A New Boundary Element Method for Accurate Modeling of Lossy Substrates with Arbitrary Doping Profiles

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Jan 26 2006

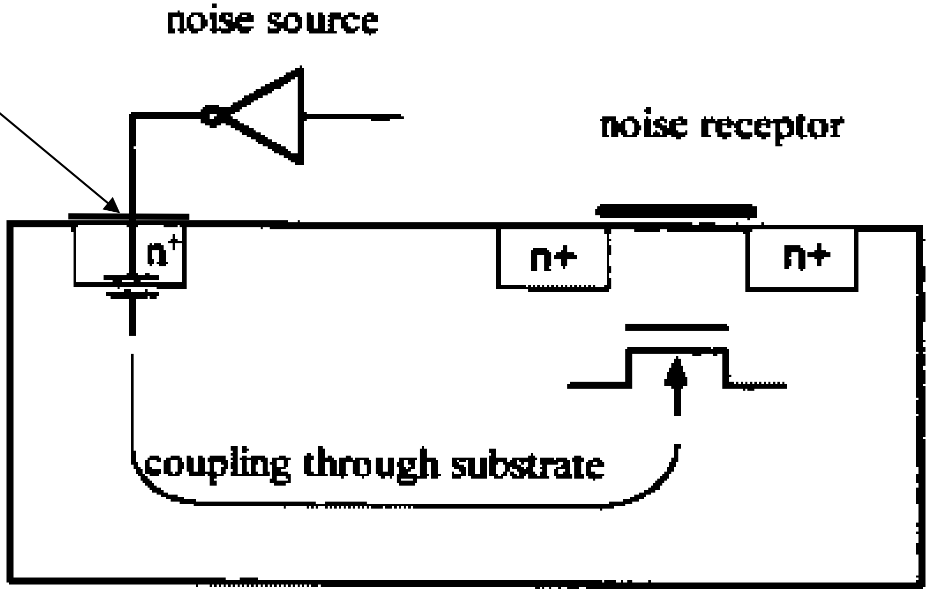
Outline

- Introduction of Substrate Coupling Problems
- Existing Numerical Methods
- Substrate Impedance Extraction Using Direct Boundary Element Method (DBEM)
- Efficient Techniques for DBEM Extraction
- Numerical Results
- Conclusions

Introduction of Substrate Coupling Problems

- There are more and more highly-integrated and high-speed circuits. For example, in SOC circuits the noisy digital components and highly-sensitive analog circuits are integrated on a single substrate.
- Despite of advantages in these circuits, e.g. low cost, a problem is that noises via substrates will:
  - Cause spurious signals
  - Degrade, even destroy, the circuit performances in many ways.

Introduction of Substrate Coupling Problems (Example)



Contact

Interface with outside

A simple example of coupling.

- At high frequency, both **resistive** and **capacitive** couplings should be considered. Niknejad [TCAD98]

A.M. Niknejad, R. Gharpurey, and R.G. Meyer, “Numerically stable Green function for modeling and analysis of substrate coupling in integrated circuits,” *IEEE Trans. CAD*, Vol. 17, pp. 305-315, 1998.

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Existing Numerical Methods

- Volume Discretization Methods,
  such as Finite Element Method (FEM),
  and Finite Difference Method (FDM)
- Advantages:
  - Extensively studied
  - Versatile for various kinds of substrate structures
    - Stratified, with multiple parallel horizontal layers
    - More complicated, e.g. those with isolating oxide wells
- Disadvantages:
  - Too many unknowns
  - Limited to small questions

Existing Numerical Methods (Cont.)

- Green's Function Based Methods (a kind of boundary element method, BEM)
- Advantages:
  - Only discretizes contact surfaces
  - Involves fewer variables than FDM/FEM
  - Some acceleration techniques proposed, such as DCT, eigendecomposition, etc.
- Disadvantages:
  - The suitable Green's function found finally involves infinite series
  - Only handle stratified (or multilayered) structures
  - Difficult, or even impossible to find a suitable Green's function for non-stratified substrates.

