Passive Interconnect Macromodeling Via Balanced Truncation of Linear Systems in Descriptor Form

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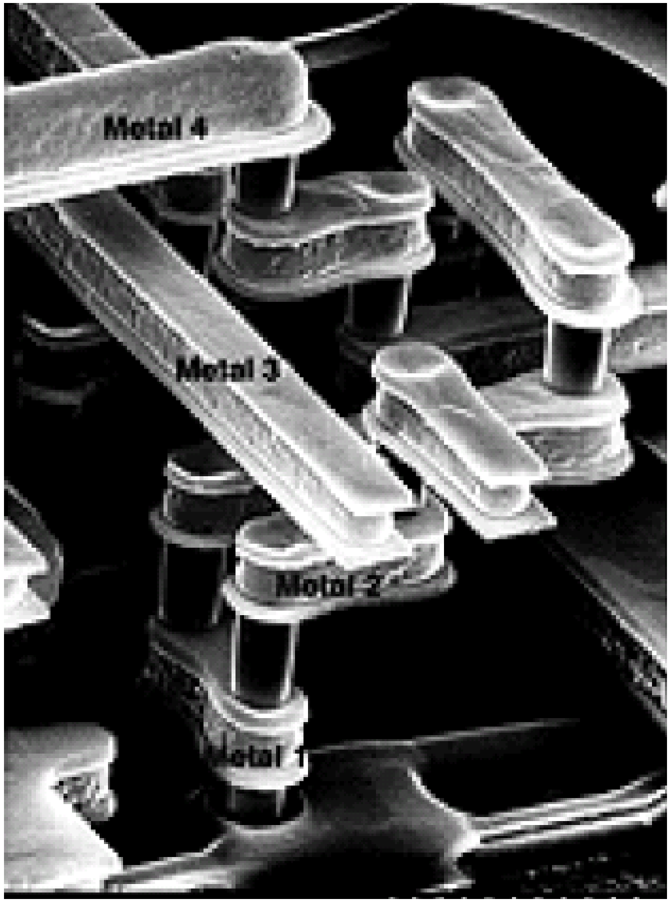
Outlook

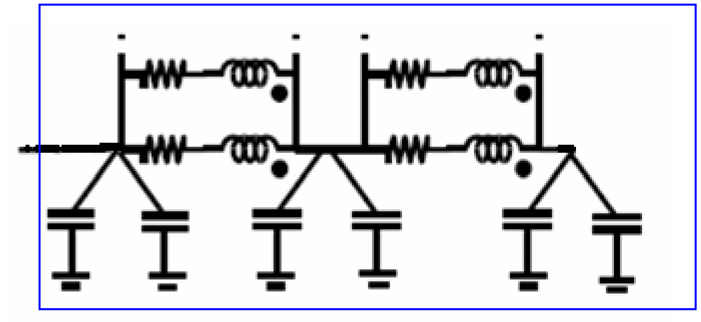
- Motivation
- RLC circuit equations
- Review of moment-matching based approaches
- Review of balanced truncation
- Projection-based balanced truncation
- Composite mode order reduction
- Examples
- Conclusion

Motivation



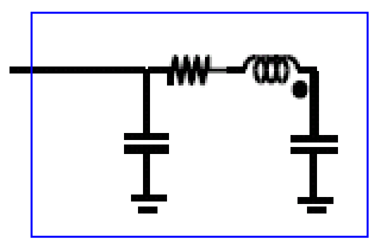
Thousands of wires

Extraction

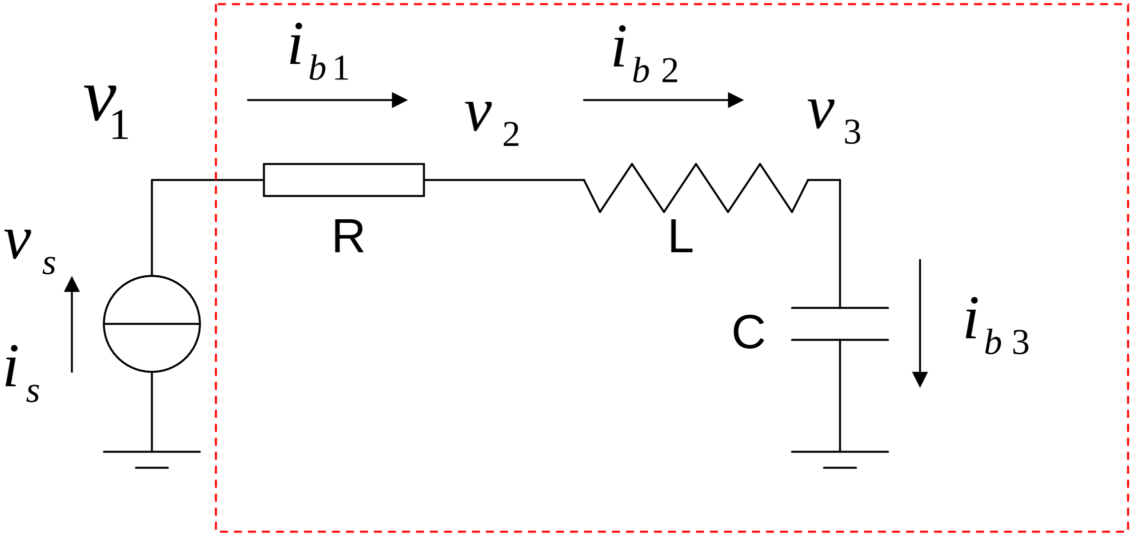


Millions of RLC elements

Model Order Reduction



RLC Circuit Equations using Modified Nodal Analysis (MNA)



$$Z(s) = \frac{V_s(s)}{I_s(s)}$$

$$i_{b1} = i_s$$
$$-i_{b1} + i_{b2} = 0$$
$$-i_{b2} + i_{b3} = 0$$

$$(v_1 - v_2)/R = i_s$$
$$-(v_1 - v_2)/R + i_L = 0$$
$$-i_L + C\dot{v}_3 = 0$$
$$v_2 - v_3 = L\dot{i}_L$$

RLC Circuit Equations in Descriptor Form

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & L \end{bmatrix} \begin{pmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{i}_L \end{pmatrix} + \begin{bmatrix} 1/R & -1/R & 0 & 0 \\ -1/R & 1/R & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ i_L \end{pmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} i_s$$

$$u_s = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ i_L \end{pmatrix}$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Standard form

