Parasitic Extraction Involving 3-D Conductors Based on Multi-layered Green's Function

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Abstract

- An algorithm for capacitance extraction based on Green's function involving 3-D conductors, considering multi-layered dielectric and lossy substrate
- An Improvement based on an algorithm implemented in ASITIC, for which the metal thickness cannot be included.
- A numerically stable and analytically integrable formula for the Green's function in vertical direction is derived
- The Green's function is integrated over the sidewalls
- The coefficient of potential is efficiently calculated by looking up the table of DCT, DCST and DST

Outline

- Background
- DCT based Green's function method
- Our algorithm
- Result and Comparison
- Conclusion

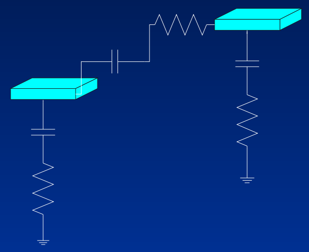
Capacitive Parasitic Extraction Based on BEM

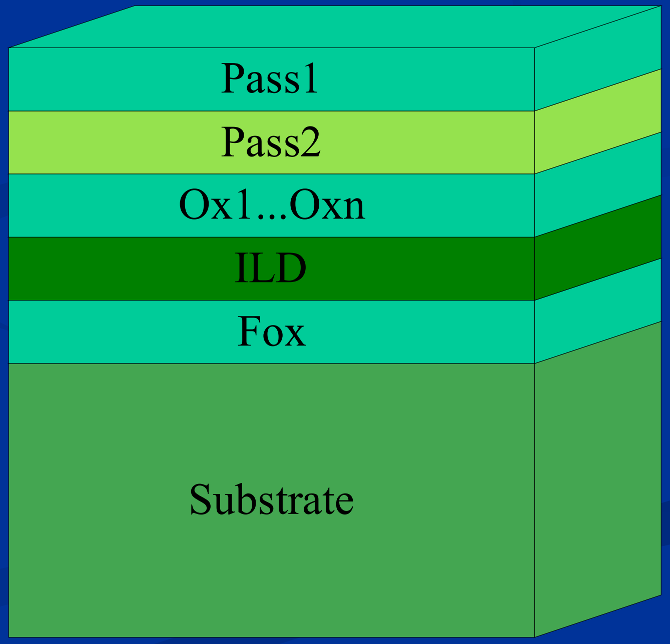
$$\mathbf{Pq = v}$$

- Computing the coefficient-of-potential p_{ij}
  - Free space method (Direct BEM)
  - Green's function method
- Solving the equation Pq=v
  - LU decomposition
  - Fast Multi-pole Method (J. White, MIT)
  - Singular Value Decomposition (S. Kapur, Bell Lab) } kernel independent
  - Hierarchical Method (W. Shi, TAMU)
  - Quasi Multiple Media (W. Yu, THU)
  - …

Advantages of Green's Function Method

- Direct BEM method
  - Discretize the interfaces between each adjacent dielectrics
  - A Large number of variables
- Green's function method
  - Layout independent Green's function for technology is solved first
  - Dielectric information is included in the Green's function
  - Only conductors need to be discretized
  - Limited to some certain profiles.





DCT Based Green's Function Method

- Based on multi-layered Green's function
- First proposed by R. Gharpurey in 1996 to analyze the effect of substrate coupling
- Numerically stabilized by A.M. Niknejad in 1998
- Developed to an academic software, ASITIC, which can simulate all kinds of passive elements.
- Conductors are assumed to be <u>infinitely thin</u>, *i.e.*, the thickness is ignored
- Direct extending to 3-D meets problems

[R. Gharpurey, *CICC'96*] [A.M. Niknejad, *TCAD'98*]

