Adaptive Admittance-based Conductor Meshing for Interconnect Analysis

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High-Frequency Effects

- Skin effect
  - Current density drops off exponentially from the conductor surface into the interior
- Proximity effect
  - Current crowds to the surface close to or far from nearby conductors, depending on the directions of current flow

High-Frequency Effects

- Consequences
  - Non-uniform current distribution across interconnects
  - Variations in resistance and inductance values
- Solution
  - Discretization such that the current density in each filament is approximately uniform
- Challenges
  - Large lumped circuit model is generated
  - Trade off between the model accuracy and the computational cost

Contributions

- Demonstrate the existence of a quantity that can be used to estimate the model accuracy of a given discretization
- Use this quantity to determine a discretization with the least computation and acceptable model accuracy

Quantity Tracking Simulation Error

- Consider a single conductor carrying a signal at a single frequency
- Assume a voltage difference of 1V is applied between the two terminals

$$\mathbf{1} = (R_i + j\omega L_i) I_i \rightarrow I_i = (R_i + j\omega L_i)^{-1}\mathbf{1}$$

$\mathbf{1}$: vector of ones

I_i : vector of filament currents at discretization i

R_i and L_i : resistance and inductance matrices at discretization i

Quantity Tracking Simulation Error

- The total current at discretization G_i is equal to the conductor-level complex admittance Y_i

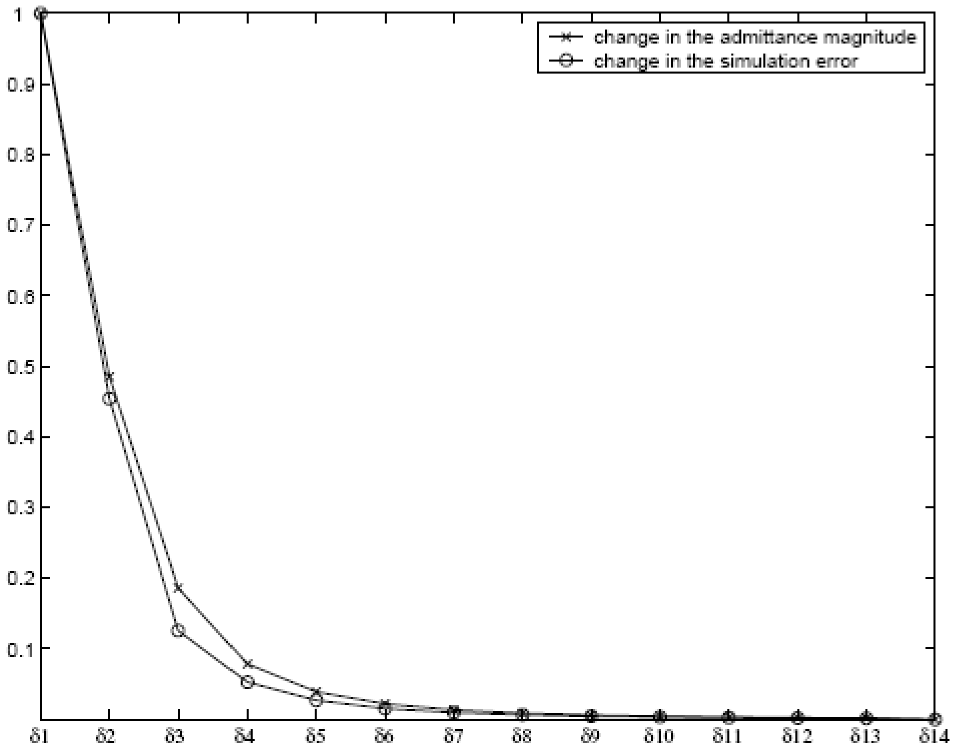
$$\mathbf{1}^T I_i = \mathbf{1}^T (R_i + j\omega L_i)^{-1} \mathbf{1} = \mathbf{1}^T y_i \mathbf{1} = Y_i$$

  - Characteristics of $|Y_i|$
    - Increases monotonically with increasing i
    - Converges to the true value of the conductor-level admittance $|Y_\infty|$
- Discretization with the higher admittance magnitude is more accurate

