Calculating Frequency-Dependent Inductance by Complete Multiple Reciprocity Method (CMRM)

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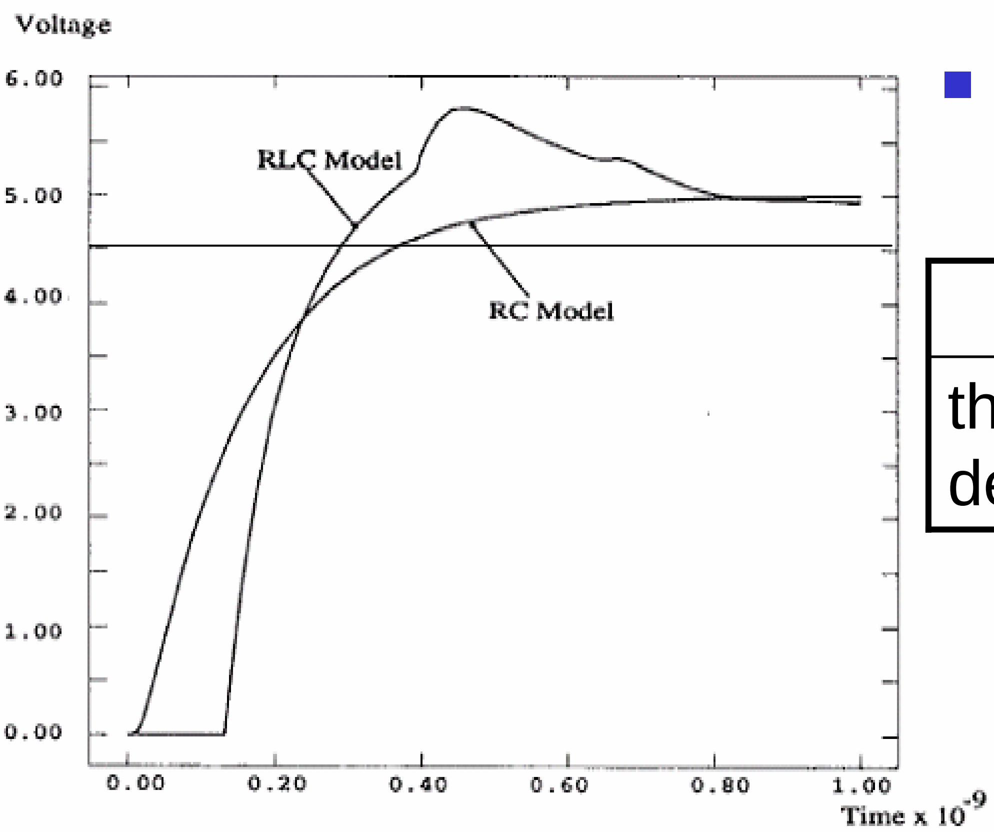
Outlines

- Introduction
- Features of CMRM
- How to apply CMRM
- Numerical results
- Conclusion

Intro 1: Inductance cann’t be neglected

- With increasing of operating frequency, **Inductance** places a more and more important role in circuit simulation and verification [Kahng TCAD’97].

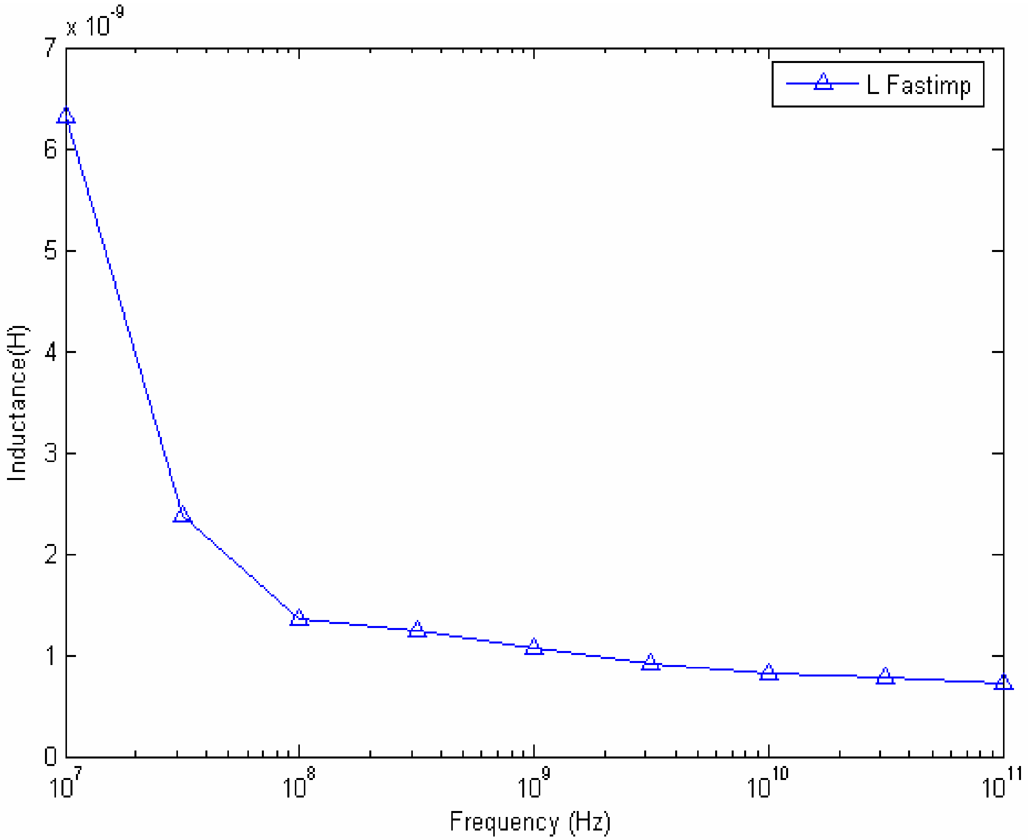


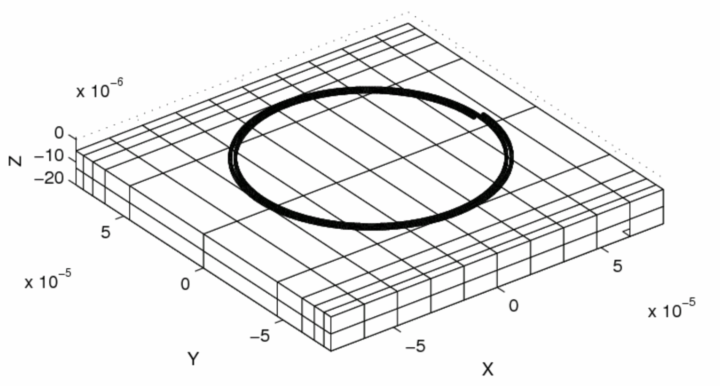
- An interconnect line is driven by a step input.

