

Flow Time Minimization under Energy Constraints

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Agenda

- ✚ Introduction
- ✚ System Models and Preliminary Results
- ✚ Our Algorithms
 - ▣ Flow Time Minimization
 - Ideal Processors
 - Non-Ideal Processors
 - ▣ Weighted Flow Time Minimization
- ✚ Performance Evaluation
- ✚ Conclusion



Introduction

- ❖ Energy-efficiency is important in system designs



Hardware Methodology for Power Saving

- ❖ Dynamic power management (DPM)
 - ❖ The operation mode of the system
 - ❖ ACPI
- ❖ Micro-architecture technique
 - ❖ Adaptive architecture
 - ❖ Cache management
- ❖ Dynamic voltage scaling (DVS)
 - ❖ Supply voltage scaling
 - Intel Xscale, StrongARM; Transmeta Crusoe
 - Intel SpeedStep, AMD PowerNow!
 - ❖ Threshold voltage scaling



Dynamic Voltage Scaling

- ✿ A higher supply voltage usually results in a higher frequency (or higher execution speed)
 - ✦ $s = k * (V_{dd} - V_t)^2 / (V_{dd})$, where
 - s is the corresponding speed of the supply voltage V_{dd} and
 - V_t is the threshold voltage
- ✿ The dynamic power consumption function $P_d()$ of the execution speeds of a processor is a convex function:
 - ✦ $P_d(s) = C_{ef} V_{dd}^2 s$, in which C_{ef} is the switch capacitance related to tasks under executions
 - ✦ $P_d(s) = C_{ef} s^3 / k^2$, when $V_t = 0$
- ✿ The static power consumption comes from the leakage current
 - ✦ A constant or
 - ✦ A linear function of the supply voltage



Energy-Efficient Scheduling versus Energy-Constrained Scheduling

- ✦ *Energy-efficient* scheduling is to minimize the energy consumption while the performance index or the timing constraint is guaranteed
- ✦ *Energy-constrained* scheduling is to maximize the performance or system rewards under a specified energy constraint



Performance Maximization

✦ Reward Maximization

- ✦ Rusu et al. (RTSS'02)
- ✦ Kang et al. (RTSS'02)
- ✦ Rusu et al. (ECRTS'03)
- ✦ Chen et al. (SAC'04)
- ✦ AlEnawy and Aydin (ECRTS'04)
- ✦ Chen and Kuo (RTSS'05)

✦ Flow Time Minimization

- ✦ Albers and Fujiwara (STACS'06)
- ✦ Pruhs et al. (SWAT'04)

✦ Completion Time Minimization



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System Models

Processor model

- ❑ Ideal processors: $s_{\min} \sim s_{\max}$
- ❑ Non-ideal processors: $(s_{\min} = s_1, s_2, \dots, s_M = s_{\max})$
- ❑ Power consumption function: $P(s)$
 - $P(s)$ is a convex and increasing function
 - $P(s)/s$ is an increasing function

Job model

- ❑ Each job J_j is associated with its computation requirement in CPU cycles: c_j
- ❑ The flow time of a job: the interval length during the release time and the completion time of the job
- ❑ Each job arrives at the same time: 0

Our objective

- ❑ Find a schedule for a given job set $J = (J_1, J_2, \dots, J_N)$ such that the energy consumption is no more than the energy constraint E_b , and the average flow time of these N jobs is minimized



Execution Behavior for Optimal Solutions

There exists an optimal schedule which executes jobs in J in a non-decreasing order of their CPU execution cycles for both ideal and non-ideal processors