Quantity Tracking Simulation Error

- Admittance error : $|Y_{\infty}| - |Y_i|$
  - Equal to the steady-state error in the current amplitude if a unit sinusoidal voltage is applied at terminals of the conductor
  - Can be used as the stopping criterion of meshing schemes if $|Y_{\infty}|$ is available
- $|Y_{\infty}|$ is seldom available
  - Use the change in the admittance magnitude $|Y_{i+1}| - |Y_i|$

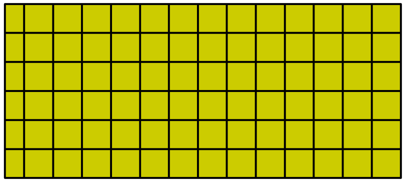
Quantity Tracking Simulation Error



- Single conductor with cross-section 3×3μm and length 1000μm
- 1V ramp with a rise-time of 10ps as the input signal
- Two quantities are compared
  - $\delta_i = |Y_{i+1}| - |Y_i|$ @ 100GHz
  - $Max(|I(t) - I_i(t)|)$

Conventional Meshing Schemes

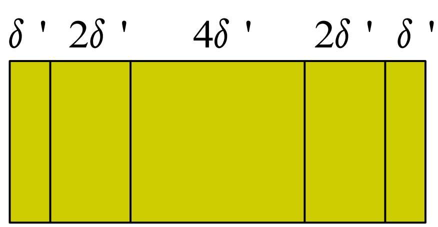
- Uniform meshing (UM)
  - Filaments are all of the same cross-sectional area
  - Width and height of filament are no larger than the skin depth



- Exponential meshing (EM)
  - Dimensions of filaments increase exponentially from outside to inside
  - Width and height of the smallest filament are no larger than the skin depth

Conventional Meshing Schemes

- EM-1
  - Width and height of the smallest filament ≦ skin depth
  - Ratio between adjacent filaments = 2
  - Number of filaments is determined automatically

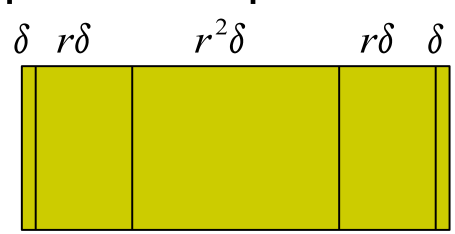


$$\delta' = \frac{W}{2(1+2)+2^2} \le \delta$$

W : width of conductor

δ : skin depth

- EM-2
  - Width and height of the smallest filament = skin depth
  - Ratio between adjacent filaments is varying
  - Number of filaments is specified a priori



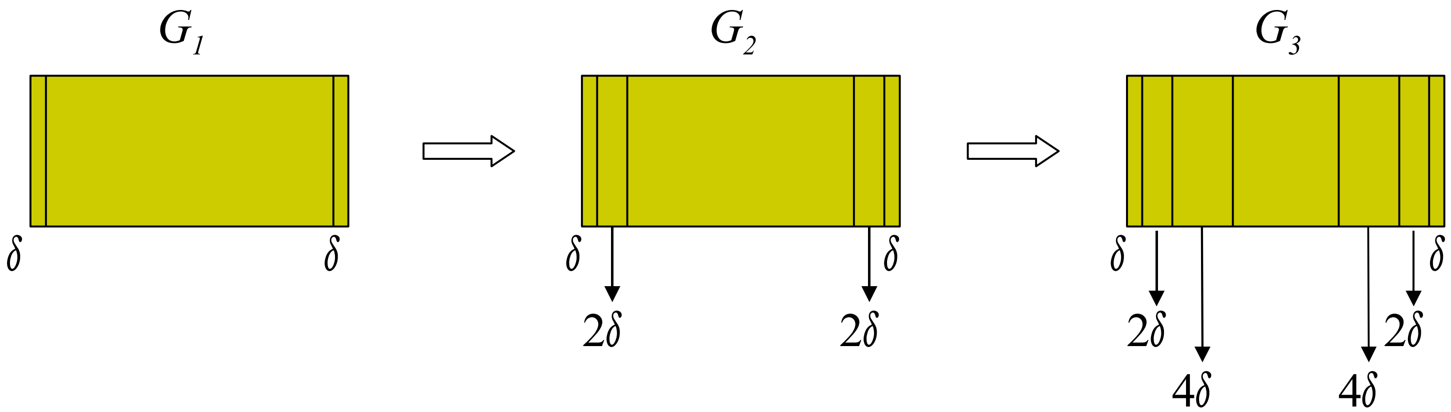
$$W = 2\delta\left(\frac{r^{(N-1)/2}-1}{r-1}\right)+\delta\, r^{(N-1)/2}$$