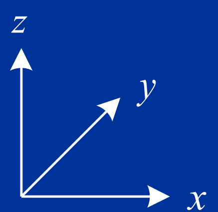
Metal thickness

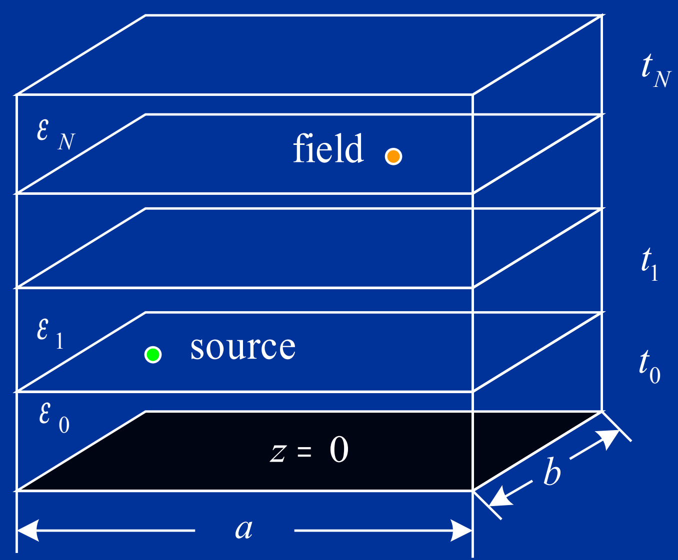
- Metal width and spacing are getting smaller and smaller, while the thickness is not reducing, sometimes even increases.
- Metal thickness should be considered.

Boundary Conditions and Green's Function

$$\nabla^2 G(r',r) = -\frac{\delta(r-r')}{\dot{\varepsilon}}$$

$$\dot{\varepsilon} = \varepsilon + \frac{\sigma}{j\omega}$$ (complex permittivity)





$\varphi = 0$ at $z = 0$

$\frac{\partial \varphi}{\partial n} = 0$ on other surfaces

$$G = X(x',x)Y(y',y)Z(z',z)$$

$$X(x',x) = \cos\left(\frac{m\pi}{a}x'\right)\cos\left(\frac{m\pi}{a}x\right)$$

$$Y(x',x) = \cos\left(\frac{n\pi}{b}y'\right)\cos\left(\frac{n\pi}{b}y\right)$$

$Z(z',z)$ subjects to:

$$Z_k^l(z') = Z_k^u(z')$$

$$\frac{dZ_k}{dz}\Big|_{z'-\delta_m}^{z'+\delta_m} = -\frac{C_{mn}}{ab\epsilon_k}$$

$$Z_k|_{z=z_k} = Z_{k-1}|_{z=z_k}$$

$$\epsilon_k \frac{dZ_k}{dz}\Big|_{z=z_k} = \epsilon_{k-1}\frac{dZ_{k-1}}{dz}\Big|_{z=z_k}$$

$$Z_0|_{z=0} = 0$$

$$\frac{dZ_N}{dz}\Big|_{z=z_{N+1}} = 0$$

[R. Gharpurey, *CICC'96*]

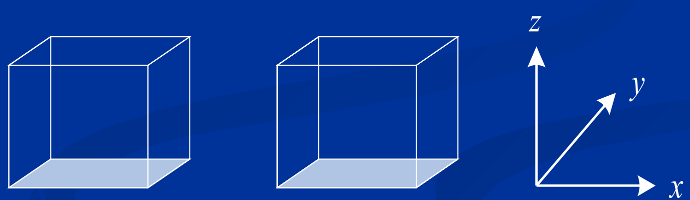
Integration for the Coefficient of Potential

- Green's function

$$G = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \cos(\delta_m x') \cos(\xi_n y') \cos(\delta_m x) \cos(\xi_n y) Z_{mn}(z', z)$$

$$p_{i,j} = \frac{1}{S_j S_i} \int_{S_i} \int_{S_j} G(x, x', y, y', z, z') \, ds_i \, ds_j$$

