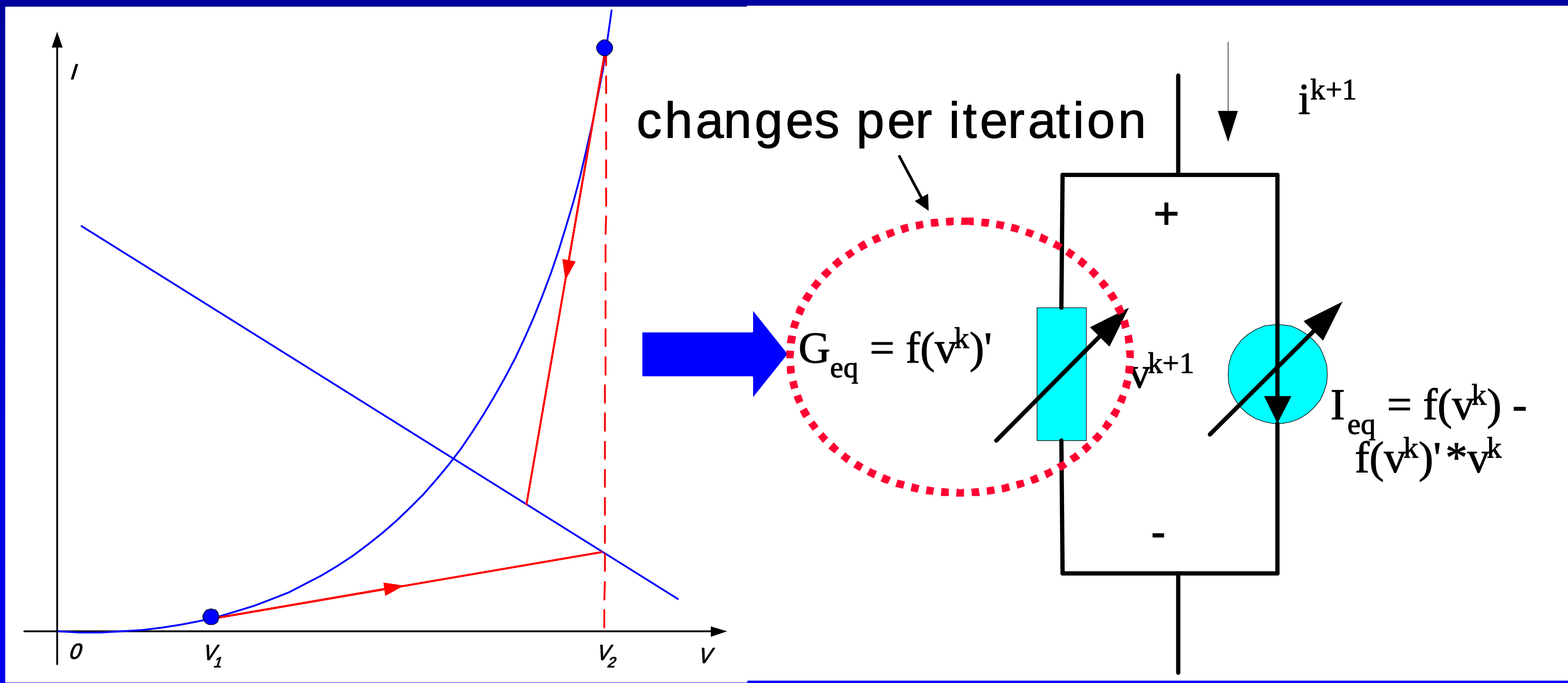
$$G_{eq} = \frac{2C}{h_n}$$

$$I_{eq} = \frac{2C}{h_n} v_{n-1} + C\,\dot{v}_{n-1}$$

- New LU factorization if h_n changes

Matrix Change due to Nonlinear Devices (Newton-Raphson Method)

$$i^{k+1} = f(v^{k+1}) \approx f(v^k)'v^{k+1} + f(v^k) - f(v^k)'v^k$$



- New LU factorization if $f(v^k)'$ changes

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Quasi-Newton Based Iterative Methods

- Solve $Ax = b$ with quasi-Newton methods

$$x^{(k)} = x^{(k-1)} + M^{-1}(b - Ax^{(k-1)})$$

single search direction per iteration

non-orthogonal

(slow convergence)

- How to achieve constant M ?
  - Fixed leading coefficient numerical integration
  - Quasi-Newton method for nonlinear iteration

Krylov Subspace Based Iterative Methods

- Solve $Ax = b$ with krylov subspace methods

$$M^{-1}Ax = M^{-1}b$$

$$x = x^{(0)} + \kappa_m(M^{-1}A, x^{(0)})$$

$$\kappa_m(M^{-1}A, x^{(0)}) = \mathrm{span}(r^{(0)}, M^{-1}Ar^{(0)}, \ldots, (M^{-1}A)^{m-1}r^{(0)}), \quad r^{(0)} = M^{-1}(b - Ax^{(0)})$$

m-dimensional orthogonal krylov subspace

(better convergence)

Krylov Subspace Based Iterative Methods (cont.)

- How to achieve constant M (preconditioner) ?
  - Quasi-Newton like time step-size control
  - Piecewise weakly nonlinear definition of nonlinear devices

- How to construct $Ax=b$?
  - Standard implicit numerical integration (better stability)
  - Original nonlinear device models (better accuracy)

Quasi-Newton like Time Step-Size Control

- G and C represent conductance and capacitance/ susceptance matrices, b is the input vector

$$Gx + C\dot{x} = b$$

$$h_n = \alpha h$$

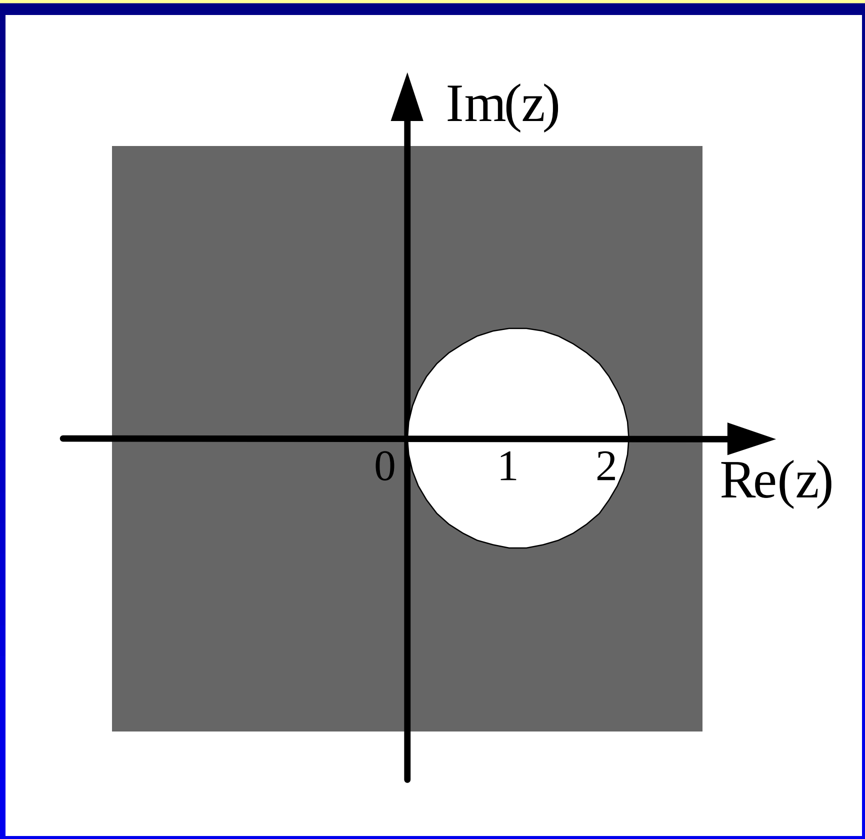
$$\left(G + \frac{2C}{\alpha h}\right)x_n^{(k)} = \frac{2C}{\alpha h}x_{n-1} + C\dot{x}_{n-1} + b$$

