Optimal TDMA Time Slot and Cycle Length Allocation for Hard Real-Time Systems

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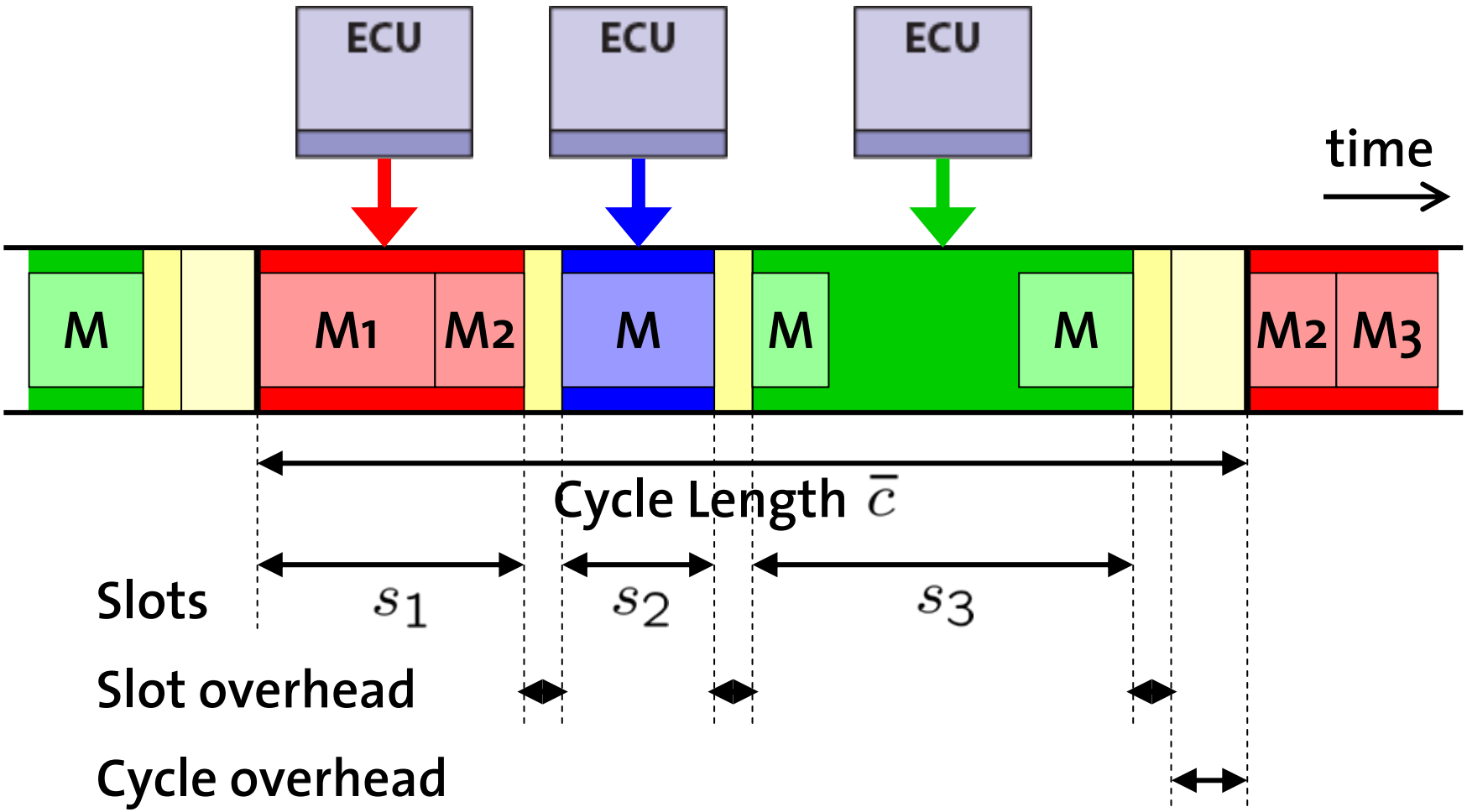
ASP-DAC, Yokohama, Japan

Ernesto Wandeler *Lothar Thiele*

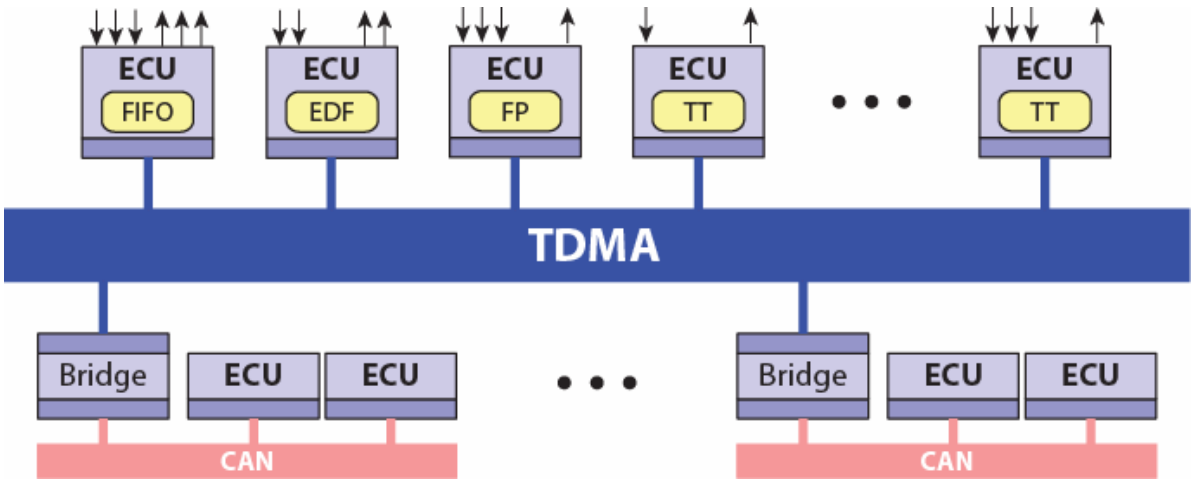
Computer Engineering and Networks Laboratory

ETH Zurich, Switzerland

Time Division Multiple Access (TDMA)

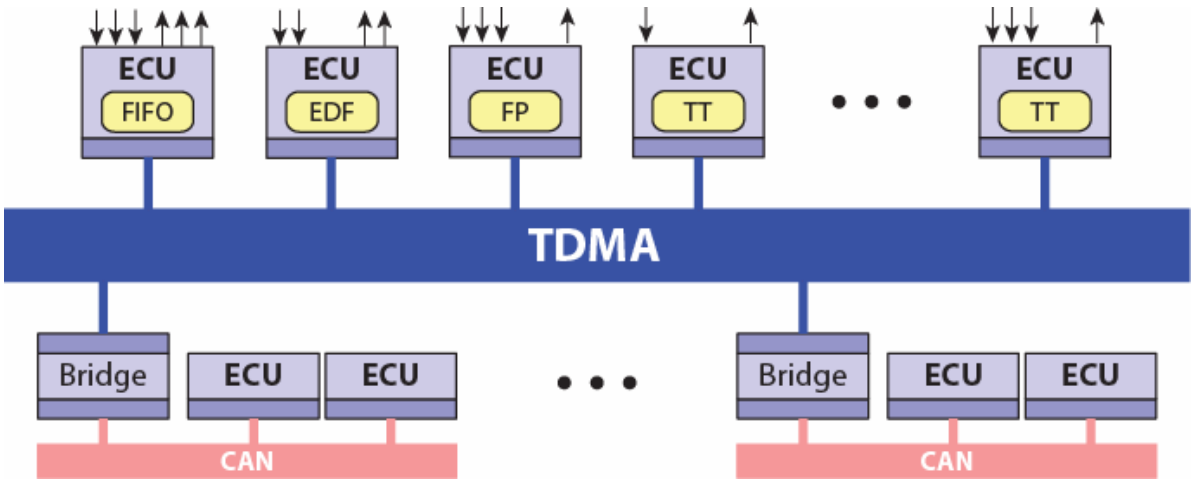


Motivation – TDMA as Backbone in DES



- Interconnect a large number of ECU's.
- Interconnect separate communication clusters.
- Serve time- & event-triggered messages.