	RC	RLC	error
threshold delay (ps)	358	288	24%

Intro 2: Inductance is Freq.-Dependent

- However, Inductance is frequency-dependent, so accurate circuit simulation needs the inductance of multiple freq. points, while not only of a single freq. point.





Circular spiral inductor over a substrate ground

Intro 3: Is there a quick solution?

- Unfortunately by now, current main inductance extractors, for example: ***Fasthenry* and *Fastimp***, simply extract inductances point by point. They didn’t consider this question seriously

Question:

How to extract the inductance of **multiple frequency points Quickly**?

This is the main topic of this paper.

CMRM 1:Integral is the Time Killer

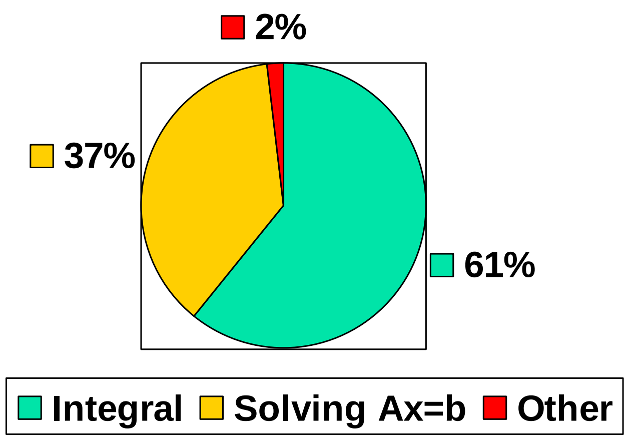
Before introducing the CMRM, it is useful to analysis the CPU time distribution:

- Fastimp, a direct Boundary Element Method (BEM), needs many CPU time on integrals.
- Even worse, the integrals are freq.-dependent, so they have to be recomputed (freq.) point by point.

$$k = -\sqrt{-i2\pi f \mu\sigma}$$

(Under MQS assumption)

CPU Time Distr. of a Spiral Inductor By Fastimp



$$\int \frac{e^{ikr}}{r} \approx ?$$

CMRM 2: Key advantage of CMRM

- Complete Multiple Reciprocity Method, as a kind of BEM, is improved from the Multiple Reciprocity Method (MRM).
- Key advantage is **transforming the freq.-dependent integral to a series of freq.-independent integral**. Compute and save the integrals at first, and we can quickly get the P(i,j) by partial summation. In other words, the integral can be reused easily.

$$P(i,j)=\int\frac{e^{ikr}}{4\pi r}ds \longrightarrow \sum_{j=1}^{N}(-1)^{j}k^{2j}\cdot\int\frac{r^{2j}}{4\pi r(2j)!}$$

CMRM transformation

k (freq.-dependent)

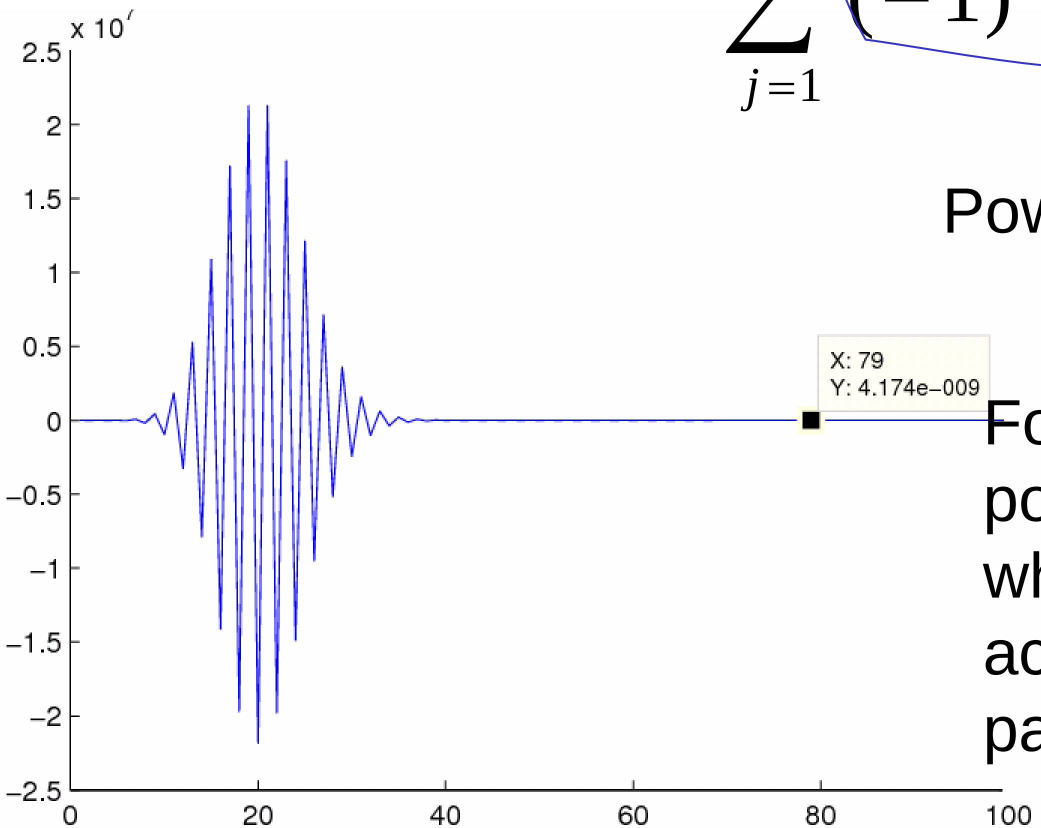
freq.-independent integral

CMRM 3: Fatal drawback of CMRM

- The CMRM formulation is power series. For large *k* or *r*, they cannot be calculated accurately. The cause of this numerical difficulty is termed **cancellation**.