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \dot{x} + \begin{bmatrix} G_1 & G_2^T \\ -G_2 & 0 \end{bmatrix} x + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} B_1^T & 0 \end{bmatrix} x$$

$$\left(G_1 = G_1^T \geq 0, C_1 = C_1^T \geq 0, C_2 = C_2^T > 0\right)$$

$$C\dot{x}(t) = -Gx(t) + Bu(t)$$
$$y(t) = B^T x(t)$$
$$(G + G^T \geq 0, C = C^T \geq 0)$$

Descriptor form

$$Z(s) = B_1^T (sC_1 + G_1 + \frac{1}{s} G_2^T C_2^{-1} G_2)^{-1} B_1 = Z^T(s)$$

reciprocity

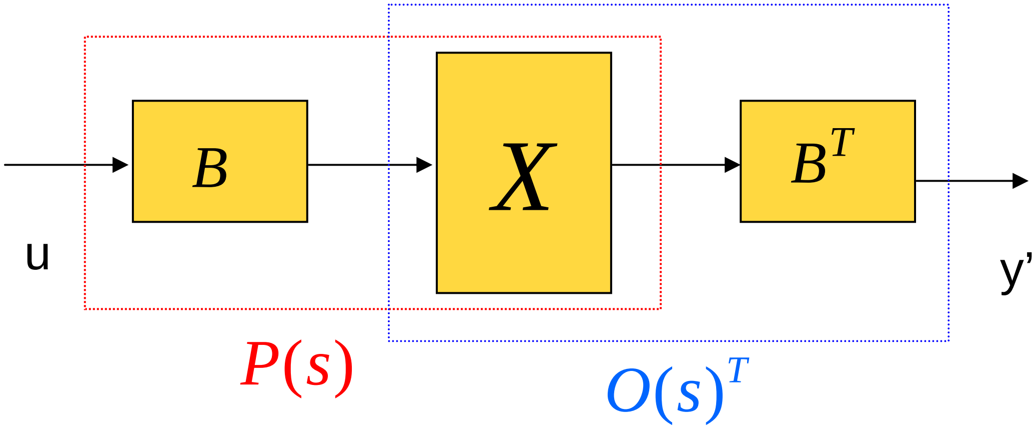
passivity

MOR Methods In a Nutshell

- Krylov subspace based projection methods
  - Fast, scalable to large problem.
  - No global error bound.
  - Less efficient for circuit with many terminals
- Balanced truncation methods
  - With global error bound.
  - High computing costs
- Node reduction method
  - Local reduction, no global error bounds.
  - Easy for realization.

Review of Moment-Matching-Based Approaches

Transfer function: $$H(s) = B^T(sC+G)^{-1}B = B^T P(s) = O(s)^T B$$



Input Moment

$$P(s) = (I + (s - s_0)A_{In})^{-1}R_{In} = \sum_{i=0}^{\infty} A_{in}^i R_{in}(s - s_0)^r$$

$$A_{in} = (s_0C + G)^{-1}C, R_{in} = (s_0C + G)^{-1}B$$

Output Moment

$$Q(s) = (I + (s - s_0)A_{out})^{-1}R_{out} = \sum_{i=0}^{\infty} A_{out}^i R_{out}(s - s_0)^r$$

$$A_{out} = (s_0C + G^T)^{-1}C, R_{out} = (s_0C + G^T)^{-1}B$$

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Review of Moment Matching Based Approaches

$$Z(s) = B^T (sC + G)^{-1} B$$

$$C_r = V^T C V, G_r = V^T G V, B_r = V^T B$$

$$C_r = W^T C V, G_r = W^T G V, B_r = W^T B$$

Orthogonal Projection

a.k.a. Congruence Projection

Oblique Projection

$$Z_r(s) = B_r^T (sC_r + G_r)^{-1} B_r$$

Match r moments

Match 2r moments

$$K_r(A_{in}, R_{in}) = colspan(V)$$
$$K_r(A_{out}, R_{out}) = colspan(W)$$

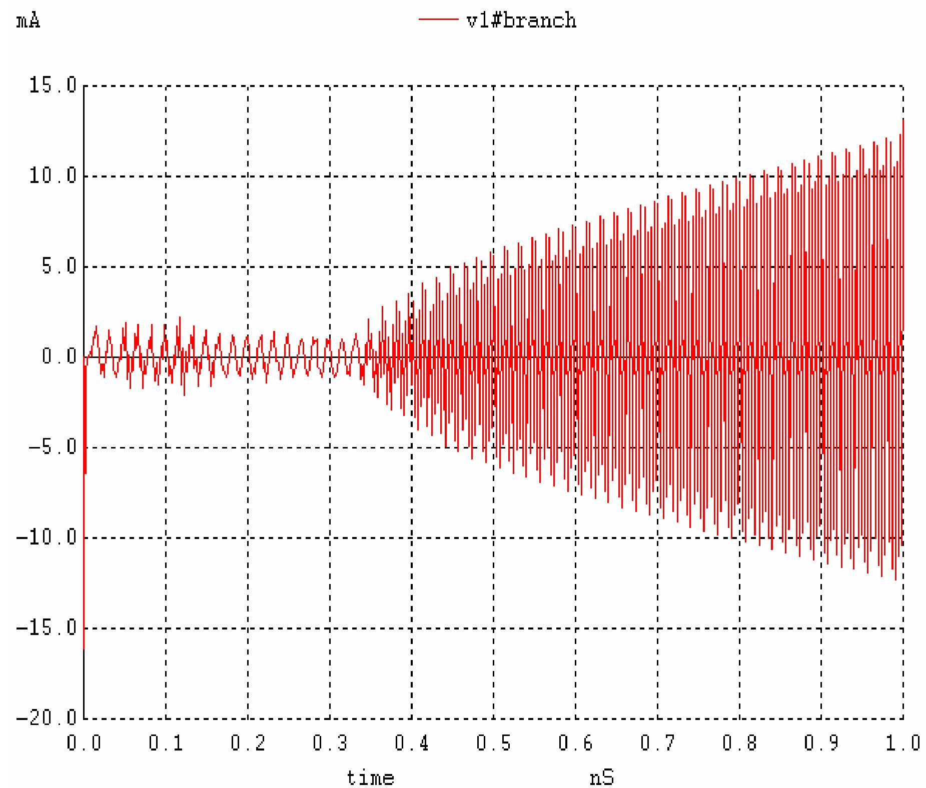
*r*th Krylov subspace $K_r(A, R) = span(R, AR, \cdots A^{r-1} R)$

Review of Moment Matching Based Approaches

$$C\,\dot{x}(t) = -Gx(t) + Bu(t)$$
$$y(t) = B^T x(t)$$

Orthogonal Projection

$$C_r\,\dot{x}_r(t) = -G_r x_r(t) + B_r u(t)$$
$$y(t) = B_r^T x_r(t)$$



$$C_r = V^T C V, G_r = V^T C V, B_r = V^T B$$

$$G + G^T \geq 0, C = C^T \geq 0$$

$$G_r + G_r^T \geq 0, C_r = C_r^T \geq 0$$

Orthogonal projection preserve positive semi-definiteness and thus passivity!