Advantages of TDMA



- Supports temporal composability.
- Has deterministic timing behavior.
- Can be made fault tolerant.
- Supports error detection and error contention.

Difficulties of TDMA

How do we optimally select the TDMA parameters?

- Optimal parameter selection:
  - Bandwidth: B
  - Cycle length: $\overline{c}$
  - Slot lenghts: s_i

Related Work

	scope	optimal (exact)	real-time messages	stream model
• [Kopetz *et al.* 1997]	TT	✓	✓	P
• [Obermaisser *et al.* 2002] (Direct share)	TT&ET	✗	✗	S
• [Pop *et al.* 2004] (Heuristic)	TT&ET	✗	✓	P
• [Hamann *et al.* 2005] (Evolutionary Algorithm)	TT&ET	✗	✓	PJD
• **This work**	TT&ET	✓	✓	any

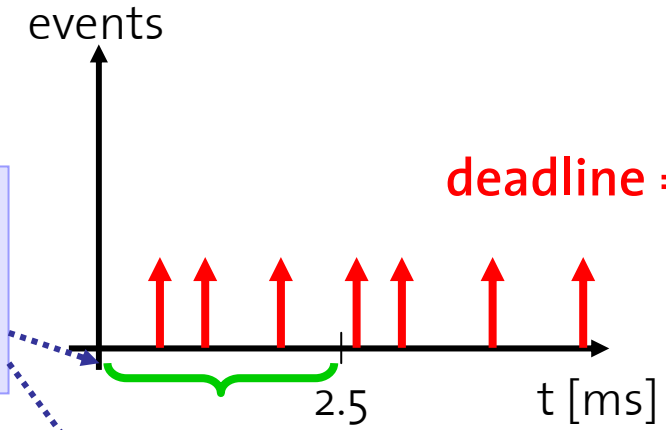
Solution Strategy

1. Determine a method to model the *service demand* of an event- or time-triggered real-time message stream.
2. Determine a method to model the *service supply* of a TDMA resource.
3. Based on the two above models, find an analytic method to determine the *provable smallest slot length* that must be allocated on a TDMA resource to serve a real-time message stream.

Service Demand: Arrival Curves

Event Stream

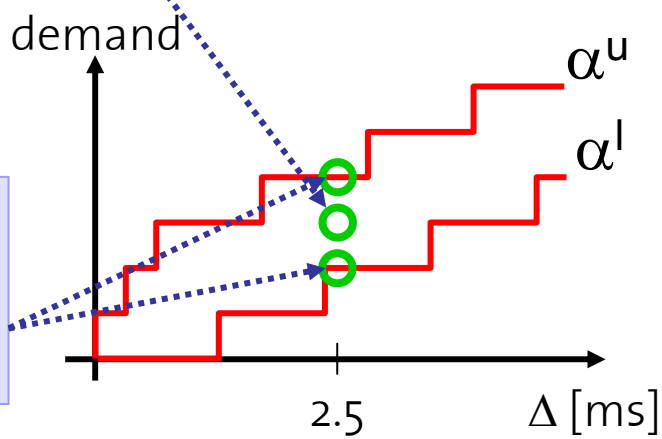
number of events in in t=[0 .. 2.5] ms



t

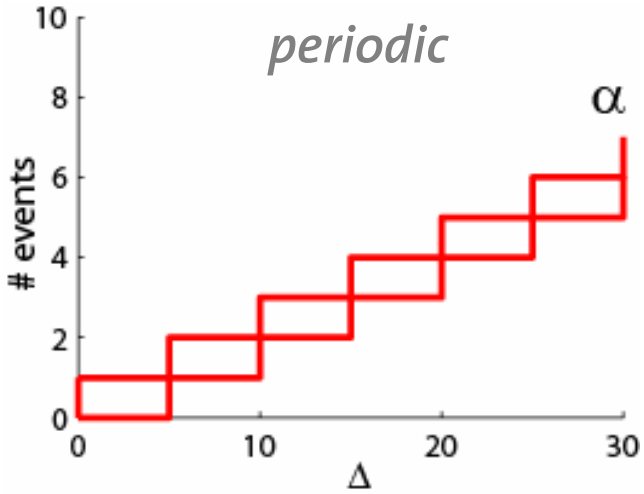
Arrival Curve α & Deadline d

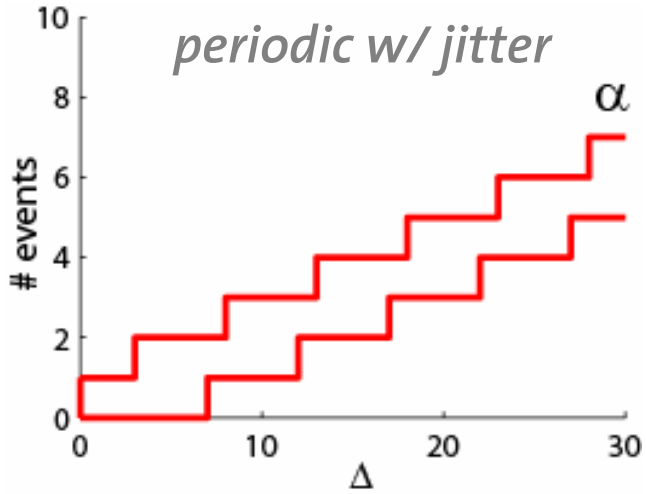
maximum / minimum arriving demand in *any interval* of length 2.5 ms