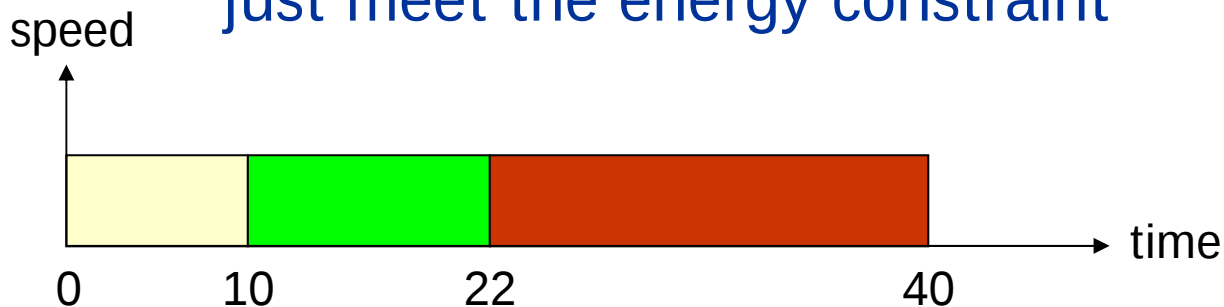


We index jobs so that the execution cycles of the jobs in J are in a non-decreasing order, i.e., shorted job first



A Motivational Example

Execute jobs at the speed that just meet the energy constraint



$$c_1 = 5$$

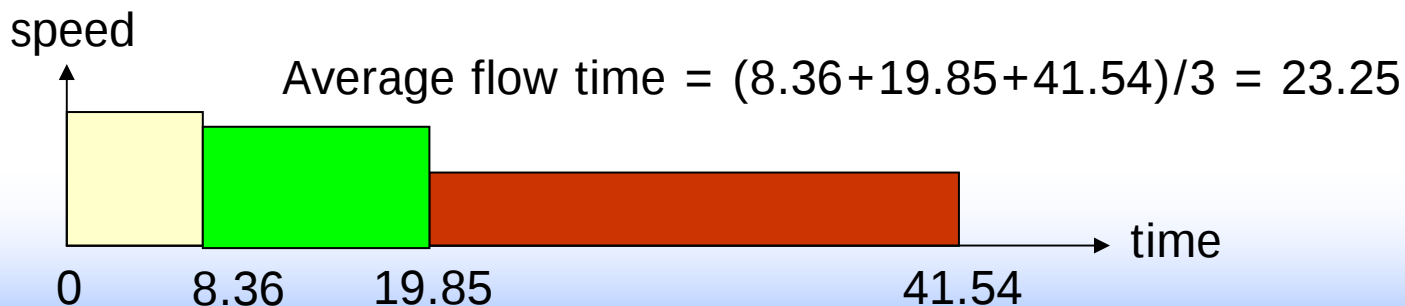
$$c_2 = 6$$

$$c_3 = 9$$

$$P(s) = s^3$$

$$E_b = 5$$

$$\text{Average flow time} = (10 + 22 + 40) / 3 = 24$$



$$\text{Average flow time} = (8.36 + 19.85 + 41.54) / 3 = 23.25$$



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Scheduling for Ideal Processors

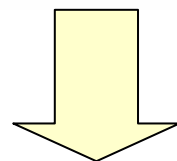
- ✚ An optimal solution executes job J_i at a speed: r_i
- ✚ The energy consumption is $\sum_{j=1}^N P(r_j) \frac{c_j}{r_j}$
- ✚ Executing jobs from J_1 to J_N consecutively leads to a solution
 - ✚ The flow time of job J_j is $\sum_{i=1}^j c_i / r_i$
- ✚ The average flow time is $\sum_{j=1}^N \sum_{i=1}^j \frac{c_i}{r_i} = \sum_{j=1}^N (N - j + 1) \frac{c_j}{r_j}$
- ✚ There exists an optimal schedule with $r_1 \geq r_2 \geq \dots \geq r_N$



Algorithm LM: Optimal Solutions

$O(N^2 \log N)$

minimize $\sum_{j=1}^N (N - j + 1) \frac{c_j}{r_j}$
 subject to $\sum_{j=1}^N P(r_j) \frac{c_j}{r_j} = E_b$, and
 $s_{\min} \leq r_j \leq s_{\max}$, for $j = 1, 2, \dots, N$.



