An Efficient Algorithm for 3-D Reluctance Extraction Considering High Frequency Effect

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Outline

- Background
- Partial reluctance extraction considering high frequency effect
- Efficient extraction of frequency-dependent reluctance
- Numerical results
- Conclusions

Background

- Ultra deep sub-micron (even nano) technology is used
- Operating frequency reaches multiple giga-hertz
- Other factors:
  - Reduction of resistance by copper
  - Reduction of capacitance by low-κ dielectric
  - High integrated density
  - Growing complexity of interconnect design

⇒ Efficient modeling of on-chip inductance effect becomes indispensable

Background

- Main difficulty of on-chip inductance modeling
  - the unknown circuit return path before extraction and simulation
- PEEC (partial element equivalent circuit) model
  - Introducing the concept of partial inductance whose return path is at infinity
  - Coupling of partial inductance is among all the conductors, the resulted matrix is extremely dense
  - Simply truncating small terms leads system to unstable

Background

- Partial reluctance (K element)
  - Attracts people's attention because of its good locality similar to capacitance
  - Ignoring small elements in matrix doesn't affect stability
  - Simulation based on it is more accurate and efficient
- Conventional partial reluctance extraction
  - Most works doesn't consider high frequency effect
  - High calculation complexity prohibits efficiency
  - Can't handle 3D complex structure well

Background

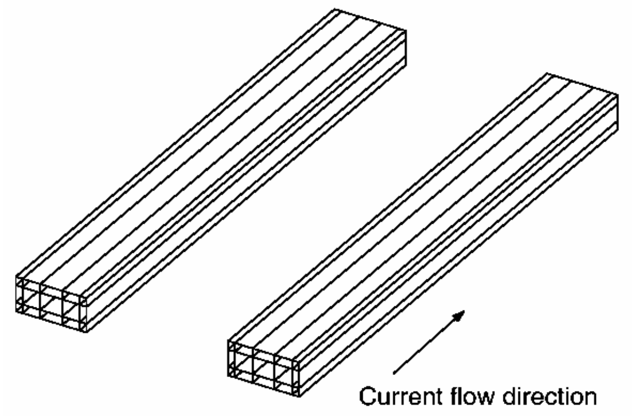
- Direct extraction method
  - In [MTT-S04], YuDu presented a new method to capture partial reluctance
  - Extracts partial reluctance directly according to its physical meaning
  - Advantage: save calculation consumption
  - Unfortunately, still doesn't consider high frequency effect, not accurate enough

Background

- Our contribution
  - Based on direct extraction method, meanwhile take high frequency effect into account
  - Applying efficient window selection strategy
  - Using other accelerating techniques
  - Achieve high accuracy and efficiency at the same time

K extraction considering high frequency effect

- Capturing high frequency effect
  - Magneto quasi static (MQS) assumption
  - Current in a conductor isn't evenly distributed according to skin effect
  - Meshing non-uniformly



two parallel conductors mashed into 5 × 3 filaments non-uniformly

- Frequency-dependent reluctance extraction
  - To use the locality property of partial reluctance, apply some window selection strategy
  - In each window,

K extraction considering high frequency effect

- Formulate $Z_m(\omega) = R_m(\omega) + j\omega L_m(\omega)$

- Set conductors to be aggressor one by one and calculate the admittance matrix $\mathbf{Y}_n$, where m and n are number of filaments and conductors in the window respectively

- Inverse Y to get Z; calculate L; inverse L again to get K

$$Z_n(\omega) = Y_n^{-1}(\omega) \Rightarrow L_n(\omega) = \mathrm{Im}\{Z_n(\omega)\}/\omega$$

$$K_n(\omega) = L_n^{-1}(\omega)$$

Twice matrix inversion

K extraction considering high frequency effect

- Merge the local partial reluctance matrix with the global one
- The calculation complexity in each window is $O(m^3)$ plus two matrix inversions of dimension n

- Window selection strategy
  - Key issue to assure the accuracy of the extraction
  - Larger window size will bring higher accuracy as well as longer running time

For more detail, please refer to :
C. Luk 's paper on ASPDAC, pp. 793-798, Jan 2004

