Wire Sizing with Scattering Effect for Nanoscale Interconnection

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Outline

- Introduction
- Model of Scattering Effects
- Wire Delay & Sizing with Scattering Effects
- Conclusion

Interconnect Delay Dominates

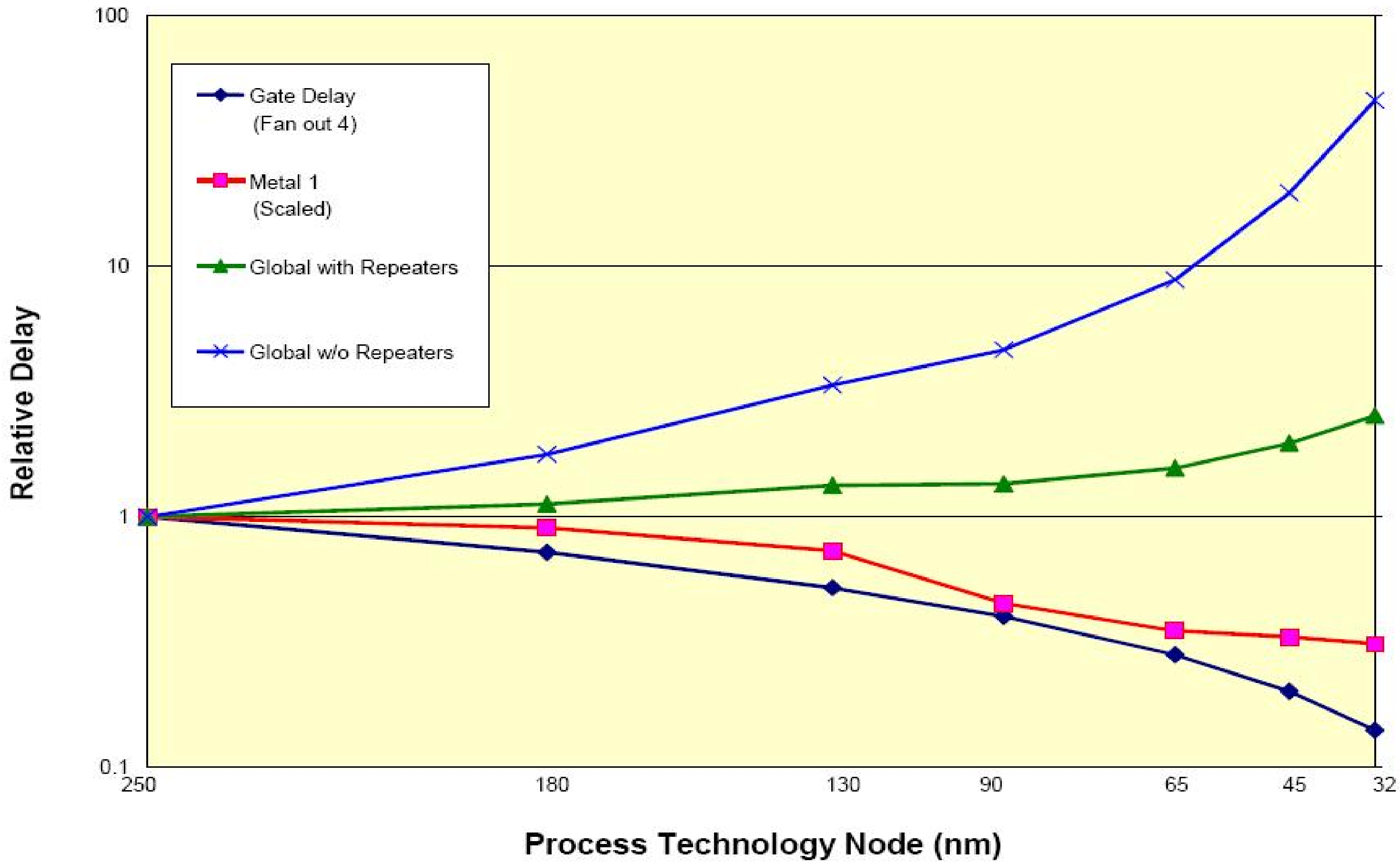


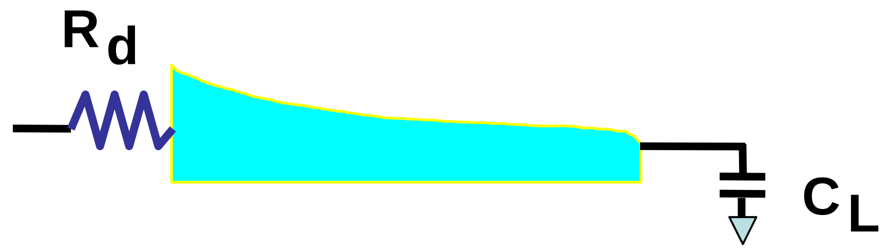
Figure 54 Delay for Metal 1 and Global Wiring versus Feature Size

(source: ITRS 2003)

What is Wire Sizing?