Relax the speed constraint

Yes! Form a
sub-problem

minimize $\sum_{j=1}^N (N - j + 1) \frac{c_j}{r_j}$
 subject to $\sum_{j=1}^N P(r_j) \frac{c_j}{r_j} = E_b$,

Apply Lagrange Multiplier Method

Return solution

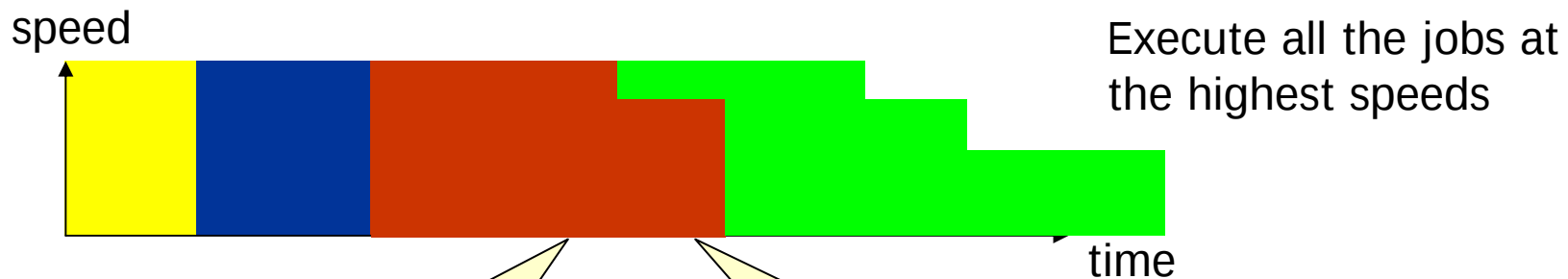
No!

Check if the solution
violates the
speed constraint



Algorithm Greedy for Non-Ideal Processors

$O(NM \log NM)$



Repeated until the energy constraint is satisfied

Slow down the job J_j that is most energy-flow-time efficient



Weighted Flow-Time Minimization

✿ If the execution order is determined

✦ Apply simple revisions of Algorithm LM or Algorithm Greedy to determine the execution speeds

✦ The weighted flow time is

$$\sum_{j=1}^N \left(\sum_{i=j}^N w_i \right) \frac{c_j}{r_j}$$

✿ Otherwise,

✦ Applying the well-known weighted shortest-job-first strategy as the execution order, i.e., c_i/w_i in an increasing order



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Evaluation Setup

✚ Processor:

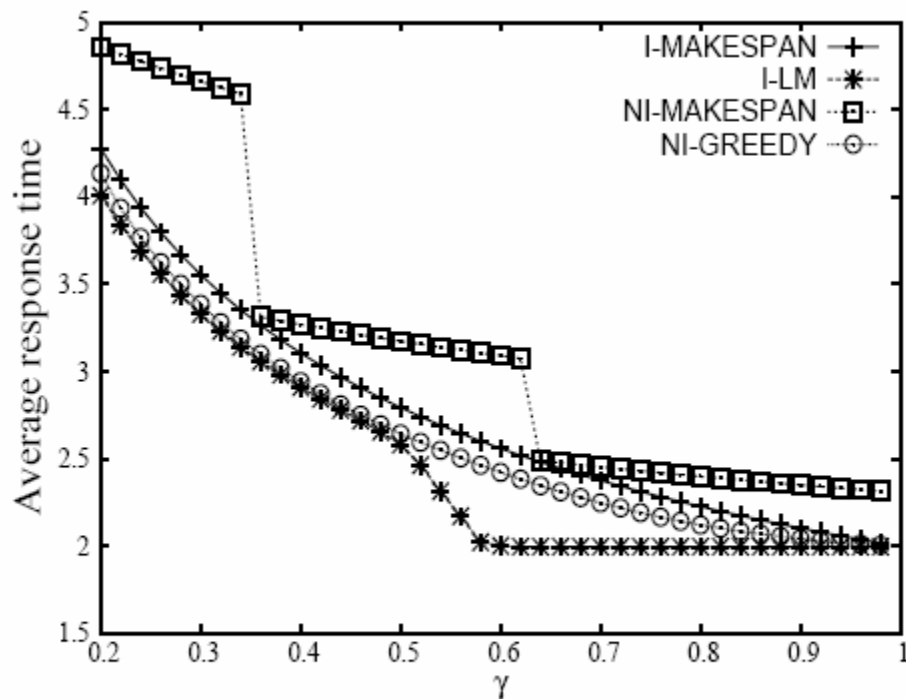
- ✚ Intel XScale: 150, 400, 600, 800, 1000 MHz with 80, 170, 400, 900, 1600 mWatt
 - Normalized so that the highest speed is 1
- ✚ Ideal processor approximation: $s_{\min}=0.15$, $s_{\max}=1$ with $P(s)=0.08 + 1.52 s^3$