- For panels both on *x-y* plane



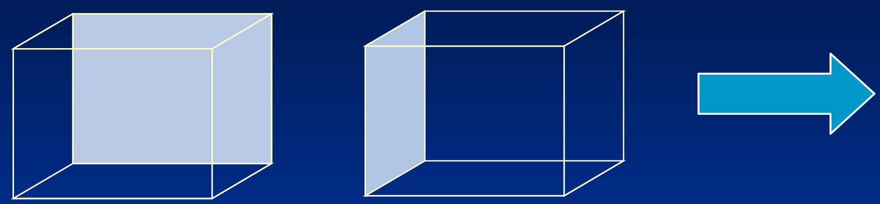
$$\int \cos(x)dx \Rightarrow \sin(x) \qquad \sin(x_1)\sin(x_2) \Rightarrow \cos(x_1 \pm x_2)$$

$$p_{ij} \Rightarrow \sum_{m,n} \underbrace{f_{mn}(z', z) \cos[\delta_m (x_{1,2} \pm x_{3,4})] \cos[\xi_n (y_{1,2} \pm y_{3,4})]}_{\text{64 terms}}$$

$\Rightarrow$ DCT (Discrete Cosine Transform)

[R. Gharpurey, *CICC'96*]

For Sidewall Panels…



$$Z_{mn}(z',z)\mathrm{d}z$$

- Gharpurey'96 (original one)

$$Z_{mn}(z',z) = \frac{f_{mn}(z')}{\beta_s^u \Gamma_s^l - \beta_s^l \Gamma_s^u} \left[\beta_f^{u,l} \underline{\cosh(\gamma_{mn} z)} + \Gamma_f^{u,l} \underline{\sinh(\gamma_{mn} z)} \right]$$

numerically instable when γ_{mn} grows

- Niknejad'98 (resolved the numerical stability)

$$Z_{mn}(z',z) = \frac{4F_k^l\left(z_f, z_s\right)}{ab\varepsilon_k \gamma_{mn}} \cdots \frac{\cosh\left(\vartheta_f - \vartheta_j\right)\cosh\left(\vartheta_f - \vartheta_{k+1}\right)}{\cosh\left(\vartheta_f - \vartheta_k\right)}$$

stable but cannot be integrated analytically

[A.M. Niknejad, *TCAD'98*]

Numerically Stable and Analytically Integrable Formula

replace the hyperbolic functions to exponential ones:

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}, \quad \sinh(x) = \frac{e^{x} - e^{-x}}{2},$$

$$Z_{mn}(z',z) = \frac{C_{mn}}{ab\varepsilon_{s}\gamma_{mn}} \times \frac{\left(\alpha_{s}^{u,l} e^{\gamma_{mn} z'} + \beta_{s}^{u,l} e^{-\gamma_{mn} z'}\right)\left(\alpha_{f}^{u,l} e^{\gamma_{mn} z} + \beta_{f}^{u,l} e^{-\gamma_{mn} z}\right)}{\alpha_{s}^{u}\beta_{s}^{l} - \alpha_{s}^{l}\beta_{s}^{u}}$$

recursive procedure for the coefficients

$$\begin{bmatrix} \alpha_{k}^{l} \\ \beta_{k}^{l} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} (1+p_{k})e^{\gamma_{mn} t_{k-1}} & (1-p_{k})e^{-\gamma_{mn} t_{k-1}} \\ (1-p_{k})e^{\gamma_{mn} t_{k-1}} & (1+p_{k})e^{\gamma_{mn} t_{k-1}} \end{bmatrix}\begin{bmatrix} \alpha_{k-1}^{l} \\ \beta_{k-1}^{l} \end{bmatrix}$$

Integration

$$\int Z_{mn}(z',z)\mathrm{d}z = \frac{X}{\gamma_{mn}} \times \left(\alpha_{f}^{u,l} e^{\gamma_{mn} z} - \beta_{f}^{u,l} e^{-\gamma_{mn} z}\right)$$

Stability-Preserving Technique

- Instability:

  $e^{\gamma_{mn} t_{k-1}}$ overflows when γ_{mn} becomes large

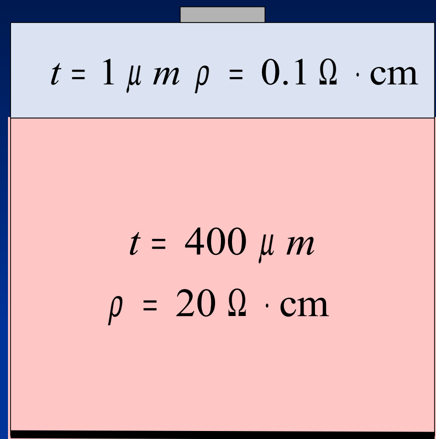
- Solution:
  - Define super complex number (hi_cplx)

    $$\text{hi\_cplx} \quad z = ae^{x} \qquad a \in \mathbf{C}, \quad x \in \mathbf{R}$$

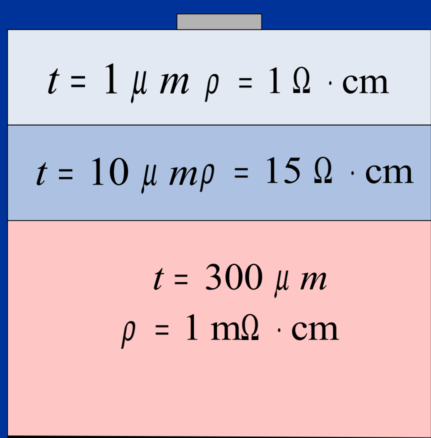
  - Re-define (overload) the operators (+,-,*,/) to avoid the overflow.

```
hi_cplx::hi_cplx(CPLX a_, double x_)
{
    a = a_;
    x = x_;
    check();
}

void hi_cplx::check()
{
    double s = abs(a);
    if (s > 100.0 || s < 1e-2)
    {
        x += log(s);
        a /= s;
    }
}

inline hi_cplx hi_cplx::operator*(hi_cplx b)
{
    hi_cplx t = *this;
    t.a *= b.a;
    t.x += b.x;
    t.check();
    return t;
}

inline hi_cplx hi_cplx::operator/(hi_cplx b)
{
    hi_cplx t = *this;
    t.a /= b.a;
    t.x -= b.x;
    t.check();
    return t;
}

inline hi_cplx hi_cplx::operator+(hi_cplx b)
{
```