- WS is an effective way to reduce distributed RC delay

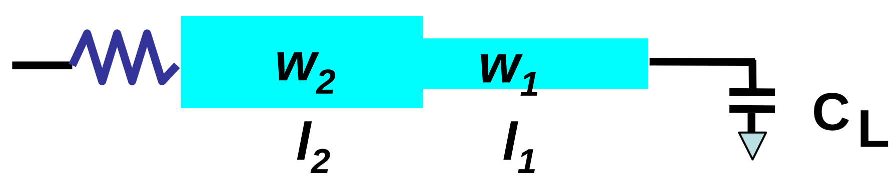
Continuous wire shaping
[Fishburn+, TCAD'96, DATE97; Chen+, DAC'96, DAC'97]



Discrete wire sizing
[Cong-Leung, ICCAD'93]

1-width sizing (1-WS)
[Cong-Pan, DAC'99]

2-width sizing (2-WS)
[Cong-Pan, DAC'99]



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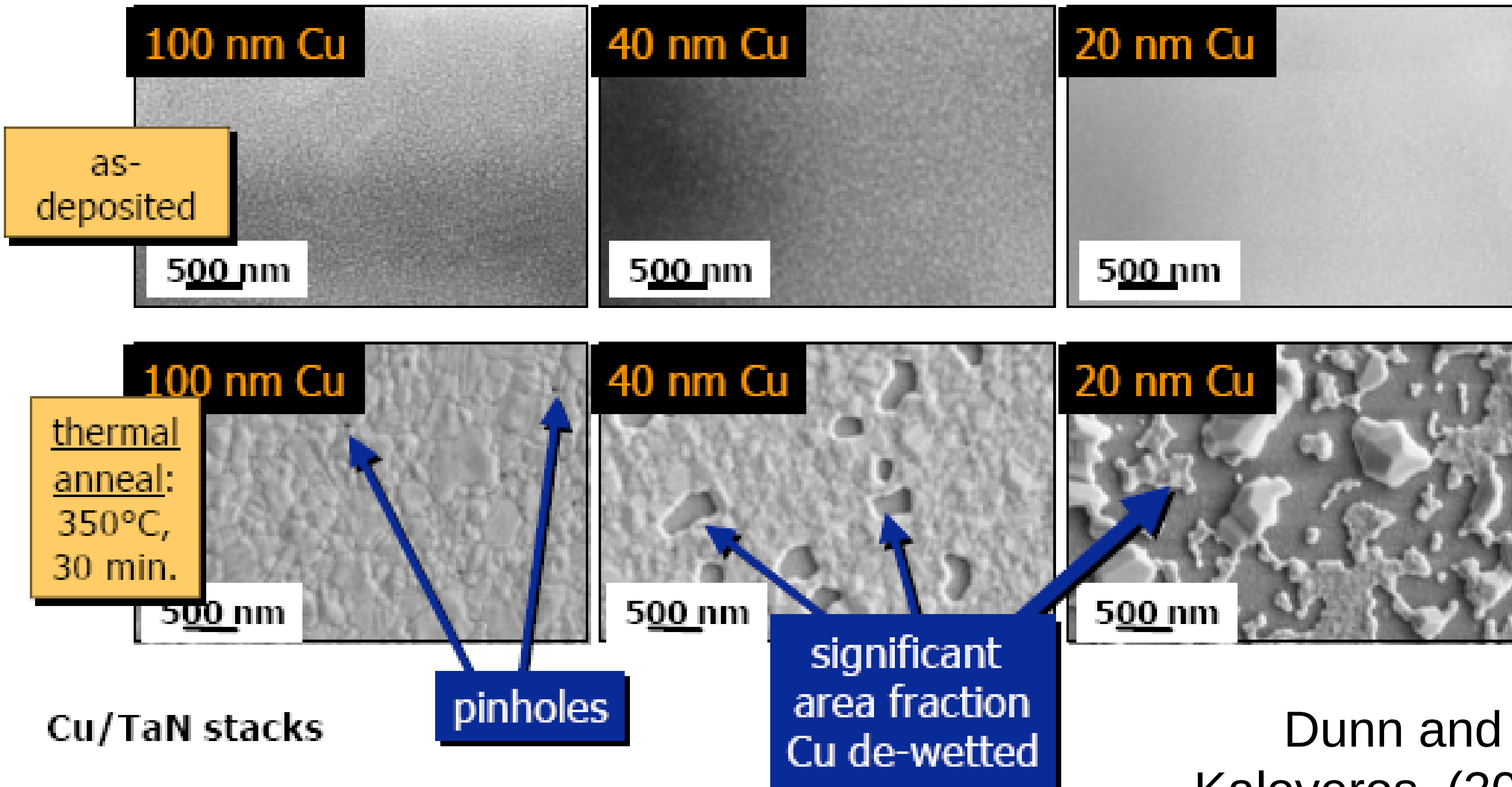
What is Scattering Effect?