K extraction considering high frequency effect

- Our method considers both shield effect and distance among conductors to determine a coupling level
- All conductors whose coupling level are less than a pre-specified 'max coupling level' form the current window
- One advantage: can handle general 3D structure well
- For general 3-D structures, perform projections of all the conductors in XOY, ZOX, and YOZ planes, respectively

Efficient extraction of frequency dependent reluctance

- With the sinusoidal-steady assumption, we have

$$(R + j\omega L)I = V \quad \xrightarrow{\text{filaments meshing}} \quad (\hat{R} + j\omega \hat{L})\hat{I} = \hat{V} \quad (*)$$

  where R, L are the resistance matrix and the partial inductance matrix of the system

- According to the definition of K matrix,

$$\mathrm{j}\omega\mathrm{I} = \mathrm{K}(\mathrm{V} - \mathrm{RI})$$

  which means the i th column of K matrix is numerically equal to the current distribution on the conductors if we set the i th entry (corresponding to the aggressor) of V-RI to $\mathrm{j}\omega$, others to 0

Efficient extraction of frequency dependent reluctance

- Physical meaning of partial reluctance
  - According to Faraday's law, $E = -j\omega A - \nabla\Phi$
    where A is the magnetic vector potential, E is the electric field,
    Φ is the scalar potential
  - Integrating both side, we have

$$V_{ab} = -(\Phi_b - \Phi_a) = j\omega\int_b^a A dl + E l_{ab}$$

  - As E contributes only to the resistance potential drop, so

$$V_{ab} - RI = j\omega\int_b^a A dl$$

    which means the ith column of K matrix is the current distribution on conductors if we set the ith entry of the vector potential drop to 1 and others to 0

Efficient extraction of frequency dependent reluctance

- Relation between filament and conductor,

$$\hat{V} = M^t V \quad \text{and} \quad I = M\hat{I}$$

$$M_{ij} = \begin{cases} 1 & \text{when filament } j \text{ is in coductor } i \\ 0 & \text{otherwise} \end{cases}$$

We use the mesh incidence matrix M to describe the relation

- Rewrite Eq.(*) (real and imaginary part respectively),

$$\begin{cases} \hat{R}\hat{I}_{re} - \omega\hat{L}\hat{I}_{im} = \hat{V}_{re} = M^t V_{re} \\ \omega\hat{L}\hat{I}_{re} + \hat{R}\hat{I}_{im} = \hat{V}_{im} = \omega M^t \int A dl \end{cases}$$

from the physical meaning, set the ith item to 1, the resulted current distribution is numerically equal to partial reluctance

Efficient extraction of frequency dependent reluctance

- Under the special vector potential drop setting, the current distribution numerically equals to one column of K matrix
- We know K matrix is real → $I_{im} = 0$
- Add this equation to the previous system, we can finally get the whole system

- Condensing the system
  - Since I is real, we don't need to know the imaginary part
  - Delete the corresponding variables from the original system

Efficient extraction of frequency dependent reluctance

original system

$$\begin{bmatrix} \hat{R} & -\omega\hat{L} & -M^t \\ \omega\hat{L} & \hat{R} & 0 \\ 0 & M & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ \hat{I}_{im} \\ V_{re} \end{bmatrix} = \begin{bmatrix} 0 \\ \omega M^t \int A dl \\ 0 \end{bmatrix}$$