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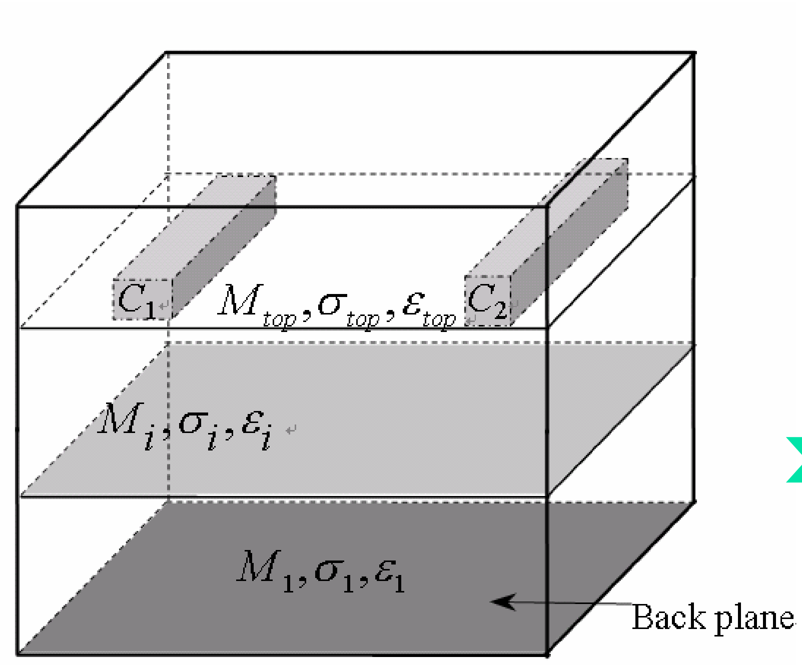
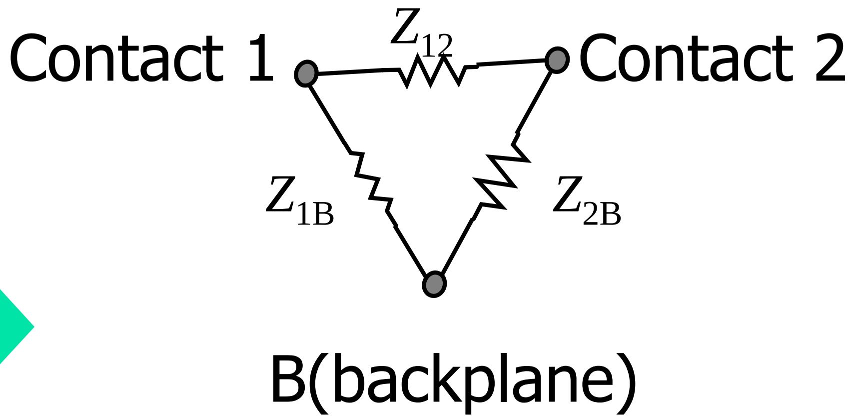
Substrate Coupling Impedance



An example substrate with a back plane. The non-zero conductivity of medium M_i is s_i, and the permittivity is e_i .

Reduced circuit model:



Contact 1

Contact 2

B(backplane)

Z denotes the coupling impedance between the contacts (including the plane).

Brief Introduction of Direct Boundary Element Method (DBEM)

- Discretizes substrate boundary (i.e. 2-D surfaces, contact surfaces + medium boundary/interface).
- Uses the free-space Green’s function only.
- Has no geometry limitation in theory.
- With improvement techniques, both the efficiency and accuracy of DBEM can be very high.
- Has been successfully applied in interconnect capacitance/resistance extraction.