N : given number of filaments

r : ratio of adjacent filaments

Adaptive Exponential Meshing Schemes (AEM)

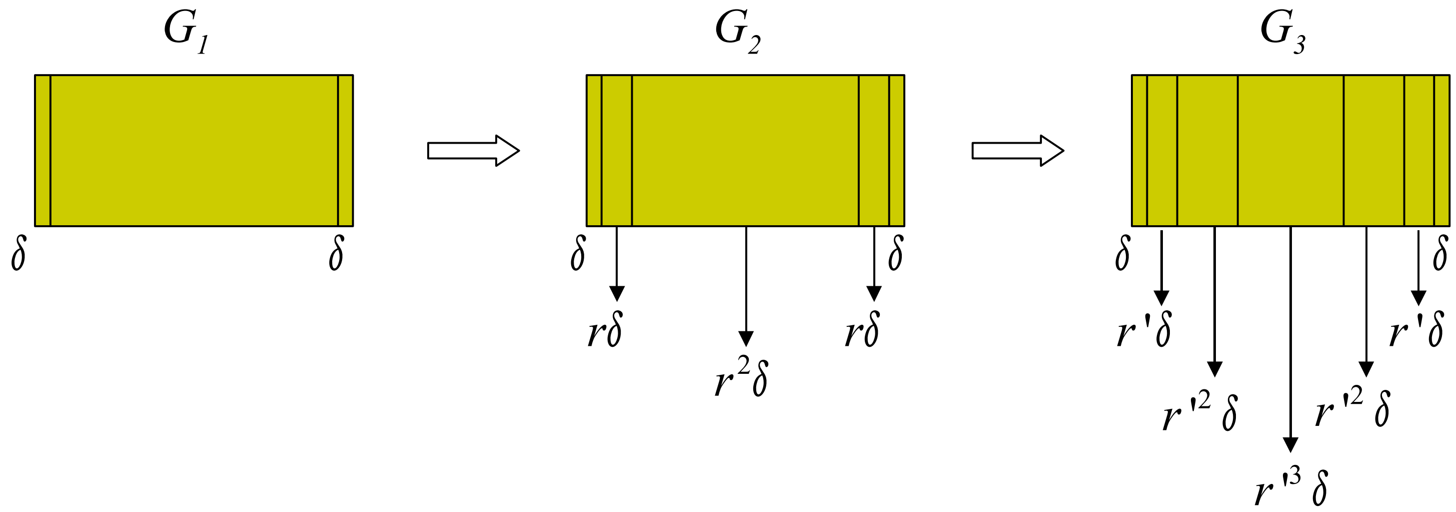
- AEM-1
  - Width and height of the smallest filament = skin depth
  - Ratio between adjacent filaments = 2
  - Add filaments until the admittance magnitude does not increase appreciably or when there is no room for adding filaments



Adaptive Exponential Meshing Schemes (AEM)

- AEM-2
  - Width and height of the smallest filament = skin depth
  - Increase the number of filaments with the ratio between adjacent filaments satisfying the equation