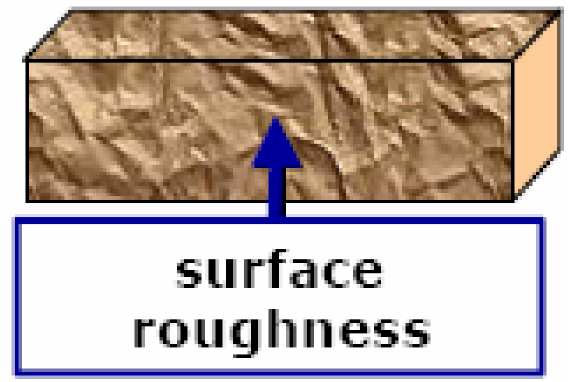
- Mesoscopic scale effect for interconnect
- Example: surface roughness

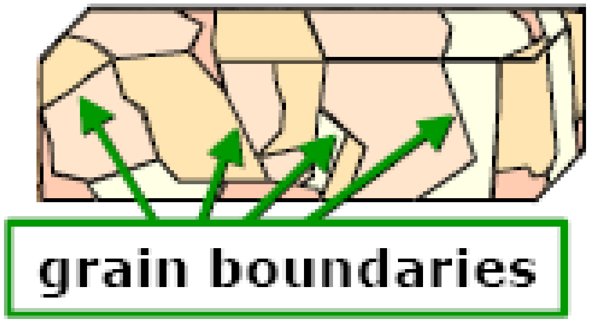


Dunn and Kaloyeros, (2000)

Two Kinds of Scattering Effects

- Surface Roughness Effect
  - Wetting and nucleation of Cu
  - FS model: [Fuchs, 1938] and [Sondheimer, 1952]
- Grain Boundaries Effect
  - Polycrystalline structure of Cu
  - MS model: Mayadas and Shatzkes [1970]
- Electrical impacts
  - Electron movement will be bumpy
  - Higher resistivity than bulk metal
- Complicated quantum mechanical effect to model them





Dunn and Kaloyeros, (2000)

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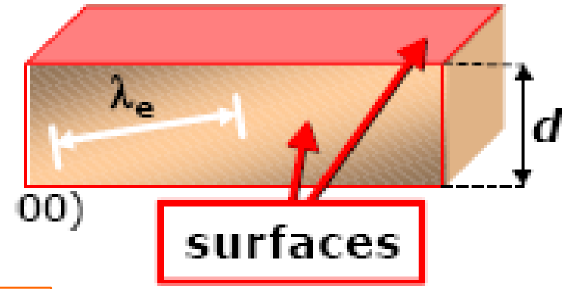
Model of Scattering Effect

- Modeling can be very complicated
- λ: bulk mean free path (39nm)



$$\rho(w) = \rho_0 \cdot \left[1 + \alpha(w) + \beta(h,w)\right] \qquad R = \rho(w) \cdot \frac{l}{w \cdot t} \neq \rho_0 \cdot \frac{l}{w \cdot t}$$

$$\frac{1}{\beta(t,w)} = \frac{3}{4\pi t w} \int_{-t/2}^{t/2} dy \int_{-w/2}^{w/2} dx \int_{-\pi+\arctan(w/t)}^{\arctan(-w/t)} d\varphi \int_0^{\pi} \sin(\theta)\cos^2(\theta) \left[1 - (1-p) \frac{\exp\left(-\frac{w}{2\lambda\cos(\theta)\cos(\varphi)}\right)}{1 - p\cdot\exp\left(-\frac{w}{2\lambda\cos(\theta)\cos(\varphi)}\right)} \right] d\theta$$

$$+ \frac{3}{4\pi t w} \int_{-t/2}^{t/2} dy \int_{-w/2}^{w/2} dx \int_{\arctan(-w/t)}^{\arctan(w/t)} d\varphi \int_0^{\pi} \sin(\theta)\cos^2(\theta) \left[1 - (1-p) \frac{\exp\left(-\frac{w}{2\lambda\cos(\theta)\cos(\varphi)}\right)}{1 - p\cdot\exp\left(-\frac{w}{2\lambda\cos(\theta)\cos(\varphi)}\right)} \right] d\theta$$

[Durkan and Welland 2000]

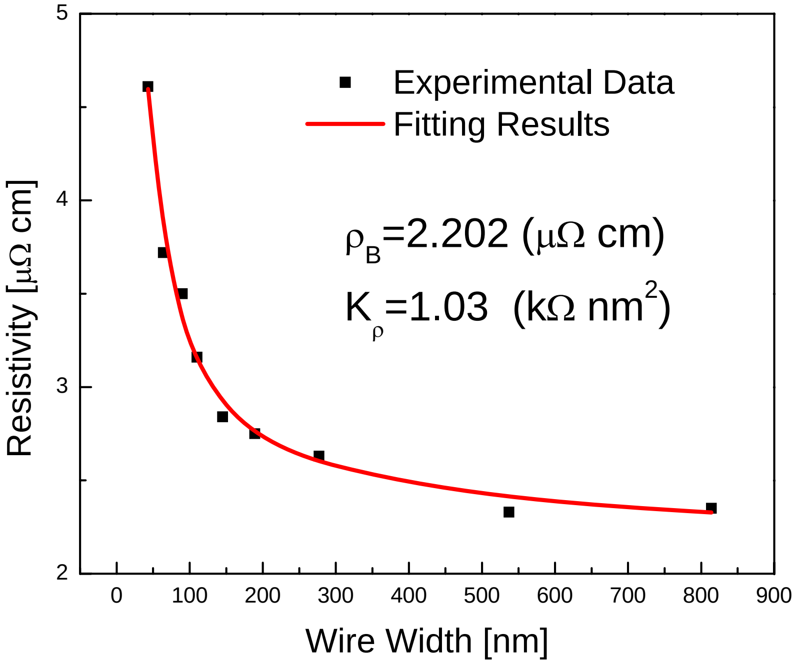
A Simple Model for Scattering Effect

- Based on the published measurement data from various sources, we obtain the following empirical resistivity model with scattering effect