Quasi-Newton Preconditioner

$$\left\|\left(G + \frac{2C}{h}\right)^{-1}\left(G + \frac{2C}{\alpha h}\right) - I\right\| = \left\|\left(G + \frac{2C}{h}\right)^{-1}\left(1 - \frac{1}{\alpha}\right)\frac{2C}{h}\right\| < \eta < 1$$

$$\left|\frac{1 - \frac{1}{\alpha}}{1 - z}\right| < \eta < 1, \quad z = -\frac{h}{2\tau}, \quad \tau = \text{eig}\left(G^{-1}C\right)$$

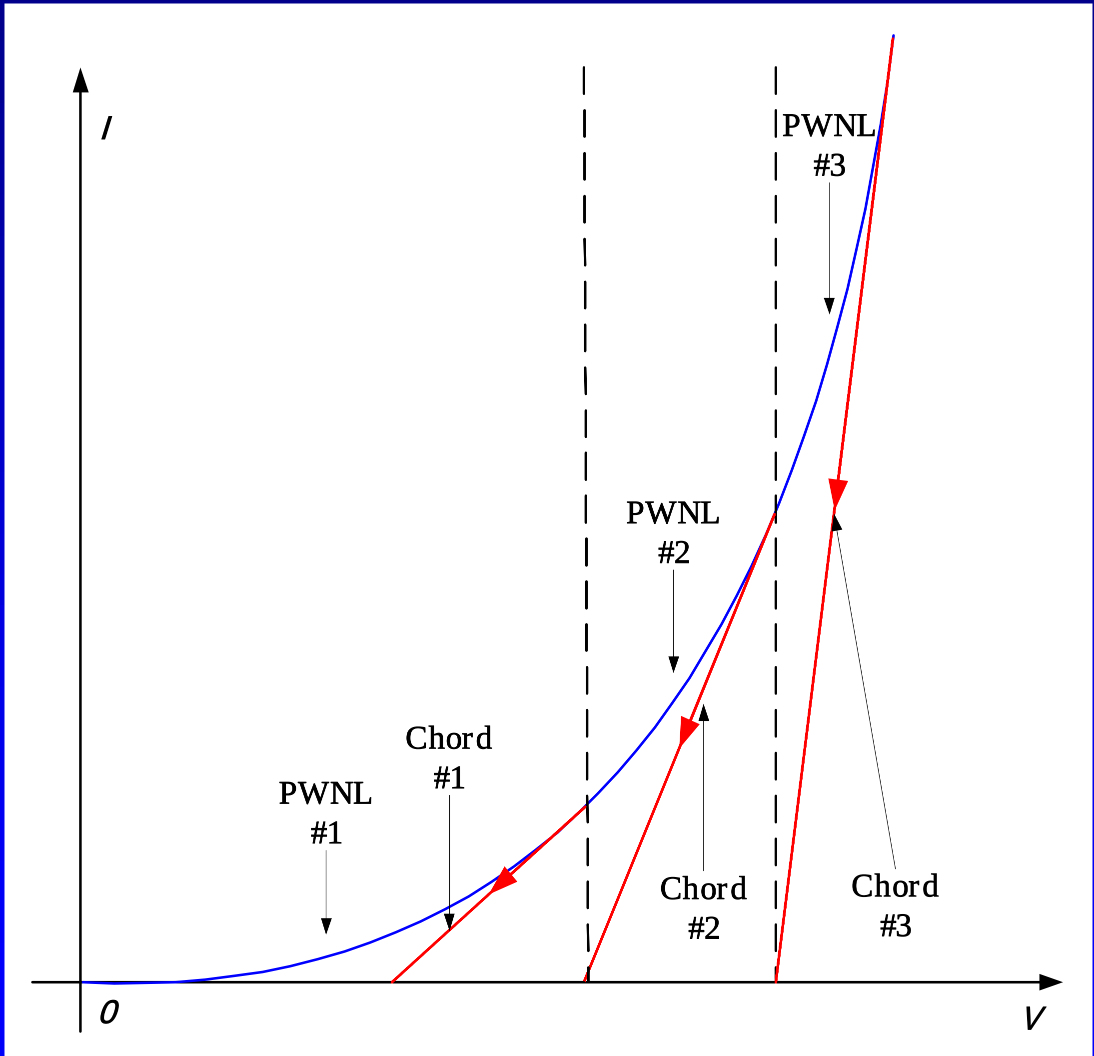
Effective Preconditioner Region



$$|z-1| > 1, \qquad \eta = \left|1 - \frac{1}{\alpha}\right| < 1 \quad \Rightarrow \quad \alpha > \frac{1}{2}$$