$$W = 2\delta\left(\frac{r^{(N-1)/2}-1}{r-1}\right)+\delta\, r^{(N-1)/2}$$

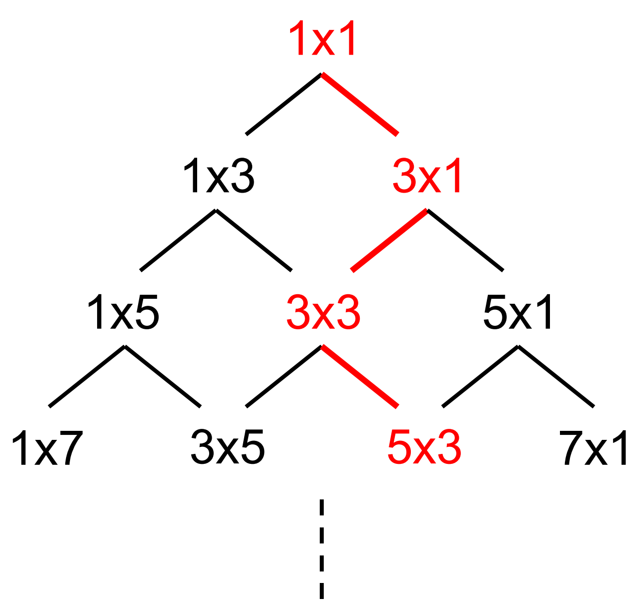


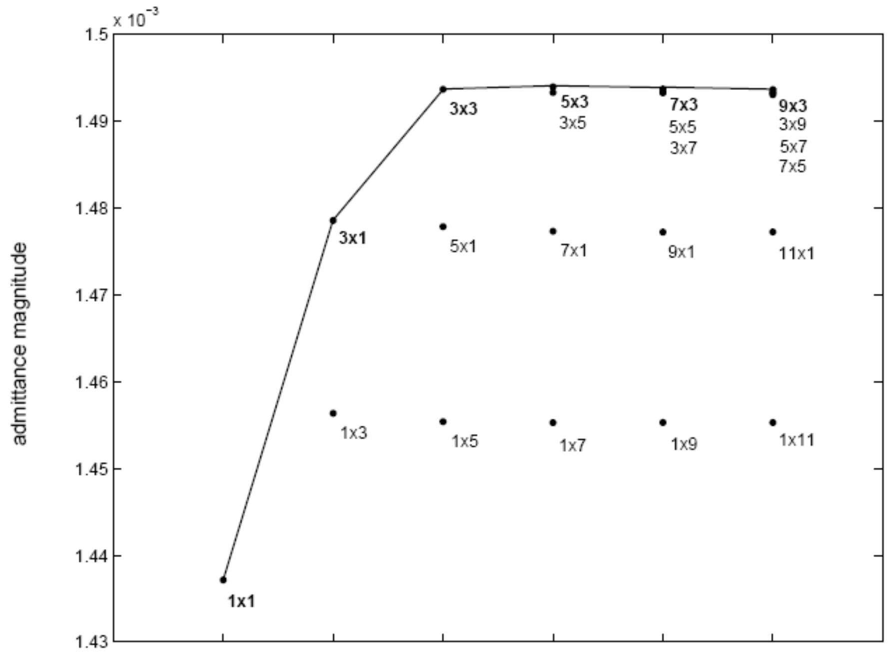
Implementations of Adaptive Meshing Schemes

- Given a discretization that needs refinement
  1. For two candidate discretizations (two more filaments along the width and height), compute the admittance magnitude at the working frequency
  2. Choose the discretization that yields a higher admittance magnitude
  3. Repeat step 1 and 2 until the change in the admittance magnitude for two successive discretizations is below a suitable threshold

Implementations of Adaptive Meshing Schemes

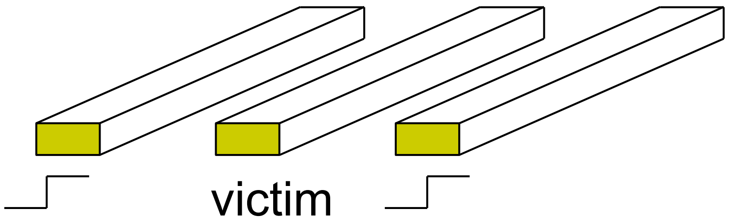
- Example
  - Single conductor of length 1000μm with cross section 10μm×3μm
  - "N×M" mesh : N and M filaments along the width and height, respectively





Non-Effect of Proximity

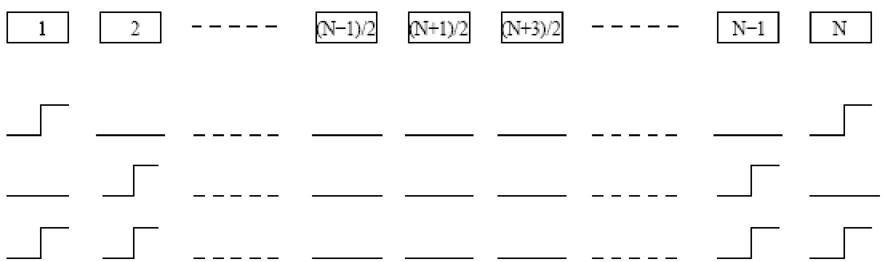
- Skin effect dominates the proximity effect
  - Interaction of filaments within a conductor overwhelms any effect from the filaments in any other conductors
  - Example - Three conductors are all of length 1000μm and cross section 10μm×3μm