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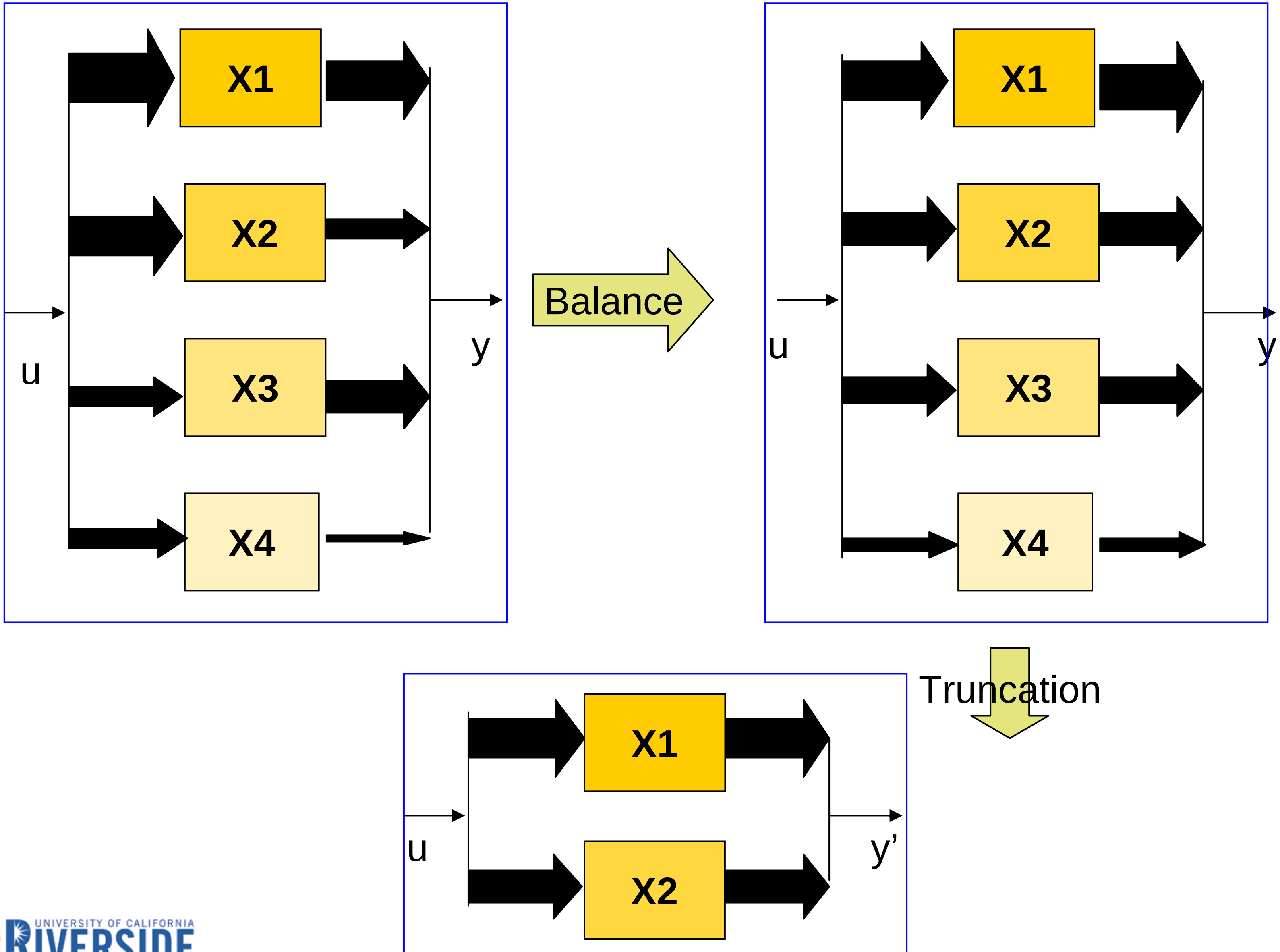
Review of Balanced Truncation



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Review of Balanced Truncation

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Balance

$$A_b = T_b^{-1} A T_b,\ B_b = T_b^{-1} B,\ C_b = C T_b$$

$$\dot{x}_b(t) = A_b x_b(t) + B_b u(t)$$
$$y(t) = C_b x_t(t)$$

Truncate

$$\begin{bmatrix} \dot{x}_{b1}(t) \\ \dot{x}_{b2}(t) \end{bmatrix} = \begin{bmatrix} A_{b11} & A_{b12} \\ A_{b21} & A_{b22} \end{bmatrix} \begin{bmatrix} x_{b1}(t) \\ x_{b2}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_{b1}(t) \\ x_{b2}(t) \end{bmatrix}$$

$$\dot{x}_{b1}(t) = A_{b11} x_{b1}(t) + B_1 u(t)$$
$$y(t) = C_1 x_{b1}(t)$$

How to find T?

Review of Balanced Truncation

State-space model in *standard* form:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Lyapunov equations (linear)

$$AW_c + W_cA^T + BB^T = 0$$
$$A^TW_o + W_oA + C^TC = 0$$

Lur'e equations (quadratic)

$$AX_c + X_cA^T = -K_cK_c^T$$
$$X_cC^T - B = -K_cJ_c^T$$
$$J_cJ_c^T = D + D^T$$

$$A^TX_o + X_oA = -K_o^T$$
$$X_cC^T - B = -K_o^TJ_o$$
$$J_o^TJ_o = D + D^T$$

$$T^{-1}W_cW_oT = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}, \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$$

Standard TBR

$$T^{-1}X_cX_oT = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}, \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$$

Positive-Real TBR (passivity)

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λ Indicates the importance of each state in terms of energy transfer

Review of Balanced Truncation

Traditional balanced truncation

$$C\dot{x}(t) = -Gx(t) + Bu(t)$$
$$y(t) = B^T x(t)$$

$$\dot{x}(t) = -C^{-1}Gx(t) + C^{-1}Bu(t)$$
$$y(t) = B^T x(t)$$

$$\dot{x}_r(t) = A_r x_r(t) + B\,u(t)$$
$$y(t) = C_r x_r(t)$$

mapping

Balanced truncation

Drawbacks

C is sparse but the inverse of C may be dense

If C is ill-conditioned, generate too much error or even an unstable system

All structure information inherent to RLC circuits is lost

Lur'e equations are quadratic matrix equations and thus expensive

Why not?