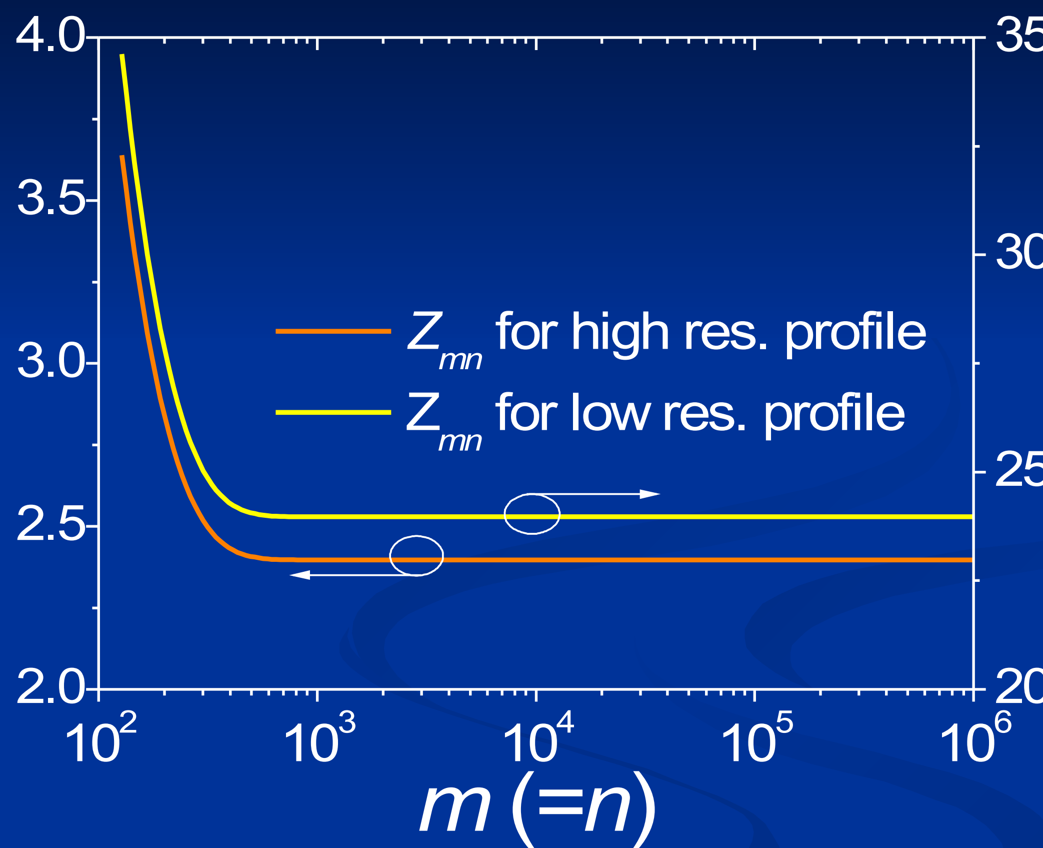
Numerical Stability



High Resistivity



Low Resistivity



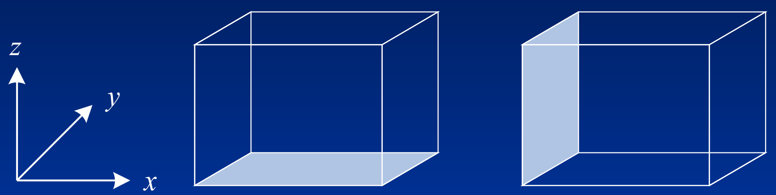
Stable even when m (=n) goes to extremely large (up to 1 million)

[J.P. Costa, TCAD'99]

Coefficient-of-Potential Involving Sidewalls: *xy-yz*

$$p_{i,j} = \frac{1}{S_j S_i} \int_{S_i}\int_{S_j} G(x, x', y, y', z, z')\, ds_i\, ds_j$$

$$ds_i = dxdy \qquad ds_j = dydz$$



source: *x-y* field: *y-z*

$$G = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \cos(\delta_m x')\cos(\delta_m x)\cos(\xi_n y')\cos(\xi_n y) Z_{mn}(z', z)$$

