A New Boundary Element Method for Multiple-Frequency Parameter Extraction of Lossy Substrates

Xiren Wang, Wenjian Yu, and Zeyi Wang

EDA Lab, Dept. of Computer Science & Technology, Tsinghua University

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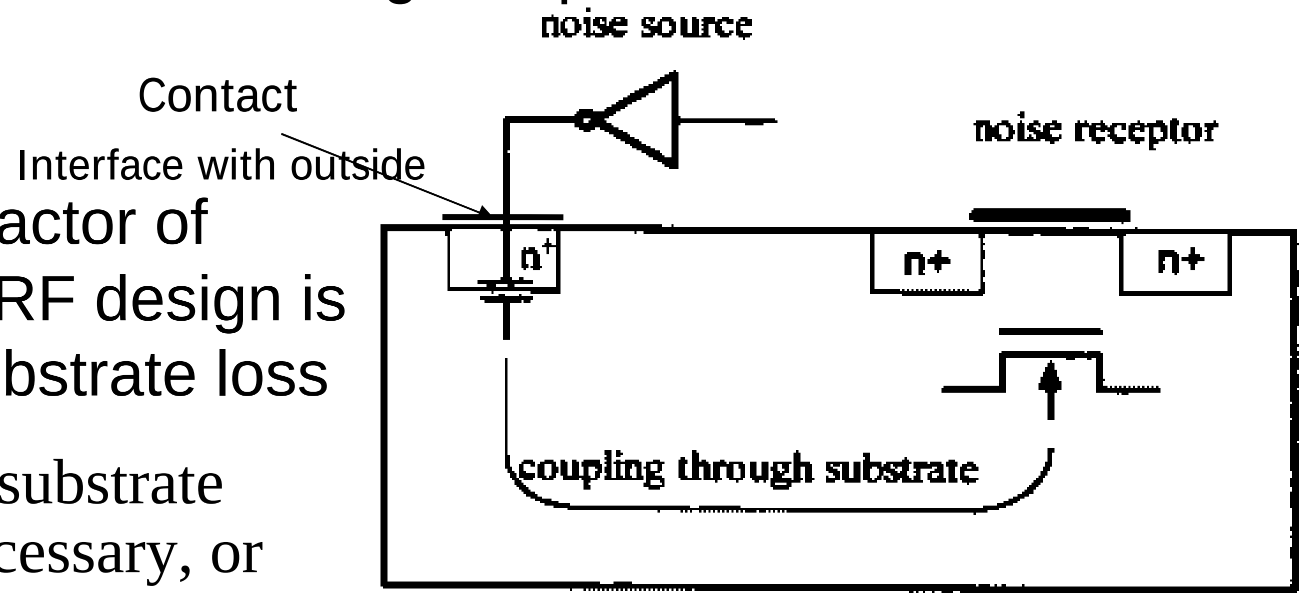
Outline

- Introduction of Substrate Coupling Problems
- High-Frequency Parameter Extraction Using Direct Boundary Element Method (DBEM)
- Efficient Techniques for Multi-Frequency Extraction
- Numerical Results
- Conclusions

Introduction of Substrate Coupling Problems

- In mixed-signal circuits
  - Digital and analog components are often built on a single lossy substrate (Si)
  - Coupling noises traveling through the substrate severely impact the sensitive analog components