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Projection based Balanced Truncation

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Balance

$$E_b = T_b^{-1} E T_b, A_b = T_b^{-1} A T_b, B_b = T_b^{-1} B, C_b = C T_b$$

$$E_b \dot{x}_b(t) = A_b x_b(t) + B_b u(t)$$
$$y(t) = C_b x_t(t)$$

$$\begin{bmatrix} E_{b11} & E_{b12} \\ E_{b21} & E_{b22} \end{bmatrix} \begin{bmatrix} \dot{x}_{b1}(t) \\ \dot{x}_{b2}(t) \end{bmatrix} = \begin{bmatrix} A_{b11} & A_{b12} \\ A_{b21} & A_{b22} \end{bmatrix} \begin{bmatrix} x_{b1}(t) \\ x_{b2}(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_{b1}(t) \\ x_{b2}(t) \end{bmatrix}$$

Truncate

$$E_{b11}\dot{x}_{b1}(t) = A_{b11}x_{b1}(t) + B_1 u(t)$$
$$y(t) = C_1 x_{b1}(t)$$

Projection based Balanced Truncation

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Balance

$$E_b \dot{x}_b(t) = A_b x_b(t) + B_b u(t)$$
$$y(t) = C_b x_t(t)$$

Truncate

$$E_{b11} \dot{x}_{b1}(t) = A_{b11} x_{b1}(t) + B_1 u(t)$$
$$y(t) = C_1 x_{b1}(t)$$

Oblique Projection

$$E_r = W^T E V$$
$$A_r = W^T A V$$
$$B_r = W^T B$$
$$C_r = CV$$

How to find W and V?

Projection Based Balanced Truncation

PriTBR

State-space model in *descriptor* form:

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Generalized Lyapunov equations:

$$EPA^T + APE^T + BB^T = 0$$
$$E^TQA + A^TQE + C^TC = 0$$

$$T^{-1}PQT = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}, \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > 0$$

$$\begin{bmatrix} W^T \\ \tilde{W}^T \end{bmatrix} PQ \begin{bmatrix} V & \tilde{V} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$$

Projection Based Balanced Truncation

PriTBR

$$C\dot{x}(t) = -Gx(t) + Bu(t)$$
$$y(t) = B^T x(t)$$

Orthogonal Projection

$$C_r\dot{x}(t) = -G_r x(t) + B_r u(t)$$
$$y(t) = B_r^T x(t)$$

$$C_r = T^T CT$$
$$G_r = T^T GT$$
$$B_r = T^T B$$

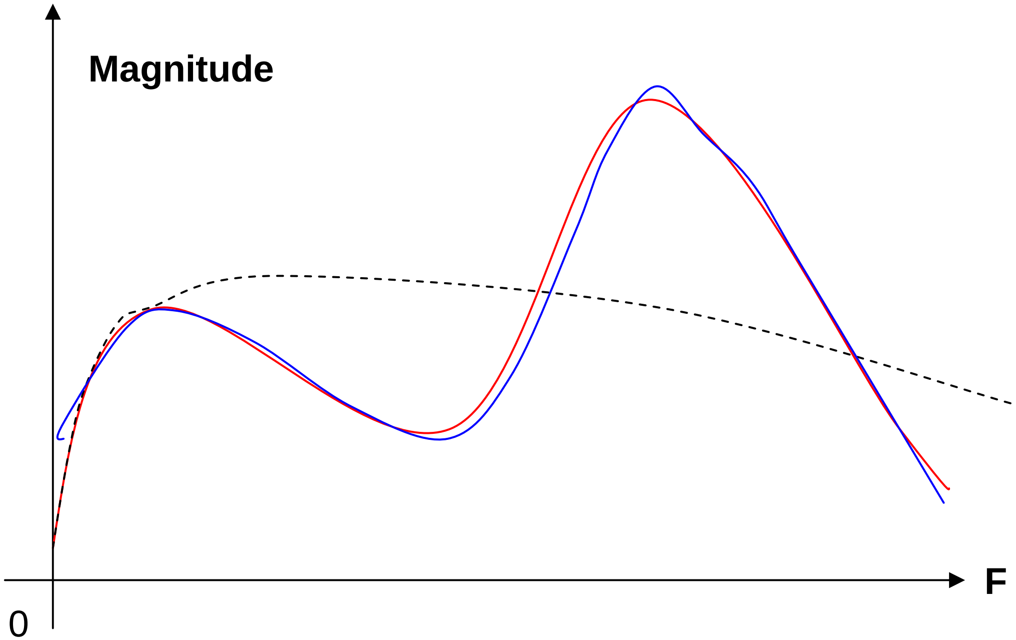
$$C_r = T^T CT, G_r = T^T CT, B_r = T^T B$$

$$G + G^T \geq 0, C = C^T \geq 0$$

$$G_r + G_r^T \geq 0, C_r = C_r^T \geq 0$$

Orthogonal projection preserve positive semi-definiteness and thus passivity!