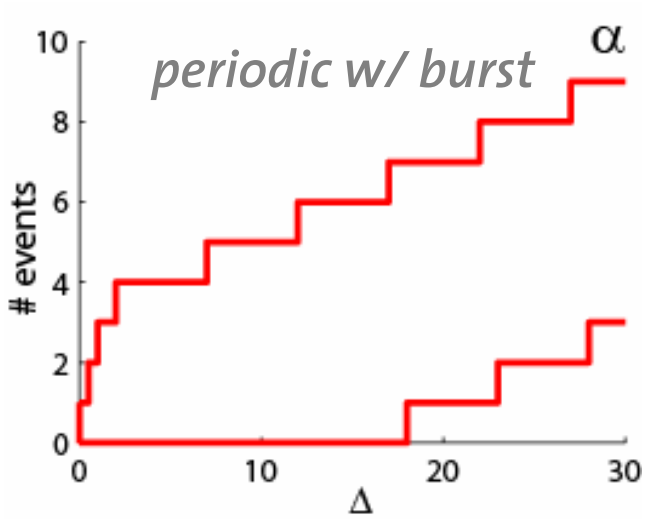


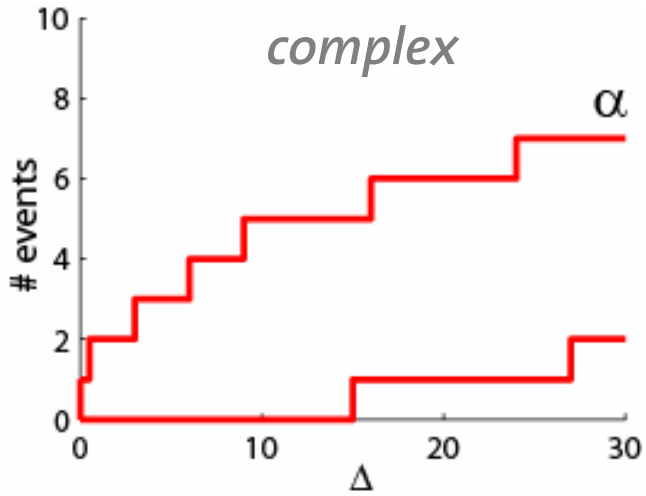
Δ

Service Demand: Arrival Curve Examples





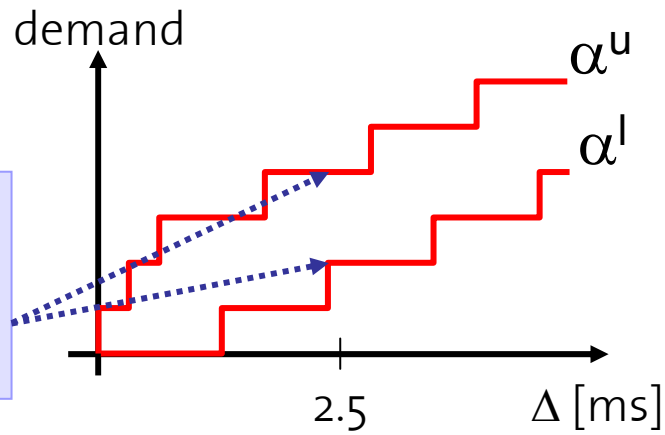




Service Demand: Service Demand Curves

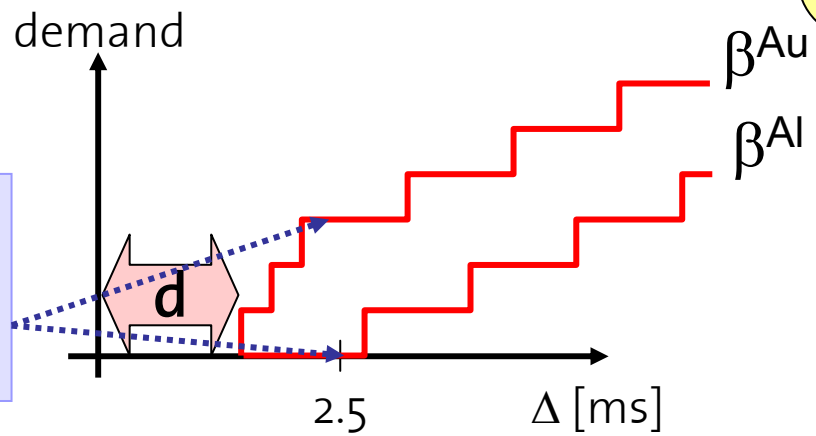
Arrival Curve α & Deadline d

maximum / minimum arriving demand in *any interval* of length 2.5 ms



Service Demand Curve β^A

maximum / minimum service demand in *any interval* of length 2.5 ms



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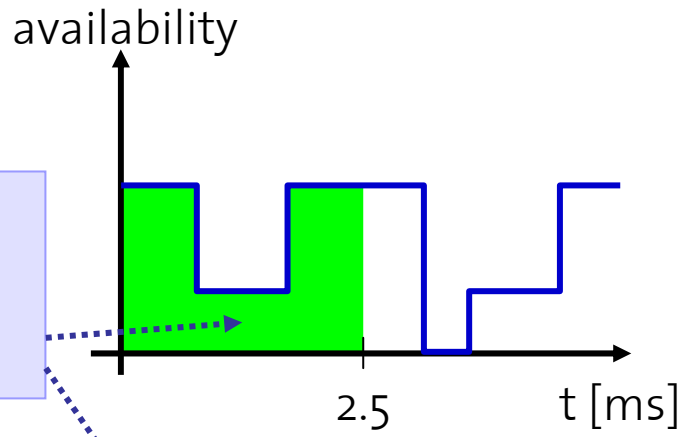
Service Supply: Serivce Curves

t

Resource Availability

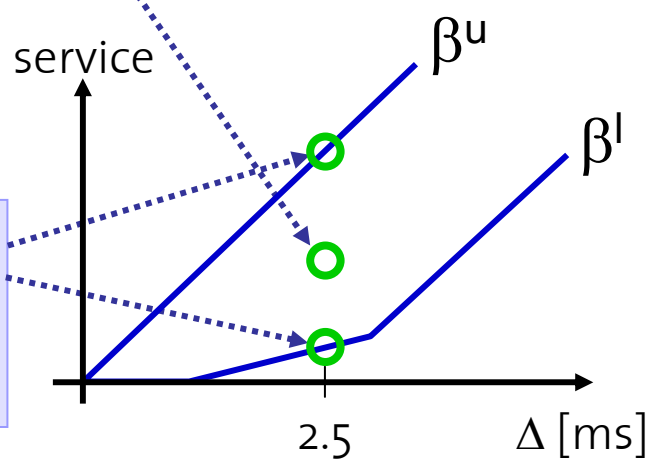
available service
in t=[0 .. 2.5] ms



Δ

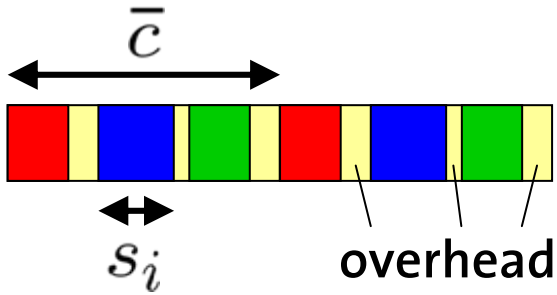
Service Curves $[\beta^l, \beta^u]$

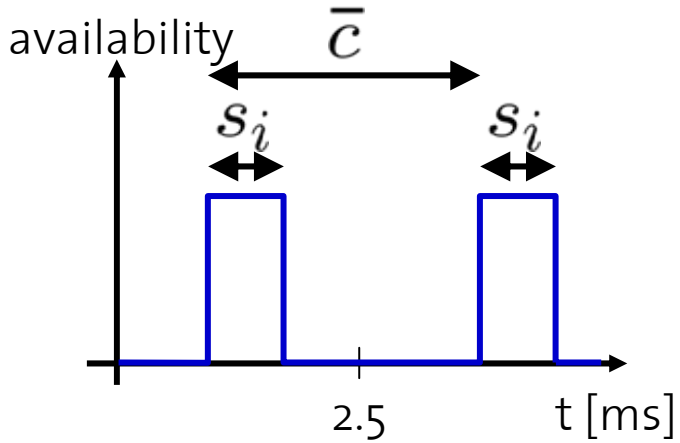
maximum/minimum
available service in *any interval* of length 2.5 ms