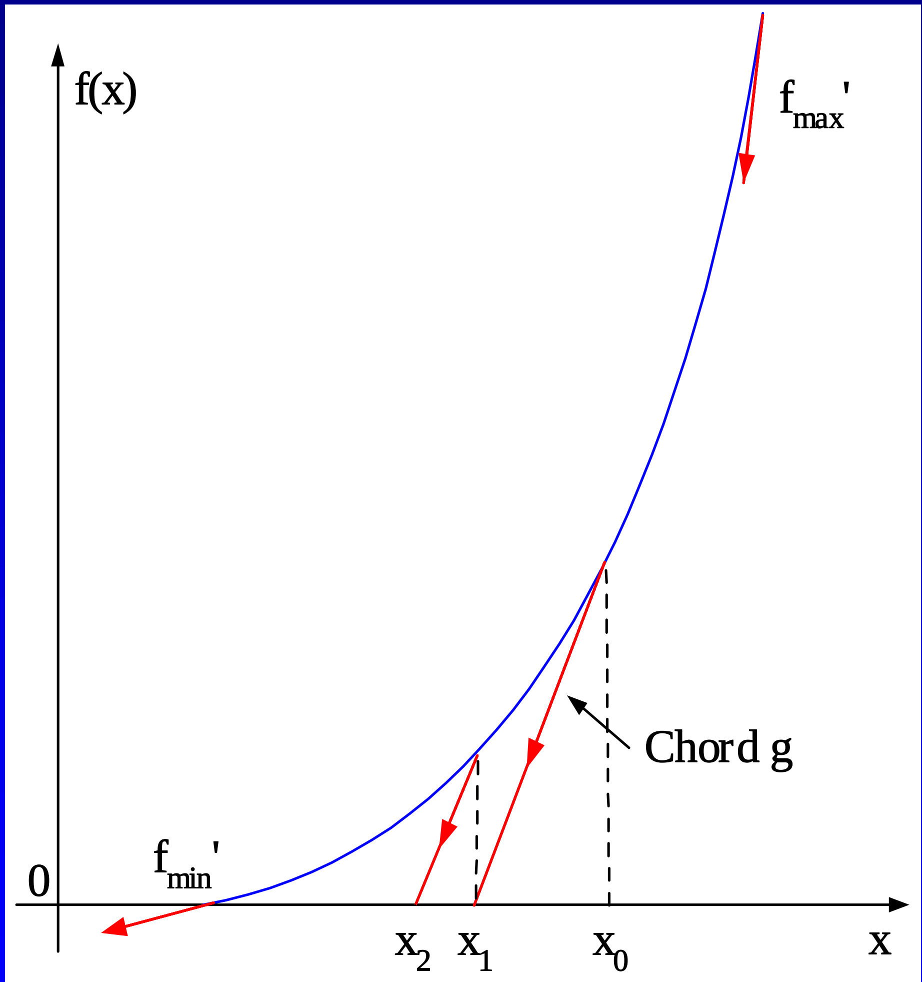
- Effective preconditioner region covers all left-half z-plane poles

Piecewise Weakly Nonlinear Definition



- Nonlinear functions are partitioned into a few PWNL regions
- Fixed chords within PWNL regions
  - Constant circuit matrices
- Chords change for new PWNL regions

Convergence of Quasi-Newton Nonlinear Iteration



- Quasi-Newton nonlinear iteration within a PWNL region

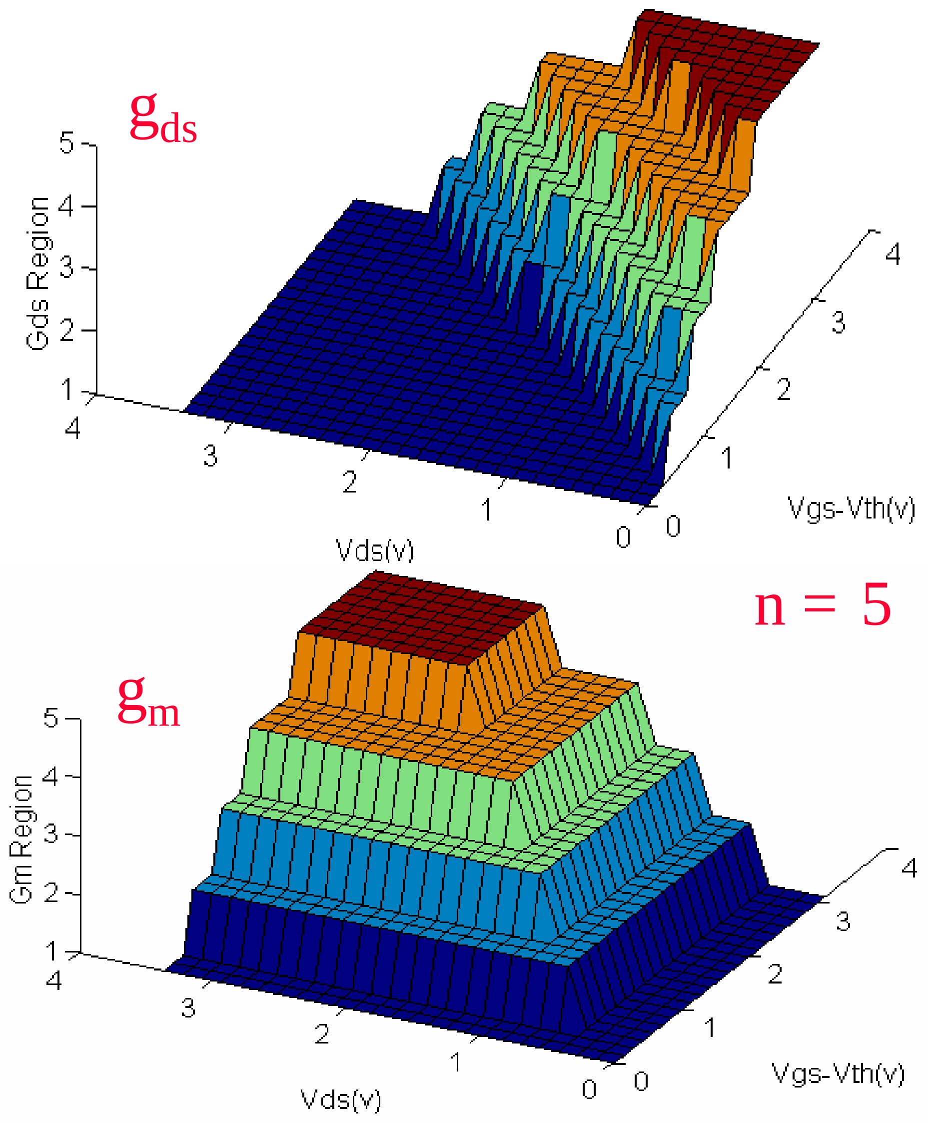
$$\varepsilon_{i+1} \approx \varepsilon_i \left(1 - \frac{f'(x_i)}{g}\right) - \varepsilon_i^{\,2} \frac{f''(x_i)}{2g}$$

$$\left|1 - \frac{f'(x_i)}{g}\right| < \delta, \quad 0 < \delta < 1$$

$$\frac{f'_{max}}{1+\delta} < g < \frac{f'_{min}}{1-\delta}$$

- Guideline for generating PWNL regions

Generalized MOSFET PWNL Definition



- PWC regions of g_{ds} and g_m are equivalent to PWNL regions of I_{ds}

$$g_n = g_{max}$$
$$g_{i-1} = (1-\delta) g_i , \quad i = n,...,2$$

- Generalized MOSFET PWNL definition used to construct quasi-Newton preconditioner M
- Original MOSFET models for $Ax = b$