Hybrid Projection



Original

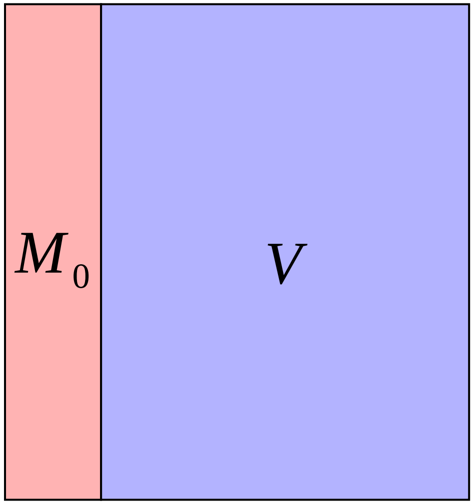
Balanced truncation

Globally accurate

Moment matching

Locally accurate (DC)

Composite Space Projection



Exact DC match

Wideband accuracy

PriTBR Reduction Algorithm

ALGORITHM 1: PROJECTION-BASED PASSIVE TBR (PRITBR)

1. Solve $EPA^T + APE^T + BB^T = 0$ for P
2. Solve $E^TQA + A^TQE + C^TC = 0$ for Q
3. Compute Cholesky factors $P = L_pL_p^T, \; Q = L_QL_Q^T$
4. Compute SVD of $L_p^TE^TL_Q$:

   $$L_p^TE^TL_Q = [U_1, U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [V_1, V_2]^T$$

5. Compute the dominant basis $T_1 = L_pU_1\Sigma_1^{-1/2}$
6. Solve $AM_0 = B$ for M_0
7. Make a union of M_0 and T_1 and orthonormalize it $X = orth(M_0, T_1)$
8. Compute the reduced system with $\tilde{E} = X^TEX; \; \tilde{A} = X^TAX; \; \tilde{B} = X^TB; \; \tilde{C} = CX$

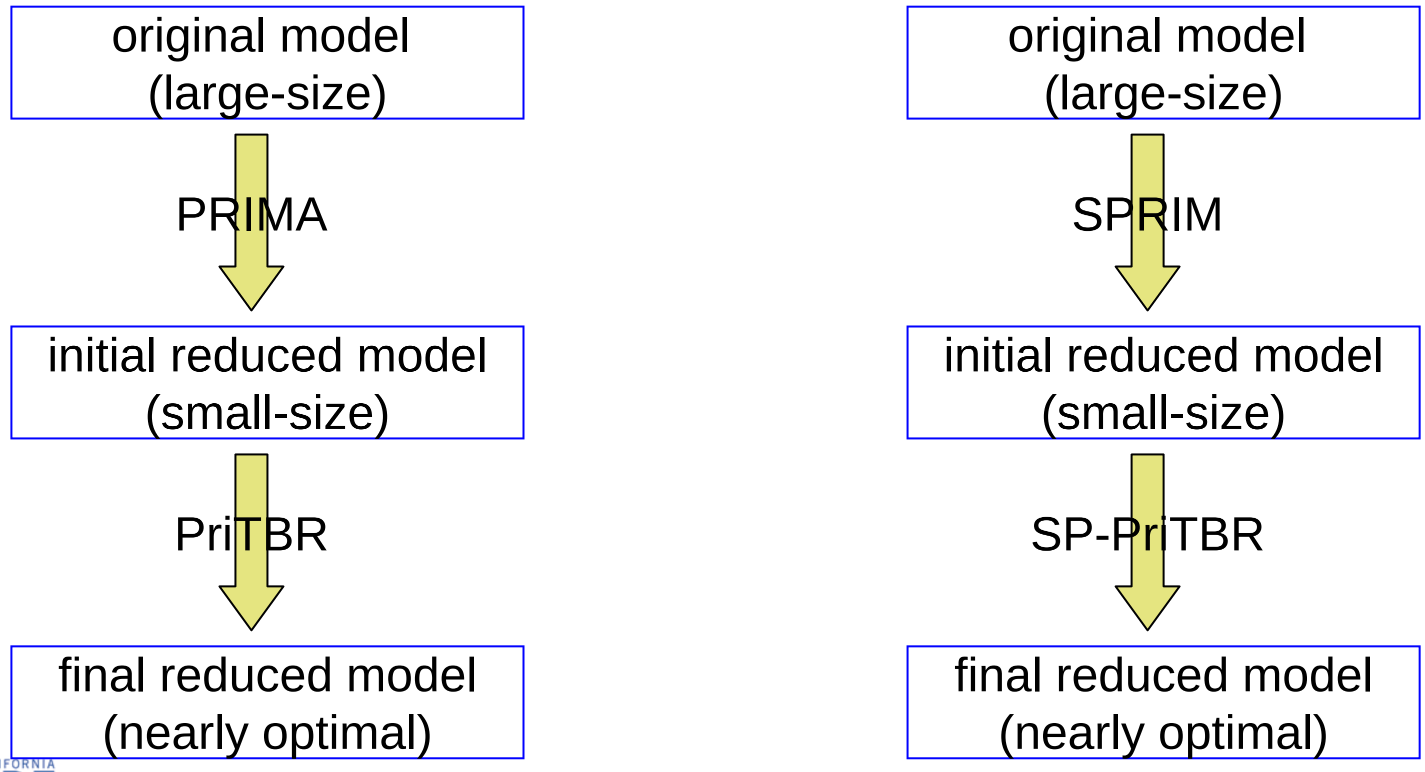
Two-Step Reduction

Moment Matching : Low Cost

Balanced Truncation: Compact

Two-Step Reduction Process

Passivity Preserved MOR | Structure Preserved MOR



Structure-Preserving PriTBR

SP-PriTBR

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\dot{x} + \begin{bmatrix} G_1 & G_2^T \\ -G_2 & 0 \end{bmatrix} x + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} B_1^T & 0 \end{bmatrix} x$$

$$\tilde{V} = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}, \left(V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}\right)$$

$$span V \supseteq span \tilde{V}$$

Orthogonal Projection

$$C_r = \tilde{V}^T C \tilde{V}, G_r = \tilde{V}^T G \tilde{V}, B_r = \tilde{V}^T B$$

$$\begin{bmatrix} V_1^T C_1 V_1 & 0 \\ 0 & V_2^T C_2 V_2 \end{bmatrix}\dot{x} + \begin{bmatrix} V_1^T G_1 V_1 & V_1^T G_2^T V_2 \\ -V_2^T G_2 V_1 & 0 \end{bmatrix} x + \begin{bmatrix} V_1^T B_1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} B_1^T V_1 & 0 \end{bmatrix} x$$

Reciprocity of RLC circuits is preserved:

$$Z(s) = B_1^T (sC_1 + G_1 + \frac{1}{s} G_2^T C_2^{-1} G_2)^{-1} B_1 = Z^T(s)$$

SP-PrTBR Reduction Algorithm

ALGORITHM 2: STRUCTURE-PRESERVING PRITBR ALGORITHM (SP-PRITBR)

