# Multithreaded SAT Solving

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# Overview

- Introduction to the Boolean Satisfiability Problem
- Sequential SAT Solving
- Distributed SAT Solving and Previous Work
- MiraXT
  - Motivation
  - Design & Implementation
- Results and Comparison to Other Solvers
- Summary & Future Work

# SAT Introduction

- What are Boolean Satisfiability Problems?
  - Contain Boolean variables
  - Problems are defined in Conjunctive Normal Form (cnf)
    - Consists of a conjunction of Clauses
    - Each clause consist of the inclusive disjunction of literals
    - If problem is SAT every clause must be satisfied
  - Complete solver must find a solution or prove unsolvable
  - Hard problems (NP-Complete)

- Example: $F(x_1,..x_n)=(\neg x_1+x_2)\bullet(\neg x_1+\neg x_2+x_3)\bullet(\neg x_1+\neg x_2+\neg x_3)\bullet\ldots$

# SAT Introduction

- What are SAT solvers used for?
  - Verification (Equivalence Checking, Bounded Model Checking)
  - Automatic Test Pattern Generation
  - FPGA routing
  - AI Planning

- Existing complete powerful SAT solvers:
  - Sequential: zChaff, MiniSat, Siege, Mira, …
  - Parallel: GridSAT, PaSAT, PSATA, ySAT, MiraXT, …
  - All these solvers are based on the classical Davis-Putnam method
  - Current solvers can handle industrial problems with millions of clauses

# Sequential SAT Solvers

$$F(x_1,..x_n)=(\neg x_1+x_2)\bullet(\neg x_1+\neg x_2+x_3)\bullet(\neg x_1+\neg x_2+\neg x_3)\bullet\ldots$$

- 1. Load the problem and perform some preprocessing
  - Elimination of unused or one sided variables
- 2. Decision
  - Select a variable and assigns it a value
- 3. Boolean Constraint Propagation Procedure (BCP)
  - Find all implications and consequences of the decision
  - Signal Conflict
- 4. Conflict Analysis Procedure
  - Finds the reason for the conflict and backtracks if possible
  - Records a conflict clause to prevent future possible conflicts

