Global Optimization of Common Sub-expressions for Multiplierless Synthesis of Multiple Constant Multiplications

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Presented by H. K. Kwan

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Outline

- Introduction
- Algorithm
  - Literature Review
  - Extended search space
  - Formulation
- Results and Comparisons
- Conclusion

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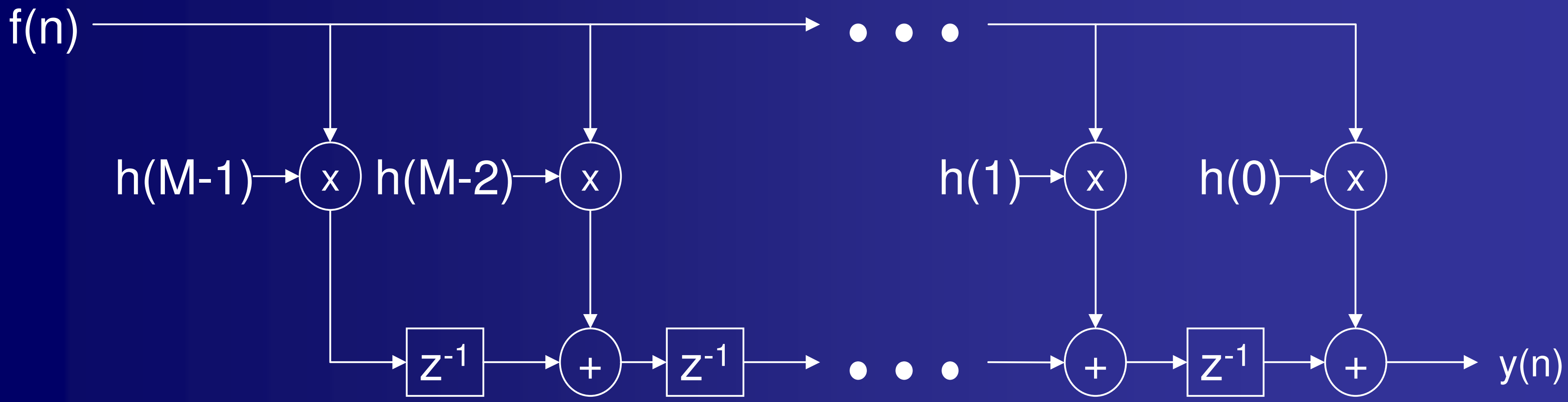
FIR filter and MCM Block

Number of taps = M, m = 1, 2.. M



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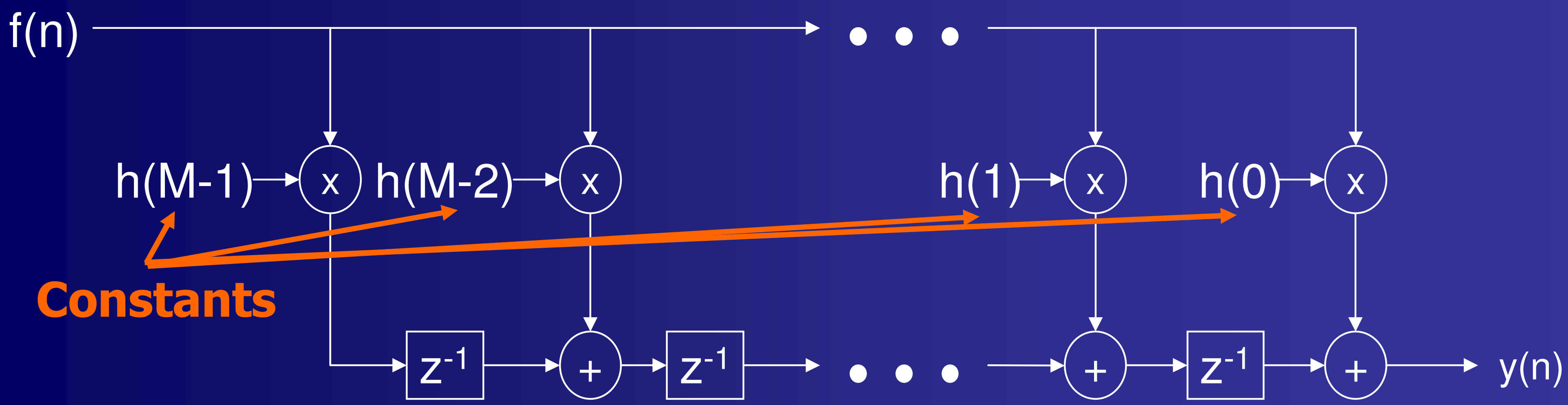