Low Rank Update

- L and U matrices of A_{new} derived from those of A_{old} if

$$A_{new} = A_{old} + cr^T$$

A_{new} and A_{old} are $n \times n$ matrices, c and r are $n \times m$ vectors

$$m << n$$

- Efficient if the number of nonlinear devices switching PWNL regions is small

Approximate Preconditioner

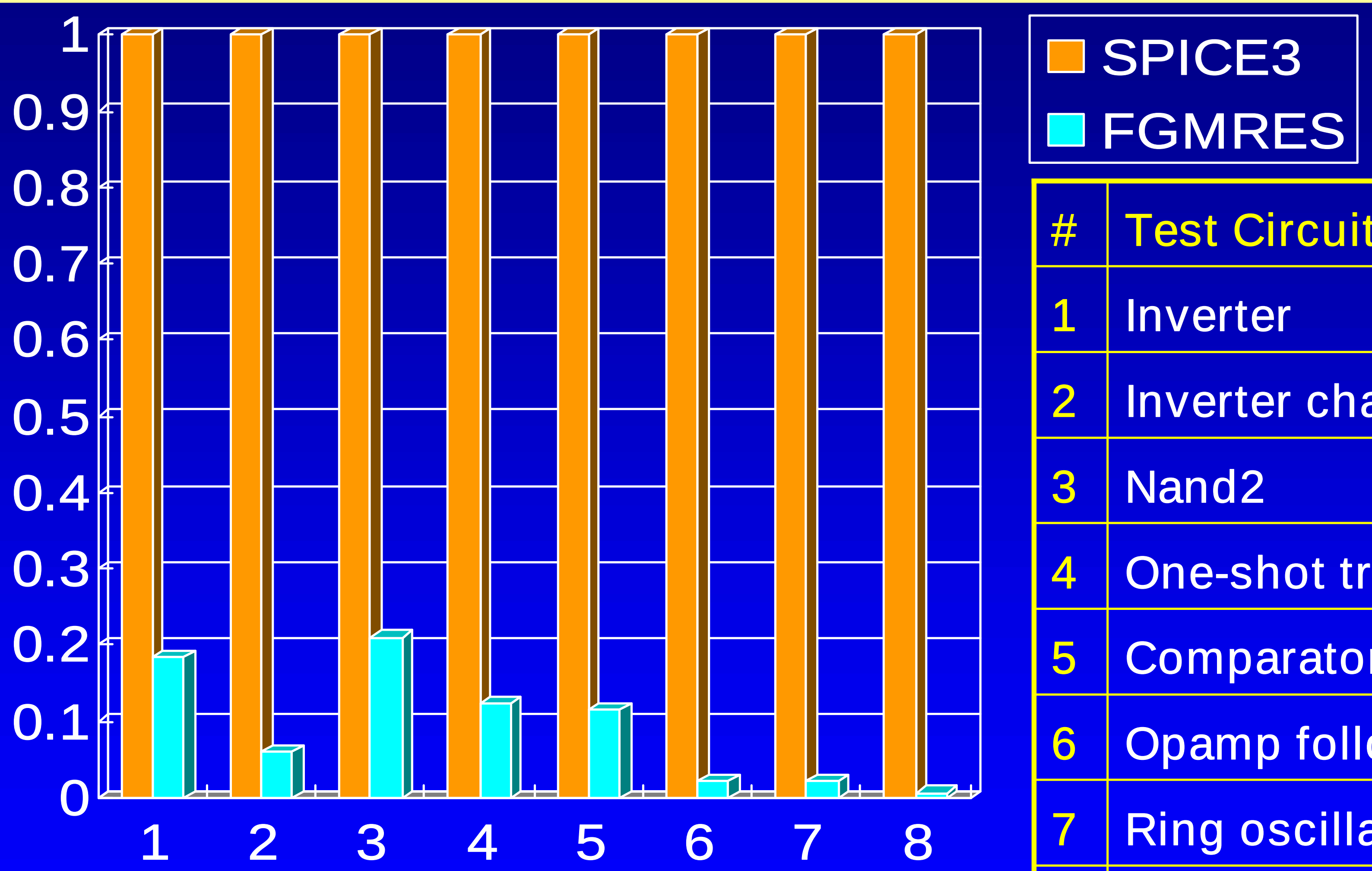
- The preconditioning process is to solve $My=c$
  - LU factorization of M
  - ILU factorization of M
  - Krylov subspace methods to solve $My=c$, such as FGMRES
- ILU preconditioner
  - Reduce the preconditioning cost of LU preconditioner
- Hybrid preconditioner
  - Solve y^0 with ILU preconditioner
  - Solve y with the FGMRES method starting from y^0
  - More preconditioning efforts for less FGMRES iterations

Incomplete LU Preconditioner

$$A = LU = \begin{bmatrix} l_{11} & & & & & & \\ \cdot & \cdot & & & & 0 & \\ \cdot & \cdot & l_{ii} & & & & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & \cdot & l_{ji} & \cdot & l_{jj} & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & l_{nn} \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & 1 & \cdot & u_{ij} & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot & \cdot \\ & & & & 1 & \cdot & \cdot \\ & 0 & & & & \cdot & \cdot \\ & & & & & & 1 \end{bmatrix}$$

- ILU preconditioner derived by removing small elements in already factorized L and U matrices
  - $|l_{ji}| < c \cdot \max(|l_{*i}|)$, $|u_{ij}| < c \cdot \max(|u_{i*}|)$, $c=0.001$
- ILU preconditioner updated by low rank update during nonlinear iteration

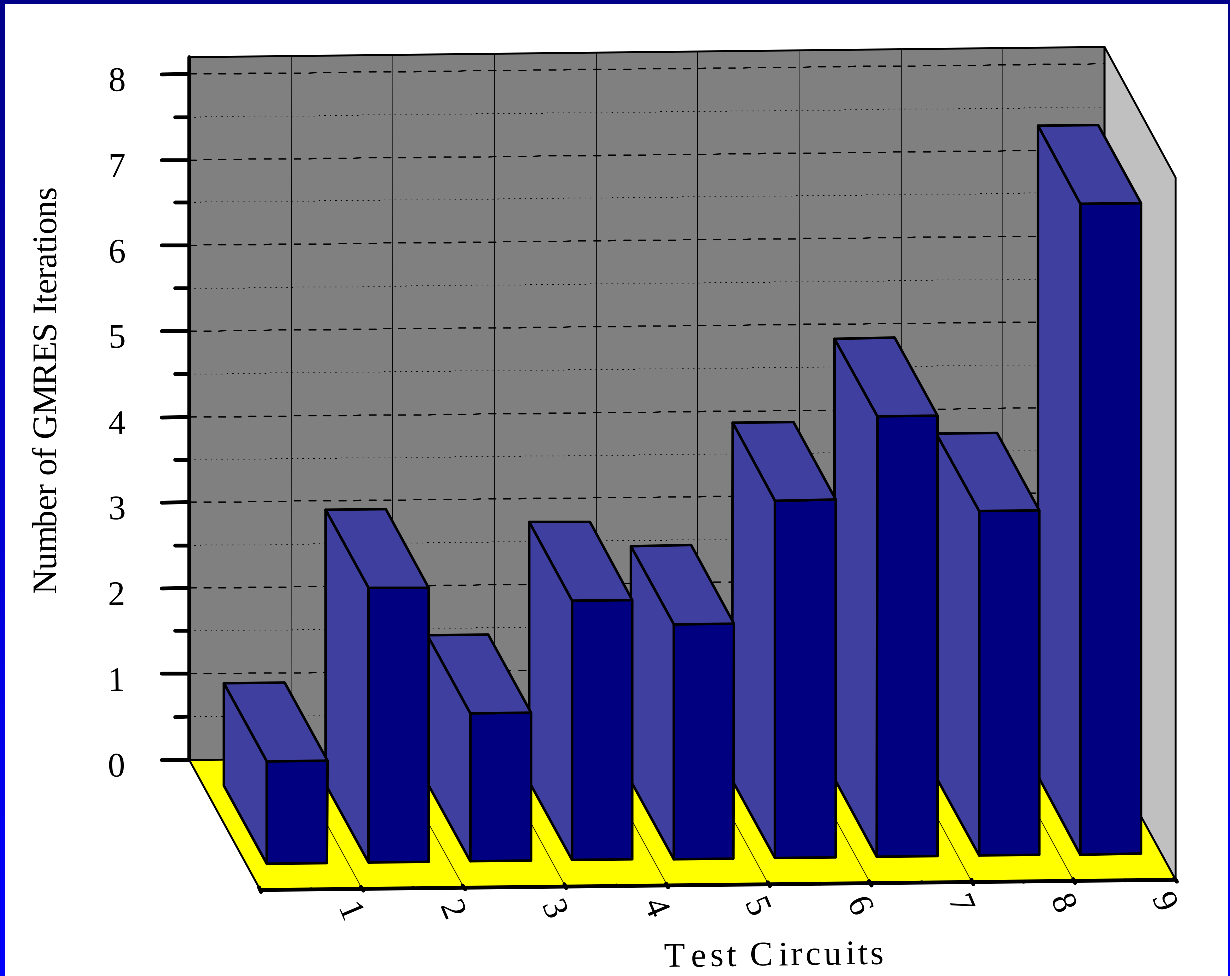
General Nonlinear Circuits – Normalized #LU