$$\sum_{j=1}^{N} (-1)^{j} k^{2j} \cdot \int \frac{r^{2j}}{4\pi r (2j)!}$$

Power series (**cancellation**)



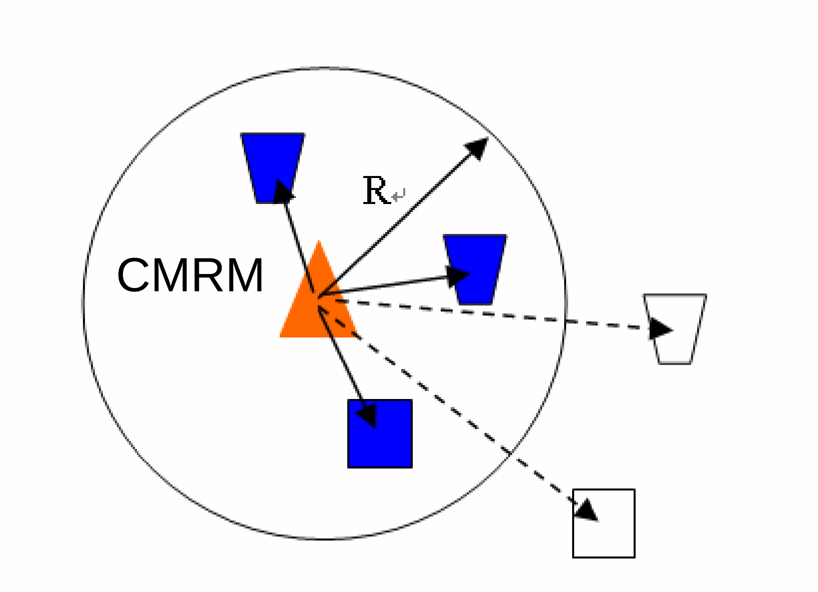
For example, partial sum of power series of exponent e^{-x}, where x=20:

accurate result : 2.0e-9,

partial sum (N=79): 4.17e-9.

How to 1: Window to Divide Far and Near Field

- In order to overcome the numerical difficulty of large *k* or *r*, a window is introduced to divide the far and near field.
  - In window (near field): apply CMRM formulae;
  - Outside window (far field): apply far field formulae.



Far field approx. formula:

$$\int \frac{e^{ikr}}{4\pi r} \approx e^{ikr_{avg}} \int \frac{1}{4\pi r}$$

How to 2: Normalization of Distance

- In order to overcome the numerical difficulty of large *r* only, we normalize the distance *r*, after introducing the **average distance** $r_{avg.}$ For example:

$$\sum_{j=1}^{N}(-1)^{j}k^{2j}\int\frac{r^{2j}}{4\pi(2j)!}=\sum_{j=1}^{N}(-1)^{j}(kr_{avg})^{2j}\int\frac{(r/r_{avg})^{2j}}{4\pi(2j)!}$$

Notes:

- Overcome the numerical difficulty of large *r* only;
- Combine the variety of *k* and *r* together;
- The criterion of window can be defined as *abs*(kr_{avg})<R(2pi).

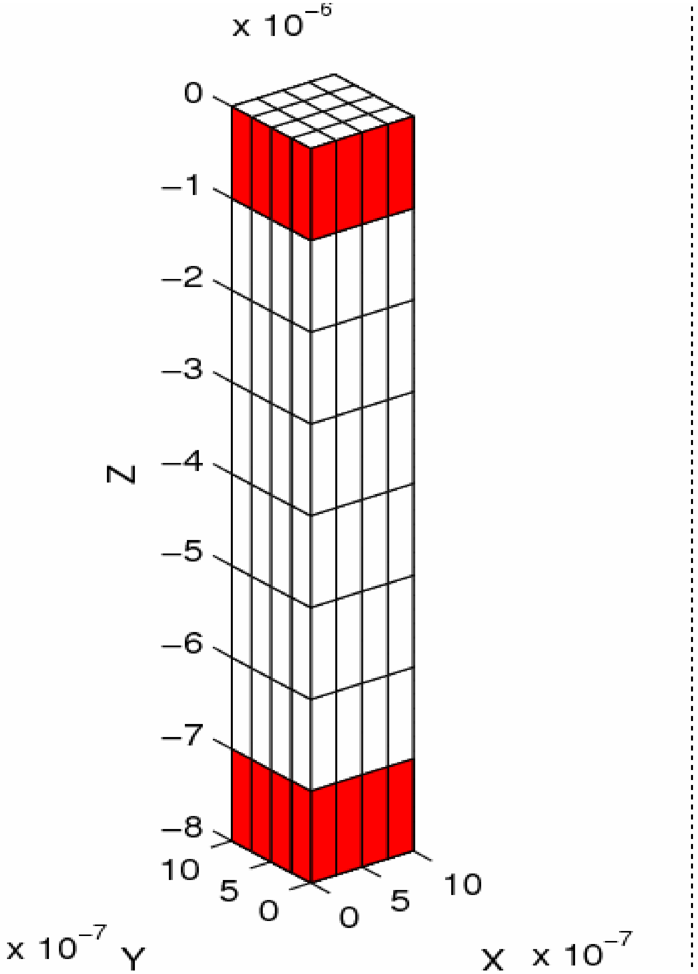
How to 3: Other problems and solutions

Applying the CMRM includes other problems:

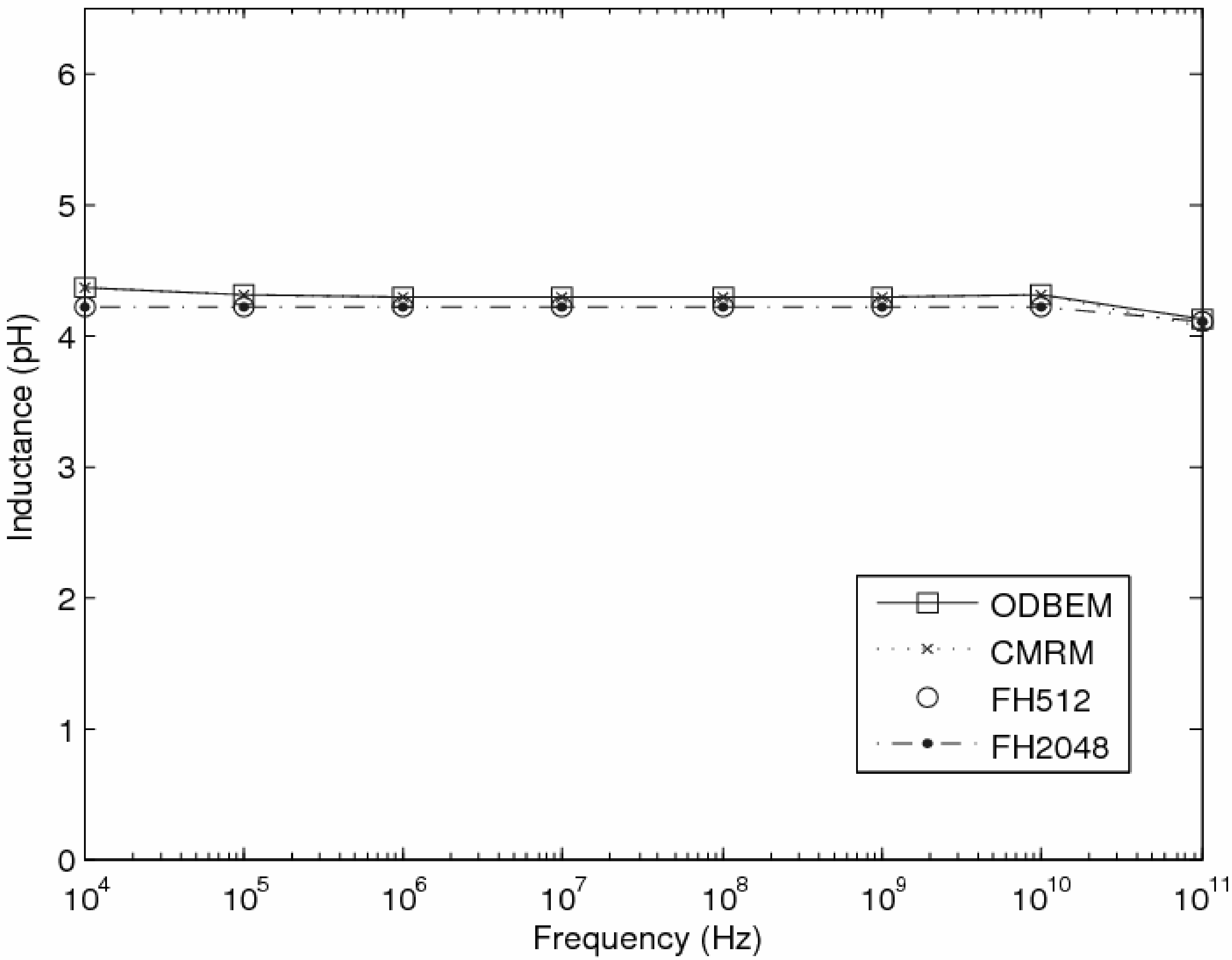
- Quick computation of integrals in series
- Criterion of window size, how many terms of series, and relationship between them.

(please refer to the paper for more details)