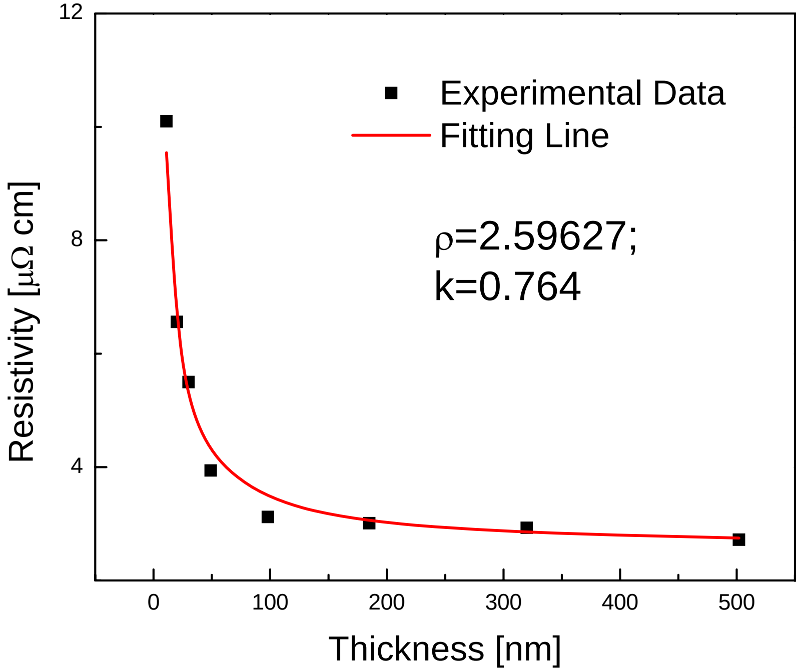
$$\rho(w) = \rho_B + \frac{K_\rho}{w}$$

- Resistivity is wire-width dependent
- K_ρ is an empirically fitting coefficient

Simple but Capture the Essence



Infineon [Steinhoegl+ 2002]