Service Supply of a TDMA Resource

Resource Availability





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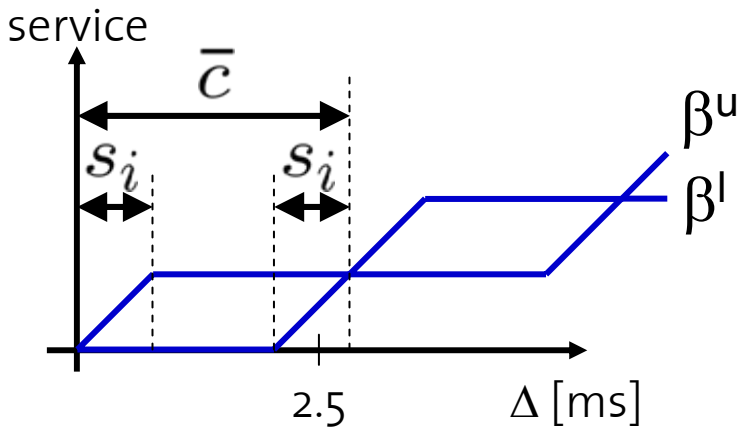
Service Curves [β^l, β^u]



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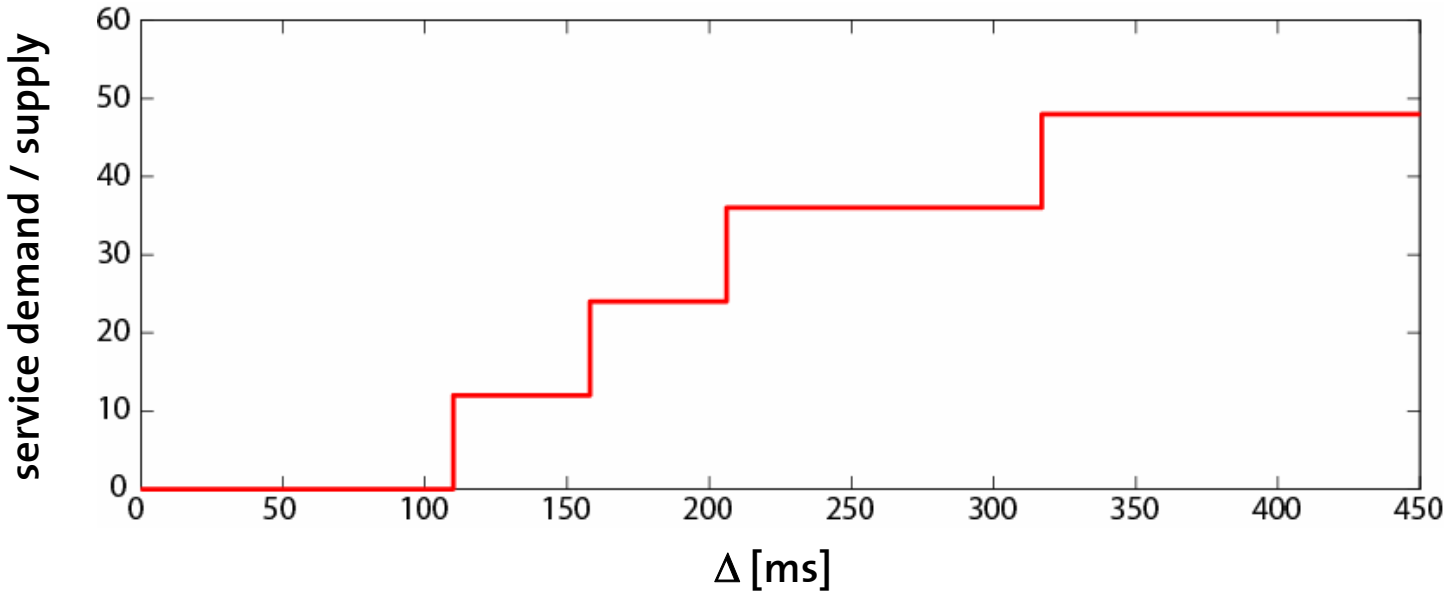
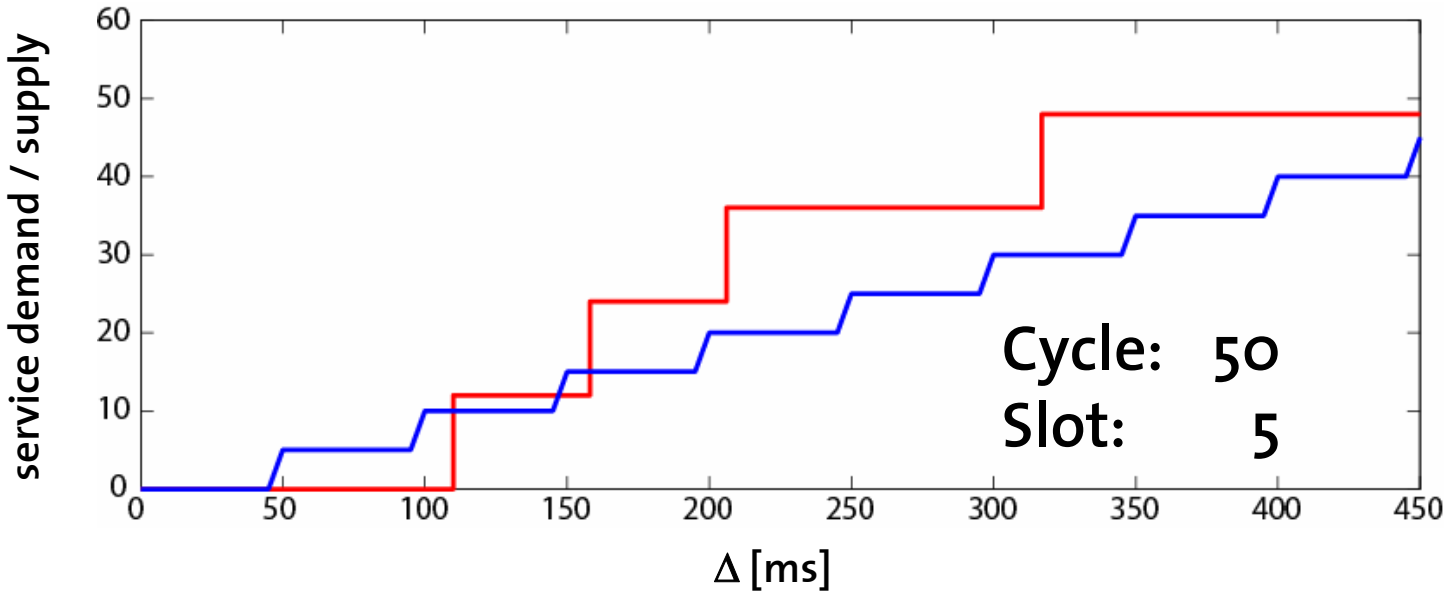
Computing the Minimum Slot Length



- Given
  - Bandwidth B
  - Cycle Length $\overline{c}$
  - Service Demand β_i^A
- Find
  - Provable smallest slot length s_i