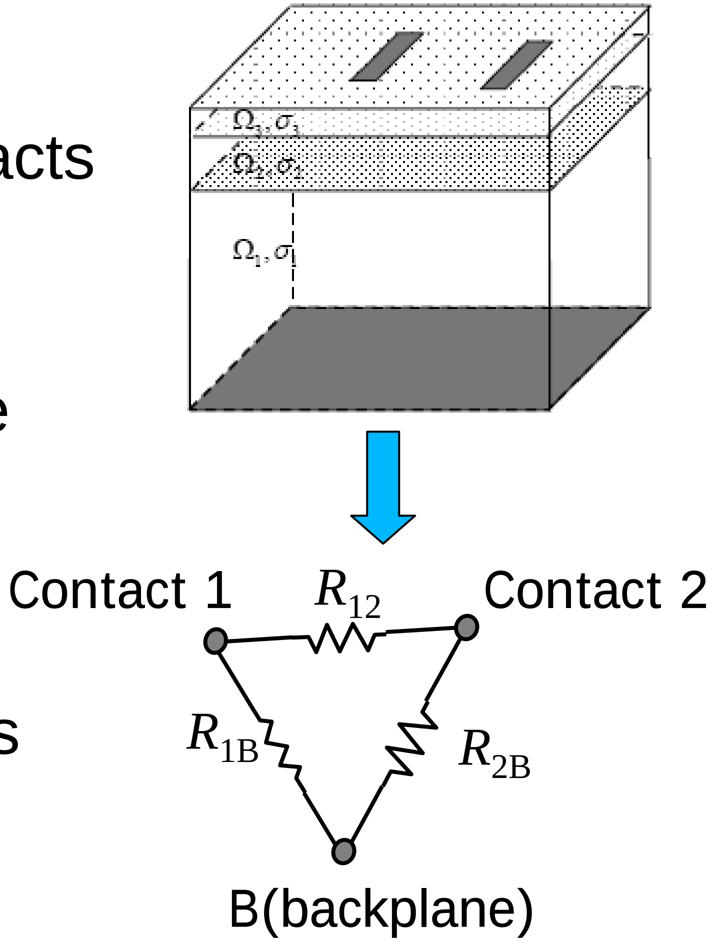


  - The quality factor of inductors in RF design is limited by substrate loss

knowledge of substrate coupling is necessary, or even critical for design

Introduction of Substrate Coupling Problems

- Modeling of substrate coupling
  - Substrate resistance among contacts
  - At higher frequency, both resistive and capacitive couplings should be considered [TCAD'98]
  - Extraction of substrate parameters becomes a challenging task



A.M. Niknejad, R. Gharpurey, and R.G. Meyer, "Numerically stable Green function for modeling and analysis of substrate coupling in integrated circuits," *IEEE Trans. CAD*, Vol. 17, pp. 305-315, 1998.

Numerical Methods for Substrate Extraction

- Volume discretization methods
  - Finite Element Method (FEM)
  - Finite Difference Method (FDM)
- Advantage:
  - Versatile for various kinds of substrate structures
    - Stratified, with multiple parallel horizontal layers
    - More complicated, e.g. those with oxide wells, trenches, sinkers, buried diffusions, shielding for noise reduction
- Disadvantage:
  - Too many unknowns
  - Then, limited to small structures

Numerical Methods for Substrate Extraction

- Green's function based methods (BEM)
  - Widely investigated
- Advantages:
  - Only discretizes contact surfaces, involves the fewest variables
  - Acceleration techniques proposed, such as DCT (discrete consine), eigen-decomposition, etc.
- Disadvantages:
  - The derivation of Green's function involves infinite series, which is expensive to calculate
  - Difficult, or even impossible to find the Green's function for non-stratified substrates

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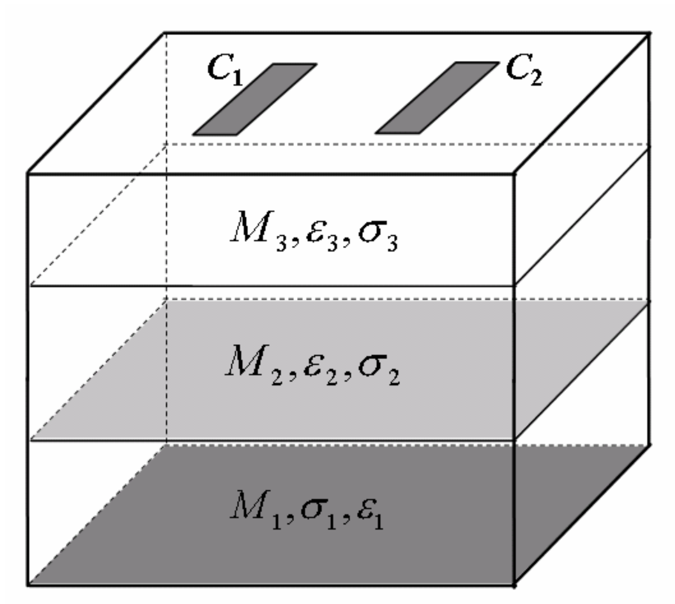
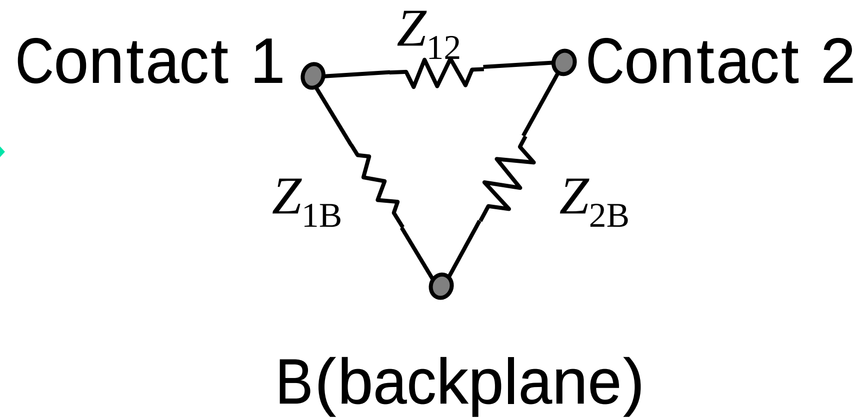
High-Frequency Model of Substrate