FIR filter and MCM Block

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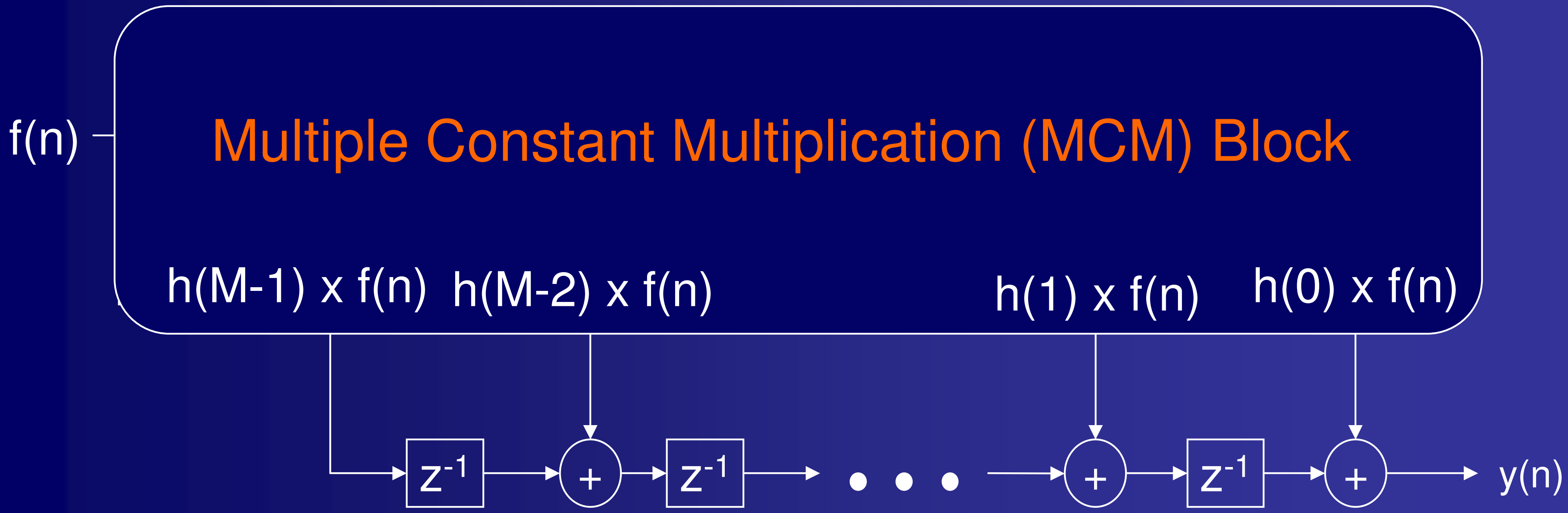
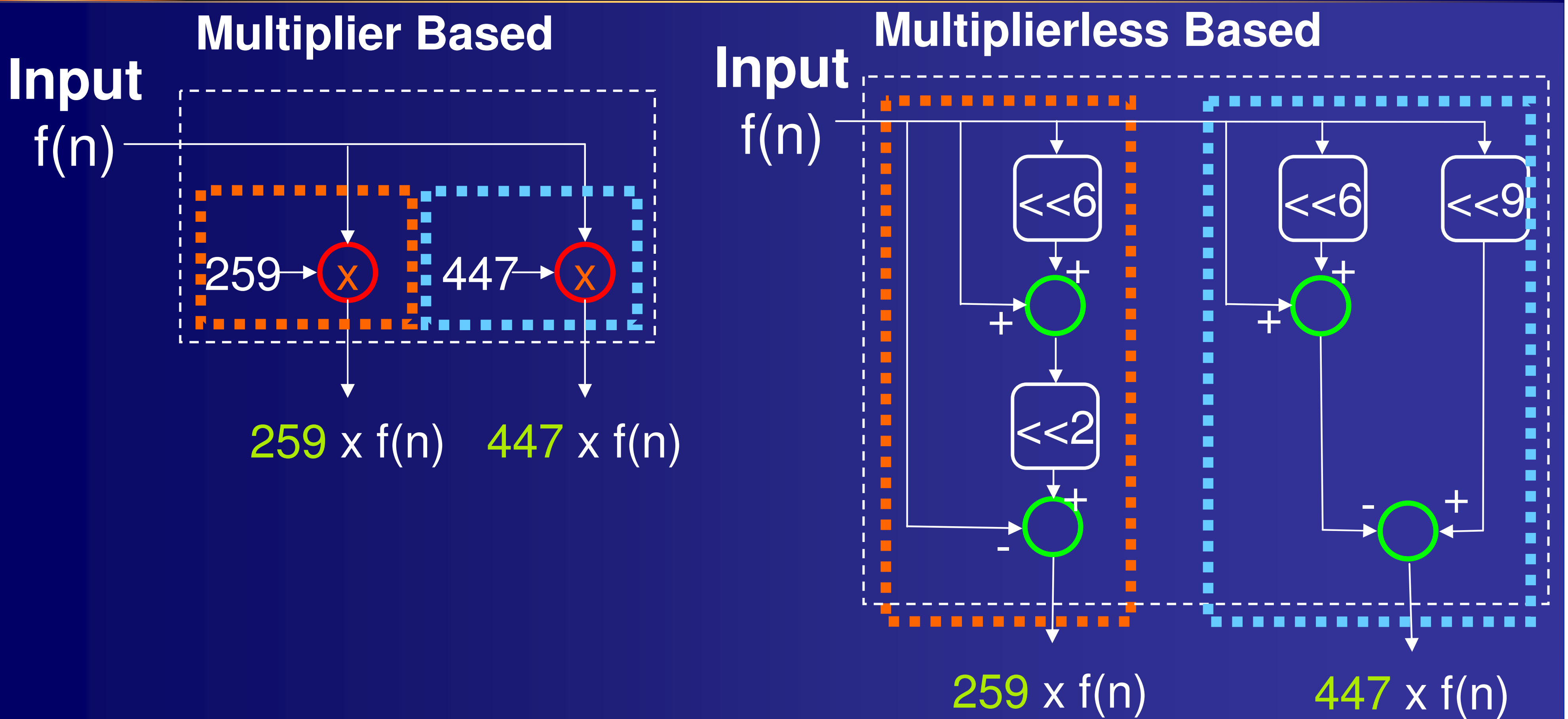