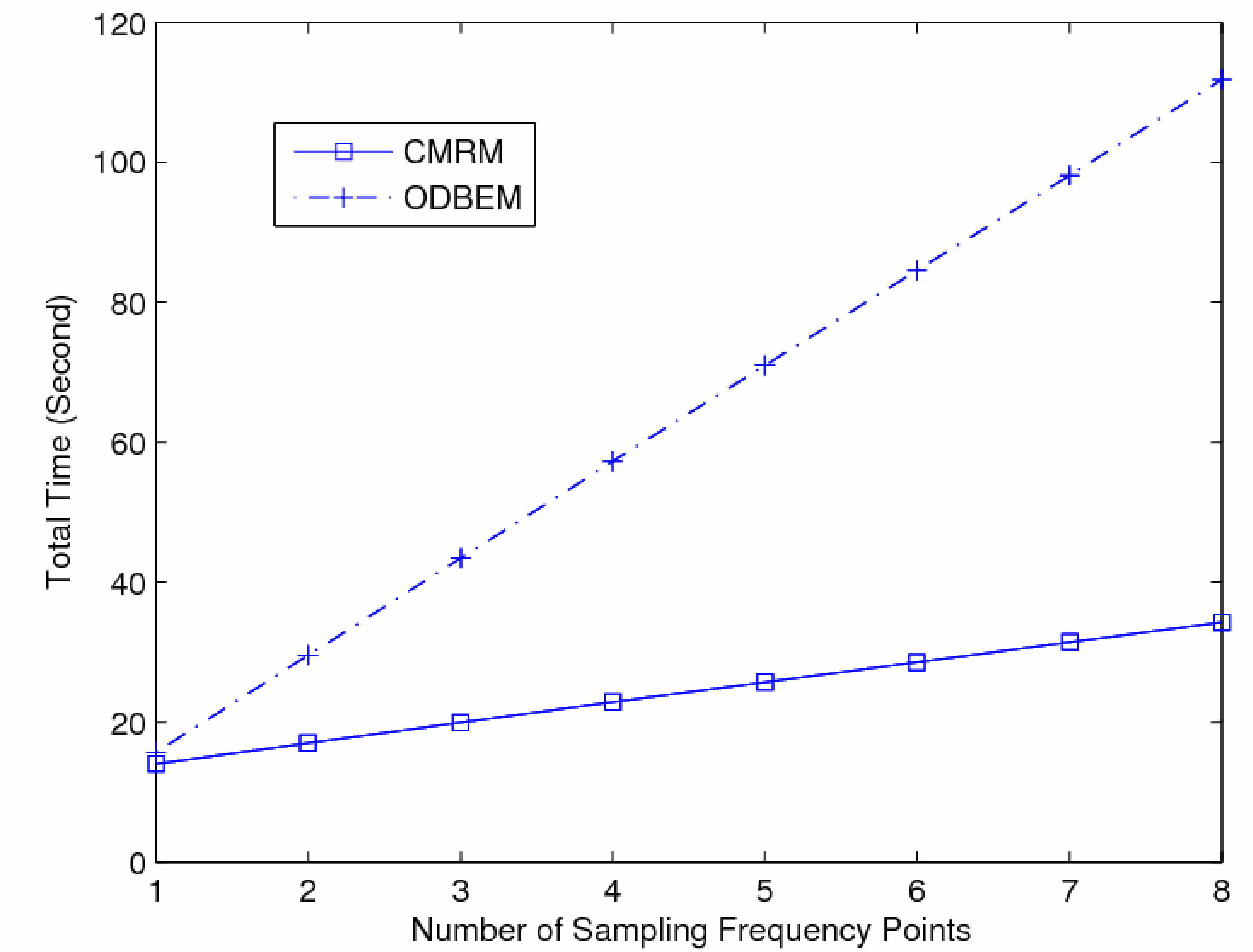
Result 1: Single wire example



Result 1: Accuracy



Result 1: CPU Time Comparison

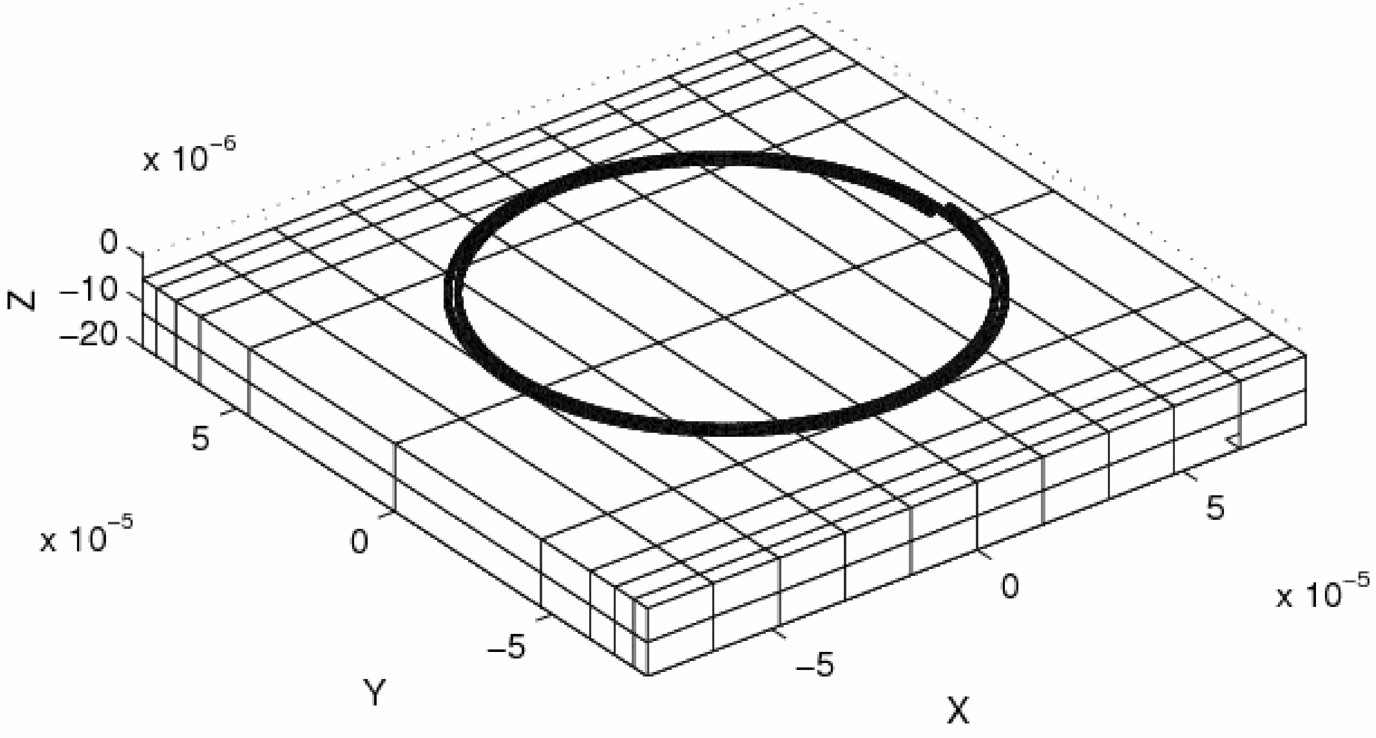


Result 2: An Improvement of CMRM

An improvement: We have combined CMRM with pFFT.

- pFFT (pre-corrected FFT) is a powerful fast method. It has been used by Fastimp. It can decrease the CPU time and memory usage at the same time.
- pFFT is a far field quick computation method, however, the CMRM is a near field quick method. Therefore, combining them together is suitable.
- This work has been done in later months of last year, after this paper is accepted.