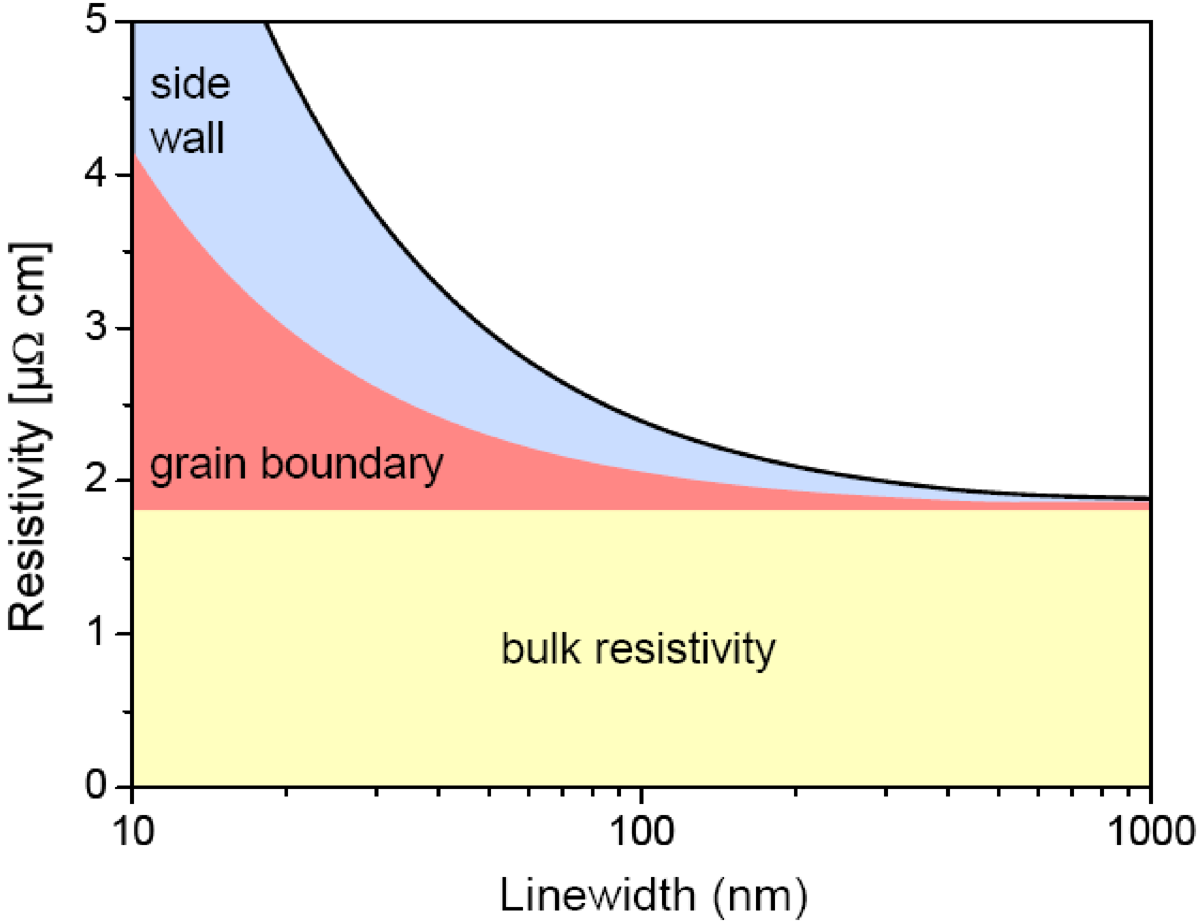


IBM [Rossnagel+ 2004]

Comparison with measurement data

Confirmation from ITRS'05

- Our simple model matches the newest ITRS'05 (just released at public.itrs.net)



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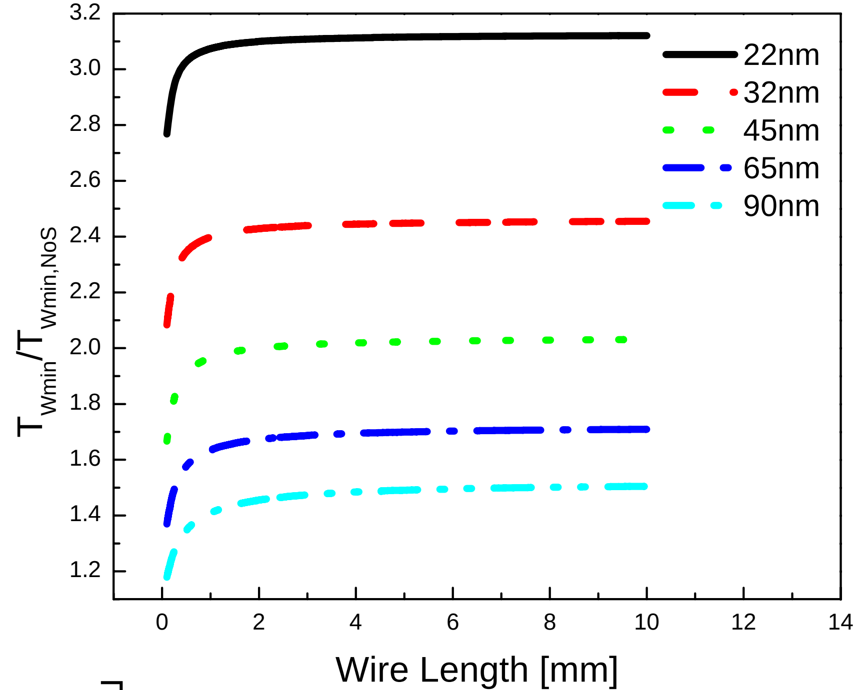
Wire Delay with Scattering

- Delay of minimum-width wire
  - Real delay (with scattering) may be up to **3x of that without scattering**



$$T_{1-WS} = R_d \cdot \left[c_a \cdot l \cdot w + c_f \cdot l + C_L \right] + \left[\frac{c_a \cdot l \cdot w}{2} + \frac{c_f \cdot l}{2} + C_L \right] \cdot \left[\rho_B + \frac{K_\rho}{w} \right] \cdot \frac{l}{w \cdot t}$$

Wire Sizing is More Effective under Scattering

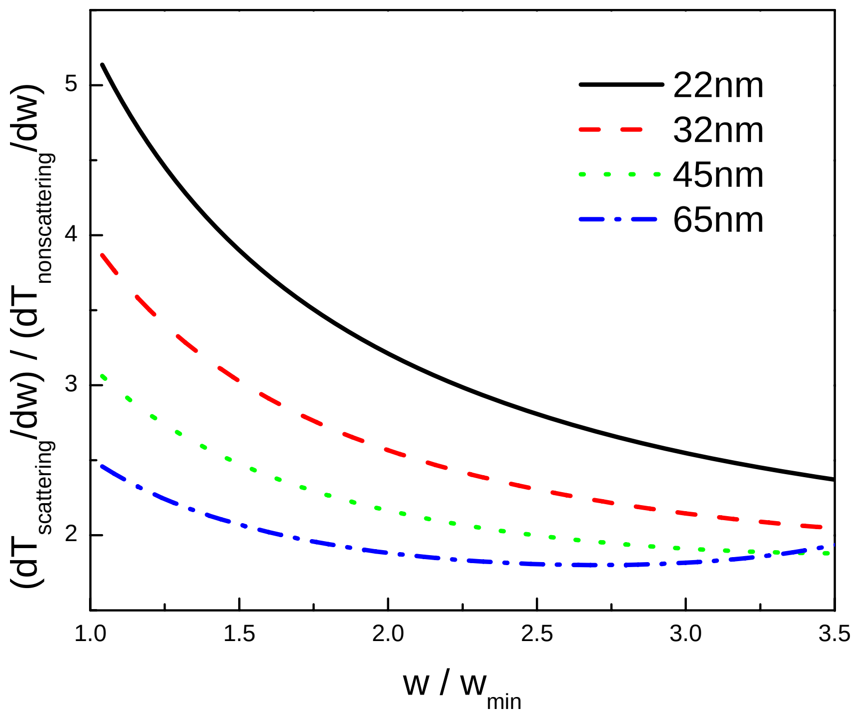
- Wire sizing sensitivity (effectiveness)
- 2-6x more effective

