An Exact Algorithm for the Statistical Shortest Path Problem

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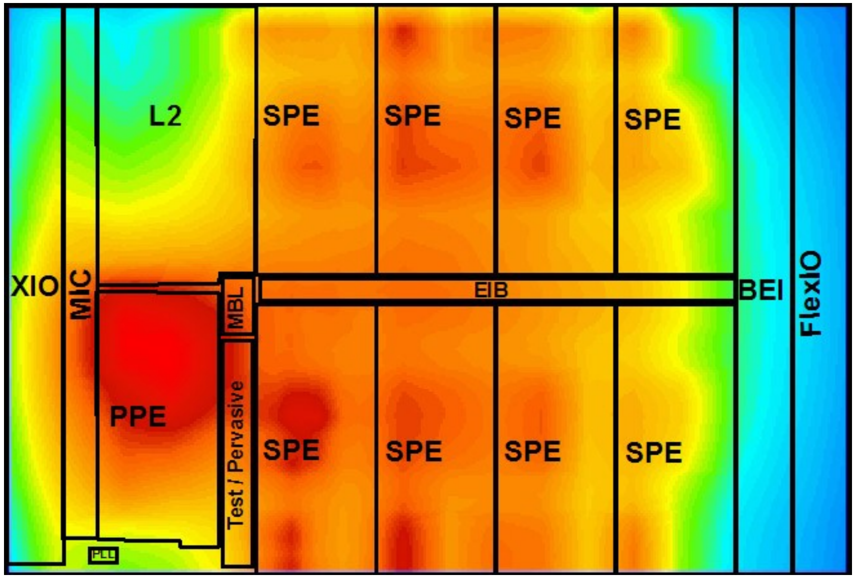
Outline

- Motivation
- Statistical shortest path (SSP) problem
- Our exact algorithm for SSP problem
- Applications
  - Maze Routing
  - Timing Analysis
  - Buffer Insertion

Why Statistical Methods?

- Intra-die variations become dominant
- Corner-based design flow leads to over design or yield loss
- Statistical methods are needed not only in simulation but also in design tools.



Temperature Variation in Cell Processor

Dac C. Pham, et al. ISSCC05

Variations, Performance and Yield

- Variation sources
  - Process variations
    - Gate length variation
    - Geometric variation in interconnection wires
  - Temperature variations
  - Supply voltage variations
- Statistical models for circuits have been proposed
- New algorithm considering variations are needed for performance/yield optimization

Statistical Model for Variations

- Use mean μ and variance σ^2 to capture the random property of variations
- Exact for Gaussian, uniform, binominal, exponential distributions and etc.
- Good approximation for arbitrary random variables

Statistical Model for Variations

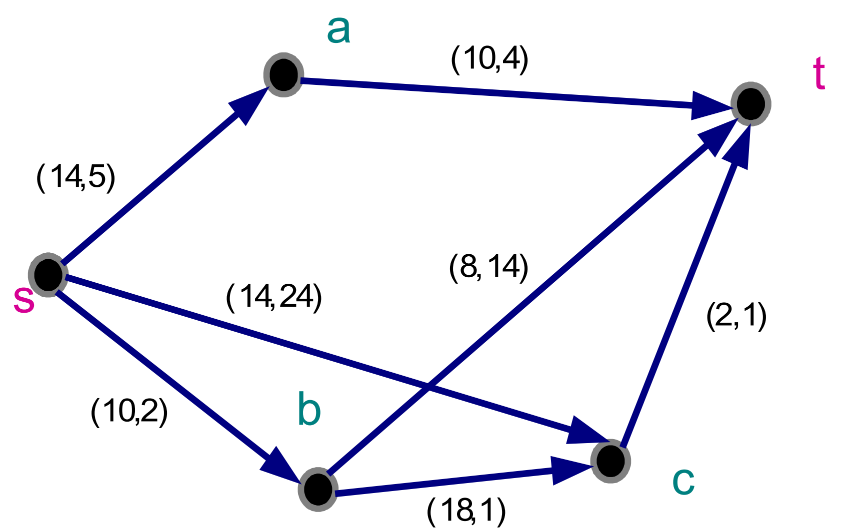
- Mean and variance are additive, but not the standard deviation σ
- Recall the Chebyshev's Inequality:

$$P(|X-\mu| \leq k\sigma) > 1-\frac{1}{k^2}$$

- The cost function $\mu+k\sigma$ is important to yield optimization
- σ not additive presents difficulties in solving statistical graph problems