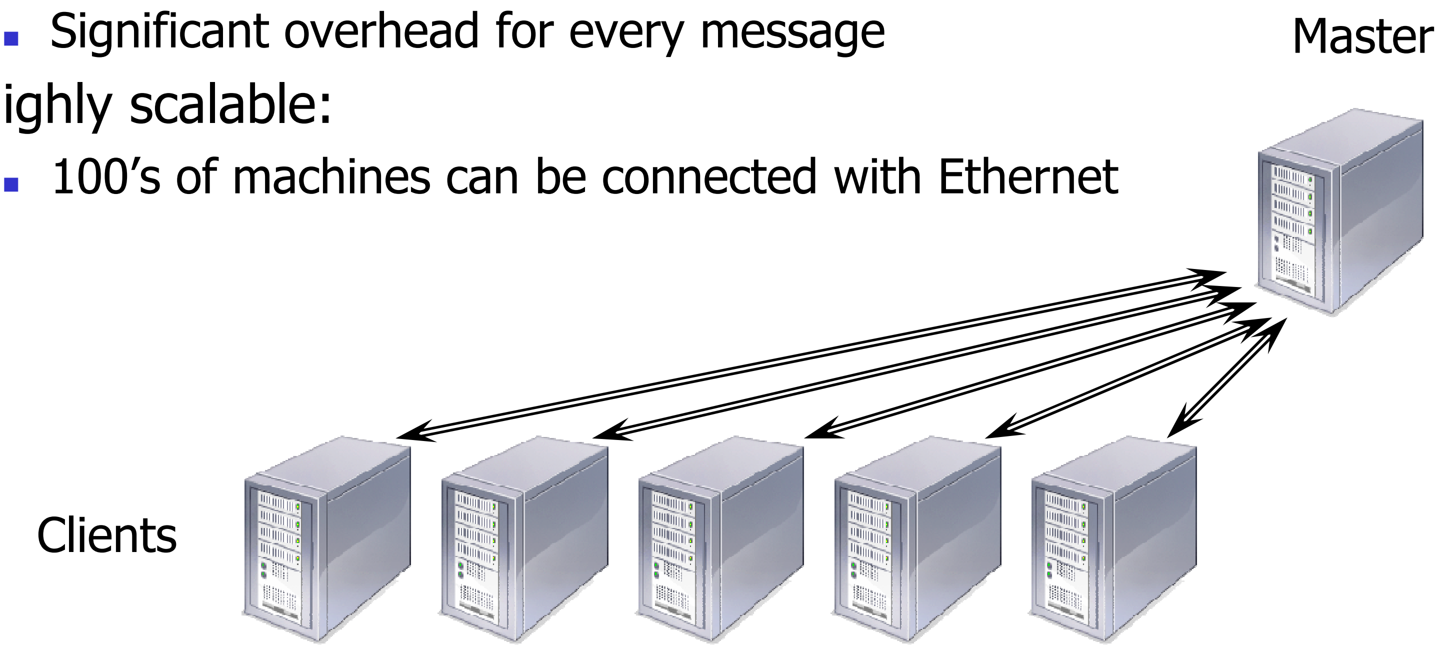
# Parallel SAT Solving

- A Parallel SAT Solver in theory is simple:
  - Divide the problem space and use multiple sequential SAT solvers

- However there are some important issues:
  - How do we divide the search space?
  - Communication
    - Control and Synchronization
    - Knowledge Sharing?

# Parallel SAT Solving – Implementation

- Previous work based on loose integration (GridSAT, PSATO, …)
  - Normally, an existing sequential SAT solver can be used
    - GridSAT uses zChaff, PSATO uses SATO
  - Communication is done with Message Passing
    - Relatively slow form of communication
    - Significant overhead for every message
  - Highly scalable:
    - 100's of machines can be connected with Ethernet