Impedance Extraction Using DBEM

- Preset bias voltage of contacts, in $u = \bar{u}e^{jwt}$ form.
- Solve for the electric field *E*, also in $E = \bar{E}e^{jwt+q}$ form.
- For realistic substrates, the sinusoidal potential *u* fulfils:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

- For interfaces between mediums *a* and *b*, the potential and normal current are continuous:

$$u_a = u_b,$$

$$(s_a + jwe_a)E_{n,a} = (s_b + jwe_b)E_{n,b}.$$

Impedance Extraction Using DBEM (Cont)

- Apply the Green's identities. Select the free-space Green's function as weighting function, and we get an integral equation holding on Γ_i (boundary of medium *i*):

$$c_s u_s + \int_{\Gamma_i} q^* u d\Gamma = \int_{\Gamma_i} u^* q d\Gamma$$

- Substitute boundary conditions, and get a linear system.
- With multiple right-hand sides, the system will be:

$$\boldsymbol{AX} = \boldsymbol{B},$$

where matrix $\boldsymbol{X}$ is consist of variables u and E_n.

Impedance Extraction Using DBEM (Cont)

- After normal field E_n is obtained, the full current flowing through contact k is

$$\int_{\Gamma_k} (J + \frac{\partial D}{\partial t}) d\Gamma = \int_{\Gamma_k} (s + jwe) E_n d\Gamma .$$

- With proper bias voltage setting, the corresponding impedance Z will be:

$$Z_{mk} = \frac{1}{\int_{\Gamma_k} (\sigma + j\omega\varepsilon) E_n d\Gamma} .$$

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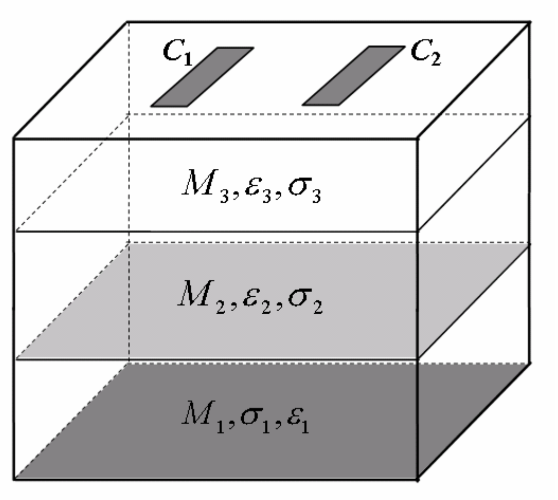
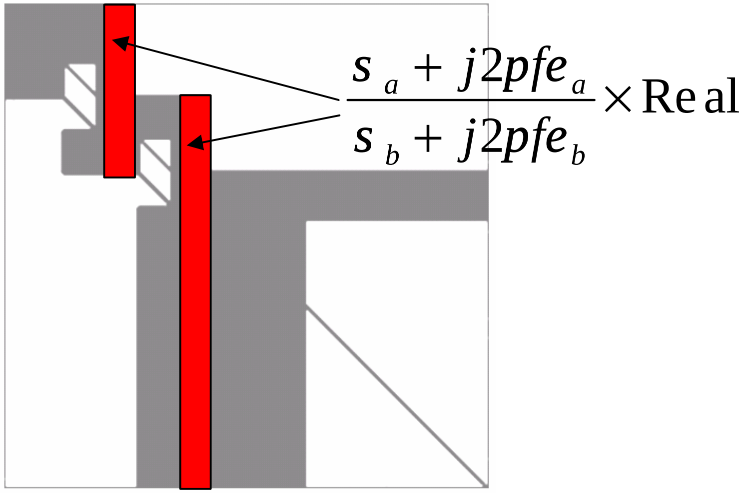
1: Matrix Reuse for Multiple-frequency Cal.

- Only the red blocks (related to interface variables) in the coefficient matrix are frequency-dependent.



Exam. Struct.