Reduced circuit model:

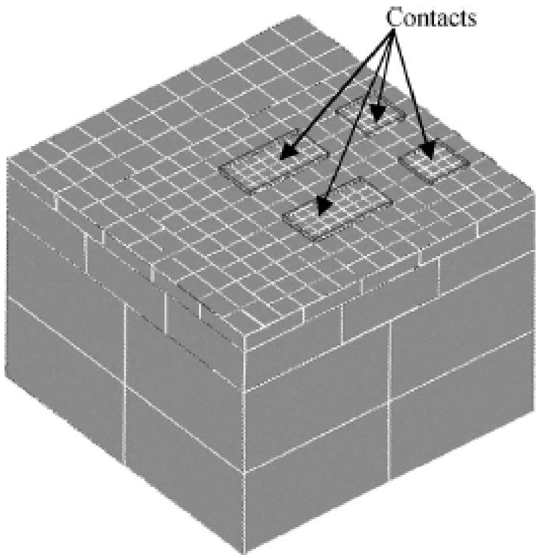


Consider both ohmic and displacement current;
Each substrate material has a conductivity σ_i , and a permittivity ε_i .

Z denotes the coupling impedance between the contacts (including the plane).

Direct Boundary Element Method (DBEM)

- Discretizes substrate boundary,
  - Contact surface
  - Medium boundary/interface
- Uses the simple free-space Green's function
- Has no difficulty in handling non-stratified substrates
- Efficient techniques of unknown reduction and matrix sparsification have been proposed for substrate resistance extraction [TCAD'06]

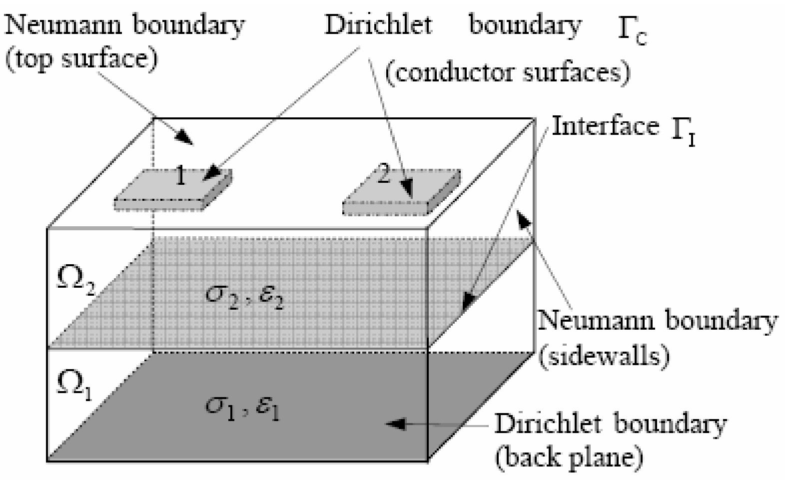


X. Wang, W. Yu and Z. Wang, "Efficient direct boundary element method for resistance extraction of substrate with arbitrary doping profiles," IEEE Trans. Comput. Aided Design, vol. 25, pp. 3035-3042, 2006.

High-Freq. Parameter Extraction using DBEM

- Within each medium

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$



- Apply the Green's identities, with the weighting function of free-space Green's function

$$c_s u_s + \int_{\Gamma_i} q^* u d\Gamma = \int_{\Gamma_i} u^* q d\Gamma$$

u is electric potential, and *q* is its normal derivative on boundary

s is a collocation point;
u^* is the free-space Green's function, and
q^* is its normal derivative on boundary

- Get linear equations with unknowns of *u* and *q* on boundaries
- *u* or *q* has known values on the outer boundaries

High-Freq. Parameter Extraction using DBEM

- Along the interface of medium a and b:

$$u_a = u_b,$$
$$(\sigma_a + j\omega\varepsilon_a)E_{n,a} = (\sigma_b + j\omega\varepsilon_b)E_{n,b}.$$

For displacement current, reflects the high-freq. model

- The linear equations for each medium can be combined together, to get an overall linear system

$$\boldsymbol{AX} = \boldsymbol{B}$$

A frequency-dependent complex-value system

To get whole impedance matrix, bias voltages are set on contacts, and ***B*** reflects these settings. ***X*** includes the unknowns of u and q on elements.