$$\frac{\partial T_{NonScattering}(w,l)}{\partial w}$$

$$= [R_d c_a l] + \left[\frac{1}{2} c_f \rho \frac{l^2}{t} + C_L \rho \frac{l}{t}\right] \cdot \frac{-1}{w^2}$$

$$\frac{\partial T(w,l)}{\partial w}$$

$$= [R_d c_a l] + \left[\frac{1}{2} c_f \rho_B \frac{l^2}{t} + C_L \rho_B \frac{l}{t}\right] \frac{-1}{w^2} - \frac{1}{2} c_a K_\rho \frac{l^2}{tw^2} - \left[c_f \frac{l^2}{t} + 2C_L \frac{l}{t}\right] K_\rho \frac{1}{w^3}$$



New Wire Sizing Function

- Delay of a single-width wire sizing
- Optimal width with width-dependent resistivity

$$T_{1-WS} = R_d \cdot \left[c_a \cdot l \cdot w + c_f \cdot l + C_L\right] + \left[\frac{c_a \cdot l \cdot w}{2} + \frac{c_f \cdot l}{2} + C_L\right] \cdot \left[\rho_B + \frac{K_\rho}{w}\right] \cdot \frac{l}{w \cdot t}$$

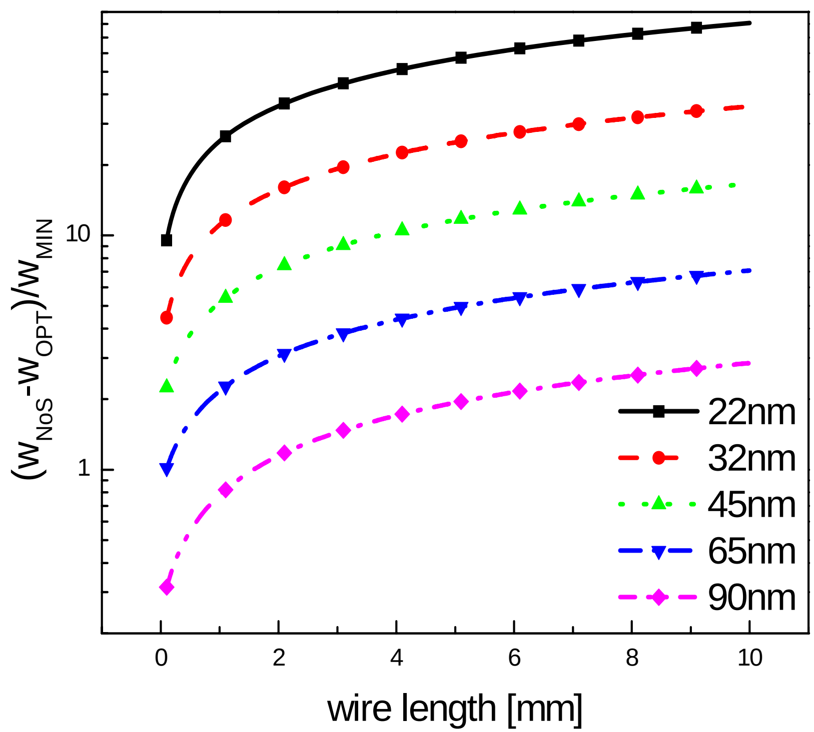
$$\partial T_{1-WS} / \partial w = 0$$

$$w_{Optimal} = w_1 = 2\sqrt{-\frac{a_1}{3}} \cos\left(\frac{\theta}{3}\right) \geq w_{\min}$$

$$\text{where } a_1 = \frac{-\left[c_a K_\rho l + c_f \rho_B l + 2C_L \rho_B\right]}{2 \cdot R_d \cdot c_a \cdot t}, \theta = \cos^{-1}\left(\frac{K_\rho \left(c_f l + 2C_L\right)\sqrt{54 R_d c_a t}}{\left(c_a K_\rho l + c_f \rho_B l + 2C_L \rho_B\right)^{3/2}}\right)$$