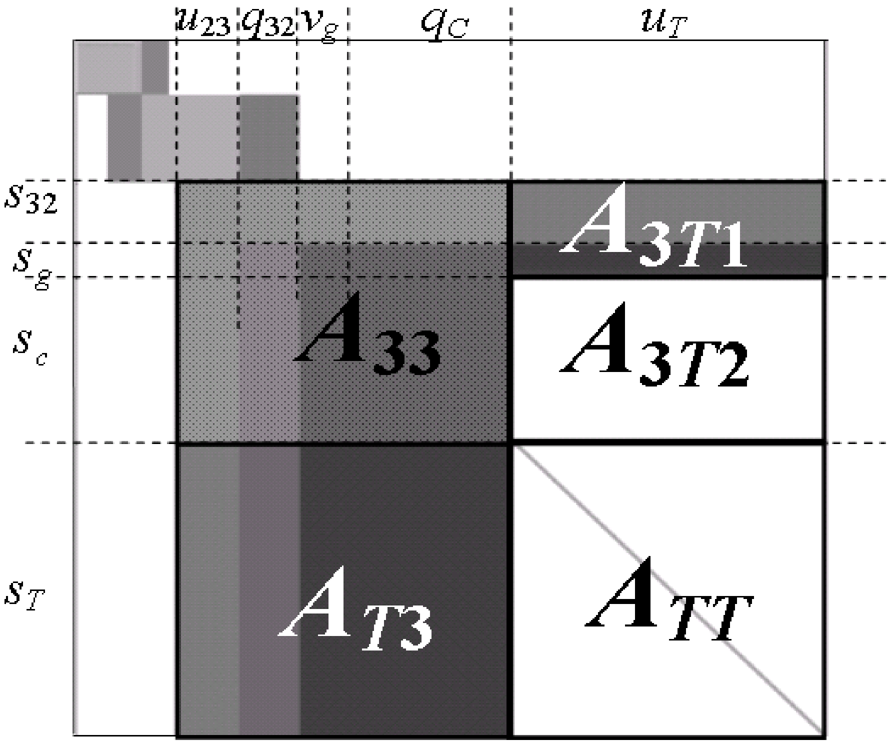


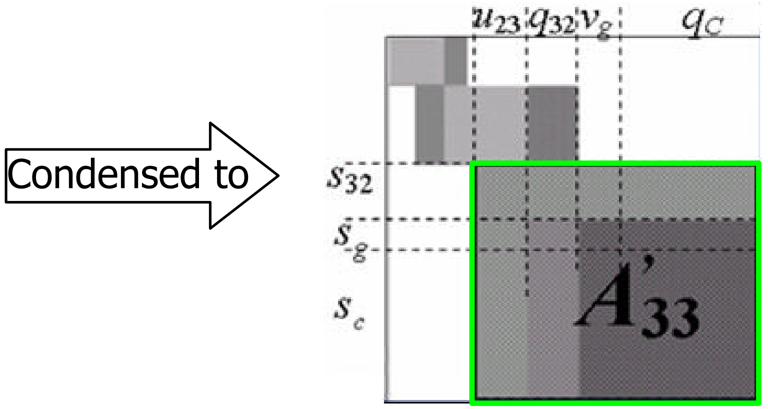
Coefficient Matrix

- When the impedances at multiple frequencies are desired, we can reuse the other blocks (in grey).

2: Matrix Reduction Technique

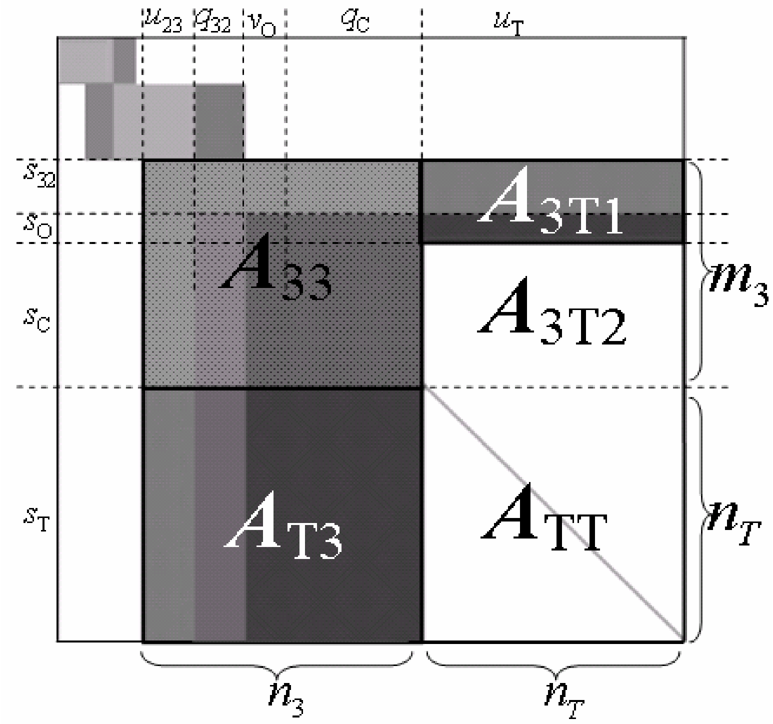
Matrix population: (White blocks are zeros)