Multiplierless implementation of MCM Block



- Multipliers are implemented by Adders (or subtractors) and Hardwired binary shift

Common Sub-expression sharing in MCM block

- Decomposition of 259:

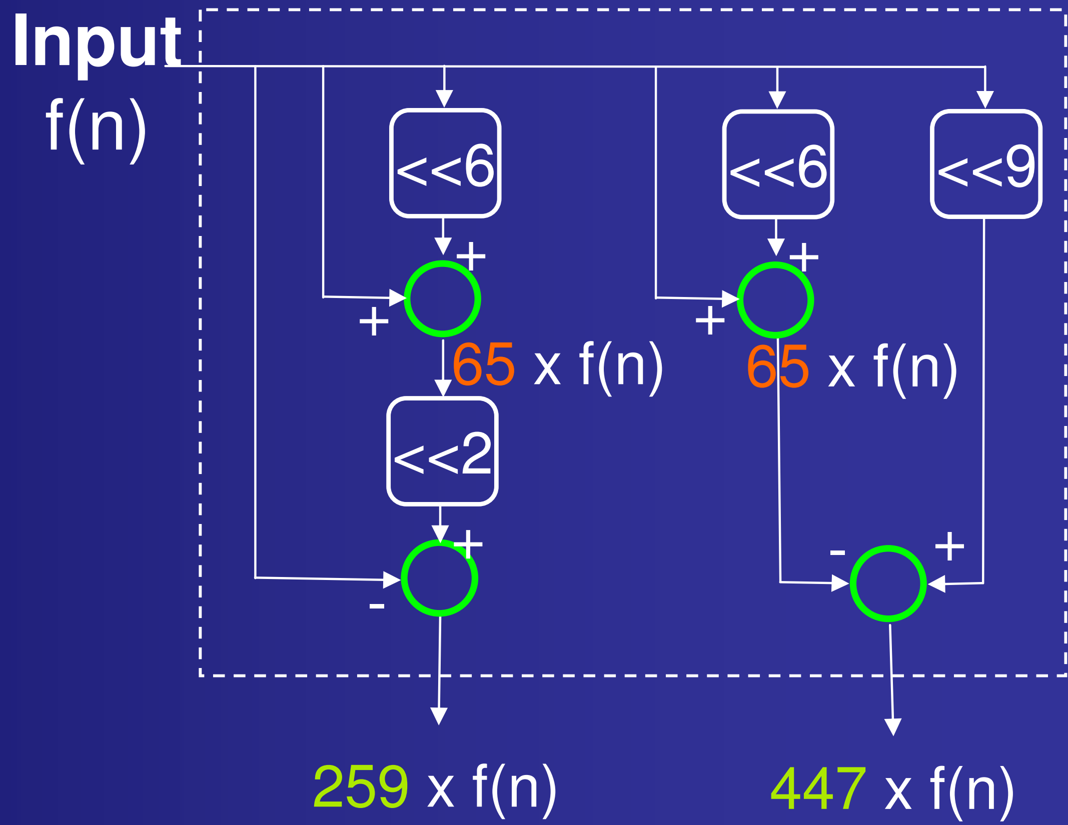
  $259 = -1 + 65 \times 2^2$

- Decomposition of 447:

  $447 = -65 + 2^9$

- Sub-expressions

Multiplierless Based MCM Block



Common Sub-expression sharing in MCM block

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- Sub-expressions
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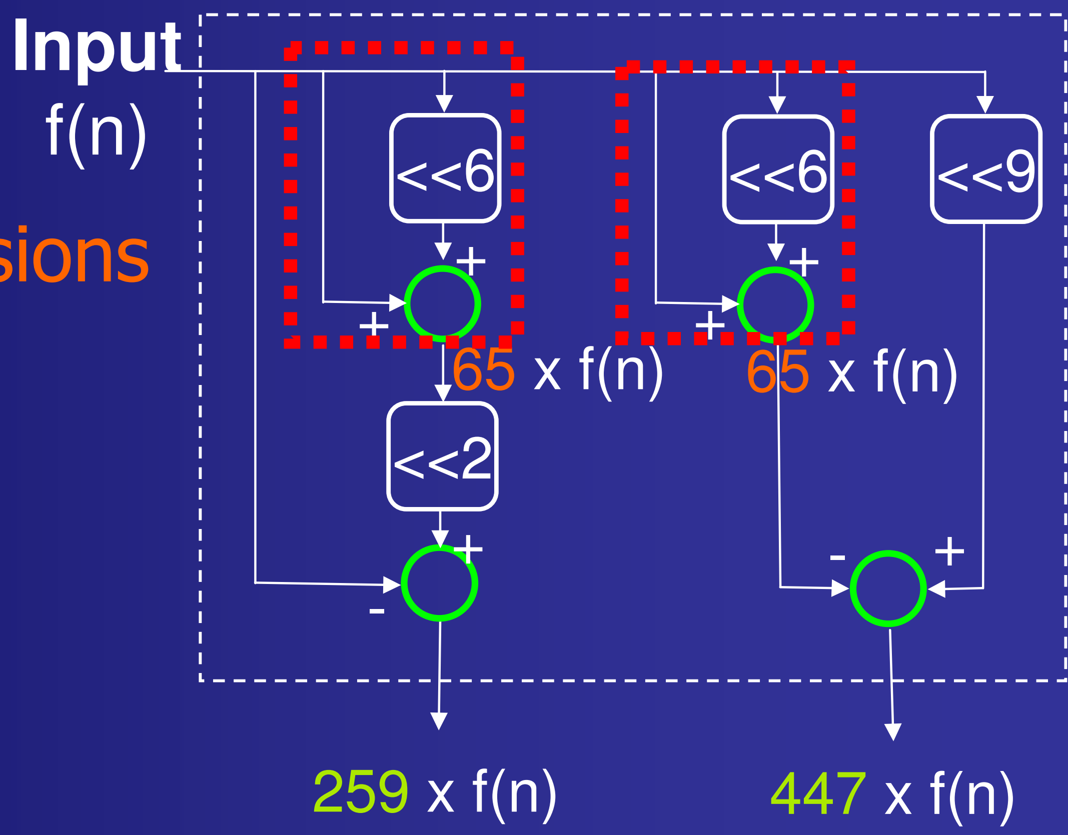
Multiplierless Based MCM Block