✚ Jobs:

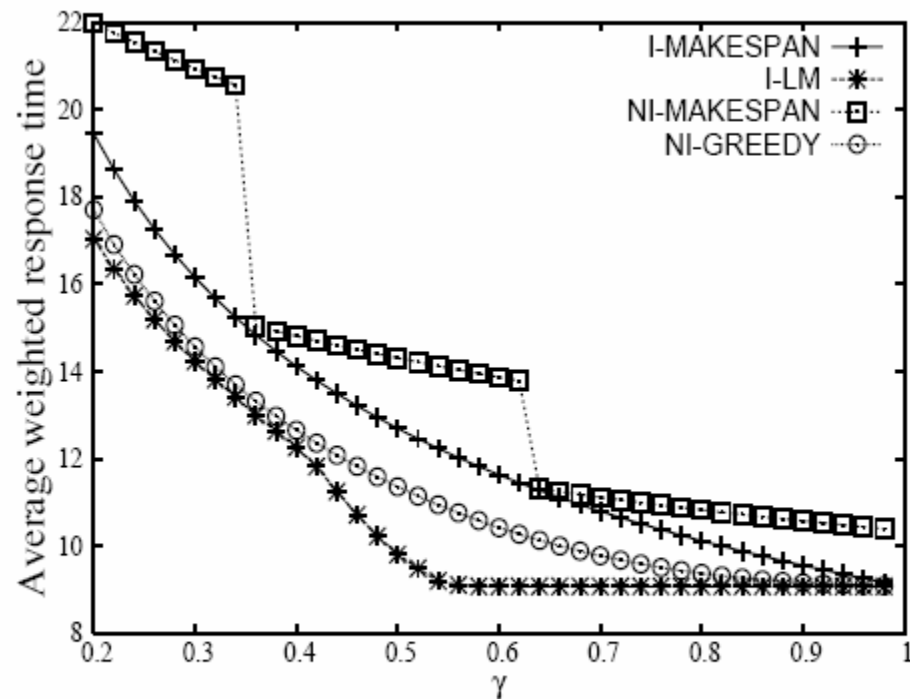
- ✚ Execution cycle is a random variable in $(0, 1]$
- ✚ The weight of a job is a random variable in $(0.1, 10.1]$



Evaluation Results(1)