$$\boldsymbol{A}'_{33} = \boldsymbol{A}'_{33} - \boldsymbol{A}_{3T}\,\boldsymbol{A}^{-1}_{TT}\,\boldsymbol{A}_{T3}$$

2: Matrix Reduction Technique (Cont.)



Mathematical properties:

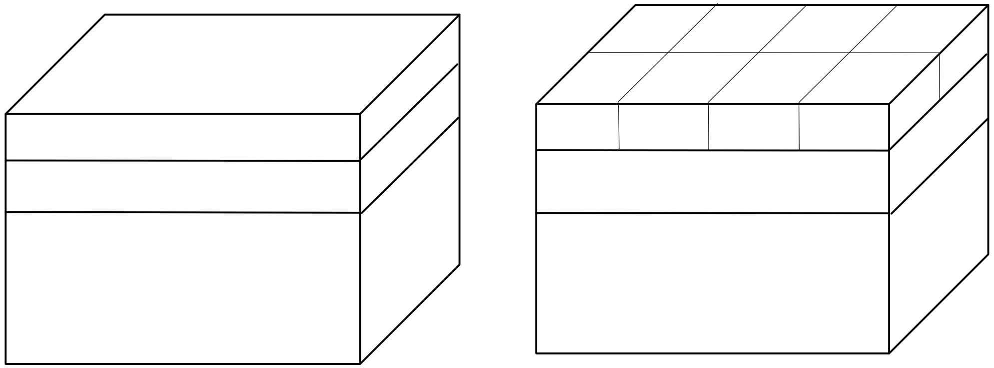
- $\boldsymbol{A}_{TT}$ is diagonal.
- $\boldsymbol{A}_{3T2}$=0.

Computational cost bound:

$$\boldsymbol{A}'_{33} = \boldsymbol{A}'_{33} - \boldsymbol{A}_{3T}\boldsymbol{A}_{TT}^{-1}\boldsymbol{A}_{T3} \quad :O(m_3 \times n_T \times n_3)$$

3: Matrix Sparsification Technique

- Matrix ***A*** is sparse for multi-medium substrates.
- To ***quasi-cut*** original physical mediums into many parts makes ***A*** sparser.