- The current flowing through contact *k* is

$$\int_{\Gamma_k} (\sigma + j\omega\varepsilon)E_n d\Gamma = \frac{1}{Z_{mk}}$$

,if contact m is with 1V, and contact k 0V

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- Introduction of Substrate Coupling Problems
- High-Frequency Parameter Extraction Using DBEM
- Efficient Techniques for Multi-Frequency Extraction
  - Extract substrate resistance (freq=0)
    - **Perform one time**
  - Revise solution of R extraction to freq.-dep. parameters
    - **With improving techniques, easy for computation**
- Numerical Results
- Conclusions

Multi-Frequency Extraction of Substrate

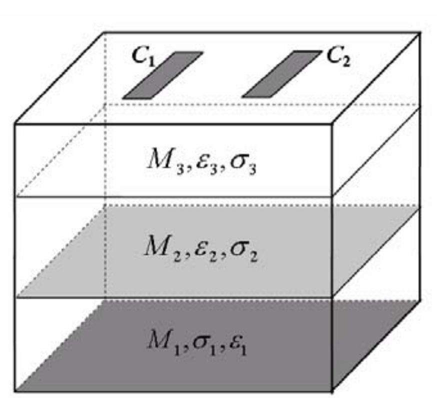
- When consider both ohmic and displacement current, the substrate coupling parameter (Z) is frequency-dependent
- In DBEM calculation
  - Interface equation includes frequency and imaginary

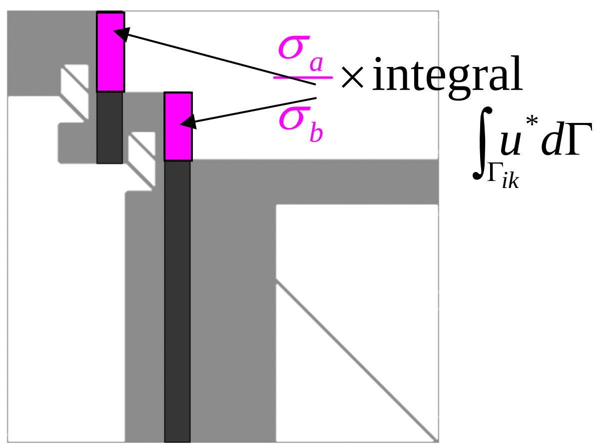
$$(\sigma_a + j\omega\varepsilon_a)E_{n,a} = (\sigma_b + j\omega\varepsilon_b)E_{n,b}$$

  - Solve a frequency-dependent, complex-valued $\boldsymbol{AX} = \boldsymbol{B}$
  - Get freq.-dependent R and C from the complex-valued Z
- The substrate parameters for many frequencies are necessary for the knowledge of substrate coupling
- The trivial approach to build and solve $\boldsymbol{AX} = \boldsymbol{B}$ repeatedly loses its efficiency with the increase of frequency points
- We propose a fast method for the multi-frequency extraction

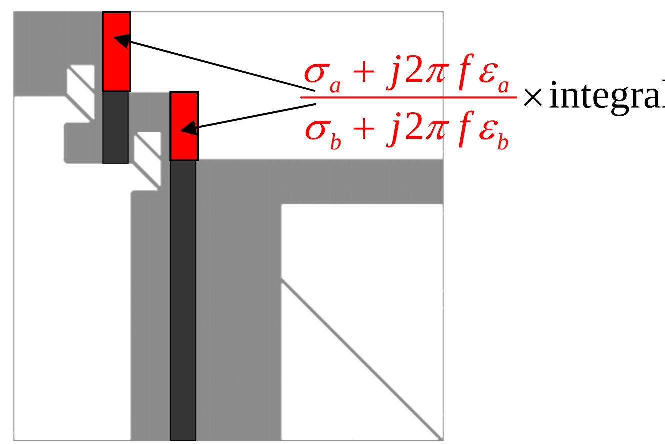
High Similarity of A_{res} and A

- Let A_{res} be the coefficient matrix for R extraction
- It's the equation for substrate resistance extraction, if we discard $j\omega\varepsilon$ items in $(\sigma_a + j\omega\varepsilon_a)E_{n,a} = (\sigma_b + j\omega\varepsilon_b)E_{n,b}$
- The differences between A_{res} and A lie on entries corresponding to q variables on interfaces