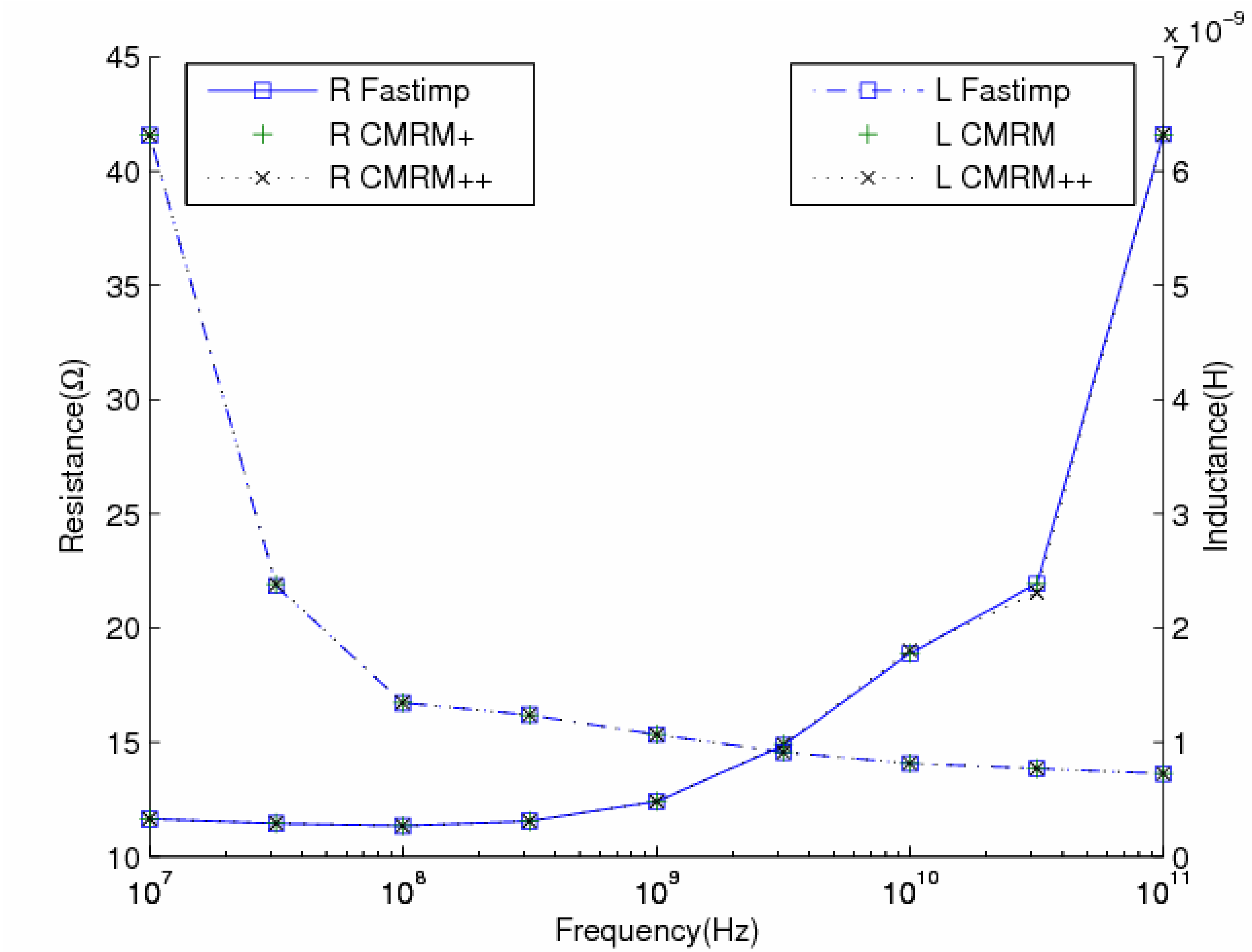
Result 2: CMRM+pFFT



Spiral inductor over substrate ground

(MQS assumption)

Result 2: CMRM+pFFT



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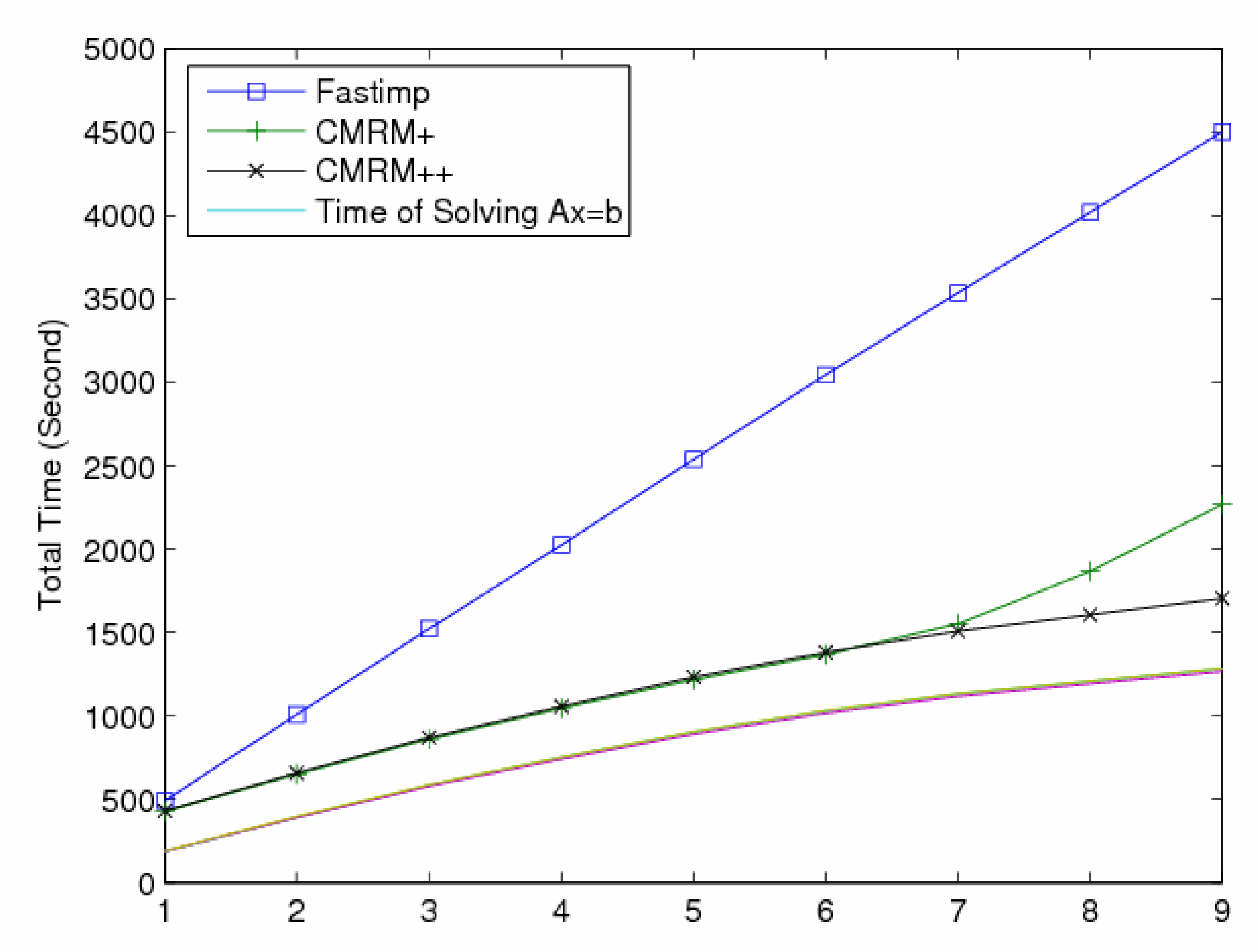