Common Sub-expression sharing in MCM block

- Decomposition of 259:

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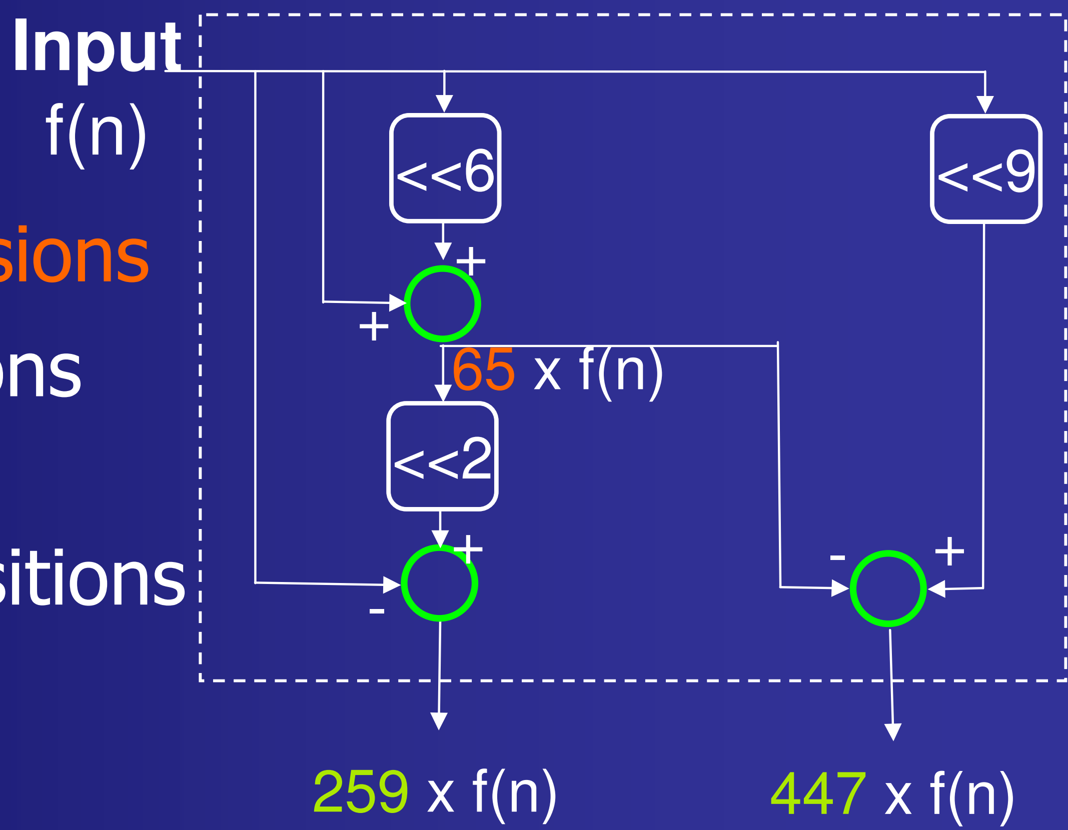
- Sub-expressions
- Common sub-expressions
- Re-use sub-expressions
  – Save hardware cost
- Coefficient decompositions are not unique
  – Search space

Multiplierless Based MCM Block



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Problem Definition

- Minimize the number of adders in the synthesis of output coefficients
  - – Finding common sub-expressions
  - – Feasible solution space of ~ $O(10^n)$
- NP-complete problem, i.e. NO known "best" way to solve

Literature Review

- Different search spaces for common sub-expression
  - Binary (0, 1) [Flores et. al. 05']
  - Signed Digit (-1, 0 ,1) [Park & Kong 02', Macleod & Dempster 05', Flores et. al. 05']
    - Canonical signed digit (CSD), Minimal signed digit (MSD)
- Existed frameworks
  - Common Sub-expression Elimination (CSE) [Yao et. al. 04', Macleod & Dempster 05']
    - Heuristic approach
    - Fast but not optimal solution
  - Binary Optimization [Flores et. al. 05']
    - Optimal within search space
    - A large number of variables and constraints → a bit long computation time