A_{res}



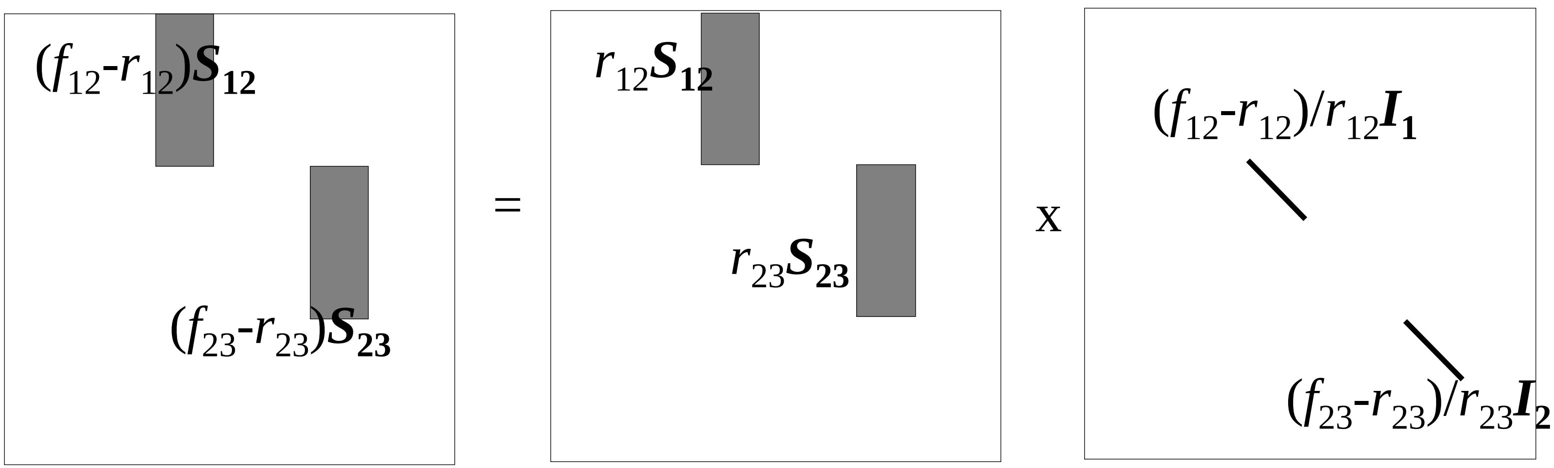
A

High Similarity of A_{res} and A (Cont.)

- In mathematics,

$$\boldsymbol{A} = \boldsymbol{A}_{res} + \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}}$$

Where $\boldsymbol{U}$ is a submatrix of $\boldsymbol{A}_{res}$, and $\boldsymbol{V}$ is a sparse diagonal matrix.



$$\boldsymbol{U}\boldsymbol{V}^{\mathrm{T}} = \boldsymbol{A} - \boldsymbol{A}_{res} \quad = \quad \boldsymbol{U} \quad \mathrm{x} \quad \boldsymbol{V}^{T}$$

$$f_{12} = (\sigma_2 + j\omega\varepsilon_2)/(\sigma_1 + j\omega\varepsilon_1), \quad r_{12} = \sigma_2/\sigma_1$$

S_{12}, S_{23} are matrices with entries calculated from the integrals of $\int_{\Gamma_{ik}} u^* d\Gamma$.
I_1 and I_2 are identity matrices.

Revise R to Frequency-Dependent Z

- According to Sherman-Morrison-Woodbury formula,

$$
\begin{aligned}
A^{-1} &= (A_{res} + UV^{T})^{-1} \\
&= A_{res}^{-1} - A_{res}^{-1}U(I + V^{T}A_{res}^{-1}U)^{-1}V^{T}A_{res}^{-1}
\end{aligned}
$$