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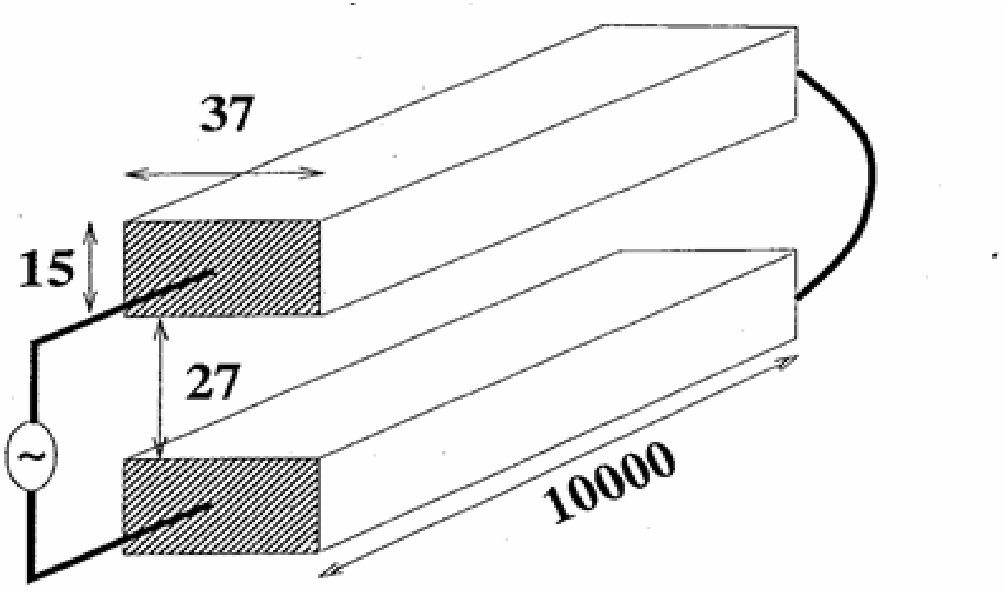
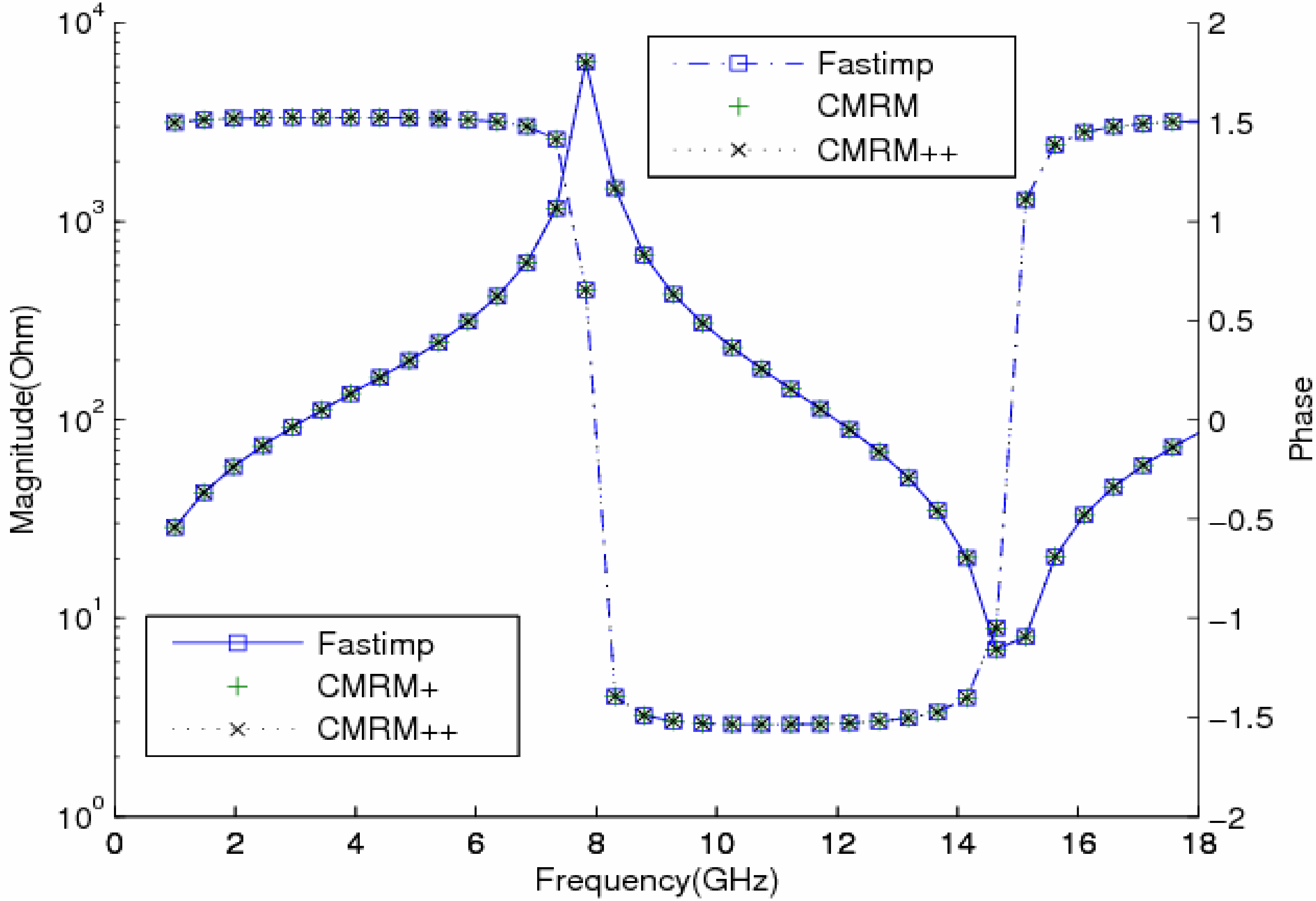