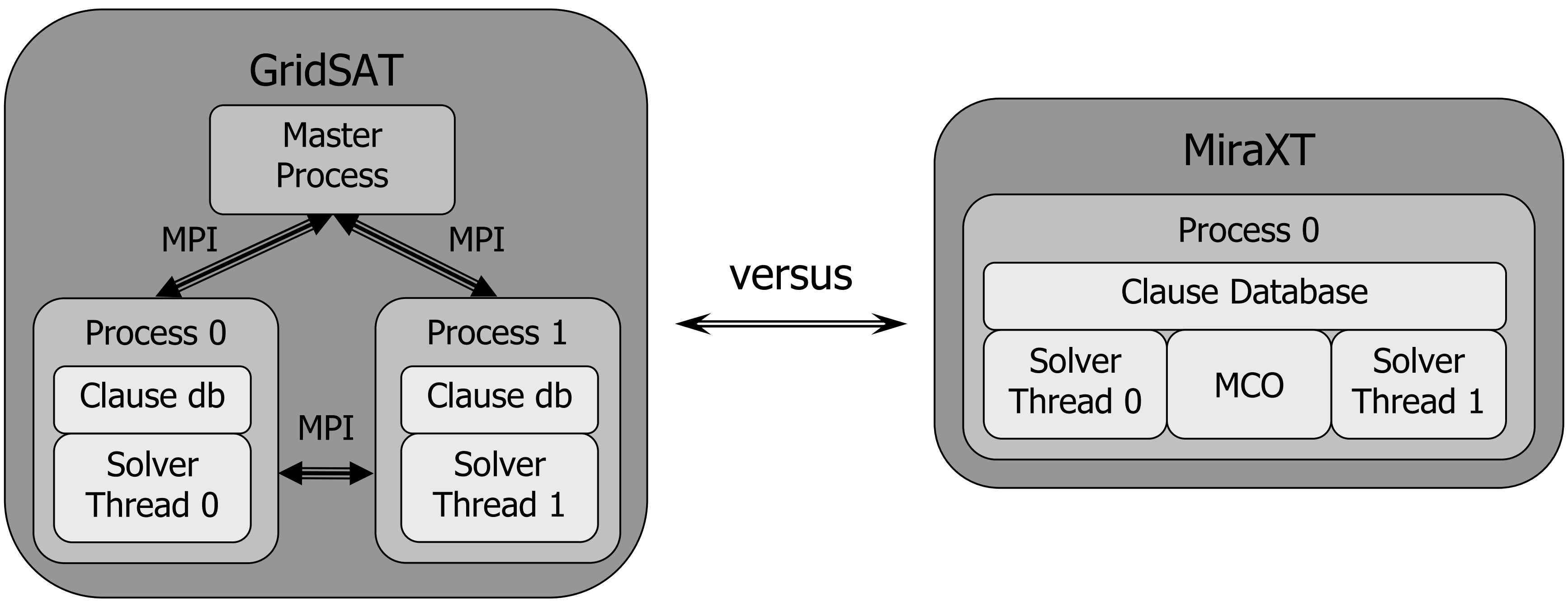


# MiraXT – Motivation

- Focus on workstations not grids
  - Multi-core and multi-CPU workstations
- Tighter integration of SAT Solver threads
  - Shared memory used for communication and clause database
    - Provides high bandwidth, low latency, low overhead communication
- Improve Knowledge Sharing
  - In GridSAT clause with 3 literals or less were shared
    - In PaSATs "Clause Store" clauses with 5 literals or less were shared
  - Due to message passing limitations clauses are sent in bundles
    - Introduces significant latency in the knowledge sharing system
  - Problem - clients choose the clauses they want to share
    - Client should select the clauses they want to use based on their current state!
- Provide competitive single threaded performance

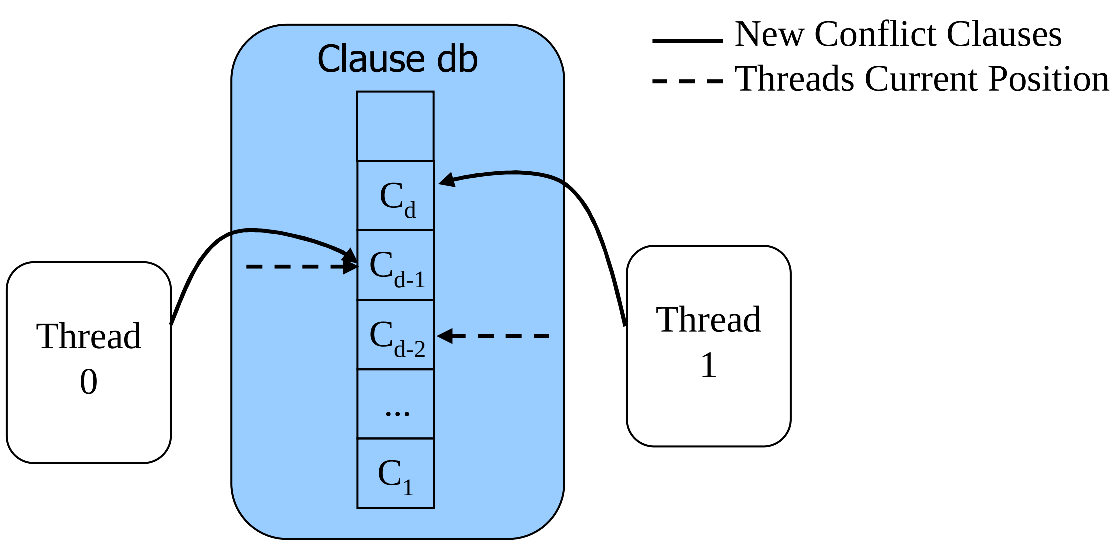
# MiraXT – Design

- Tight integration of SAT Solver threads
- Use of shared memory clause database
- No Master Process
  - Shared object used for control signals and communication



# MiraXT – Shared Clause Database

- What we need from the Clause Database:
  - Fast insertion of new clauses
    - Very little or No Lock Contention (only a pointer insertion)
  - Unimpeded use of clauses by BCP procedure
    - Clause are read only
  - Fast independent deletion of unused clauses