⇒ sin cos sin sin $Z^{(1)}_{mn}(z', z) \left(= \int Z_{mn} dz \right)$

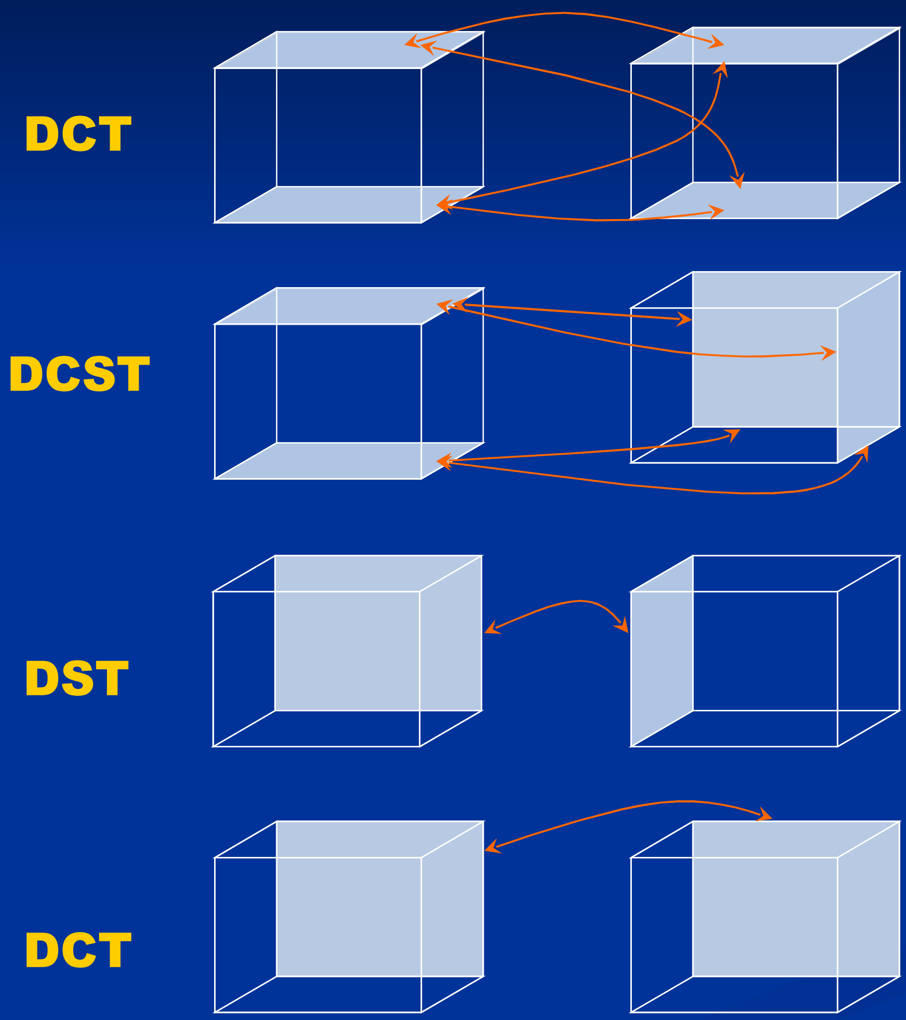
⇒ sin cos

$$p_{ij} \approx \sum_{m,n} f^{(1)}_{mn}(z', z) \sin\left[\delta_m (x_{1,2} - x_3)\right] \cos\left[\xi_n (y_{1,2} - y_{3,4})\right]$$

32 terms

⇒ DCST (Discrete Cosine-Sine Transform)

All Positional Relations

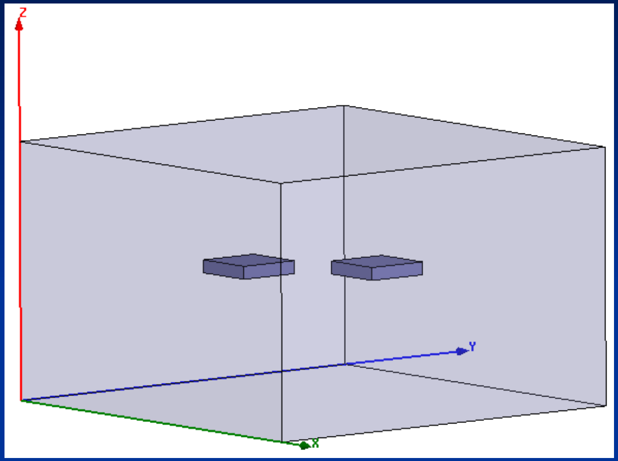


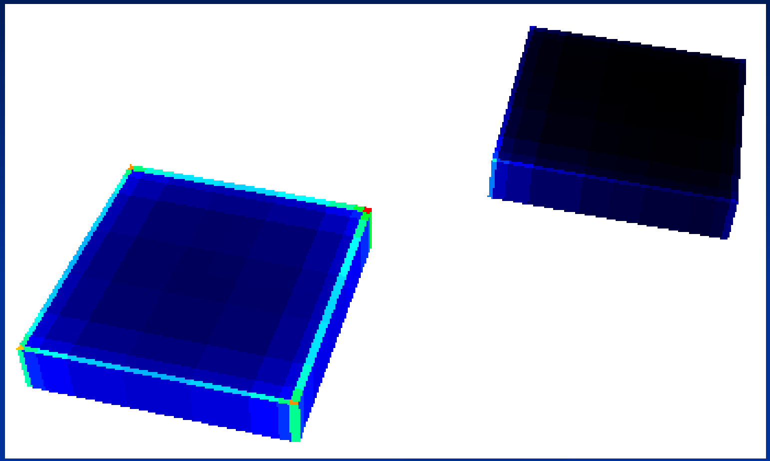
- Consider totally 10 situations classified to 4 types
- For each situation, a Green's function should be computed and stored
- It required about 10X of that cost by ASITIC
- Can be reduced by applying non-uniform grid