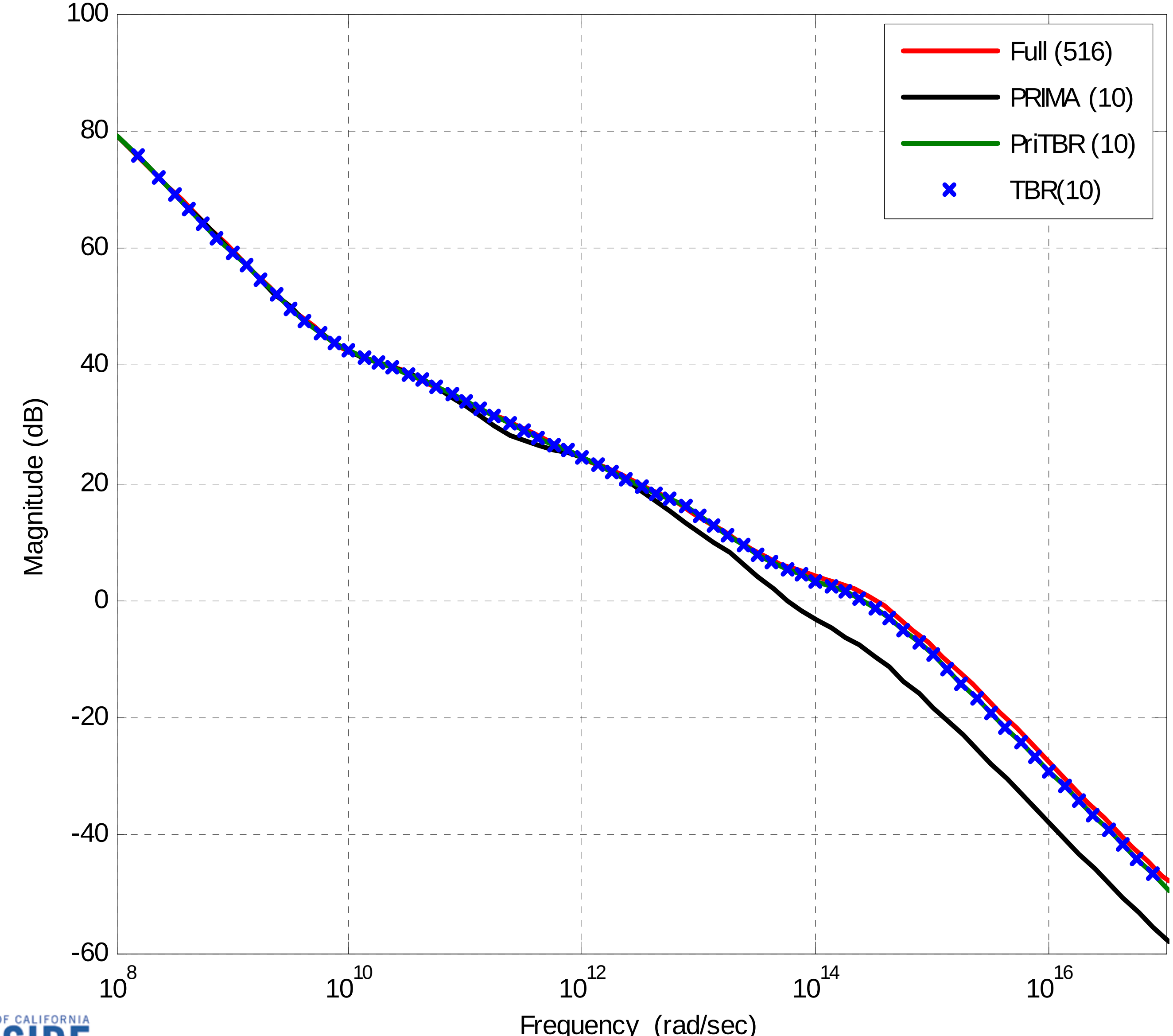
1. Perform Algorithm 1 (step 1- step 6) for $V = [M_0, T_1]$
2. Partition V corresponding to the block sizes of G, C

   $$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

3. Set $\tilde{V} = \begin{bmatrix} orth(V_1) & 0 \\ 0 & orth(V_2) \end{bmatrix}$
4. Obtain the reduced model by projection:

   $$\tilde{G} = \tilde{V}^T G \tilde{V}, \tilde{C} = \tilde{V}^T C \tilde{V}, \tilde{B} = \tilde{V}^T B$$

Examples



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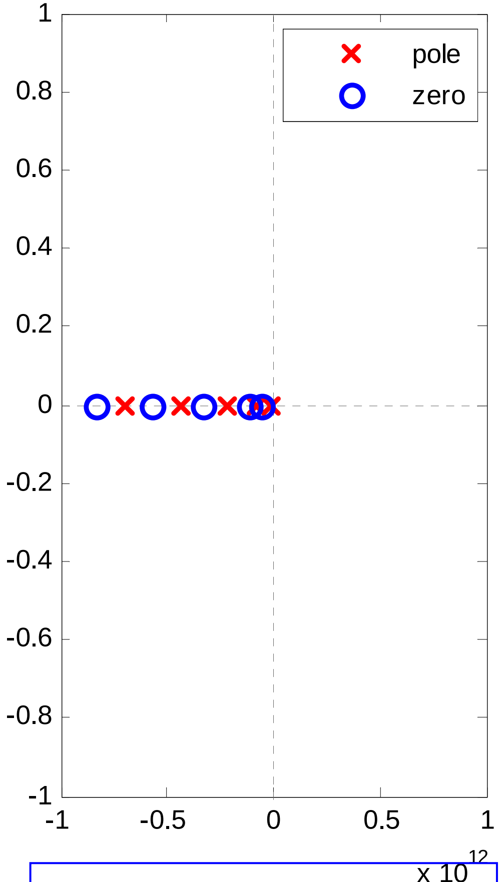
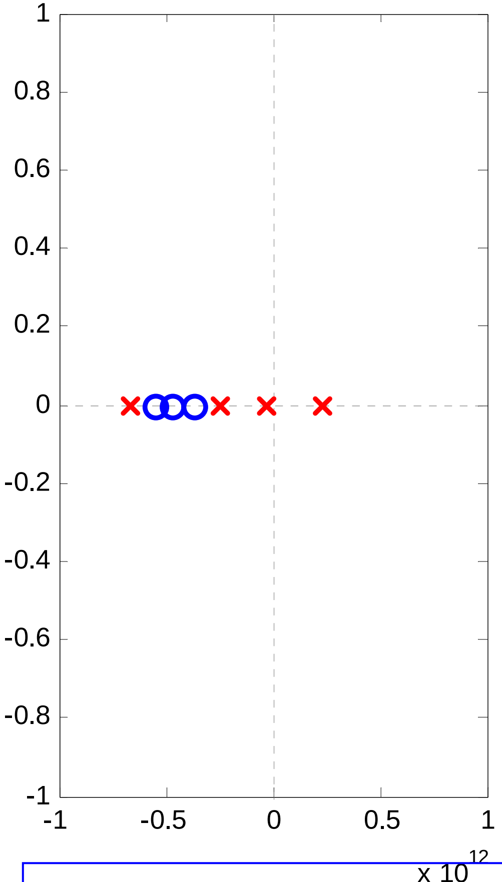
PriTBR is as accurate as standard TBR and superior to PRIMA