Computing the Minimum Slot Length



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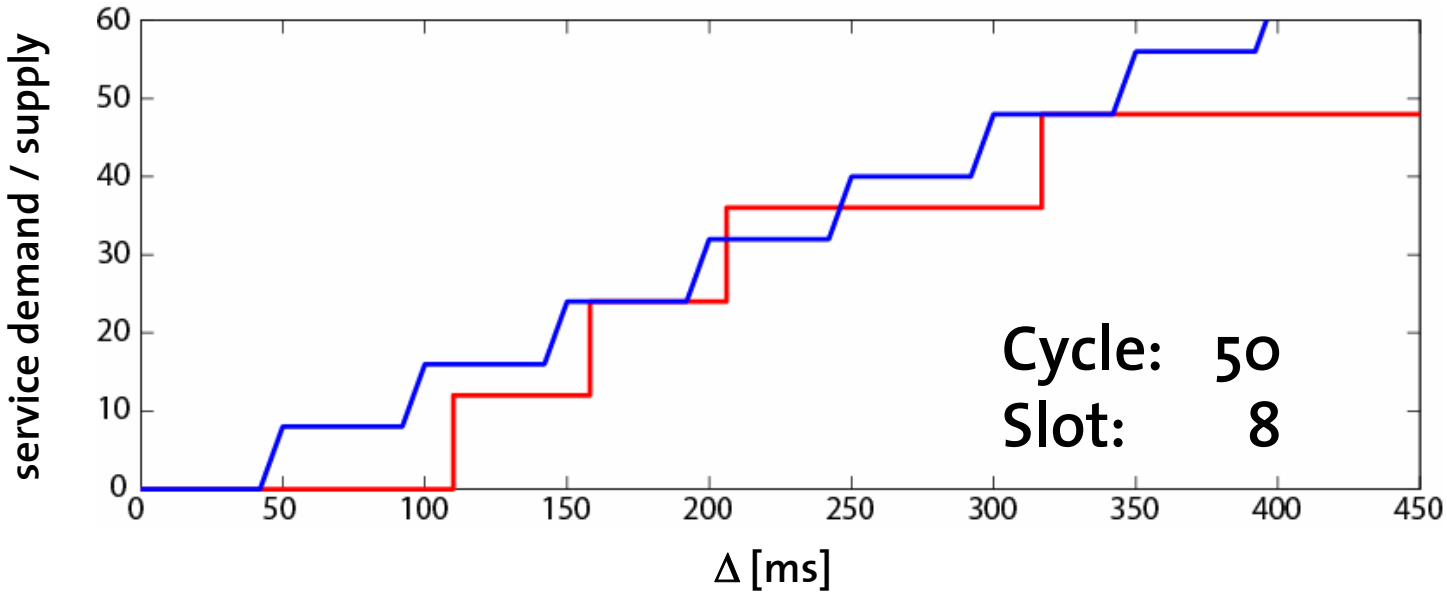

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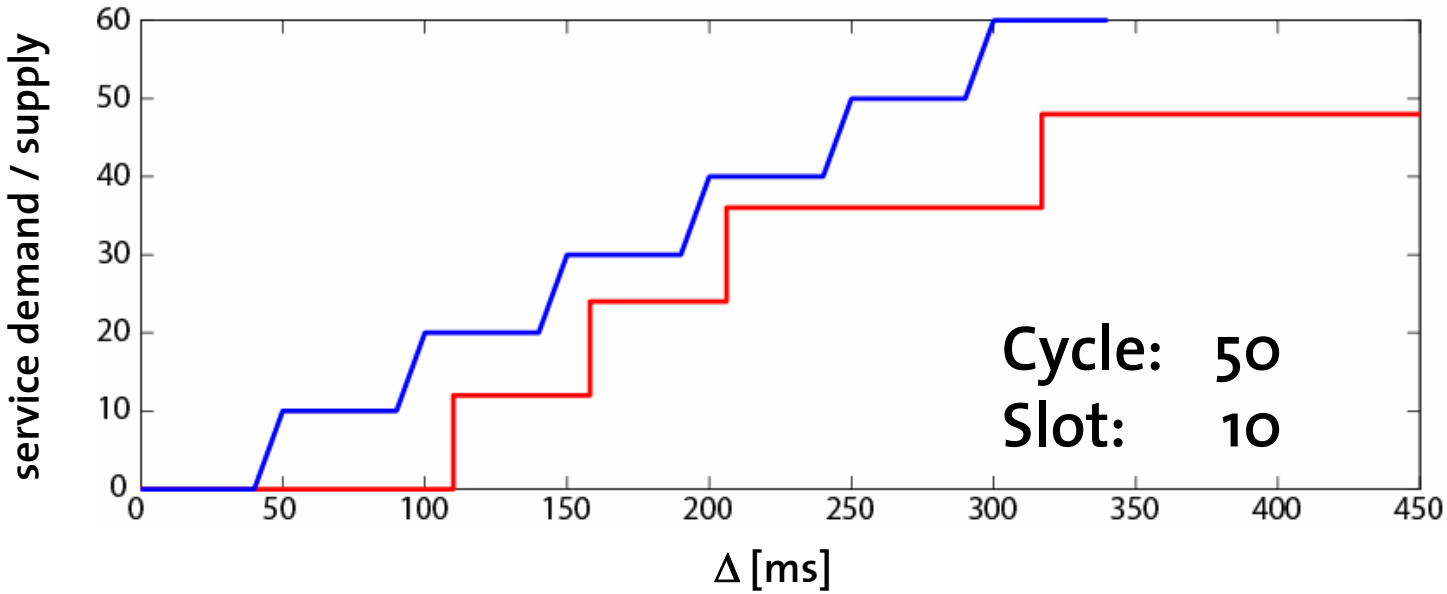

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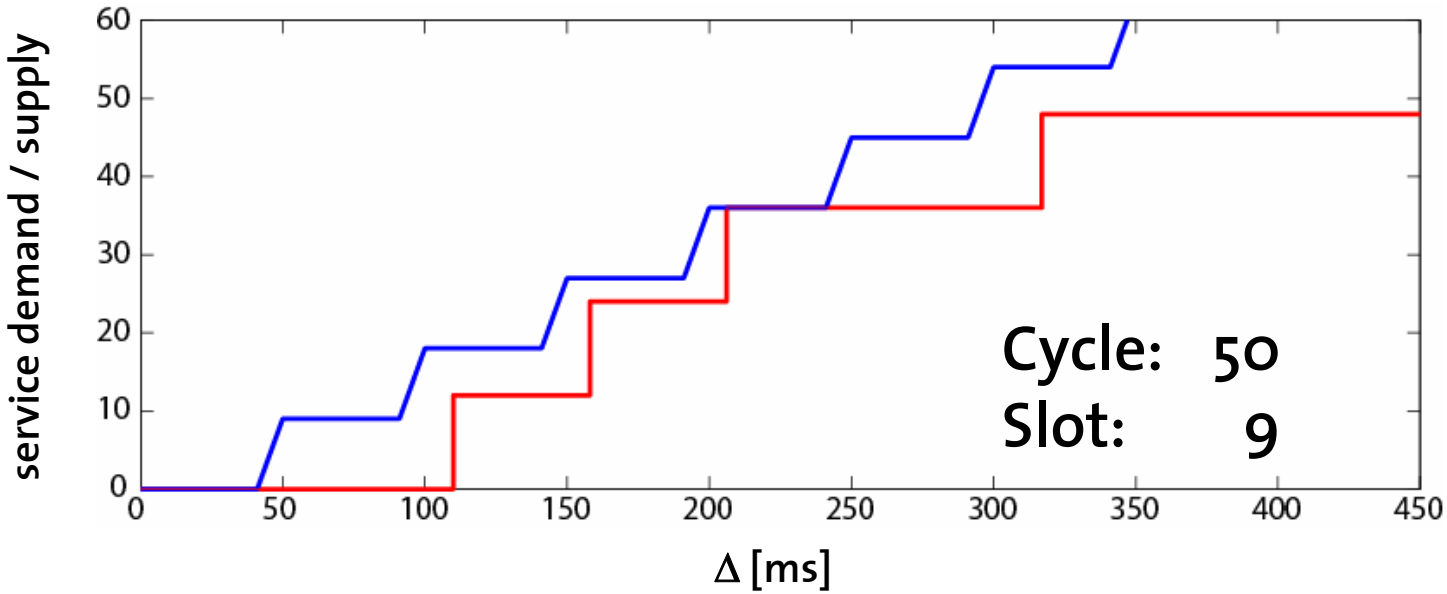