Statistical Shortest Path Problem

- Edge weights are random variables
- To find a path with minimum $\mu + \Phi(\sigma^2)$ value
- Existing methods cannot solve this problem



Edge weight : (mean, variance)

From Deterministic to Statistical

Deterministic	Statistical
Edge weight w	Edge weight (μ, σ^2)
w is additive	μ, σ^2 are additive
Path weight Σw	Path weight (μ_P, σ_P^2)
Minimize Σw	Minimize $\mu_P + \Phi(\sigma_P^2)$

Statistical Shortest Path Problem

- Given a directed graph G
  - Not necessarily a DAG
- Find a path p from source vertex s to sink vertex t such that
  - $\mu_P + \Phi(\sigma_P^2)$ is minimized
  - Path weight of p is a random variable with mean μ_P and variance σ_P^2

Practical Observations for EDA problems

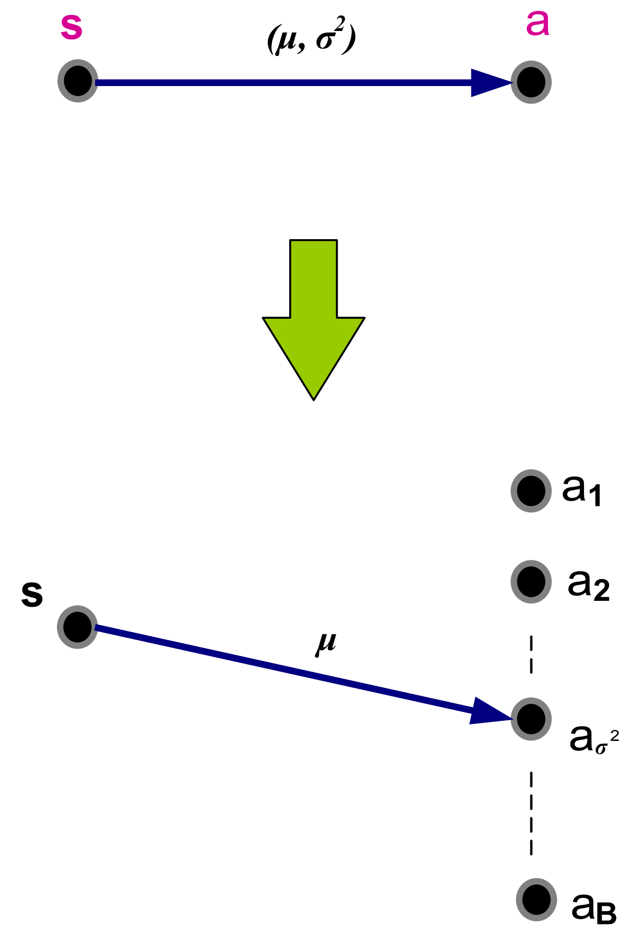
- μ, σ^2 are additive
- For yield optimization problems
  - σ^2 is bounded
  - σ^2 can be discretized without introducing much error
- We may assume the variance σ^2 of path weight are integers upper bounded by *B*, i.e., $\sigma^2 \leq B$

Algorithm for Solving SSP Problem

- Vertex splitting for μ, σ^2
- Graph expansion to generate a new graph *G'*
- *G'* has real numbers as its edge weights
- Each vertex *u* in G is split into a set of vertices in *G'*: $\{u_1, u_2, \ldots, u_B\}$