#	Test Circuits
1	Inverter
2	Inverter chain
3	Nand2
4	One-shot trigger
5	Comparator
6	Opamp follower
7	Ring oscillator
8	VCO

- #Tran LU is reduced to below 1/5
- #Tran Iter is kept almost the same as that of SPICE3

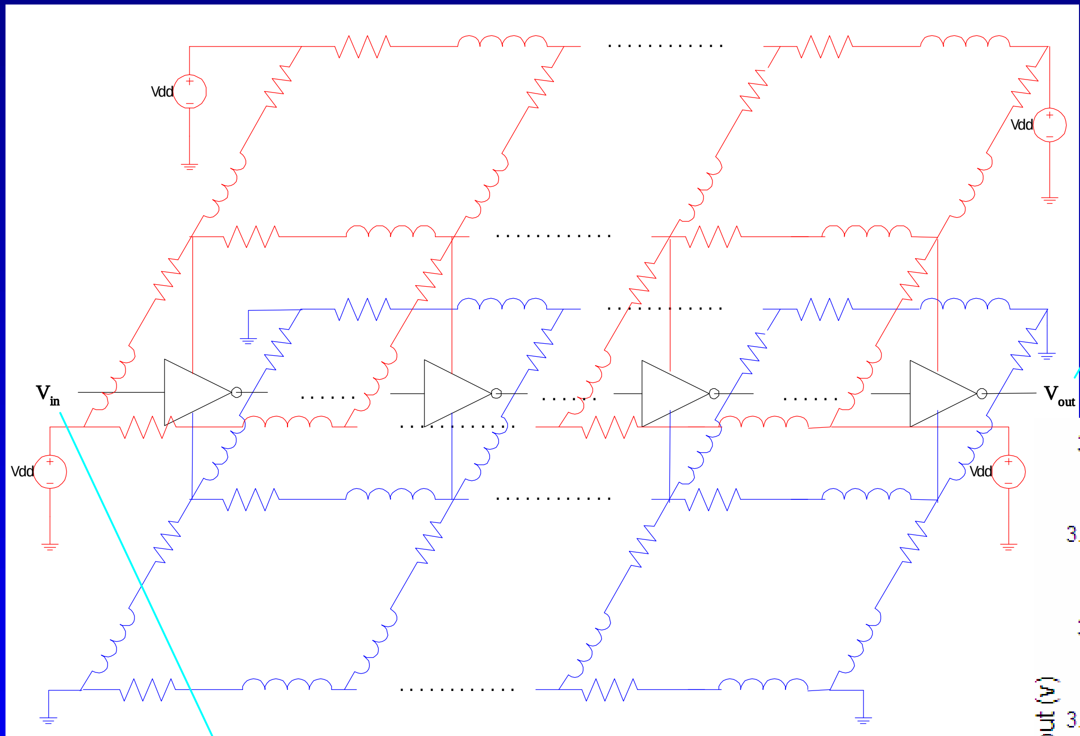
General Nonlinear Circuits - #FGMRES Iteration



#	Test Circuits
1	Inverter
2	Inverter chain
3	Nand2
4	One-shot trigger
5	Comparator
6	Opamp follower
7	Ring oscillator
8	VCO
9	Power Amplifier

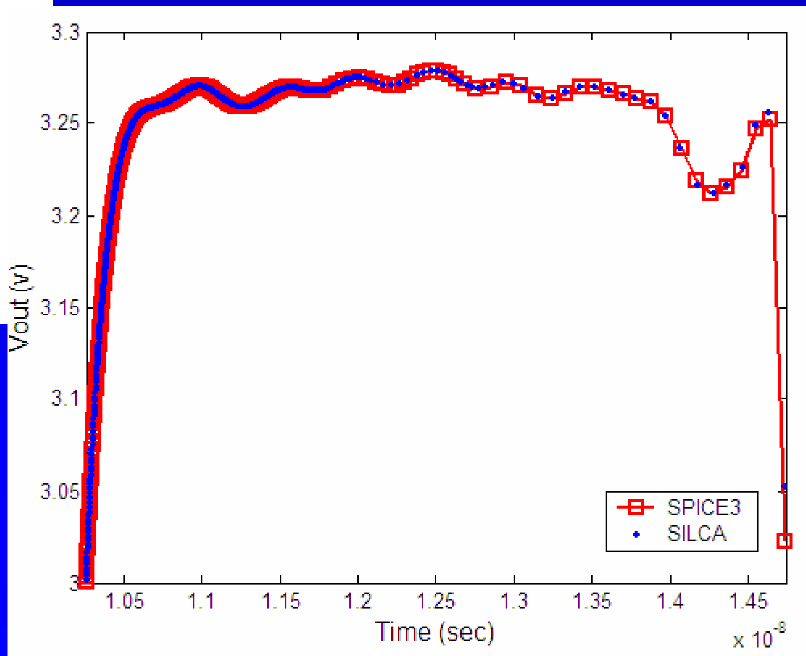
- #FGMRES Iter per FGMRES call is generally below 5 with $\varepsilon = 1e\text{-}10$