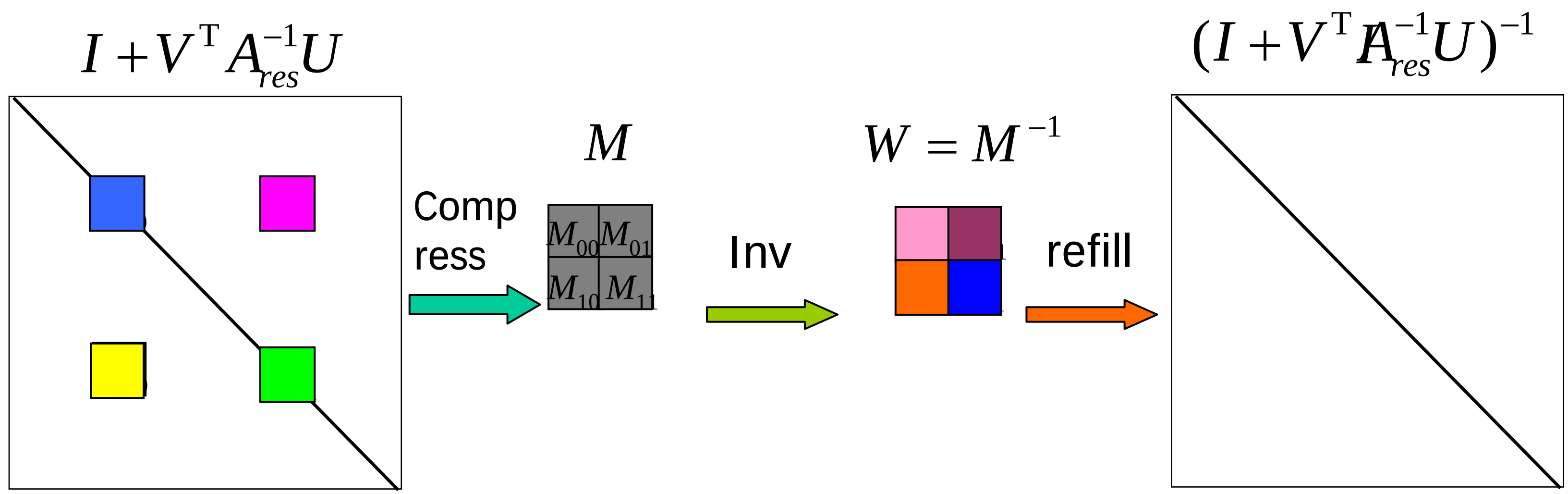
$$
\begin{aligned}
\Rightarrow X = A^{-1}B &= (A_{res} + UV^{T})^{-1}B \\
&= A_{res}^{-1}B - A_{res}^{-1}U(I + V^{T}A_{res}^{-1}U)^{-1}V^{T}A_{res}^{-1}B \\
&= X_{res} - \{A_{res}^{-1}U(I + V^{T}A_{res}^{-1}U)^{-1}V^{T}\}X_{res}
\end{aligned}
$$

- Thus, freq.-dept. solution (X) can be obtained through revising the solution for R extraction (X_{res})

Efficient Technique for $(I+V^{\mathrm{T}}A_{res}^{-1}U)^{-1}$

$$X = X_{res} - A_{res}^{-1}U(I+V^{T}A_{res}^{-1}U)^{-1}V^{T}X_{res}$$

- Usually, difficult to get $(I+V^{\mathrm{T}}A_{res}^{-1}U)^{-1}$
- *U* and *V* are sparse matrices, and so is $I+V^{\mathrm{T}}A_{res}^{-1}U$
- For the three-medium example:



Algorithm Flow

1. Generate A_{res} in (5). Select some entries to form U;
2. Solve for $X_{res} = A_{res}^{-1}B$ as well as for $A_{res}^{-1}U$;
3. For each frequency point, calculate the corresponding Z:
   a) Create such factors as $(f_{12}/r_{12}\text{-}1)I_1$ and $(f_{23}/r_{23}\text{-}1)I_2$, so as to form matrix V;
   b) Compress $I + V^T A_{res}^{-1} U$ into small matrix M;
   c) Inverse matrix M, and get W;
   d) Refill W to get matrix $(I + V^T A_{res}^{-1} U)^{-1}$;
   e) Get $X = X_{res} - A_{res}^{-1} U (I + V^T A_{res}^{-1} U)^{-1} V^T X_{res}$.
   f) Get the desired Z parameter through (9).

Solve the real-valued problem of R extraction; Runs only once.

The order of ***M*** is # of interface elements, which is much less than total # of unknowns