Literature Review

Extended search space

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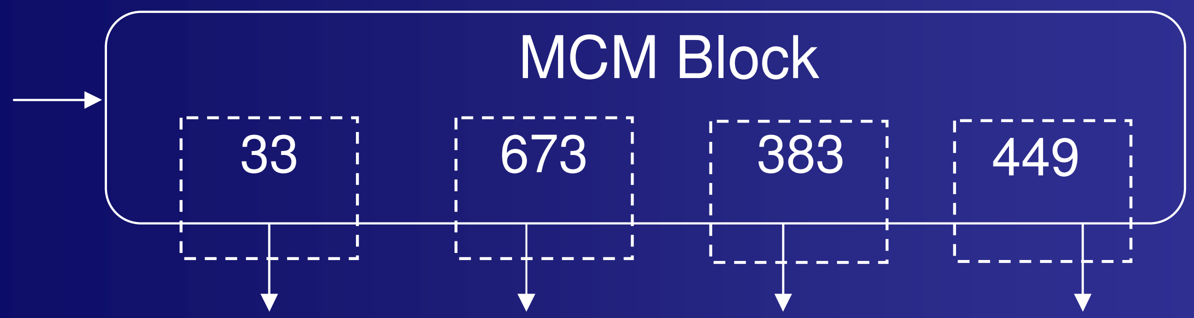
New framework

Outline

- Introduction
- Algorithm
  - Literature Review
  - Extended search space
  - Mixed Linear Integer Programming
- Results and Comparisons
- Conclusion

Search Space

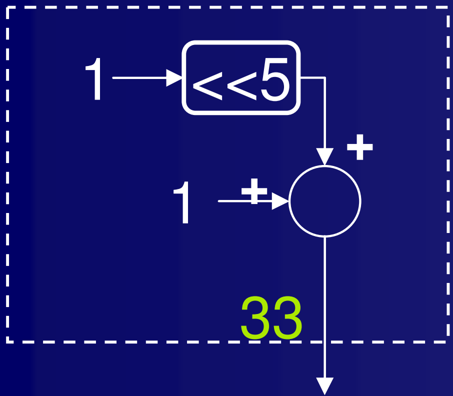
- A simple case
  - Output Set, O_{set}: {33, 673, 383, 449}



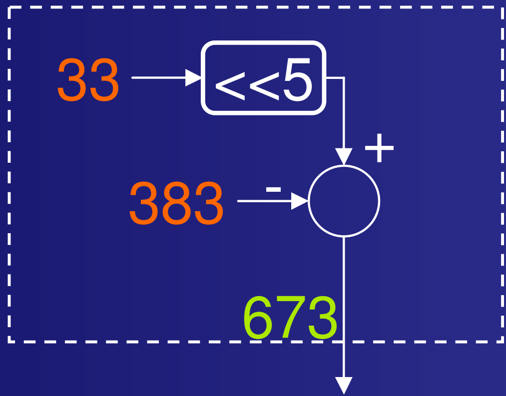
Extended Search Space

- Based on Search space of MSD representation
  - Flores et. al. 2005 (ICCAD)
    - Binary, CSD, MSD representation

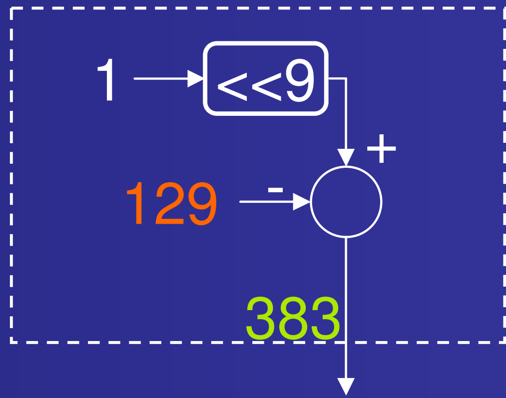
$33 = 1 \times 2^5 + 1$



$673 = 33 \times 2^5 - 383$



$383 = 1 \times 2^9 - 129$



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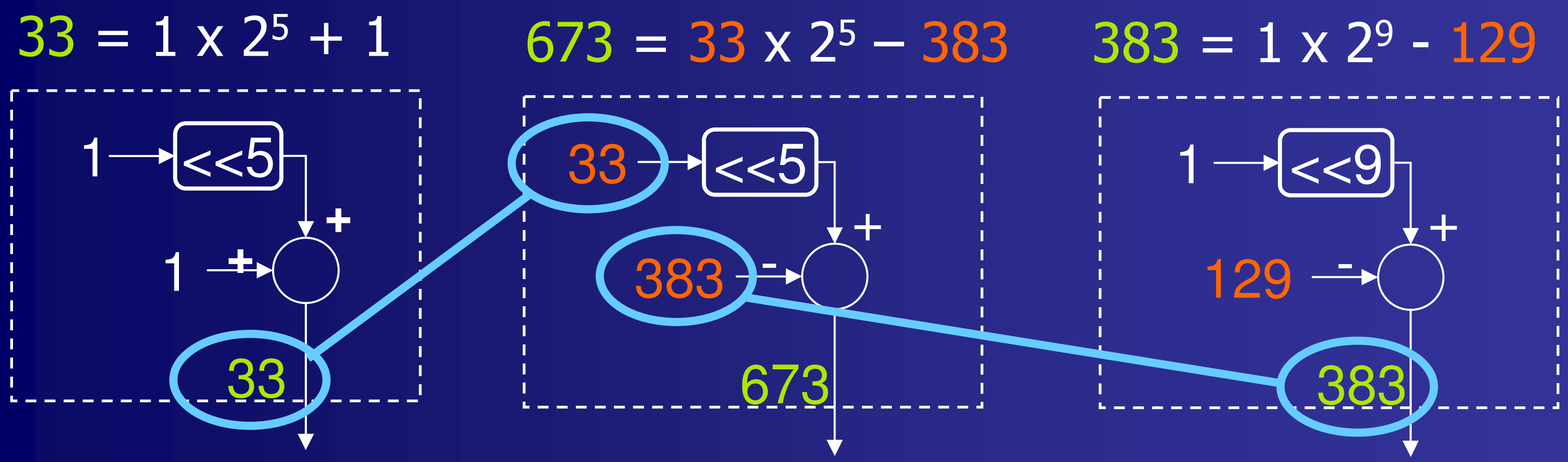