Power/Ground Example



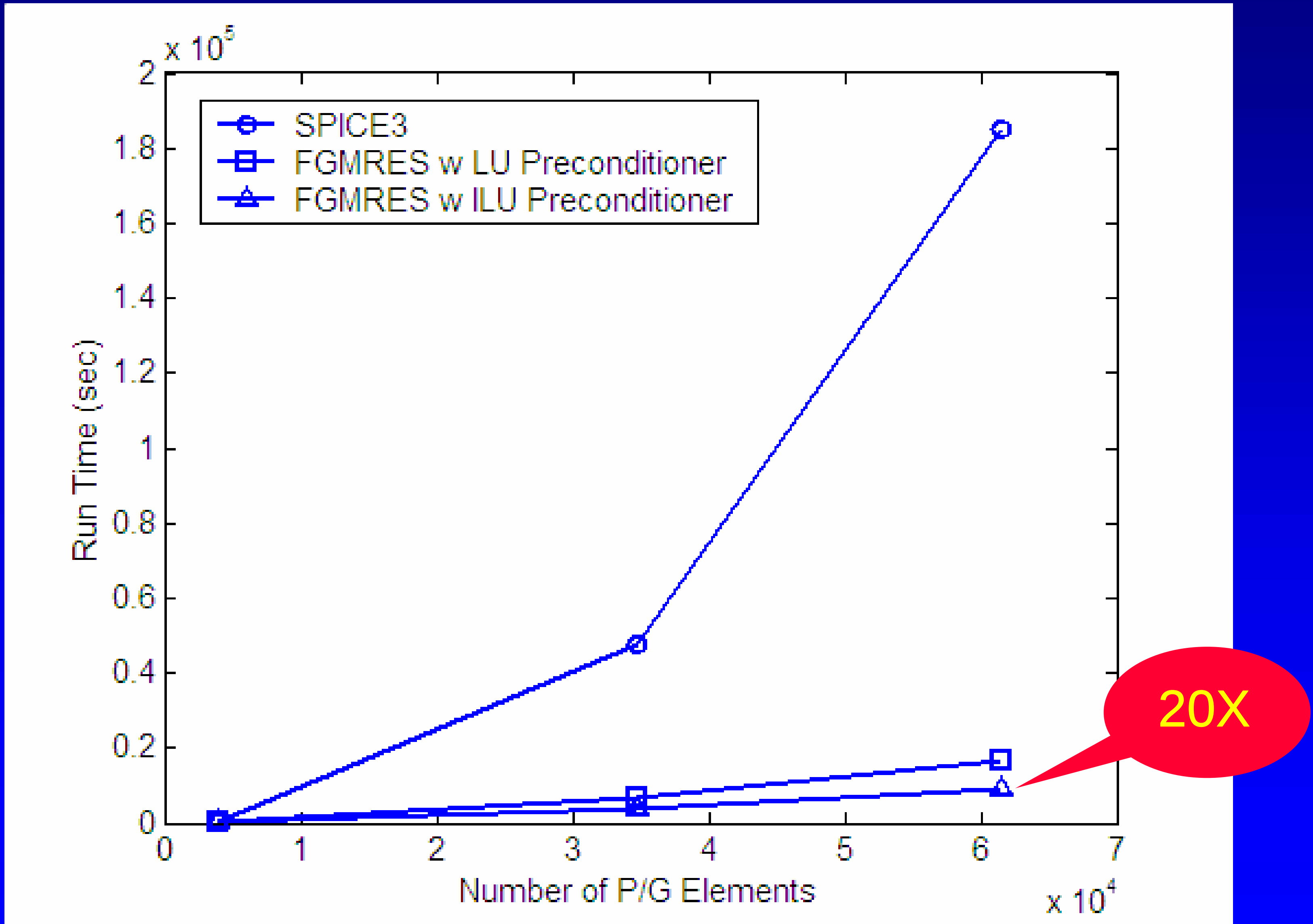
Output

Input

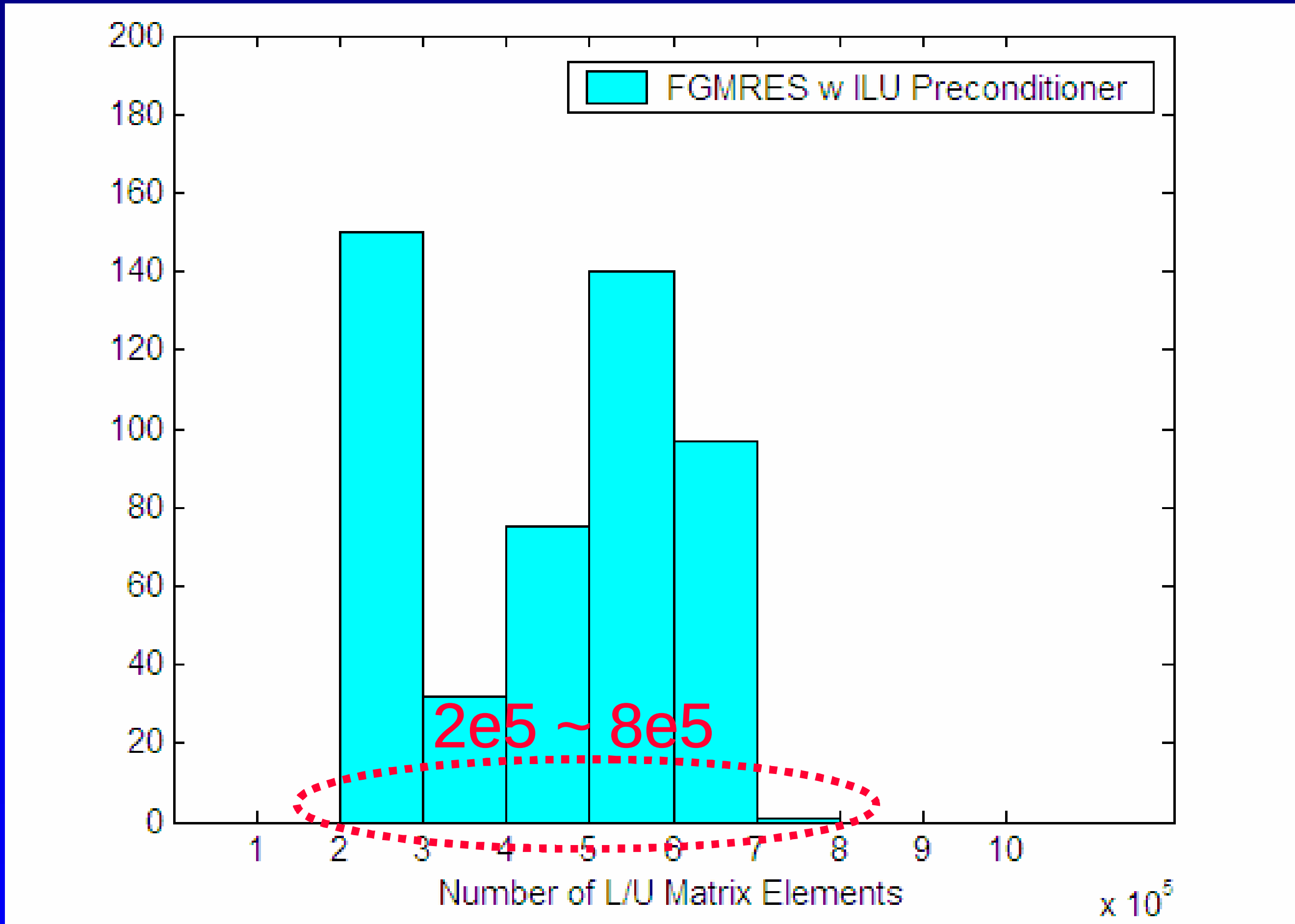


IR drop and L*dI/dt effects

Run Time vs. #Elements

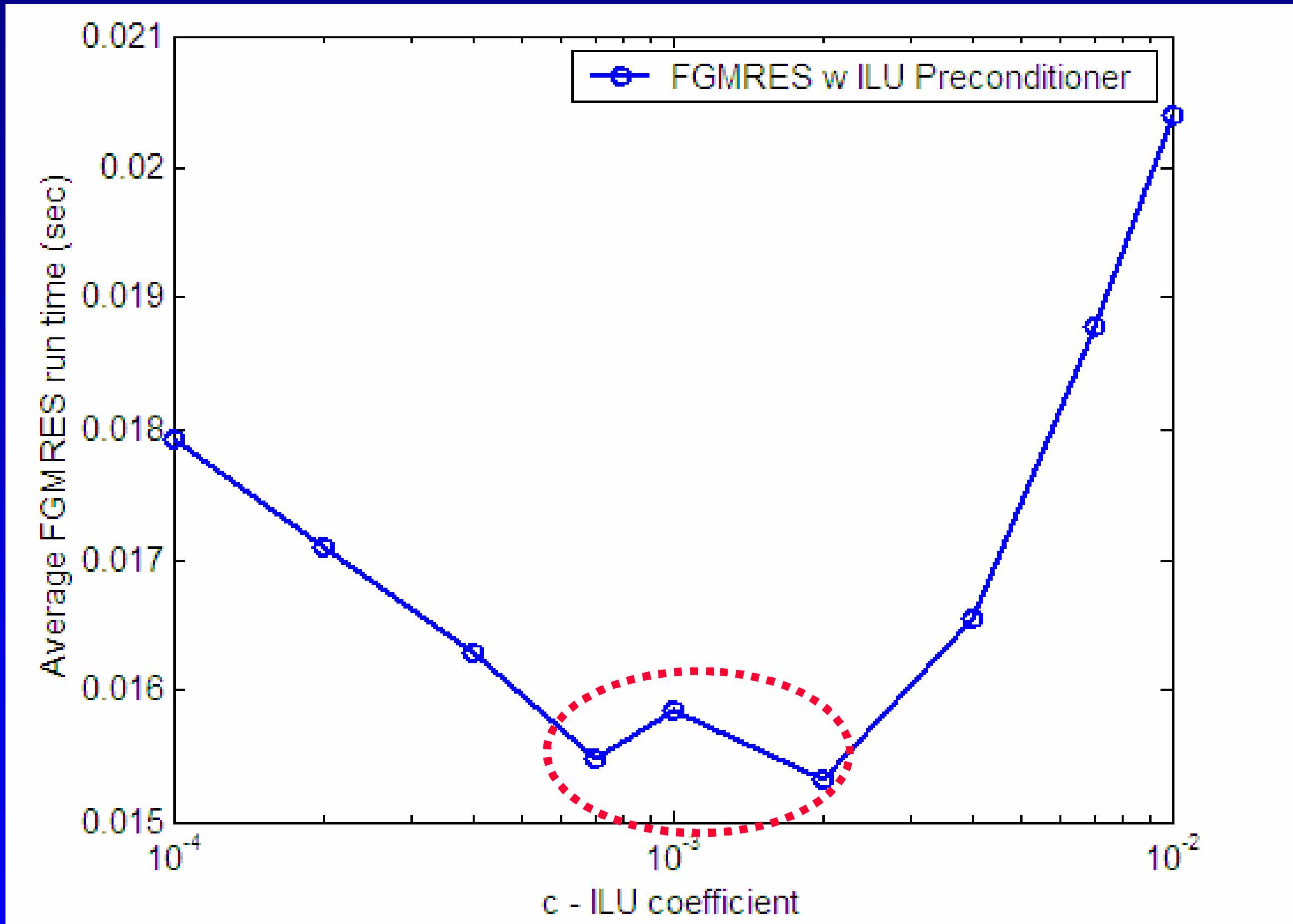