Examples



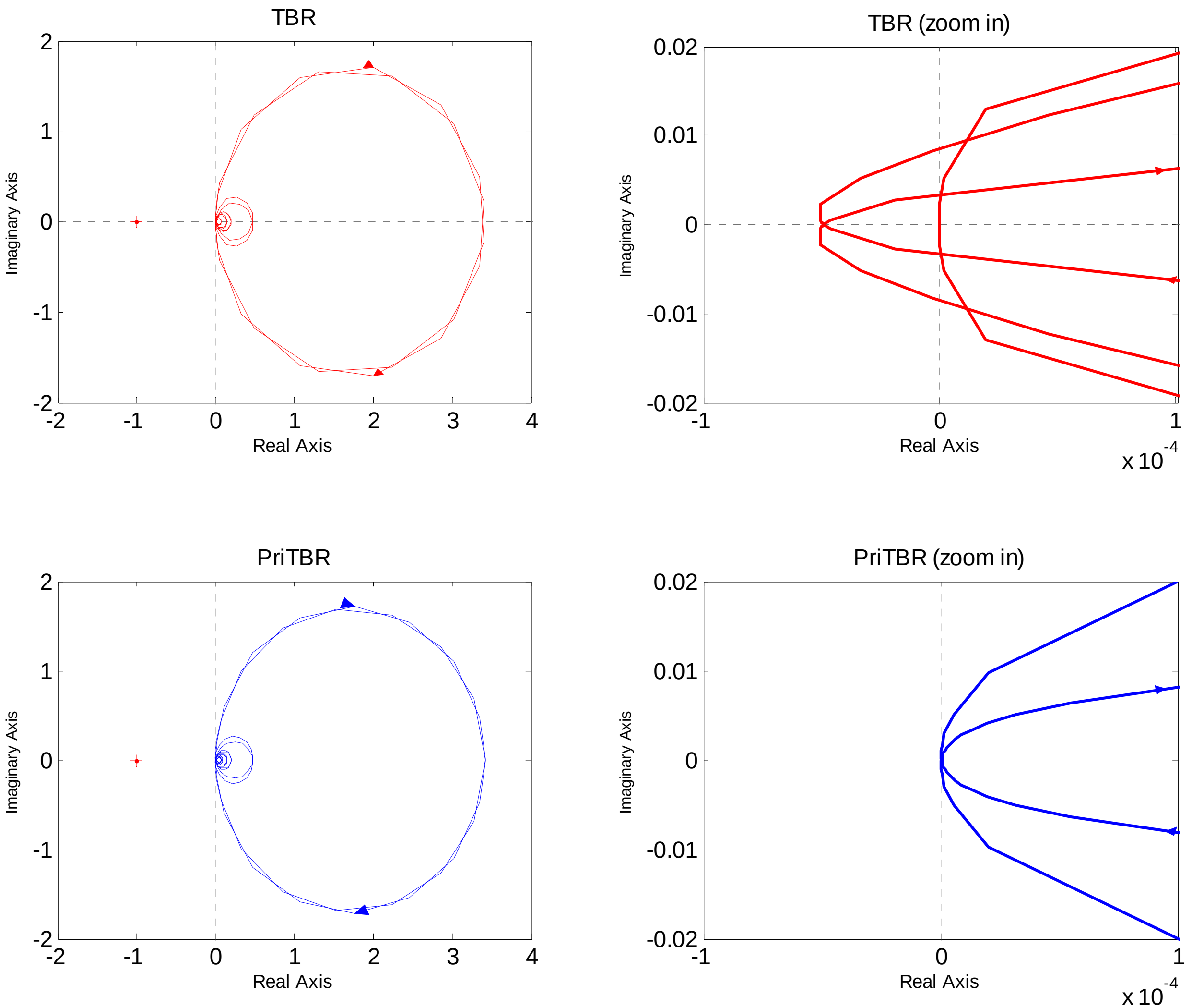


$$C\dot{x}(t) = -Gx(t) + Bu(t)$$
$$y(t) = B^T x(t)$$

$$\dot{x}(t) = -C^{-1}Gx(t) + C^{-1}Bu(t)$$
$$y(t) = B^T x(t)$$

The mapping will generate unstable system

Examples



PriTBR can preserve passivity while standard TBR cannot.