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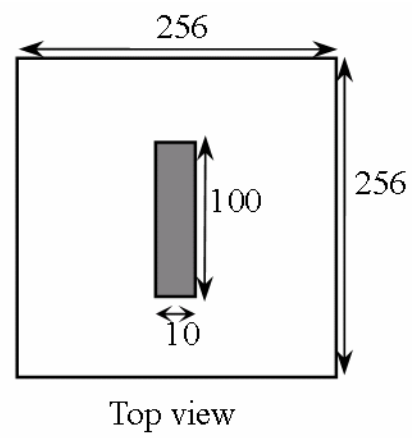
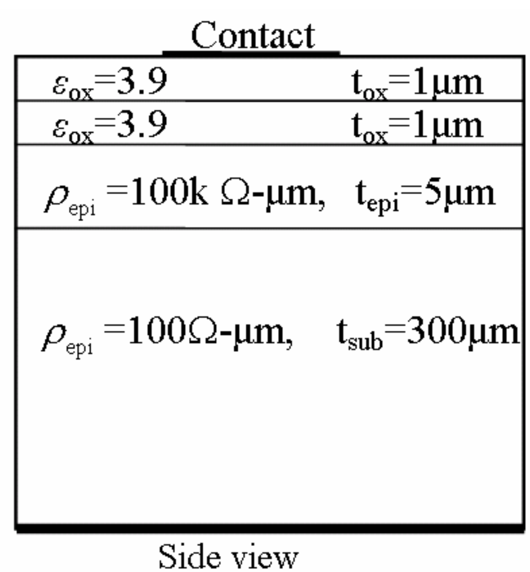
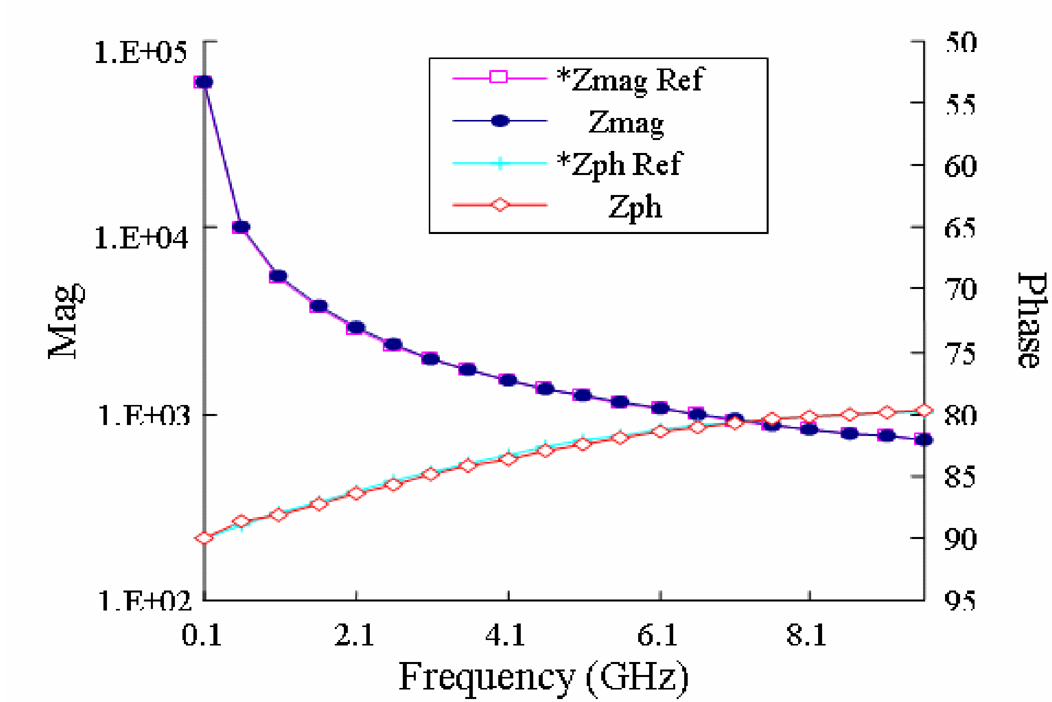
Numerical Results -- Accuracy

Case from [TCAD'98]





The discrepancy between our results and those from [TCAD'98] < 1.0%



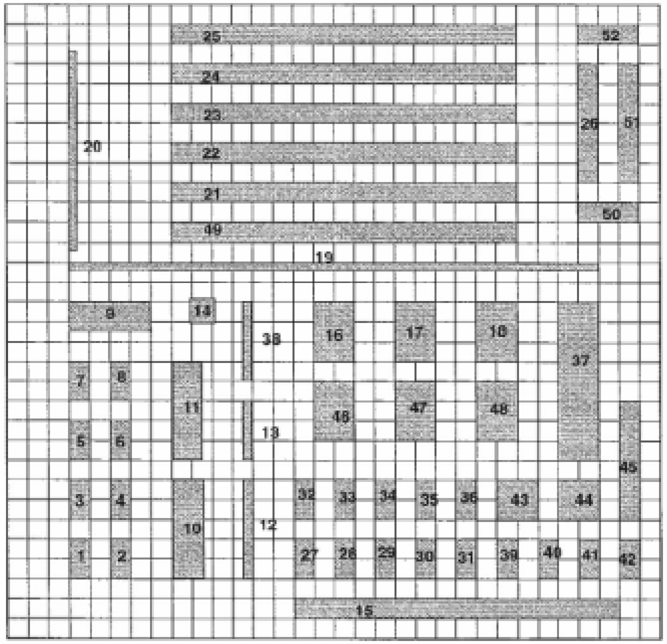
Since displacement current increases with frequency, the Z's magnitude decreases with frequency.