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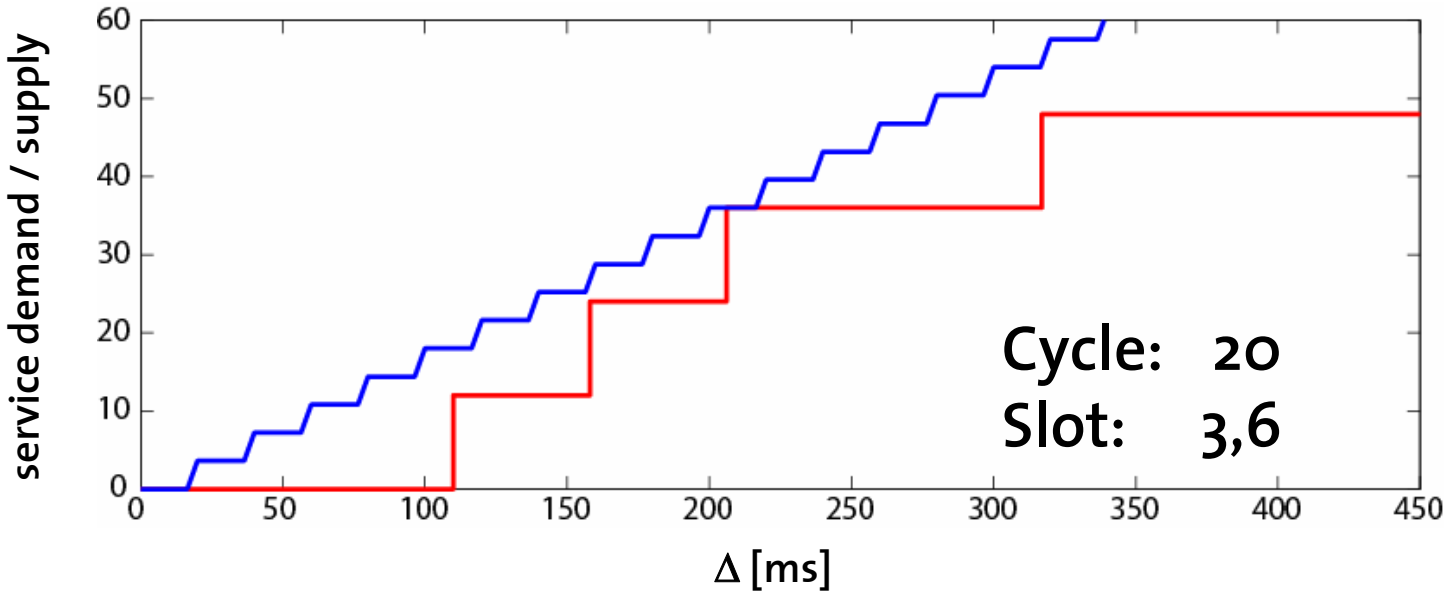

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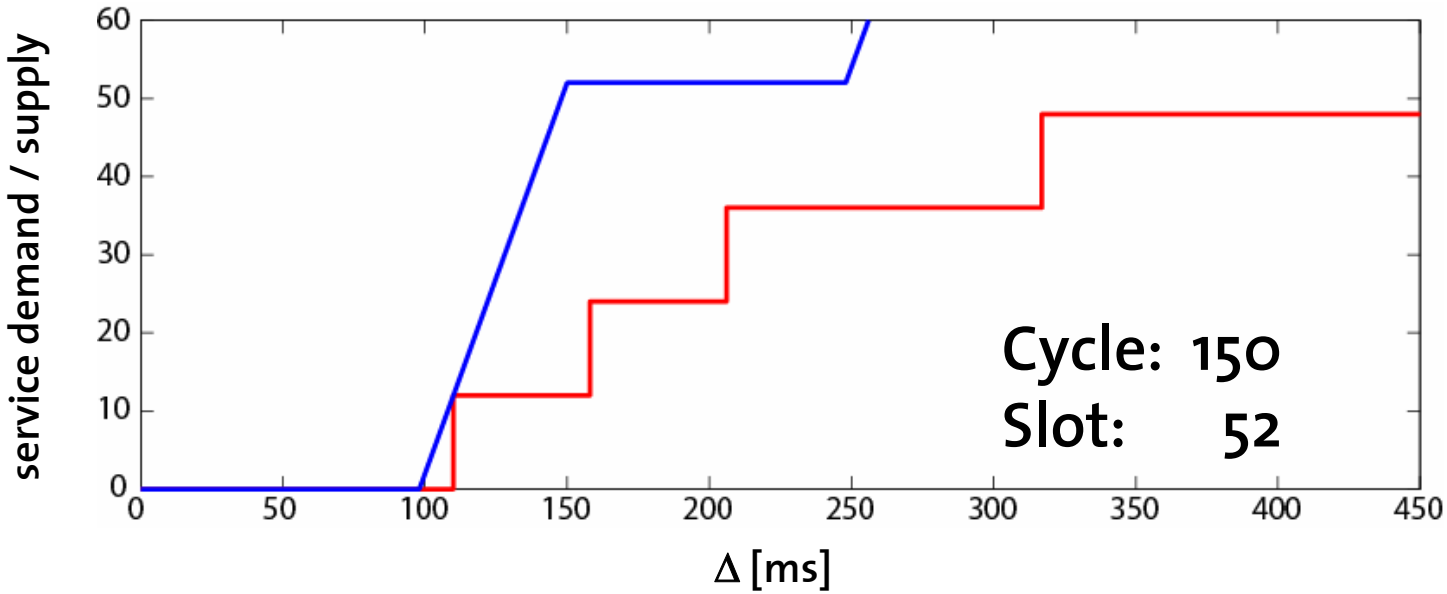

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Computing the Minimum Slot Length

$$s_i = \sup_{\Delta \geq 0} \left\{ \min \left(\frac{\beta_i^A}{B \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor}, \frac{\beta_i^A - B\Delta + B \left\lceil \frac{\Delta}{\bar{c}} \right\rceil \bar{c}}{B \left\lceil \frac{\Delta}{\bar{c}} \right\rceil} \right) \right\}$$