delete $\hat{I}_{im}$ from the original system

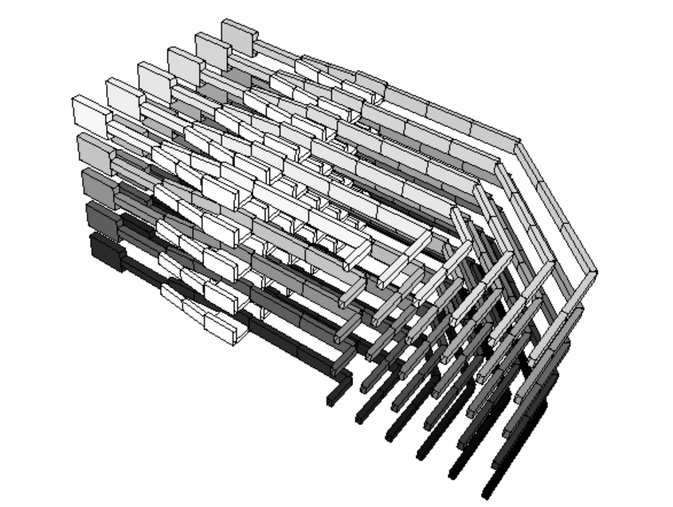
condensed system

$$\begin{bmatrix} I+(\omega\hat{R}^{-1}\hat{L})^2 & -\hat{R}^{-1}M^t \\ M\hat{R}^{-1}\hat{L} & 0 \end{bmatrix} \begin{bmatrix} \hat{I}_{re} \\ V_{re} \end{bmatrix} = \begin{bmatrix} \omega^2\hat{R}^{-1}\hat{L}\hat{R}^{-1}M^t \int A dl \\ M\hat{R}^{-1}M^t \int A dl \end{bmatrix}$$

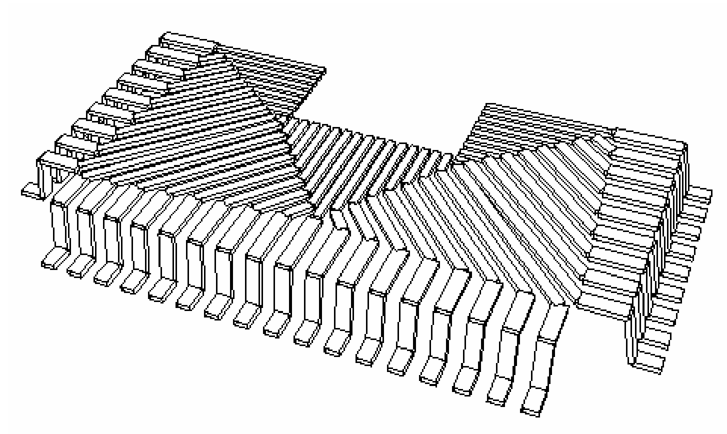
- The dimension of coefficient matrix is decreased from 2m+n to m+n
- Since m>>n, condensing can cut solving time a lot
- With little extra consumption

Numerical results

- Three typical structures (two from real industry)
  - Example 1 A randomly generated structure, containing 300 parallel conductors with unequal length



Example 2 A 30 pins real industry package structure contains 260 conductors
Numbers partitioned level: 3×3



Example 3 A 35 pins real industry package structure contains 175 conductors
Numbers partitioned level: 3×5

Numerical results

- All examples are extracted under operating frequency of 10GHz
- We compare our result with FastHenry
- First extract partial reluctance; then reverse it to get the partial inductance matrix
- Use loop error distribution to measure the accuracy

TABLE I Computational Time for the Examples (Unit in Second)

	FastHenry	Our algorithm	Speed-up ratio
example 1	855.2	9.12	94
example 2	5650.2	9.83	575
example 3	5796.3	122.51	47

The results of FastHenry are based on default preferences.

CPU time of all experiments are recorded on a Sun Ultra V880 server (frequency 750MHz).

Numerical results

TABLE II Error Distribution of Loop Inductance (Compared with FastHenry)

Error of loop inductance(%)	<3%	3-6%	6-9%	9-12%	12-15%	>15%
example 1	95.5%	4.2%	0.3%	0.0%	0	0
example 2	46.4%	26.4%	12.3%	6.8%	2.5%	5.1%
example 3	72.7%	20.7%	5.7%	0.8%	0.1%	0.0%

- The result indicates the accuracy of example 2 is not as good as the others. Because the locality of partial reluctance in example 2 is not so good
- Partial reluctance has ‘good locality’ relative to partial inductance. It doesn’t means the locality is very good in any structure

Conclusions

- Conclusions
  - Less accuracy (due to without high frequency effect consideration), too much calculation consumption, are shortcomings of most conventional methods
  - An efficient algorithm based on direct partial reluctance extraction is proposed
  - Taking high frequency effect into account, the algorithm shows high accuracy and several 10X ~ 100X speedup over FastHenry
- Future work
  - Further improvement of equation solution
  - Apply this algorithm in partial reluctance based circuit simulation

Thank you !

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