Extended Search Space

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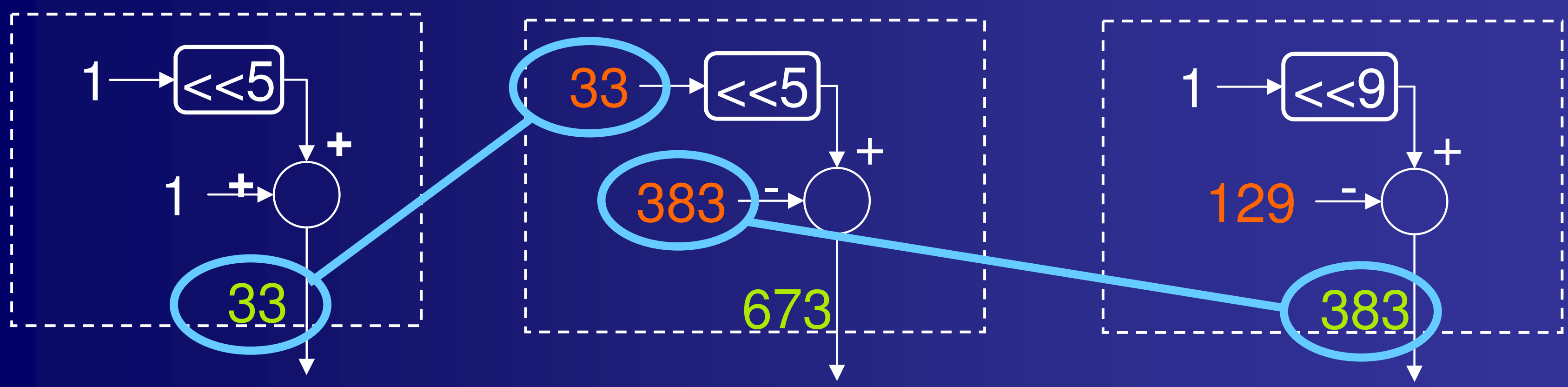
$33 = 1 \times 2^5 + 1$ $673 = 33 \times 2^5 - 383$ $383 = 1 \times 2^9 - 129$



- Proposed Expansion
  - Exploit the relationship between O_{set} and sub-expression

Search Space

$O_{set} = \{33, 673, 383, 449\}$

Search Space

O_{set} = {33, 673, 383, 449}

MSD Representation

1 0 1 0 1 0 0 0 0 1

#nzbit of 673= 4

Digit Pattern Extraction

Search Space

O_{set} = {33, 673, 383, 449}

MSD Representation

1 0 1 0 1 0 0 0 0 1

#nzbit of 673= 4

Digit Pattern Extraction

LT_{673}

(1)	673 =	$2^9 + 161$
(2)	673 =	$2^7 + 545$
(3)	673 =	$2^5 + 641$
(4)	673 =	$21 \times 2^5 + 1$
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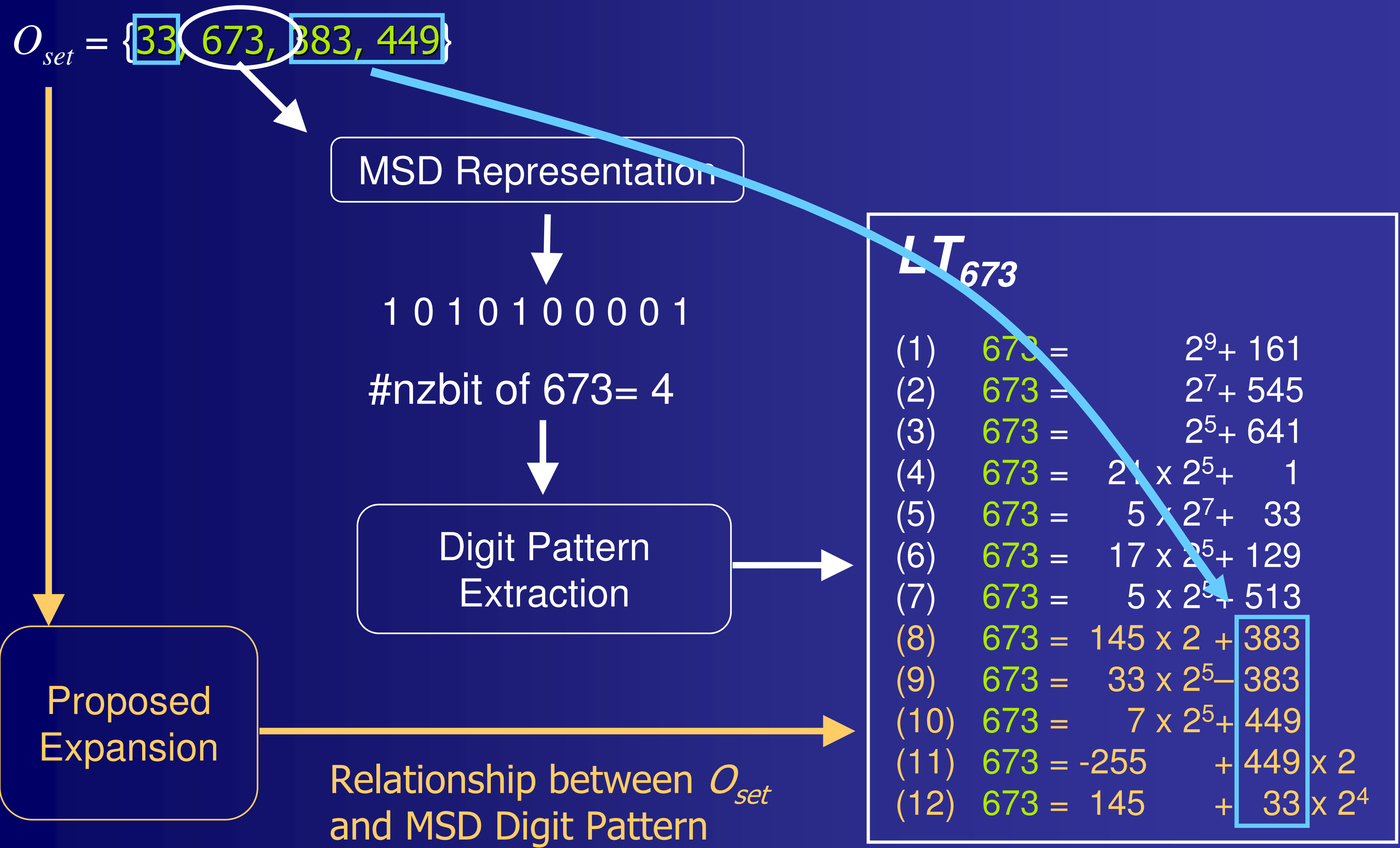
Search Space