Non-uniform Grid Method for Computing Green's Function

- FFT ⟺ uniform grid
  - Computational amount $O(N^2 \log(N^2))$
  - Memory required: $O(N^2)$ (N: FFT size, ~2000, increases with shrinking metal width)
- Non-uniform grid method have been developed (but not presented in the proceedings)
  - Computational amount $O(M^2 N_c^2)$
  - Memory required: $O(N_c^2)$
  - *Nc:* non-uniform grid size, <40; *M*: ~64 (constant)

Test Case I: Two contacts

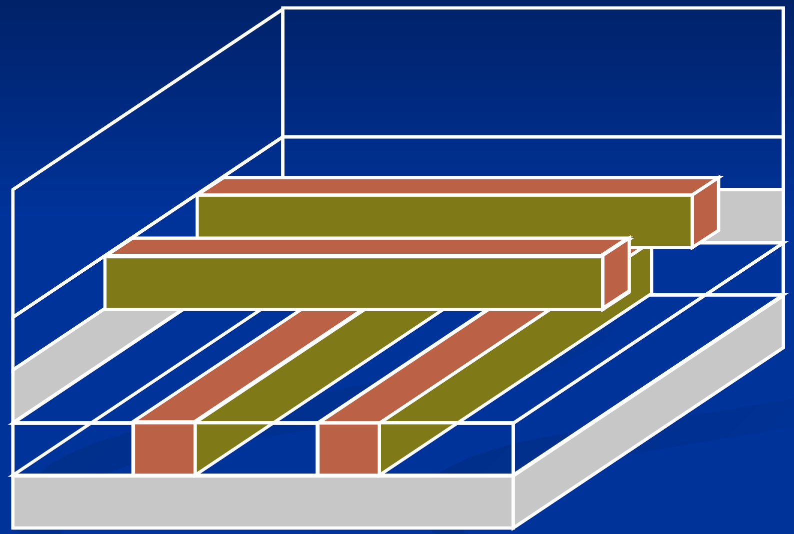




	C11 (F)	Error (%)	C12 (F)	Error (%)
Ansys	6.2520e-15	--	-1.5668e-15	--
ASITIC	5.0121e-15	-19.8	-9.7422e-16	-37.8
SCAPE	6.2573e-15	-0.08	-1.5752e-15	0.54