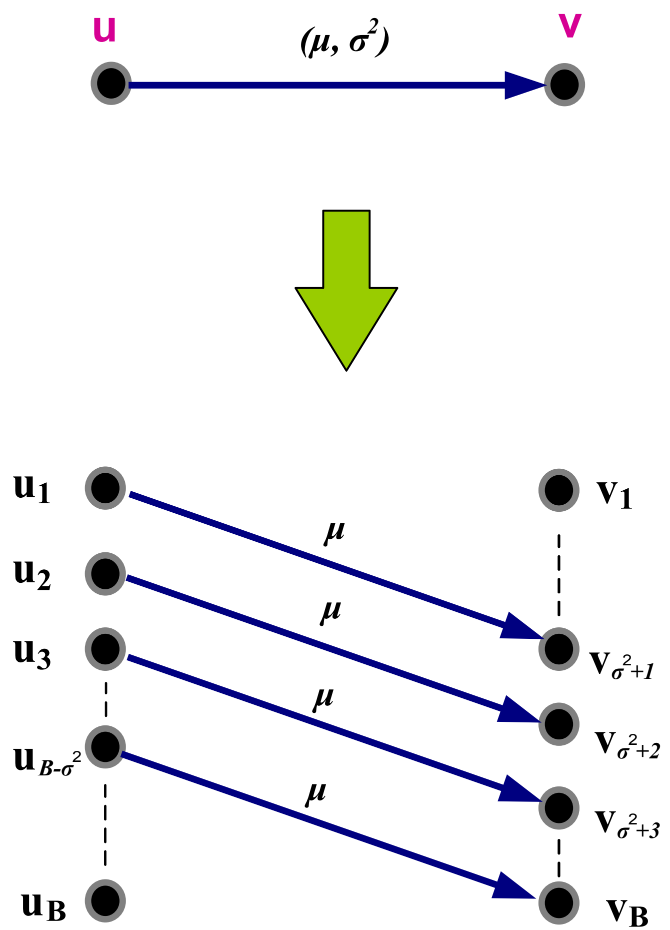
Graph Expansion – Source Node

- From source to other vertices
- Only expand vertex **a**
- Each new vertex $\mathbf{a_i}$ corresponds to **a** with variance *i*
- Edge weight is μ



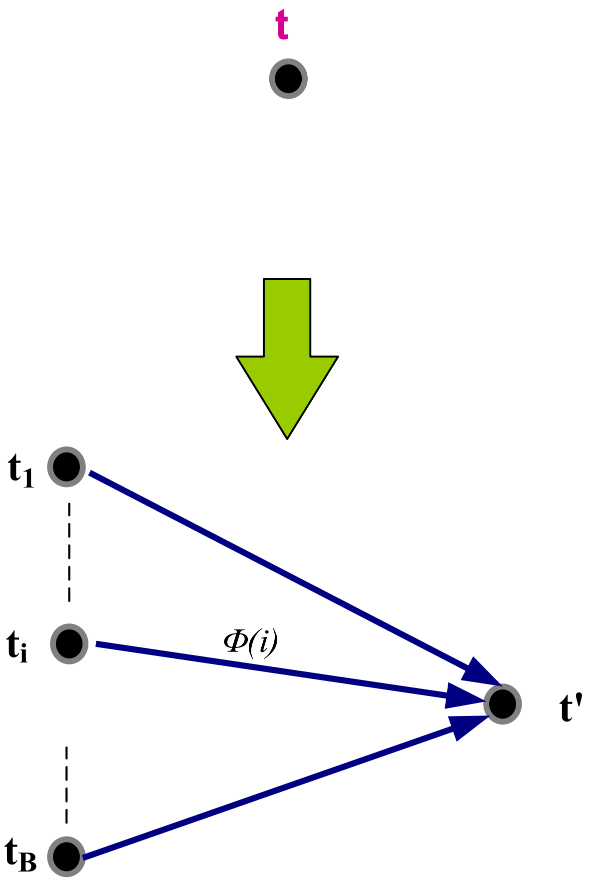
Graph Expansion – Internal Nodes

- Assuming vertex *u* is already split
- Its neighbor *v* will be also split
- Edges are connected according to σ^2 of path weight
- Edge weight are μ