Search Space

Expanded search space: Shifted Sum and Difference of O_{set} coefficients , (SSD search space)

O_{set} = {33, 673, 383, 449}

MSD Representation

1 0 1 0 1 0 0 0 0 1

#nzbit of 673= 4

Digit Pattern Extraction

Proposed Expansion

Relationship between O_{set} and MSD Digit Pattern

LT_{673}

(1) 673 = 2^9 + 161
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(11) 673 = -255 + 449 x 2
(12) 673 = 145 + 33 x 2^4

Possible decompositions of O_{set} = {33, 673, 383, 449}

Similarly, ….

One adder without sub-expressions → Optimal → Directly synthesized

LT_{33}

(1) $33 = 2^5 + 1$

LT_{673}

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(12) $673 = 145 + 33 \times 2^4$

LT_{383}

(1) $383 = 1 \times 2^9 - 129$
(2) $383 = -1 \times 2^5 + 511$
(3) $383 = -1 + 3 \times 2^5$
(4) $383 = 1 \times 2^8 + 127$
(5) $383 = 1 \times 2^7 - 255$

LT_{449}

(1) $449 = 1 \times 2^9 - 63$
(2) $449 = -1 \times 2^6 + 513$
(3) $449 = 1 + 7 \times 2^6$

Possible decompositions of O_{set} = {33, 673, 383, 449}

Similarly, ….

One adder without sub-expressions → Optimal → Directly synthesized

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LT_{383}

(1) 383 = 1 x 2^9 – 129
(2) 383 = – 1 x 2^5 + 511
(3) 383 = – 1 + 3 x 2^5
(4) 383 = 1 x 2^8 + 127
(5) 383 = 1 x 2^7 – 255

LT_{449}

(1) 449 = 1 x 2^9 – 63
(2) 449 = – 1 x 2^6 + 513
(3) 449 = 1 + 7 x 2^6

C_{set} :the set of all sub-expressions in all look-up tables of all O_{set} coefficients

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    - Objective function
    - Coefficient decomposition constraints
    - Logic depth constraints
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Formulation

- Mixed Integer Linear Programming (MILP) solved by the solver LINGO
- Define binary variables C_n for each sub-expression / output coefficients n
  - 1 for synthesized, 0 for not synthesized
- Objective function
  – minimizing the No. of sub-expressions required

$$\min \sum_{\forall n \in Cset} C_n \text{ subject to } \begin{array}{ll} C_n \in \{0,1\} & \forall n \in Cset \\ C_m = 1 & \forall m \in Oset \end{array}$$

- $Oset$: all MCM output coefficients (Required)
- $Cset$: all sub-expressions in all the possible decompositions of output coefficients (Flexible)

Decomposition Constraints in MILP

$O_{set} = \{33, 673, 383, 449\}$

- One of the decomposition expressions in LT_{673} has to be implemented

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\begin{aligned}
C_{673} \le\ & C_{161} + C_{545} + C_{641} + C_{21} \\
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And $C_{673} = 1$