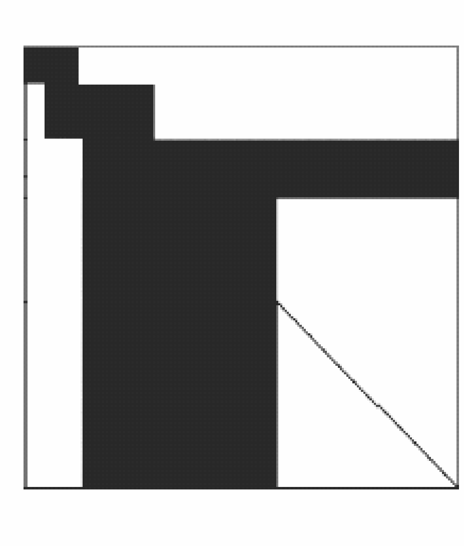


Original Substrate

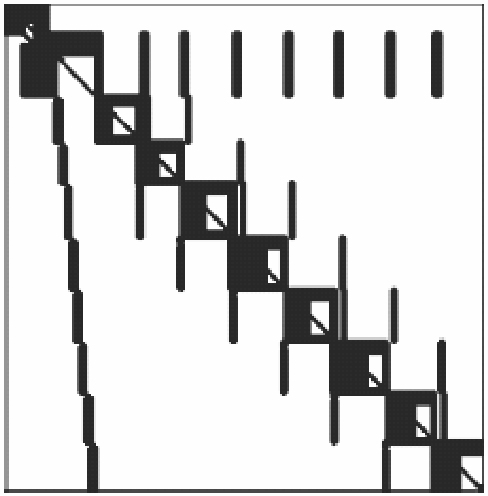
Cut Top layer into 4 x 2 *Quasi-mediums*

- This technique will speedup the consequent system solution using an iterative solver.

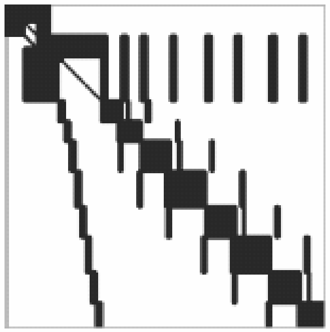
Matrix Sparsification Technique plus Matrix Condensation Technique (Exam.)



Original Matrix.
8002 x 8002



Matrix after QMM is used. 8814 x 8814.



After QMM and reduction applied. 5886 x 5886.

- The final matrix (right most) is much sparser and smaller than the original one.
- Then iterative solvers can be much more efficient.

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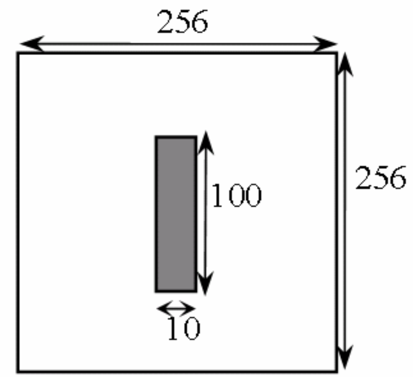
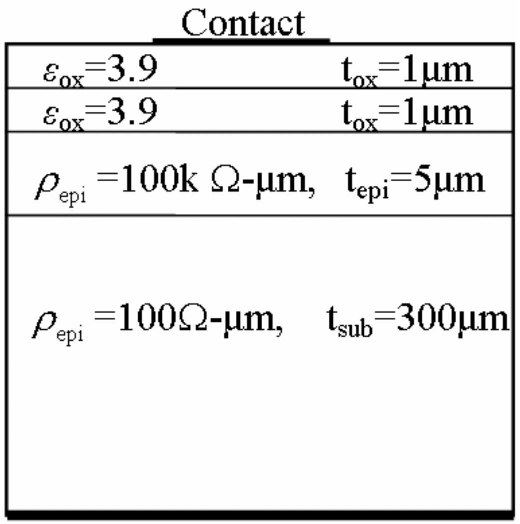
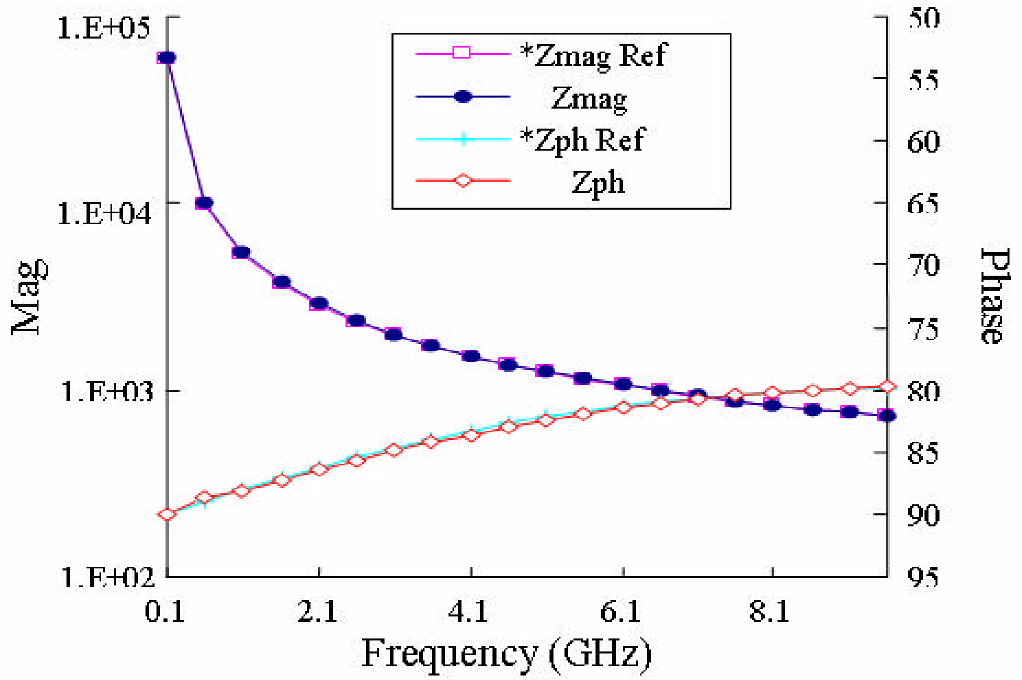
Numerical Results --Accuracy