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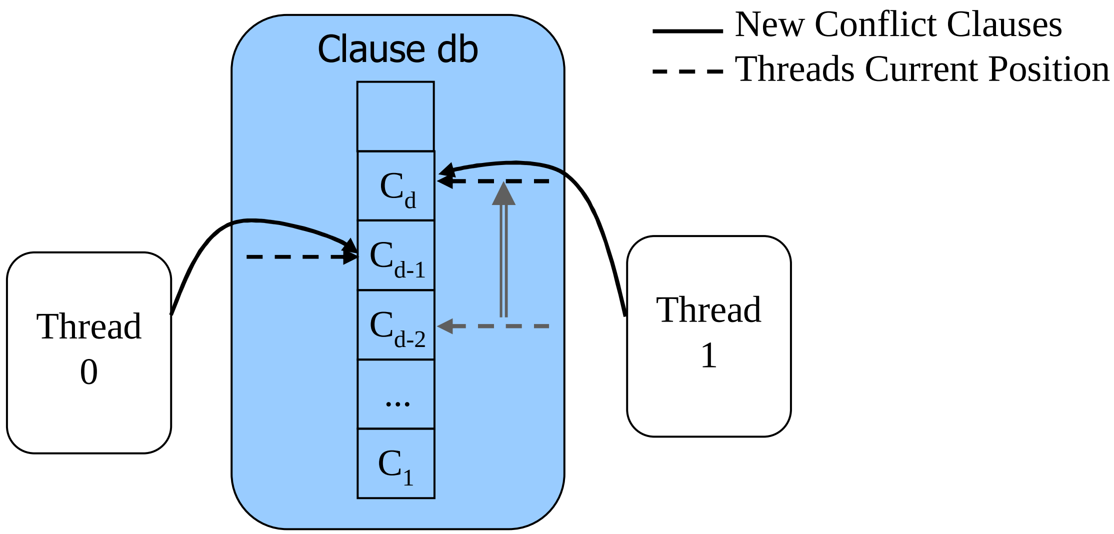
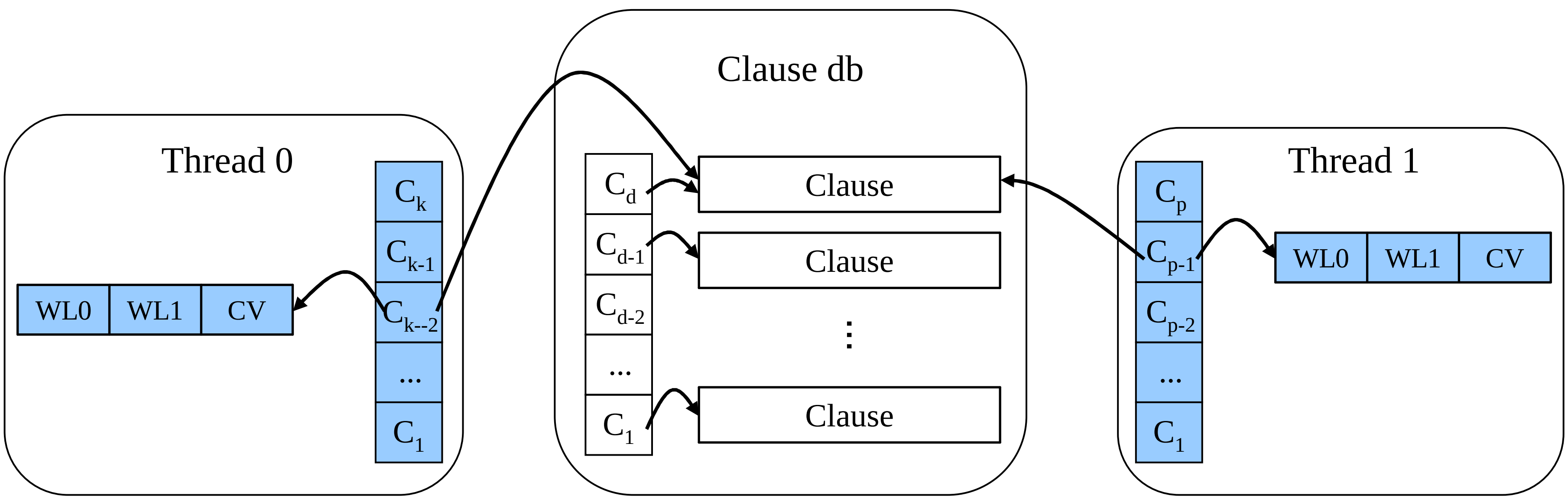


# MiraXT – Shared Clause Database

- Problem:
  - Clauses are read only (cannot easily mark WL)
  - Where are the clauses physically located?
    - Remote memory is slower than local memory
- Solution: Watched Literal Reference List (WLRL)
  - Basic idea: each thread has a partial local copy of every clause
    - Contains and provides fast access to both WL (fast BCP)
    - Contains a cache variable
    - CV is easy to calculate, it is the old WL if the WL is replaced
  - Unit, binary, and ternary clauses are fully contain within structure