- Given
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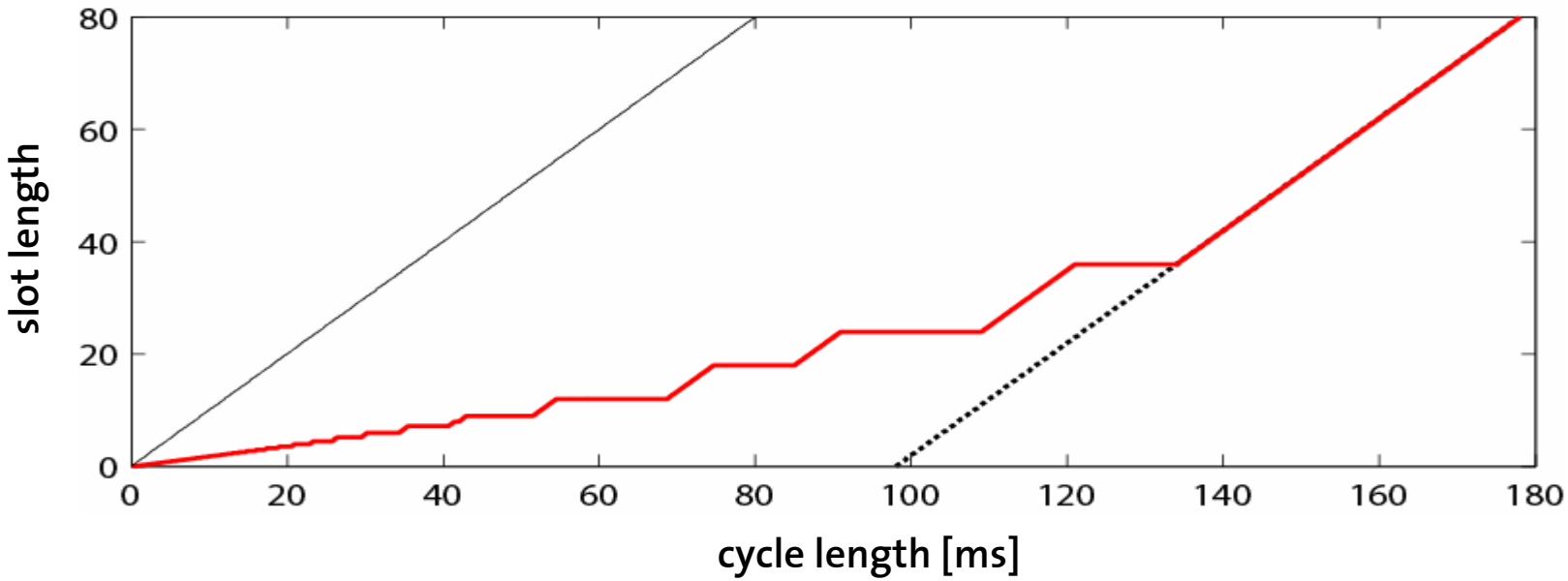
Minimum Slot Lengths vs. Cycle Length



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Finding Feasible Cycle Lengths

	M0	M1	M2	M3	M4	M5	M6	M7	M8	M9
p	198	102	283	354	239	194	148	114	313	119
j	387	70	269	387	222	260	91	13	302	187
d	48	45	58	17	65	32	78	-	86	89
e	12	7	7	11	8	5	13	14	5	6
D	110	140	115	145	180	140	200	120	140	100

- Given
  - Bandwidth B
  - Service Demand β_i^A
- Find
  - Slot lengths s_i
  - Feasible Cycle Lengths $\bar{c}$

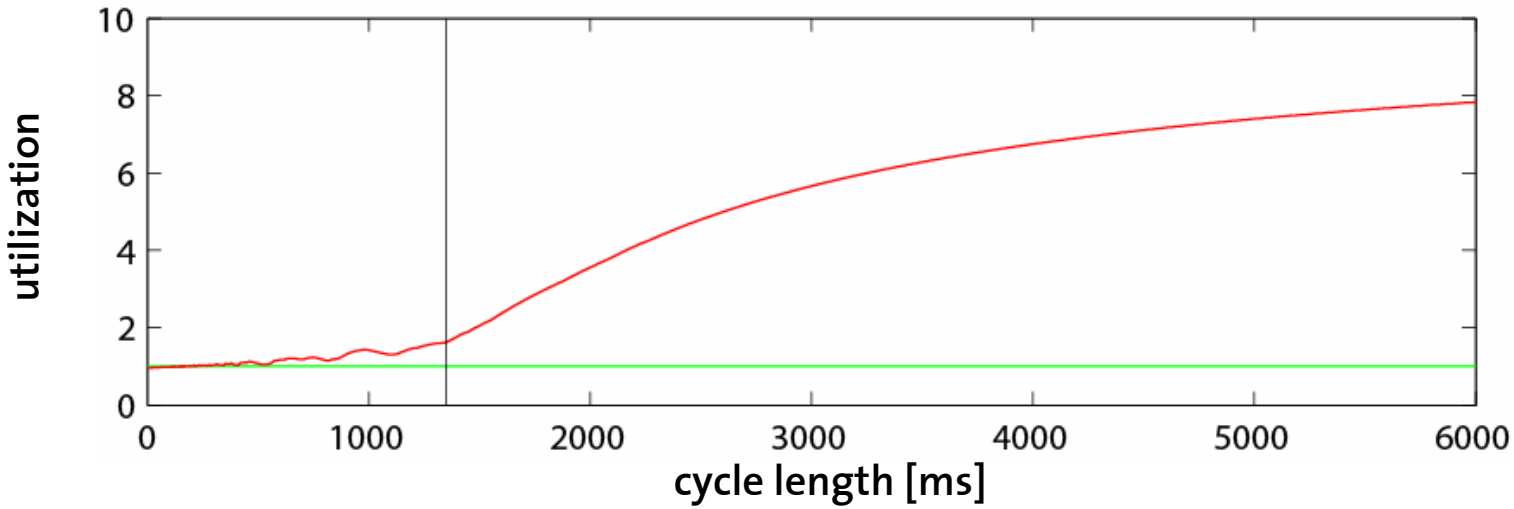
Finding Feasible Cycle Lengths

- For all cycle lengths $\bar{c}$ compute the minimum slot lengths $s_i(\bar{c})$ for all message streams.
- Compute total resource utilization: $\sigma(\bar{c}) = \frac{\sum s_i(\bar{c})}{\bar{c}}$
- If and only if $\sigma(\bar{c}) \leq 1 \rightarrow \bar{c}$ is feasible

- Given
  - Bandwidth B
  - Service Demand β_i^A
- Find
  - Slot lengths s_i
  - Feasible Cycle Lengths $\bar{c}$

Finding Feasible Cycle Lengths

Computation Time up to Upper Bound: 1,1 second!
(Pentium Mobile 1.6 GHz)



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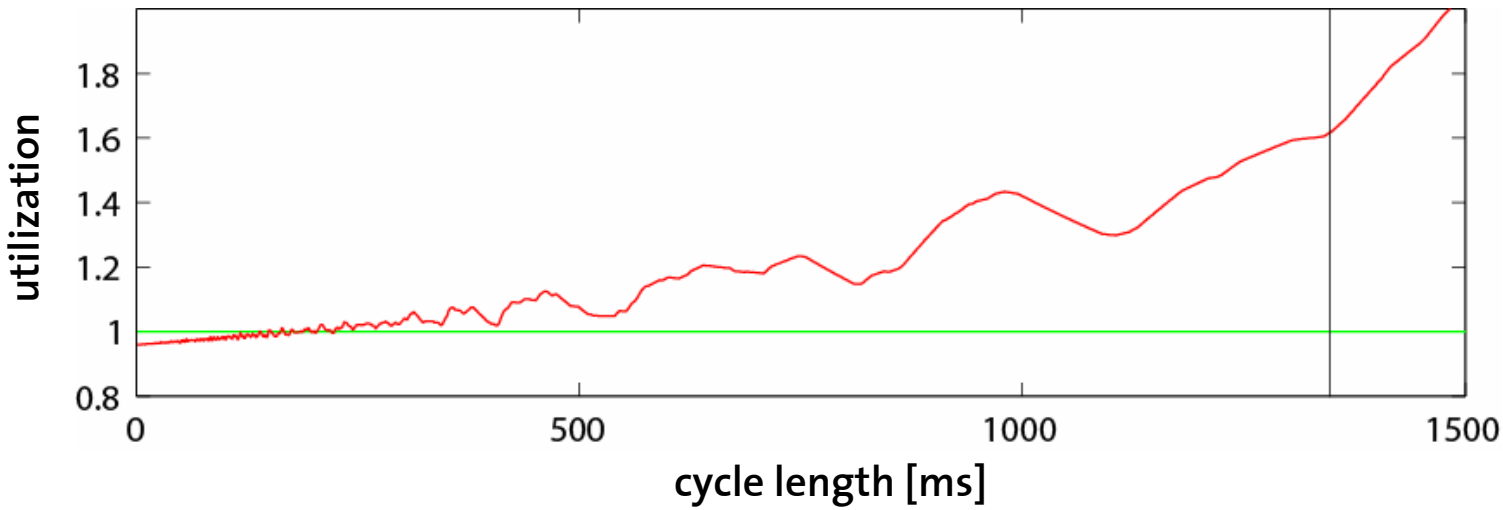