Mesh size	spacing (μm)									
	0.1		0.5		1.0		1.5		2.0	
	$\|Y\|(\times 10^{-2})$	error	$\|Y\|(\times 10^{-2})$	error	$\|Y\|(\times 10^{-2})$	error	$\|Y\|(\times 10^{-2})$	error	$\|Y\|(\times 10^{-2})$	error
5×3	2.5094	0.9793%	1.5437	0.8933%	1.1063	0.8442%	0.8973	0.8155%	0.7724	0.8188%
7×3	2.5166	0.8609%	1.5465	0.7788%	1.1078	0.7296%	0.8984	0.6995%	0.7732	0.8188%
5×5	2.5077	0.9375%	1.5421	0.8622%	1.1050	0.8196%	0.8962	0.7946%	0.7715	0.8188%
9×3	2.5176	0.8143%	1.5470	0.7425%	1.1082	0.6992%	0.8987	0.6720%	0.7734	0.8188%
7×5	2.5135	0.9400%	1.5440	0.8639%	1.1060	0.8209%	0.8970	0.7957%	0.7720	0.8188%

Experimental Results

- Simulation of wires with uniform cross-section
  - N parallel conductors, each with a cross-section of 5x1µm, length of 1000µm and inter-conductor spacing of 1um
  - Ramp signals with a rise-time of 10ps as aggressor inputs



number of conductors	discretization							
	EM-1		EM-2		AEM-1		AEM-2	
	8×4		8×4		5×3		5×3	
N	error	runtime	error	runtime	error	runtime	error	runtime
5	0.1492%	1.00	0.1101%	1.07	0.5936%	0.67	0.5346%	0.67
7	0.1571%	1.00	0.1160%	1.07	0.6835%	0.58	0.5742%	0.60
9	0.1581%	1.00	0.1183%	1.03	0.7202%	0.44	0.5841%	0.44
11	0.1576%	1.00	0.1184%	1.01	0.8279%	0.29	0.5865%	0.29
13	0.1635%	1.00	0.1231%	0.97	1.0085%	0.35	0.5993%	0.35

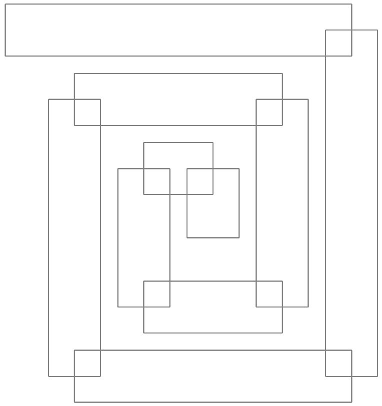
Experimental Results

- Simulation of wires with non-uniform cross-sections
  - Consider the cases of five parallel conductors and seven parallel conductors
  - Length = 1000μm, height = 3μm
  - Widths of each conductor are randomly chosen from the set {3μm, 5μm, 7μm}

number of conductors	EM-1		EM-2		AEM-1		AEM-2	
N	error	runtime	error	runtime	error	runtime	error	runtime
5	0.2121%	1.00	0.1263%	1.04	0.5682%	0.42	0.8481%	0.44
7	0.2133%	1.00	0.1311%	1.07	0.5521%	0.14	0.7421%	0.14

Experimental Results

- Extraction of frequency-dependent inductance
  - Spiral inductor composed of segments with a uniform cross-section of 3μm×1μm and the longest segment of 20μm long
  - Separation between any adjacent parallel segments is 1μm
  - Use FastHenry to extract the frequency-dependent inductance
  - Frequency points is taken from the 10GHz ~ 100GHz range



meshing scheme	EM-1	EM-2	AEM-1	AEM-2
error	0.0913%	0.0885%	0.6063%	0.1726%
runtime(second)	12.60	12.33	4.01	7.60
speedup	1.00	1.02	3.14	1.66

Conclusion

- Conductor-level admittance magnitude can be used to predict simulation errors due to discretizations
- Observations can be applied to determine the coarsest discretization with an acceptable model accuracy by the admittance magnitude
- Significant savings in computation can be realized with little sacrifice in accuracy