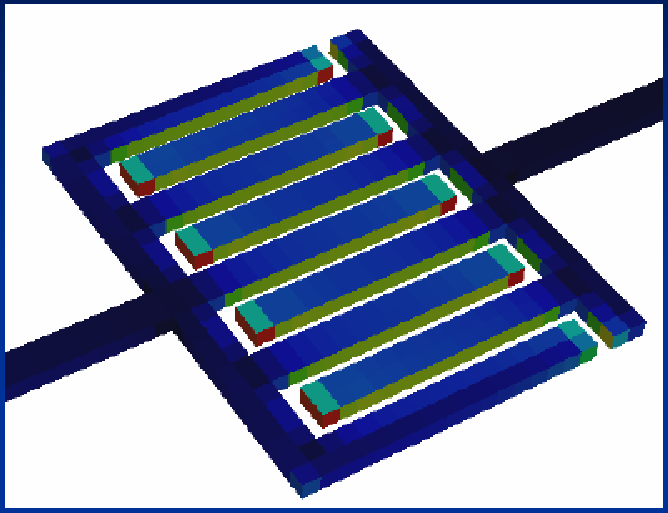
Test Case II: *k*-by-*k* buses

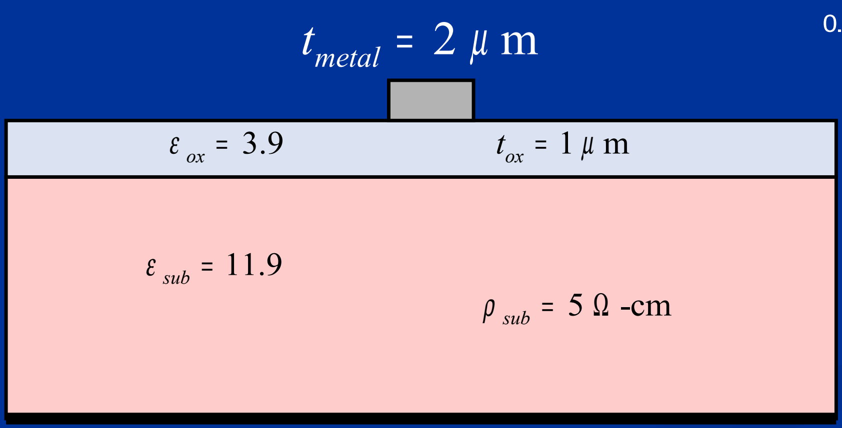
	Test problem			
	2×2	3×3	4×4	5×5
FastCap I (with fine mesh)				
CPU Time (s)	54	120	218	349
Memory (MB)	111	131	160	197
Panel #	7812	8724	9948	11492
FastCap II (with coarse mesh)				
CPU Time (s)	11.6	19.1	31.1	46.6
Memory (MB)	56	64	75	89
Panel #	3628	4080	4684	5448
Error (%)	2.89	2.84	2.85	2.76
ASITIC				
Panel #	50	84	144	220
Error (%)	23.7	24.8	25.8	25.4
SCAPE				
CPU time (s)	0.32	1.14	3.41	8.39
Memory (MB)	19	20	21	25
Panel #	146	276	464	700
Error (%)	0.99	0.91	1.60	2.38

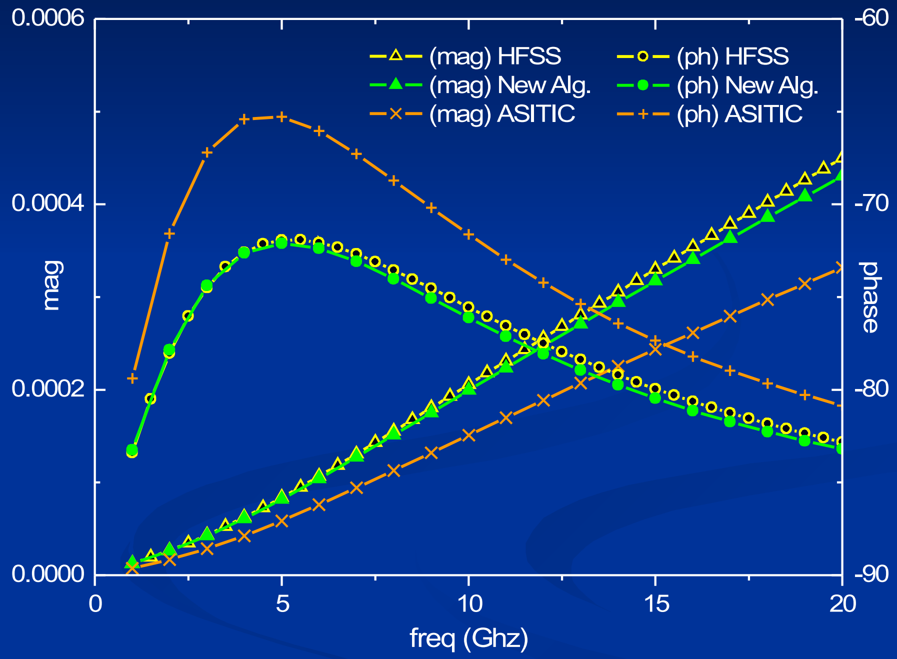


Green's function method does not need to discretize the interfaces of dielectrics

Test case III: Interdigital Capacitor (IDC) over Lossy Substrate







Frequency dependence of the complex capacitance of the IDC.

Conclusion

- 3-D capacitance extraction based on Green's function
- Stable and analytically integrable formula for the Green's function in *z*-direction
- Accurate result compared with well tested solvers (FastCap, Ansys, HFSS)
- Limitation: layered dielectric, Manhattan conductor

Thank you