Optimal Wire Sizing with and without Scattering

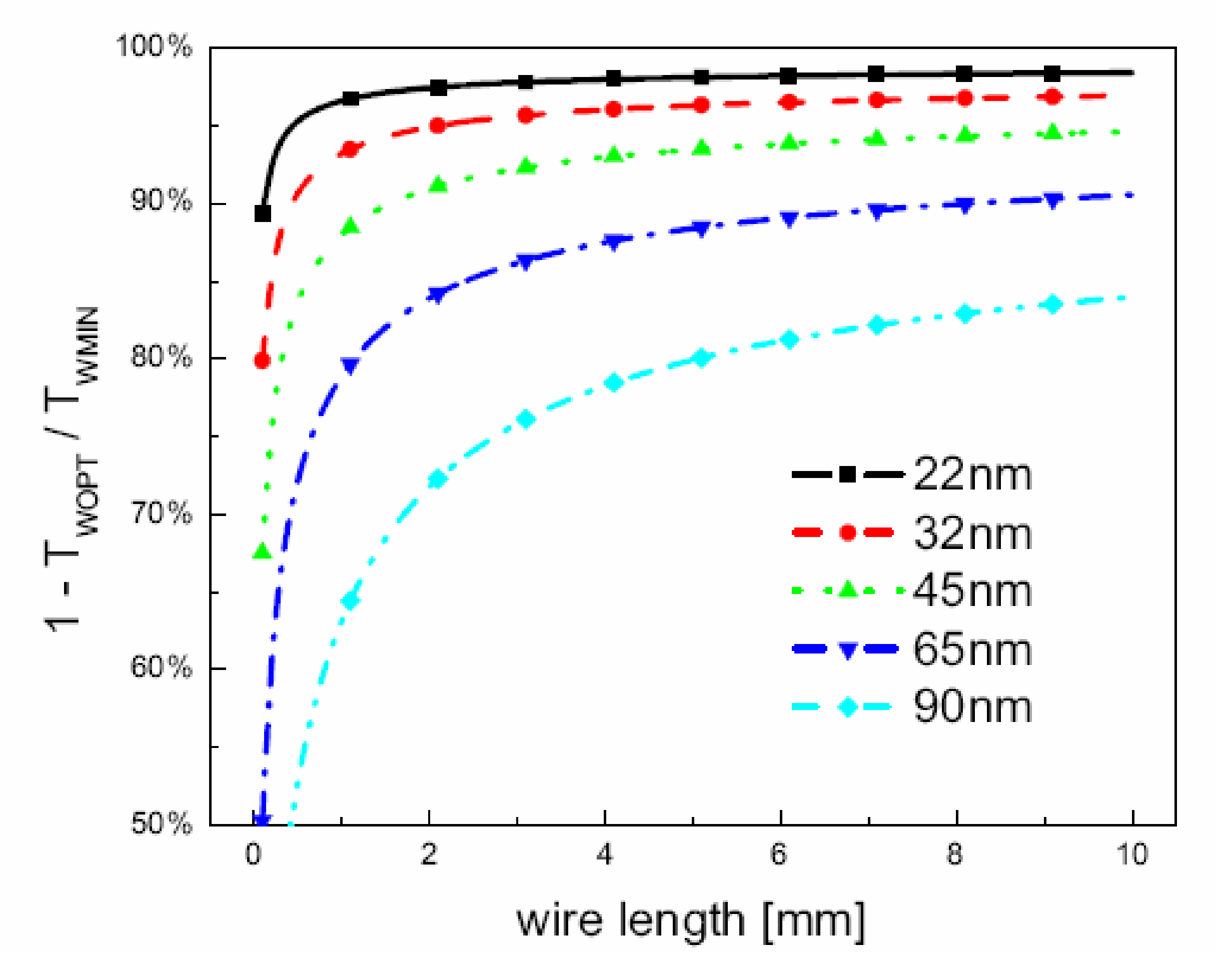
- Previous works with fixed resistivity
  - Either too small (bulk)
  - Or too big (conservative)
- An example of fixed (not width-dependent) resistivity
  - The optimal wire size will be overestimated (by 10 x min width)
  - This will cause area waste and routability problem
- Can also be undersized…



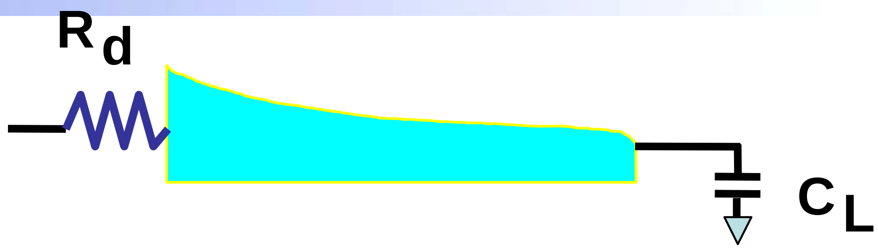
Comparison of optimal wire sizing (normalized by min width)

Delay Reduction with WS

- The optimal wire sizing delay reduction increases as technology scales (more scattering effects)



New Model for Wire shaping



- Revisit the wire shaping problem [Fishburn'97, Chen'97]

$$T = \int_0^L R_d \cdot \left[c_a \cdot dx \cdot f(x) + c_f \cdot dx + C_L \right] + \int_0^L \left[C_L + \int_0^x \left(c_a f(l) + c_f \right) dl \right] \left[\frac{\rho_B}{f(x)} + \frac{K_\rho}{f^2(x)} \right] \frac{dx}{t}$$

$$= R_d \cdot C_L + \int_0^L F(x, u, u') \cdot dx$$

- Euler's Differential Equation

$$\text{if:} \quad I = \int_{x_0}^{x_1} F(x, u(x), u'(x))\, dx \ \text{ is minimized,}$$

$$\text{then: } F_u(x, u(x), u'(x)) = \frac{d}{dx} F_{u'}(x, u(x), u'(x))$$

Polynomial Expansions

$$f(x) = u'(x) = \sum_{n=1}^{\infty} n a_n \cdot x^{n-1} = a_1 + 2a_2 \cdot x + 3a_3 \cdot x^2 + 4a_4 \cdot x^3 + \cdots$$