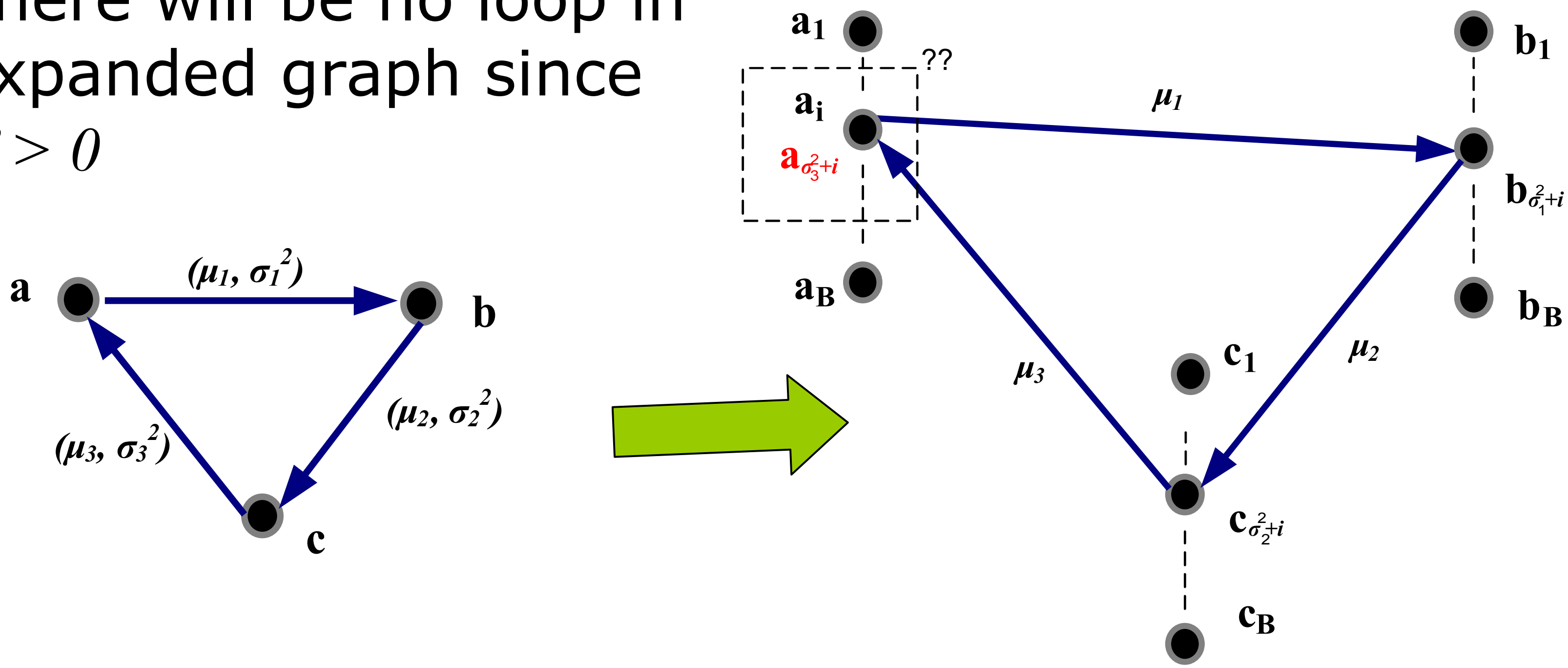
Graph Expansion – Sink Node

- Original sink node is already split according to previous steps
- Add a super sink node *t′*
- Edge weight for edge *ti* to *t′* is *Φ(i)*
- Note that any path from source to *ti* has variance equals to *i*