Top view



Side view



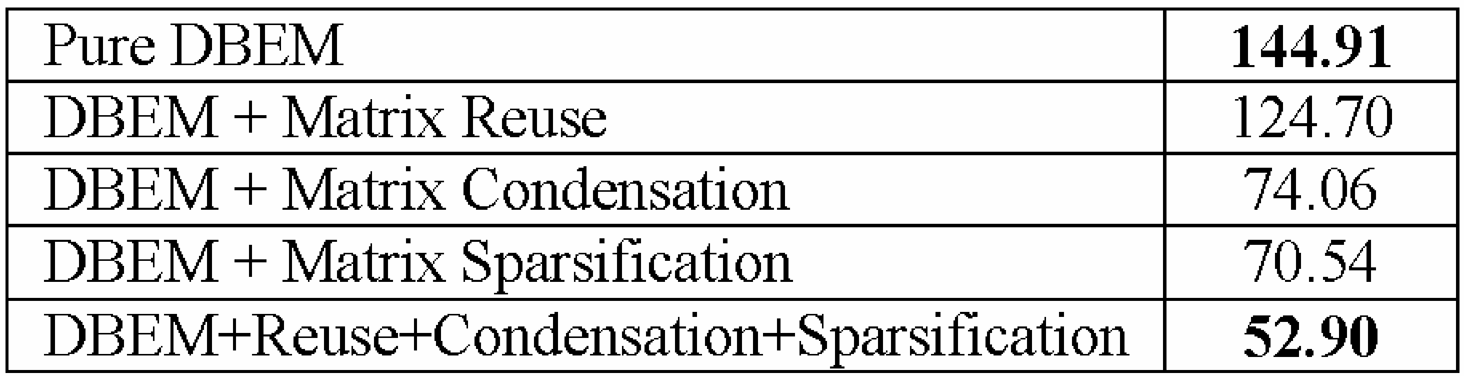
Accuracy:

Almost equal to Niknejad [TCAD98].

[TCAD98] A.M. Niknejad, R. Gharpurey, and R.G. Meyer, “Numerically stable Green function for modeling and analysis of substrate coupling in integrated circuits,” *IEEE Trans. CAD*, Vol. 17, pp. 305-315, 1998.