FIGURE 7-18: A long shorted transmission line

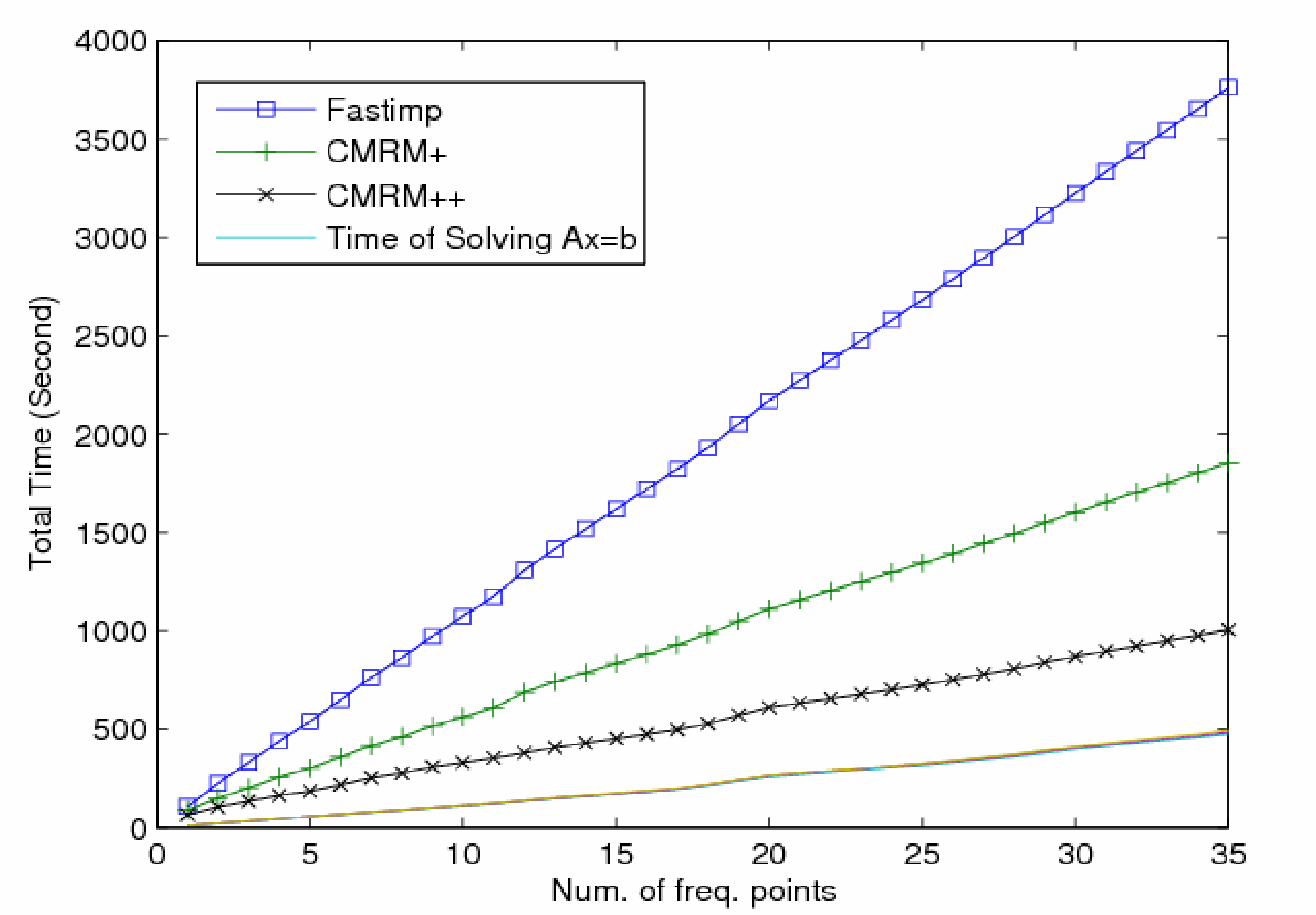
Long shorted transmission line

(Fullwave analysis)

Result 2: CMRM+pFFT



Result 2: CMRM+pFFT



Conclusion

- Inductance of multiple freq. points is important, and it lacks consideration by now.
- CMRM can transform the integrals from the freq.-dependent one into the freq.-independent one. So it can reuse the integrals easily. However because of the power series, it also introduce the numerical difficulty.
- A set of methods, including window, far field formulation and distance normalization and so on, is proposed for applying the CMRM in practice.
- CMRM can be combined with pFFT successfully. This improvement has gained speed advantage over the leading extractor, Fastimp on multiple freq. points.

Thank you !

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