Histogram of Number of L/U Matrix Elements in ILU Preconditioner



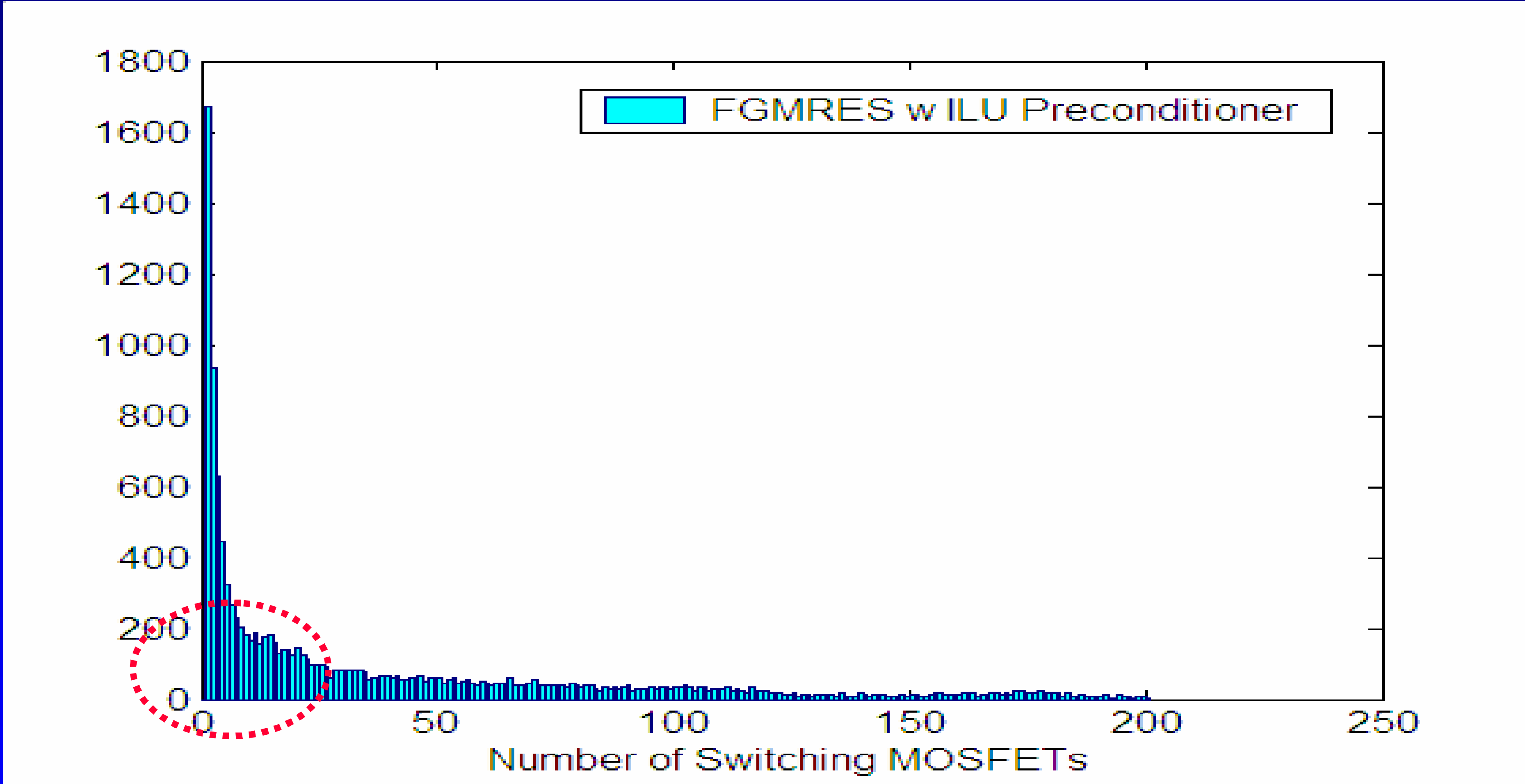
- #L/U matrix elements is reduced to 1/4~1/15 of that in LU preconditioner (~3e6)