[TCAD'98] A.M. Niknejad, R. Gharpurey, and R.G. Meyer, "Numerically stable Green function for modeling and analysis of substrate coupling in integrated circuits," *IEEE Trans. CAD*, Vol. 17, pp. 305-315, 1998.

Numerical Results -- Efficiency

Case with 52 contacts

Our method is compared with ASITIC, a shared program using DCT-accelerated Green's function method



For each frequency, our method solves a 587x587 ***M*** matrix to calculate Z from the precalculated X_{res}

Our method is much faster than ASITIC

	DBEM	ASITIC	
# Contacts	52	52	
# Variables	7252	860	4352 (default)
Memory (MB)	60	32	310
Pre-process (seconds)	**420.9***	360**	360 **
Extraction (for 1 freq.)	**9.0**	9000	(Not obtained)

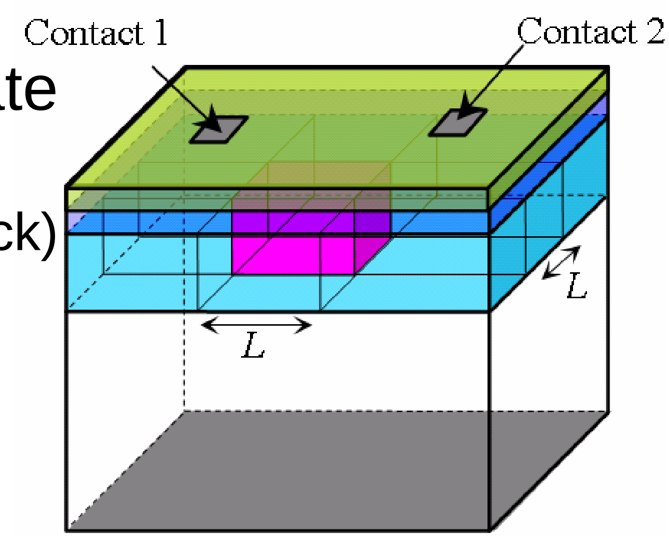
*: Calculate resistance

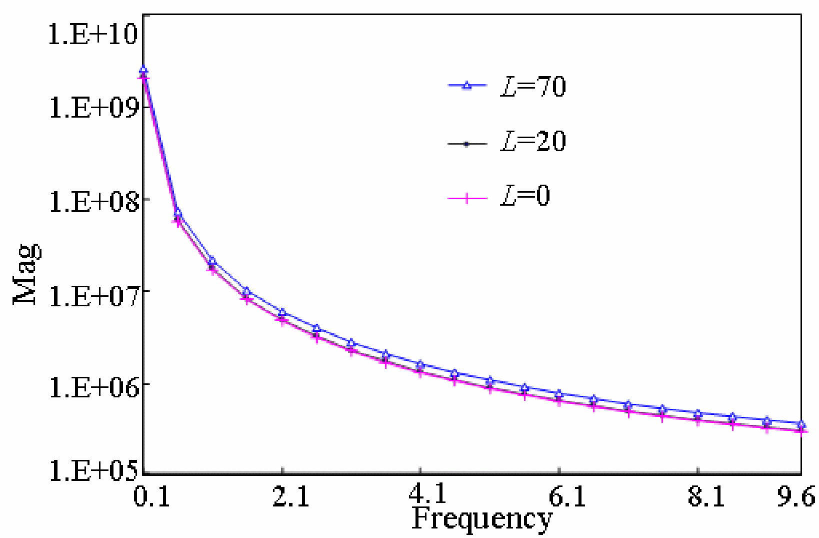
**: Calculate Green's function

Numerical Results -- Versatility

- An example with non-stratified substrate
  - The central block region has a distinct resistivity, called LVB (lateral variation block)
  - LVB's resistivity is 1000 times larger
  - Size of LVB is set to be 0, 20, and 70μm
- Green's function based method is not able to handle it, while our DBEM has no difficulty
- Right figure shows the plot of Magnitude vs. frequency, for three settings

Since LVB obstructs ohmic current flow, the Z for L=70 is larger than those corresponding to other two settings





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Conclusions

- For frequency-dependent substrate modeling, we propose an efficient method for multi-frequency extraction
  - Based on DBEM, has ability to handle non-stratified structure
  - Do not directly solve the complex linear system, but firstly solve the R extraction problem with real linear system
  - For each frequency, the result of R extraction is revised to Z parameter through solving a smaller linear system
- The proposed method
  - Is efficient
  - Does not sacrifice accuracy
  - Has large versatility

Thank you!

For more information, please contact:

yu-wj@tsinghua.edu.cn