(a) unweighted cases



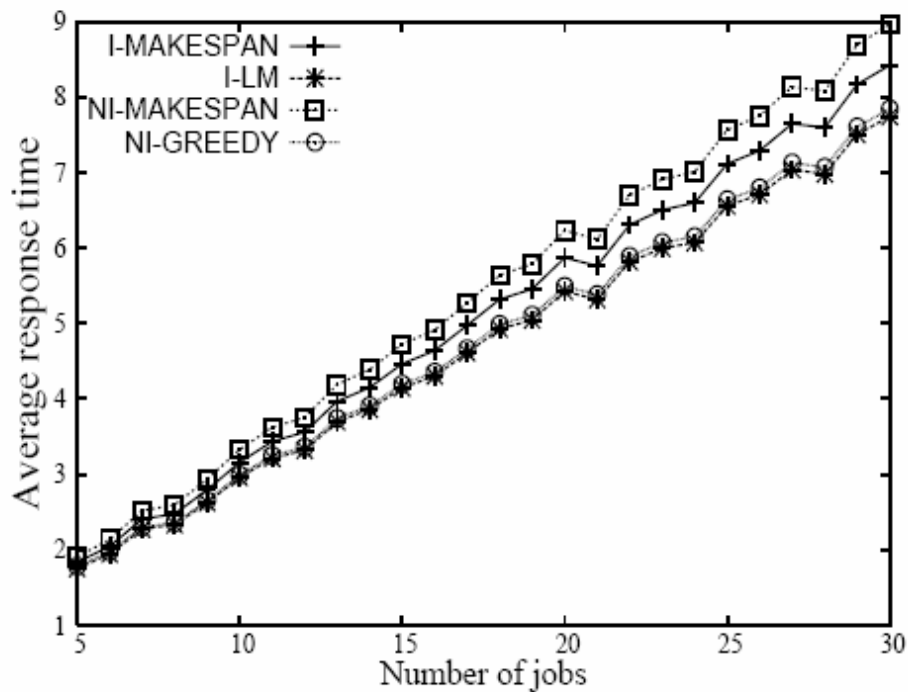
(b) weighted cases

$E_{\max}$ ($E_{\min}$, respectively) is the energy consumption by executing all the jobs at speed $s_{\max}$ ($s_{\min}$, respectively).

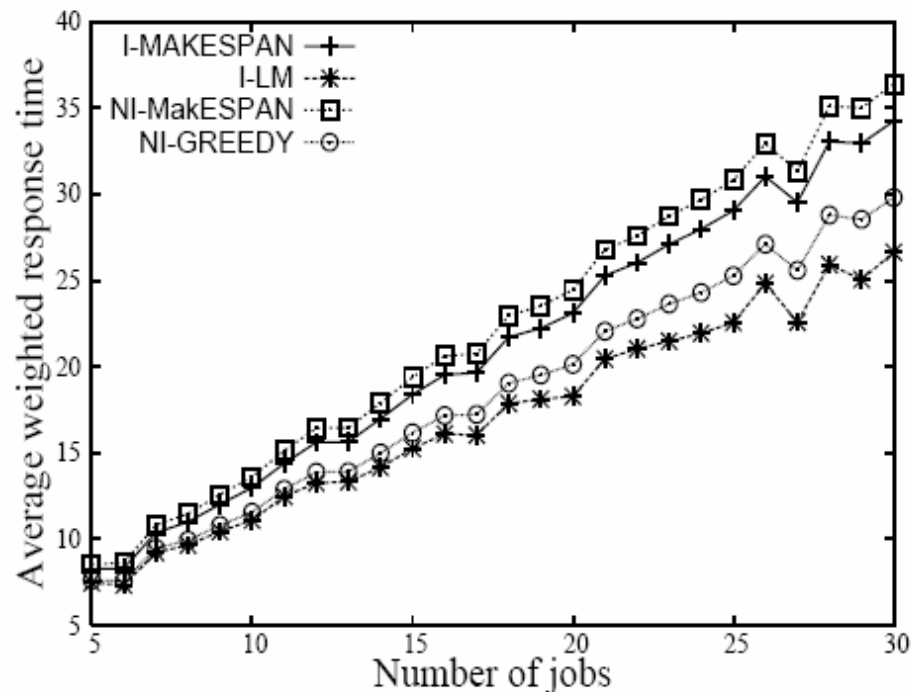
Energy consumption constraint E_b is set as $E_{\min} + \gamma (E_{\max} - E_{\min})$.



Evaluation Results(2)



(a) unweighted cases



(b) weighted cases

γ is 0.4



Conclusion and Future Work

✦ Conclusion

❑ Optimal scheduling algorithms

- Flow time minimization under energy constraints in ideal and non-ideal processors

❑ Heuristic scheduling algorithms

- Weight flow-time minimization under energy constraints

✦ Future work

❑ Minimization of the average flow time for jobs with different arrival times

❑ Worst-case analysis for the minimization of the average weighted flow time under a given energy constraint



Questions and Suggestions?