Run Time vs. ILU Coefficient c



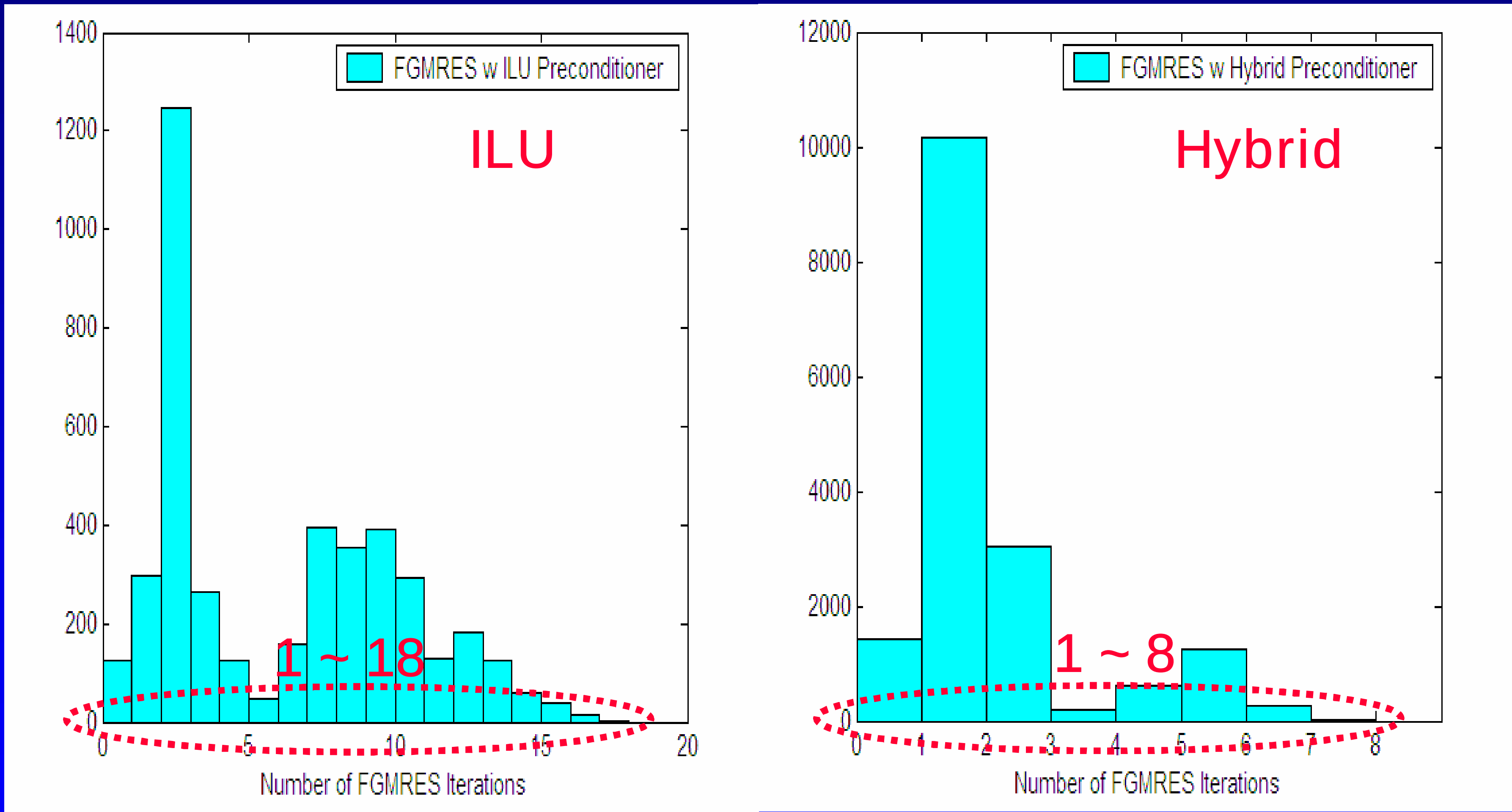
- c = 0.001 is optimal for tested power/ground examples

Histogram of #Switching MOSFETs Per Iteration



- In most nonlinear iterations, #switching MOSFETs is small – low-rank update is efficient

Histogram of Number of FGMRES Itreations Per FGMRES Call



- Less #FGMRES Iter per FGMRES call with hybrid preconditioner, however more FGMRES calls

Power/Ground Simulation Results

#Elems	SPICE3	FGMRES (LU)		FGMRES (ILU)	
	#Tran Iter	#Tran Iter	#Tran LU	#Tran Iter	#Tran LU
4002	4023	4106	54	4241	52
34802	4006	4087	55	4199	53
61602	4377	4253	53	4254	56

- #Tran LU is reduced to about 1/80
- #Tran Iter is kept almost the same as that of SPICE3

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Conclusions

- Quasi-Newton Preconditioned Newton-Krylov Method
  - Quasi-Newton like time step-size control for preconditioner construction
  - PWNL definition of nonlinear devices for quasi-Newton preconditioner
  - Low-rank update for fast L/U matrix update
  - Incomplete LU preconditioner

☞ Orders of magnitude speedup for circuits where the number of linear parasitic elements dominates the number of nonlinear devices

Thank you!