$$a_1 = w_{\min}$$

$$a_2 = \frac{c_a \cdot a_1^{\,2} \cdot \left(2\rho_B \cdot a_1 + 3K_\rho\right) + c_f \cdot a_1 \cdot \left(\rho_B \cdot a_1 + 2K_\rho\right)}{4 \cdot C_L \cdot \left(\rho_B \cdot a_1 + 3K_\rho\right)}$$

$$a_3 = \frac{2 \cdot a_2 \cdot \left[c_a \cdot a_1 \cdot \left(\rho_B \cdot a_1\right) - c_f \cdot K_\rho - C_L \cdot \rho_B \cdot a_2\right]}{3 \cdot C_L \cdot \left(3K_\rho + \rho_B \cdot a_1\right)}$$

$$a_4 = \frac{c_a \cdot \left[\rho_B \cdot \left(6 \cdot a_1 \cdot a_2^{\,2} + 3 \cdot a_1^{\,2} \cdot a_3\right) - 9 \cdot K_\rho \cdot a_1 \cdot a_3\right]}{C_L \cdot \left(36K_\rho + 12\rho_B \cdot a_1\right)} - \frac{c_f \cdot \left[\rho_B \cdot \left(2 \cdot a_2^{\,2} + 3 \cdot a_1 \cdot a_3\right) + 15 \cdot K_\rho \cdot a_3\right]}{C_L \cdot \left(36K_\rho + 12\rho_B \cdot a_1\right)} - \frac{C_L \cdot \left[18 \cdot \rho_B \cdot a_2 \cdot a_3\right]}{C_L \cdot \left(36K_\rho + 12\rho_B \cdot a_1\right)}$$

$\ldots$

Wire Shape f(x) Expansion

$$g1(x) = a_1;$$

$$g2(x) = a_1 + \left(a_2 x + a_3 x^2\right)/2;$$

$$g3(x) = a_1 + a_2 x + a_3 x^2 + \left(a_4 x^3 + a_4 x^4\right)/2;$$

$$\cdots$$

$$gn(x) = \sum_{i=1}^{2n-3} a_i x^{i-1} + \frac{1}{2}\left(a_{2i-2} x^{2i-3} + a_{2i-1} x^{2i-2}\right)$$

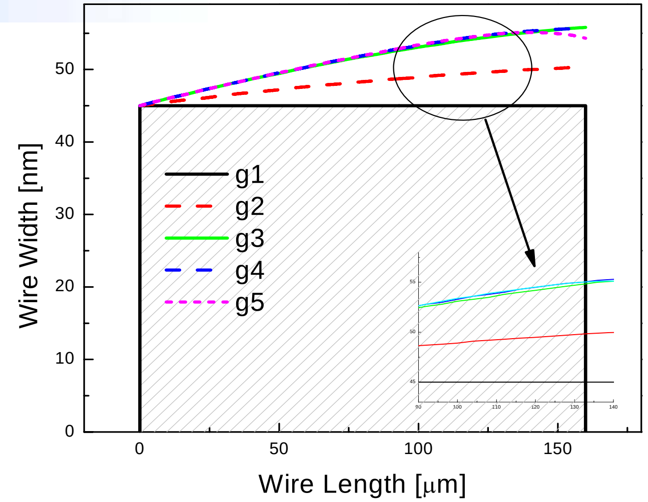
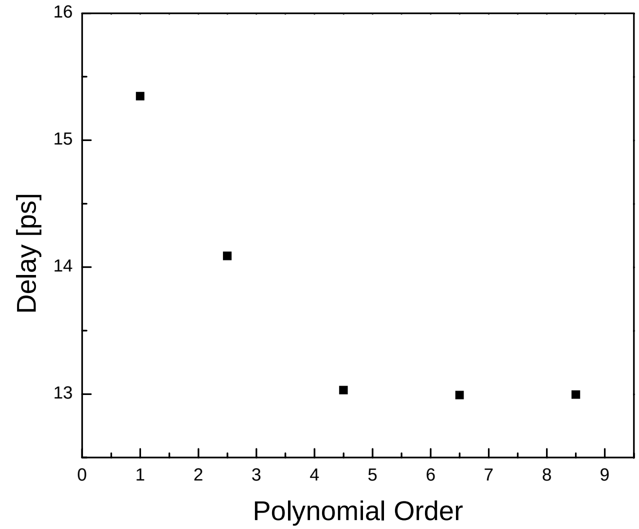


Fig. 6 Wire shaping with different order polynomial approximation

- $g_3(x)$ is good enough
- $g_3(x)$ is different from [Fishburn'97]
- Can be used for delay estimation modeling



Conclusion

- Scattering effect should be considered for interconnect of nanoscale/mesoscale IC design
  - The most recently released ITRS'05 has a lot of coverage on scattering effect
- A semi-empirical model is developed
  - It fits the measurement data well
  - Matches ITRS'05
  - Suitable for interconnection optimization
- Wire delay & sizing impacts are studied
- Future work:
  - Variational modeling (surface roughness…)