From Arbitrary Graph to DAG

- There will be no loop in expanded graph since $\sigma^2 > 0$



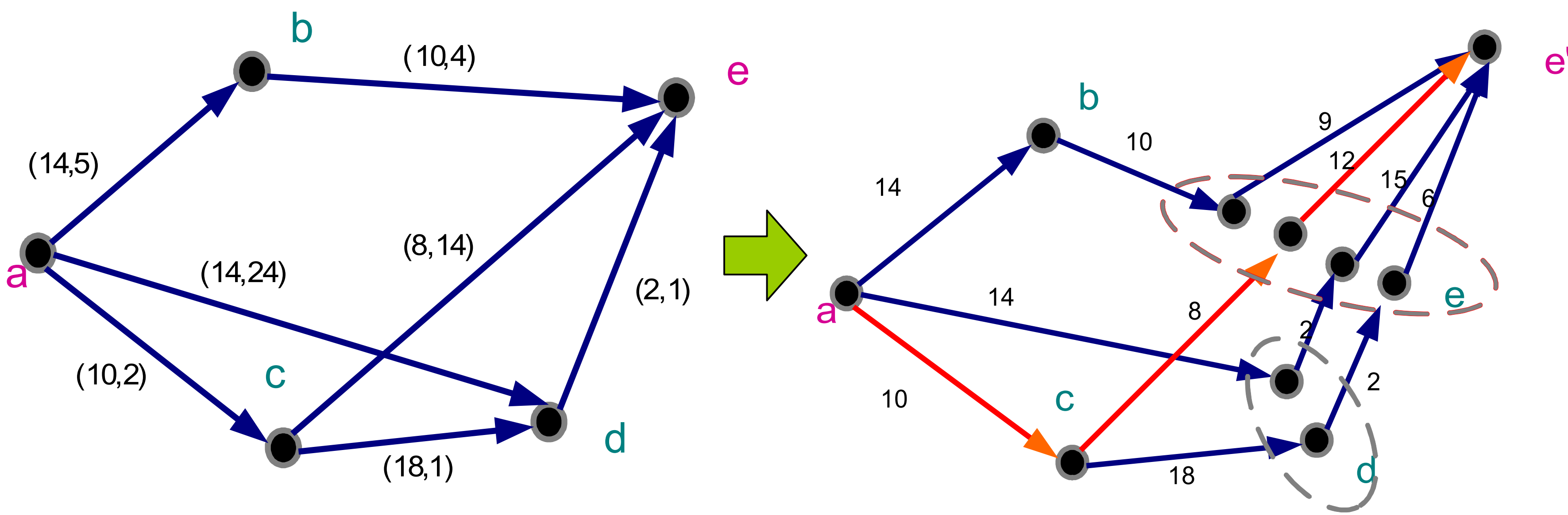
SSP Algorithm

- The expanded graph G' is a DAG
- Shortest path in G' can be found by existing deterministic shortest path algorithms for DAG
- This path corresponds to a path in G that minimizes $\mu_P+\Phi(\sigma_P^2)$
- Time complexity is $O(B(V+E))$

Improvement

- Only split a vertex whenever it is necessary; don't split all vertices
- Remove redundant vertices during splitting
  - If paths have same variance, then the one with larger mean is redundant
  - If $\Phi(\sigma_P^2)$ is a monotonically increasing function, paths with larger mean and variance are redundant

Example



Edge weight: (mean, variance)

$$\Phi(x) = 3\sqrt{x}$$

SSP Algorithm Improved

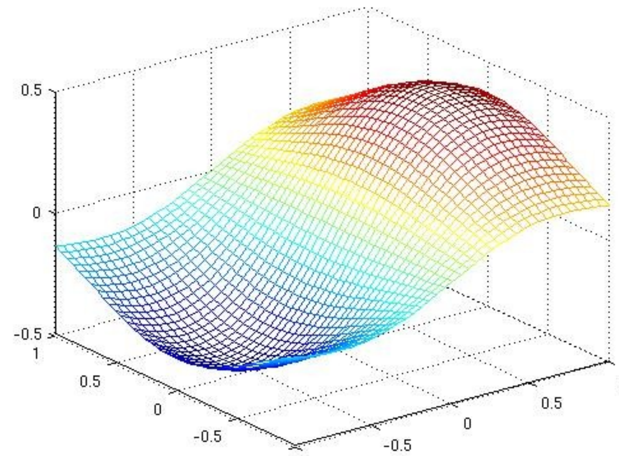
- Much less vertices are generated
  - 100 vertices needed for previous example with original approach
  - 10 vertices used with improved algorithm
- Expand graph simultaneously with searching the shortest path
- Much faster with less memory requirement