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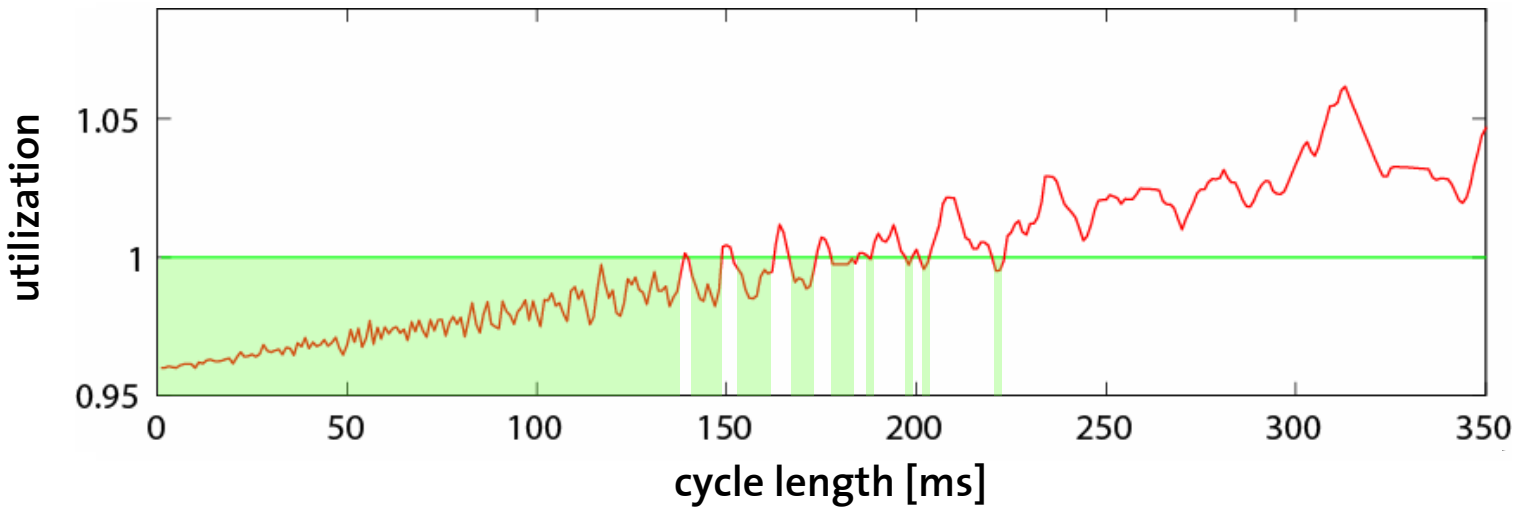

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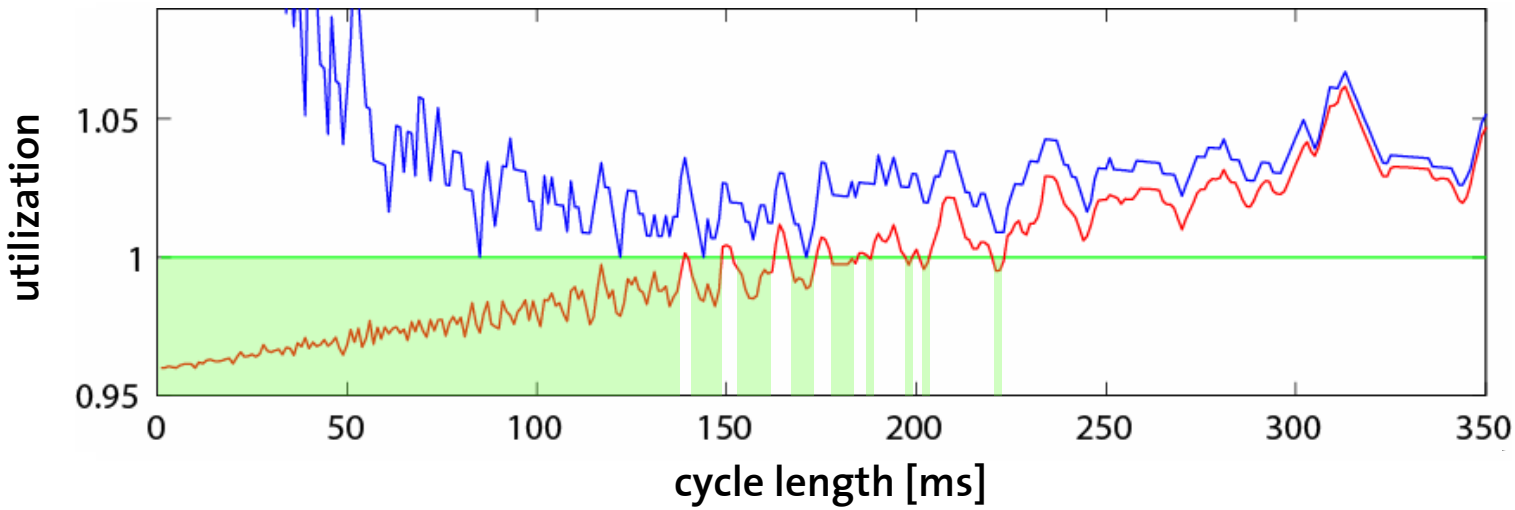

- Given
  - Bandwidth B
  - Service Demand β_i^A
- Find
  - Slot lengths s_i
  - Feasible Cycle Lengths $\bar{c}$

Finding Optimal Cycle Lengths

- Define an optimality criterion:
  - E.g. average remaining bandwidth, …
- Compute all feasible cycle lengths and select the optimal cycle length.

- Given
  - Bandwidth B
  - Service Demand β_i^A
- Find
  - Slot lengths s_i
  - Optimal Cycle Length $\bar{c}$

Finding Minimum Total Bandwidth

- Perform a binary search until all feasible cycle lengths lead to zero remaining bandwidth.

- Given
  - Service Demand β_i^A
- Find
  - Slot lengths s_i
  - Optimal Cycle Length $\overline{c}$
  - Bandwidth B

Conclusions

- We presented an *analytic method* to determine *provably smallest possible slot lengths* for TDMA.
- The presented analytical method is *computationally very efficient*.
- Based on the computational efficiency, we presented constructive methods to find the *optimal cycle length* and the *minimum required bandwidth* for a TDMA resource.

Thank you!

Ernesto Wandeler

wandeler@tik.ee.ethz.ch