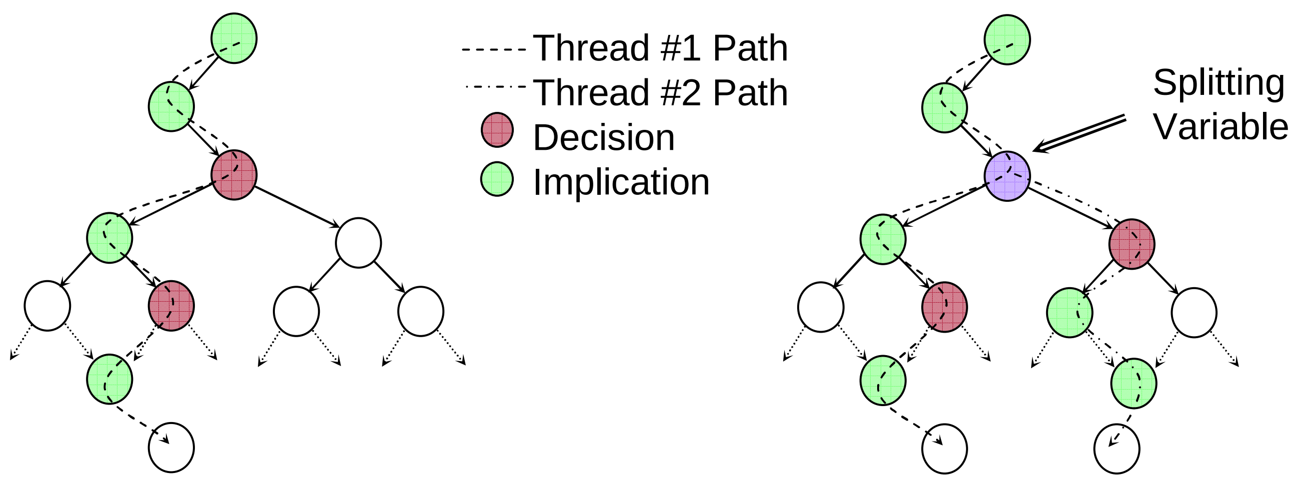
# MiraXT – Watched Literal Reference List

- WLRL – Contains a partial reference copy of every clause



# MiraXT – Sub-problem Generation

- First come first serve (not served)
  - Optimal first DL splitting variable taken with 2 threads
  - Very low overhead technique
- Pre-processing important
  - Elimination of many possible splitting variables
  - Reduces redundancy in the problem



# MiraXT – Master Control Object

- All communication is done in a passive way
  - Threads poll a simple Boolean flag to check for new events
  - If there are new events a more complicated procedure is run
    - This procedure requires locks
- What object does it contain?
  - Decision Stack Queue
    - First client thread to respond to message donates a decision stack
    - Once a sub-problem is added, a sleeping thread is signalled
  - Queue of sleeping threads
    - Idle threads are put to sleep so they don't waste CPU cycles
    - Sleeping threads are served randomly with new sub problems
  - Statistics