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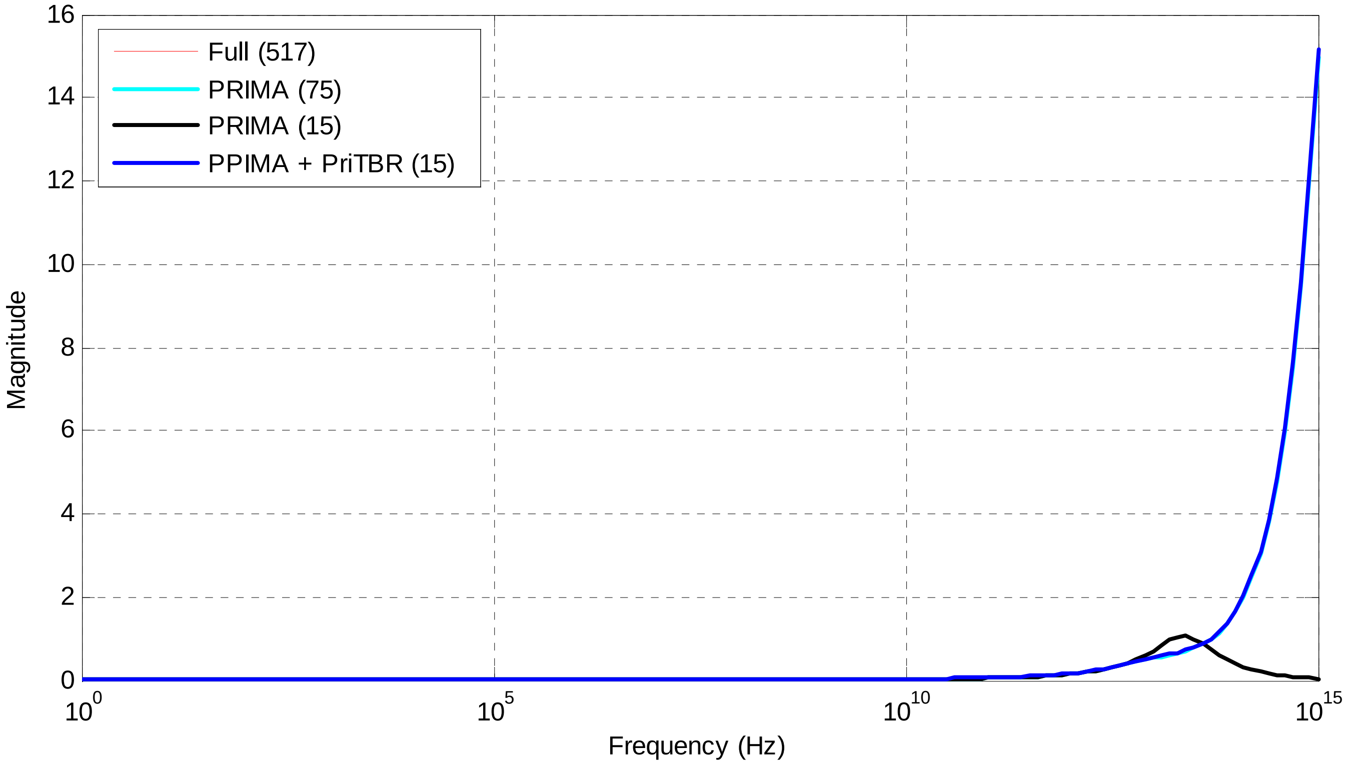
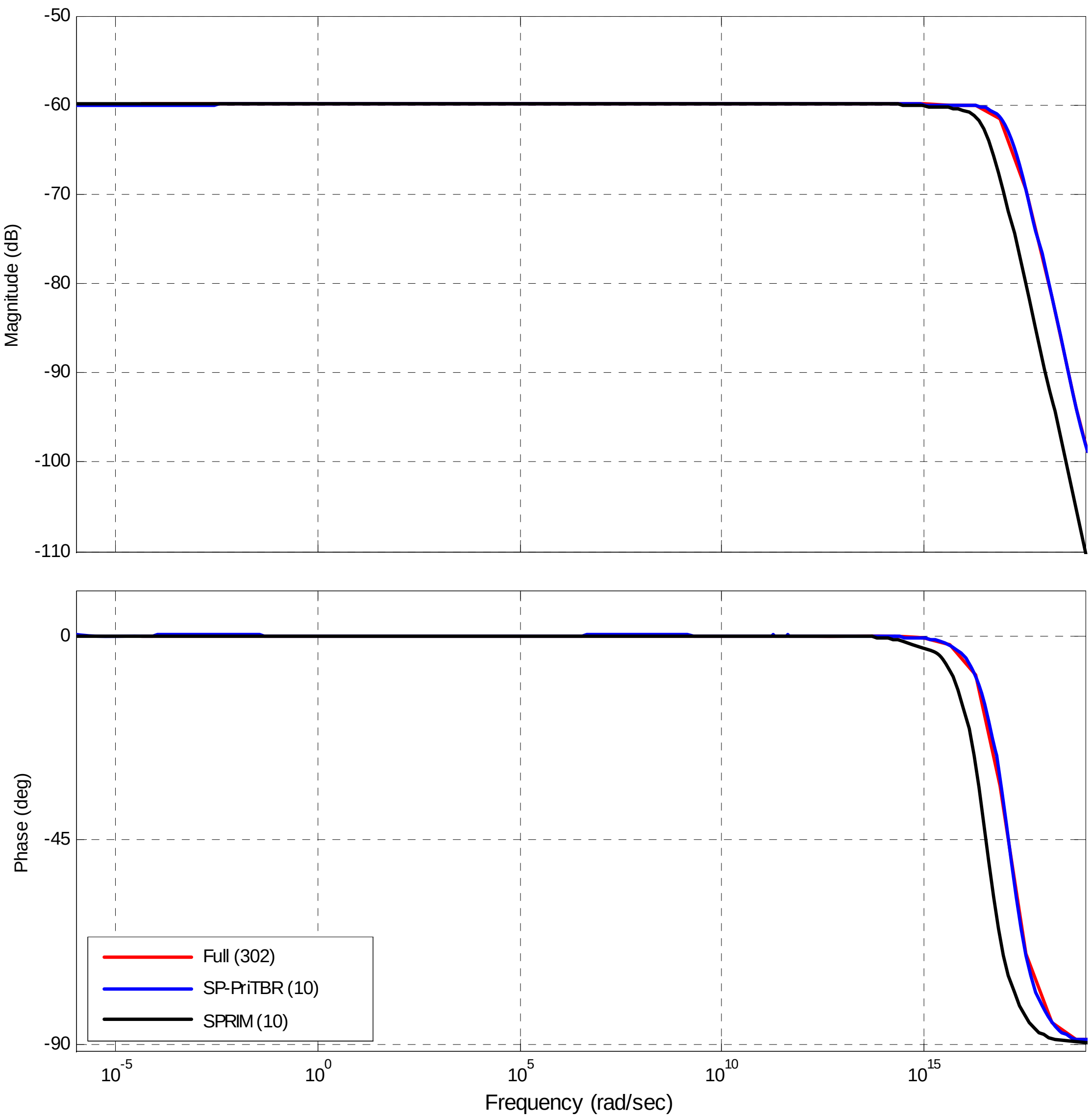
Examples



Two-step reduction process generates optimal reduced model with low cost

Examples

SP-PriTBR is superior to moment matching based structure preserving method SPRIM



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Conclusion

- Proposed new projection-based balanced truncation method, PriTBR, which combines balanced truncation with projection framework.
- Guaranteed passivity, which is less expensive and numerically more stable.
- Propose balanced truncation based approach SP-PriTBR to preserve structure information inherent to RLC circuits in addition to passivity.
- Propose hybrid projection to generate optimal reduced model with both wideband accuracy and exact DC behavior for large-scale RLC circuits.
- Can work with Krylov projection method to scale to large problems.

Questions?