Numerical Results --Efficiency

- Running time of DBEM accelerated by the three techniques



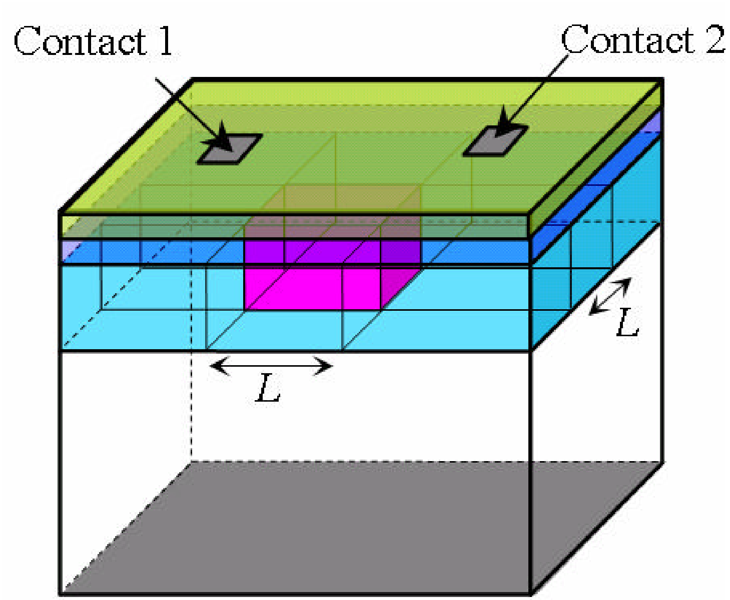
Pure DBEM	**144.91**
DBEM + Matrix Reuse	124.70
DBEM + Matrix Condensation	74.06
DBEM + Matrix Sparsification	70.54
DBEM+Reuse+Condensation+Sparsification	**52.90**

- Matrix reuse, reduction or sparsification can bring enhanced efficiency. Their combination brings a speed-up of 2.7 over pure DBEM.

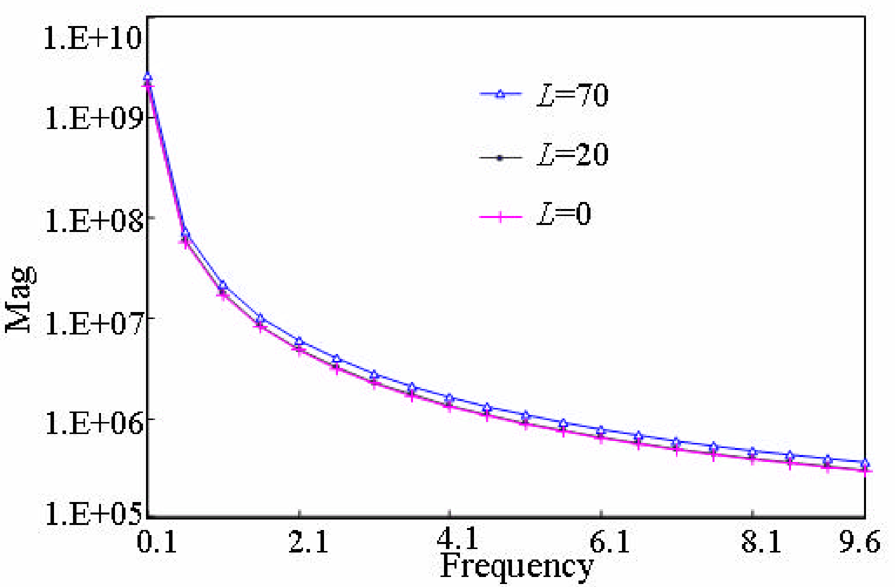
Numerical Results --Versatility

- Realistic substrates are not always stratified.
    - For example, there are many lateral variation components in substrates, such as oxide trenches and wells.
- Green function based methods meet much difficulty in simulating these irregular substrates.
- Since DBEM only uses the free-space Green's function, it does not have the above geometry limitation.

A Substrate with Complex Doping Profile



A substrate with lateral resistivity variation. The center block is of 1000 times resistivity of its neighbors.



Impedances obtained with DBEM. As *L* increases, the impedance also increases.

The curve is reasonable and understandable.

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- Improvements presented for the DBEM
  - Coefficient matrix reuse
  - Coefficient matrix reduction
  - Coefficient matrix sparsification
- Experiments show that
  - The above techniques are effective.

  And the proposed DBEM
  - Gives impedances with high accuracy and efficiency
  - Can easily simulate more complex than uniformly stratified substrates.

Thank you!

For more information, please contact:

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