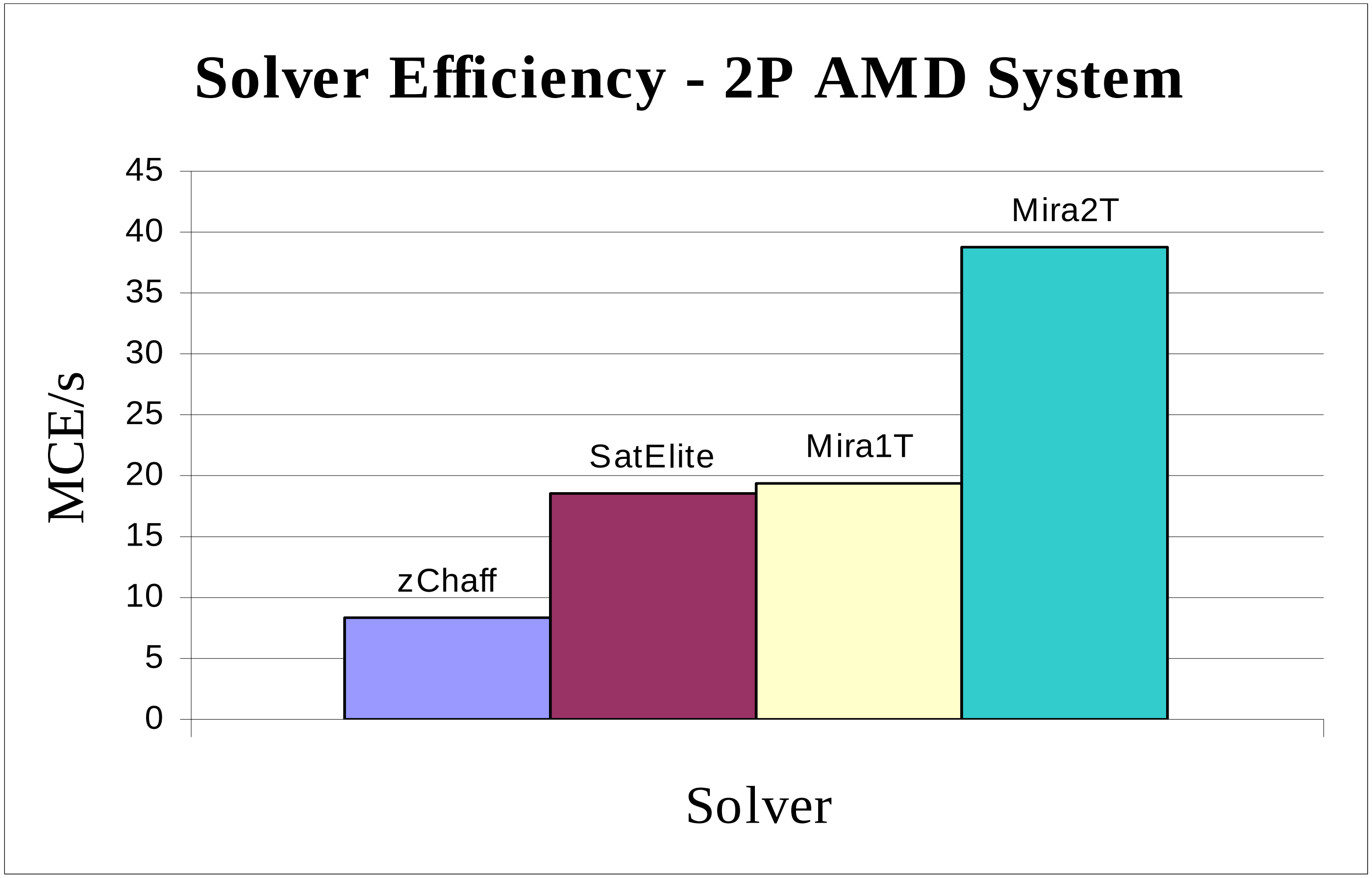
# MiraXT Solver Threads

- Sequential MiraXT solver Highlights:
  - Pre-processing
    - Variable and Clause Elimination (satELite)
    - Unit Propagation Look Ahead (Berre)
  - Advanced Decision Heuristic with Random Restarts
    - Modified Variable State Independent Decaying Sum (zChaff)
  - Boolean Constraint Propagation (BCP)
    - Use of watched literals (zChaff)
    - Early Conflict Detection Based BCP with IQS (Mira)
  - Conflict Analysis
    - First UIP based, with non-chronological backtracking and conflict clause recording (zChaff/Grasp)
    - Conflict Clause Deletion (Berkmin)

# MiraXT Design Results

- Almost No Lock Contention
  - Only a few contention per second (2-3/s with 2 threads)
  - Problem with ySAT (up to 10% of time spent waiting)
- Cache Variable Performance
  - 84% of clauses are evaluated with only the WLRL
  - Can store entire unit, binary, and ternary clauses
- Excellent BCP/Sec Scaling (Industrial example)

# MiraXT Design Results – BCP



# MiraXT – Comparison to Other Solvers

- Benchmarks
  - 2004 IBM Bounded Model Checking Benchmarks (over 1000)
  - Industrial 2005 Benchmarks from the SAT 2005 Competition
- All benchmarks were pre-processed by SatELite

- Used an AMD Dual Processor Linux System
  - 2 Opteron processors running @ 2.6 GHz
  - 2 GB of memory per processor (4 GB total)
  - SMP Linux Kernel 2.6.*

# Results

| Comparison of Solvers | | | | |
|---|---|---|---|---|
| Solver | IBM BMC 2004 | | Industrial 2005 | |
| | $T^2$ | #S | $T^2$ | #S |
| Mira2Ta | 81.4 | 923 | 18.4 | 183 |
| Mira2Tb | 86.5 | 923 | 20.7 | 182 |
| Mira1T | 120.2 | 900 | 26.4 | 178 |
| SatELite | 116.9 | 901 | 28.5 | 176 |
| zChaff | 327.4 | 784 | 35.9 | 175 |
| ySat2T | 509.6 | 709 | 99.4 | 136 |
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# Summary

- Single threaded performance is Competitive
  - Competitive with SatELite, 2005 & 2006 SAT competition winner
- Introduced efficient data structures for multithreaded SAT solving
  - Excellent BCP scaling
- Threaded speedup of ≈ 45% on BMC and Industrial benchmarks
  - 51 % and 41% speedup for SAT and UNSAT benchmarks

- Future Work
  - Combining MiraXT with message passing to allow better scalability