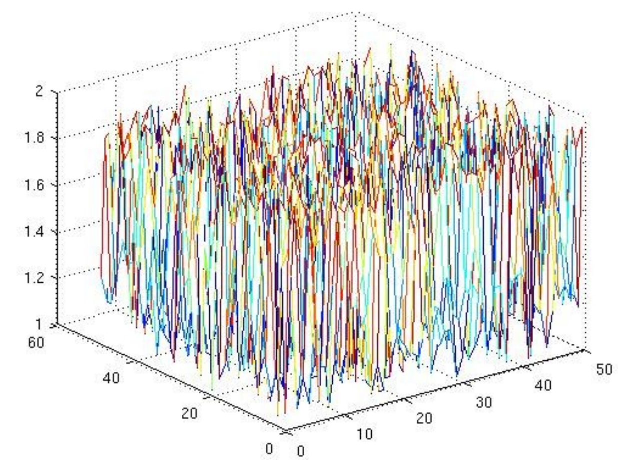
EDA applications

- Maze Routing
- Timing Analysis
- Buffer Insertion

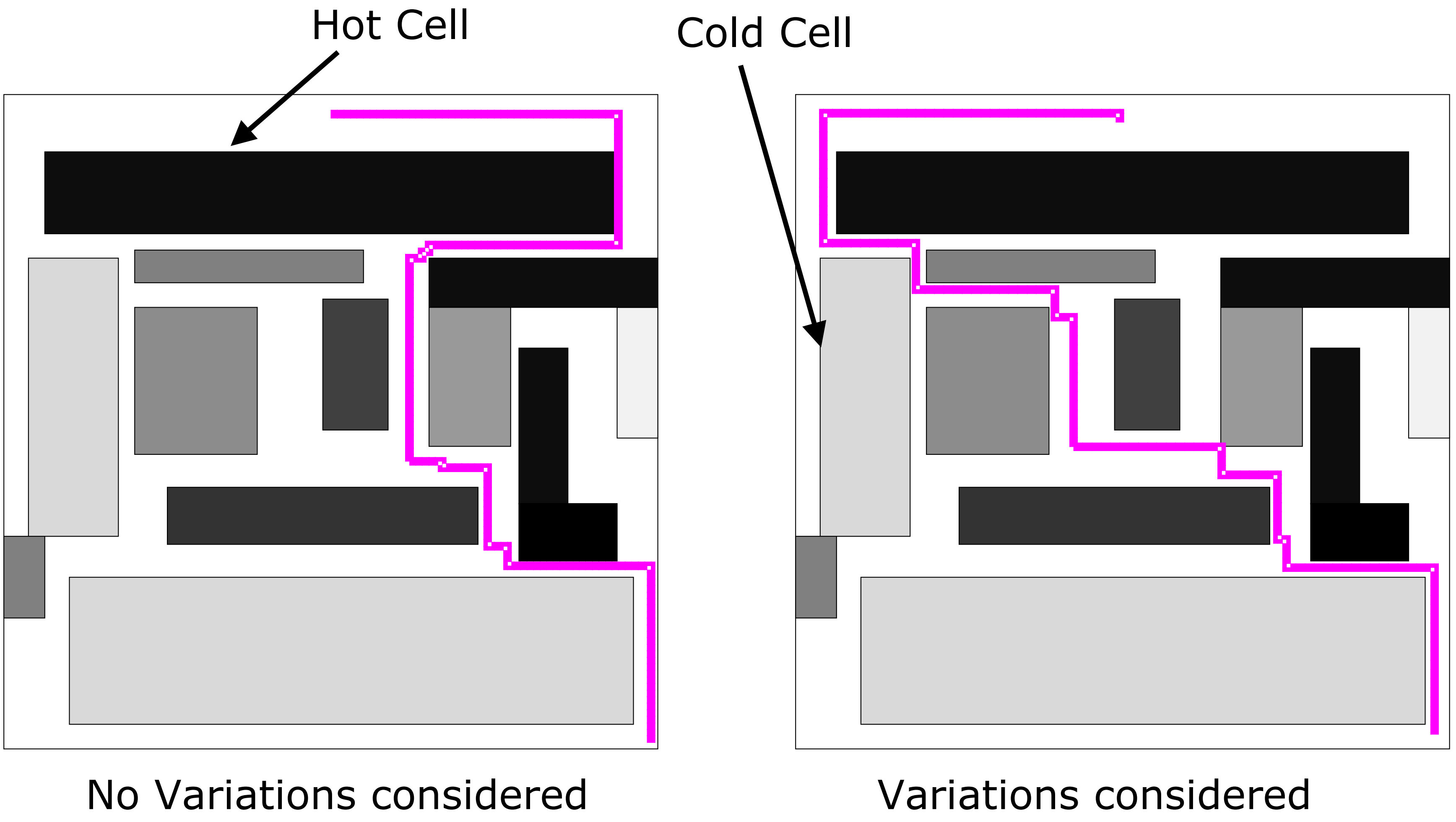
Maze Routing

- Timing-driven maze routing
- Process Variations
  - Systematic variations
  - Random variations
  - Temperature variations
- Find the shortest path to improve the performance





Maze Routing



Timing Analysis

- Find the longest delay path considering intra-die variations
- Large circuits with several logic levels
  - Gaussian distribution for the path delay
  - $\mu_P + 3\sigma_P$ is used to measure the timing-yield
- Our algorithm can also find the (path) candidates with longest delay

Timing Analysis

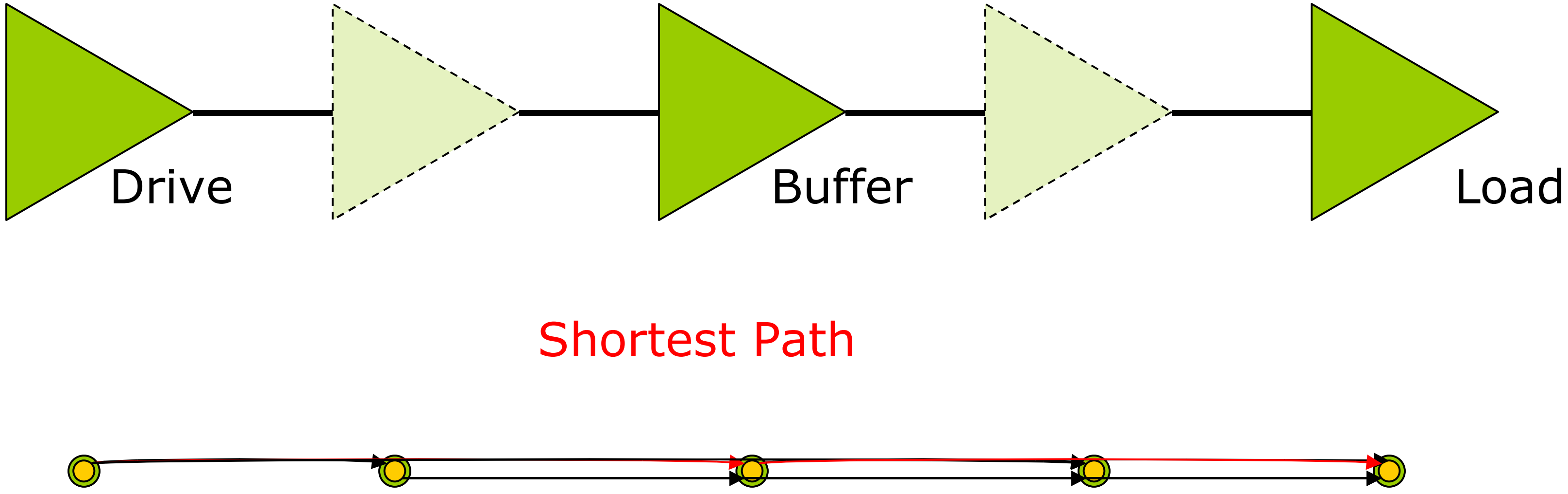
- ISCAS benchmarks
- Cell delays are not necessarily Gaussian
- 40X—1000X runtime improvement over Monte Carlo simulation
- Very little error compare to Monte Carlo method

Buffer Insertion

- Buffer insertion in 2-pin net can be formulated into shortest path problem
- With variations from both devices and interconnections, it should be formulated into statistical shortest path problem
- Our algorithm can solve this buffer insertion with variations consideration

Buffer Insertion

- Graph based approach
- Formulated as a shortest path problem



Conclusion

- Exact algorithm to solve the statistical shortest path problem
- Arbitrary graph, arbitrary cost function Φ
- Efficient implementation
- Can be used in varieties of applications in nanometer design