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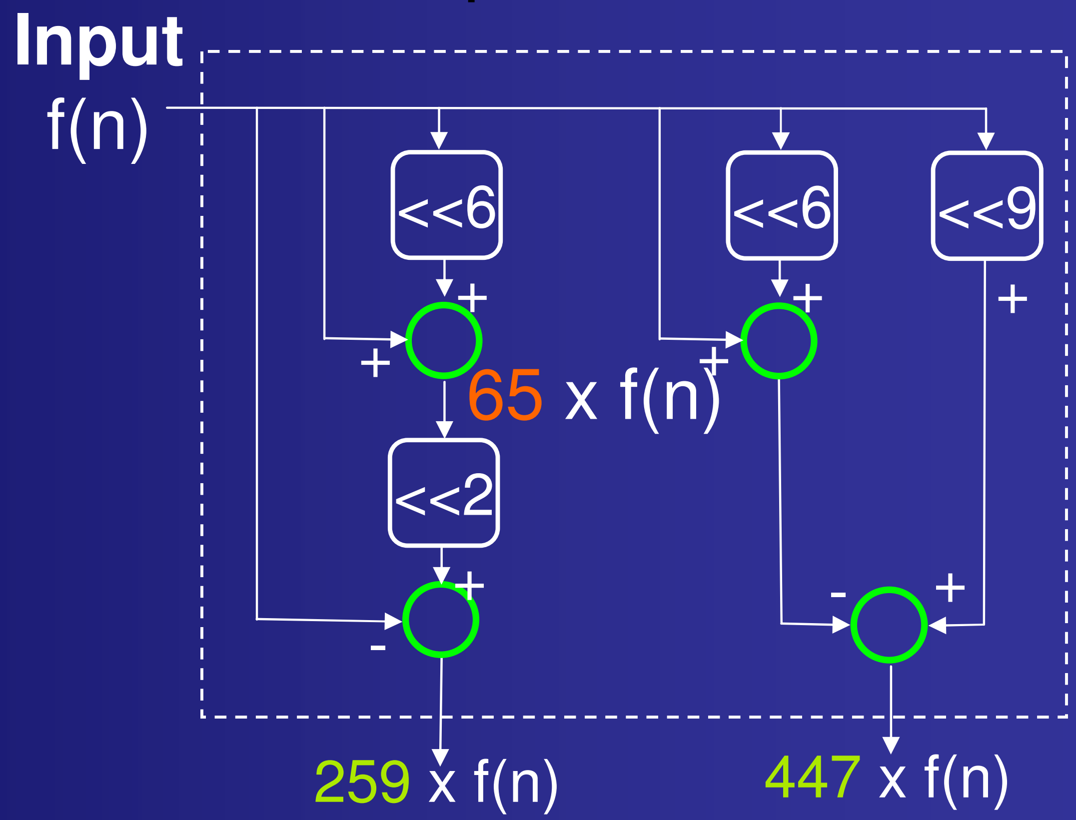
Logic Depth

- Delay of the MCM block:
  - determined by the max. No. of adders that the signal goes through from input to output
- Logic depth :
  - the No. of adders from MCM input to the coefficients



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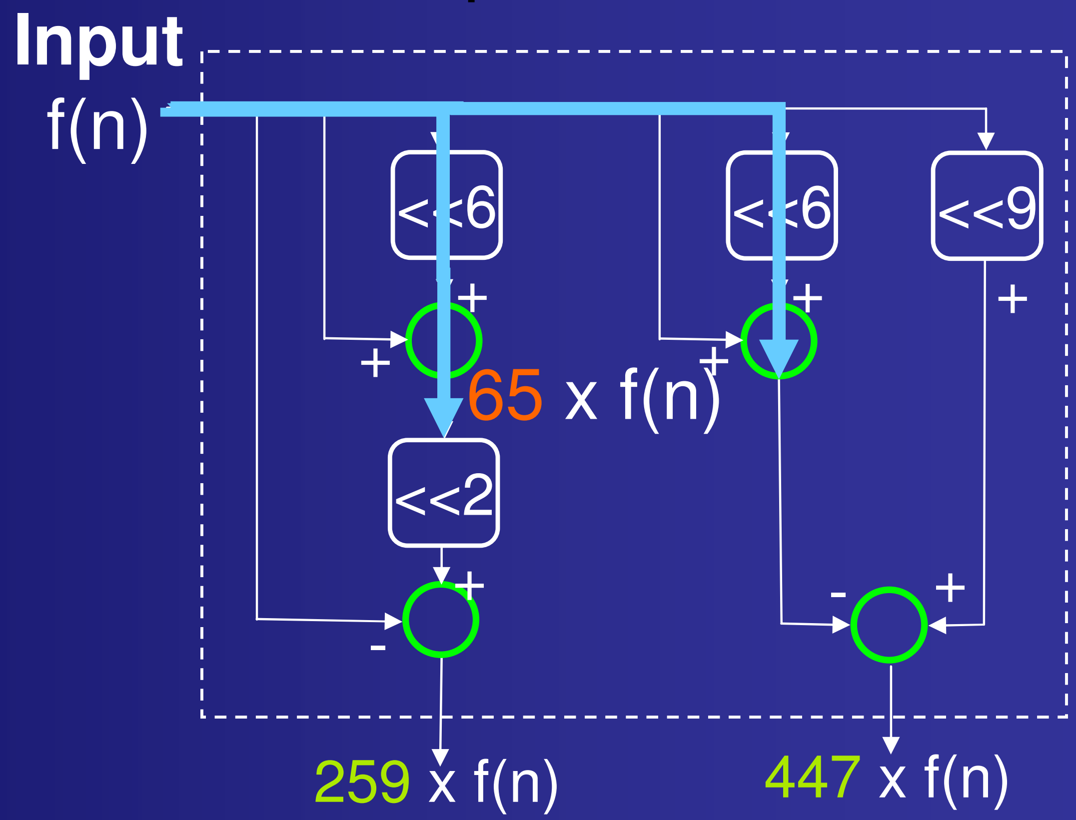
logic depth:

$D_{65} = 1$



Logic Depth

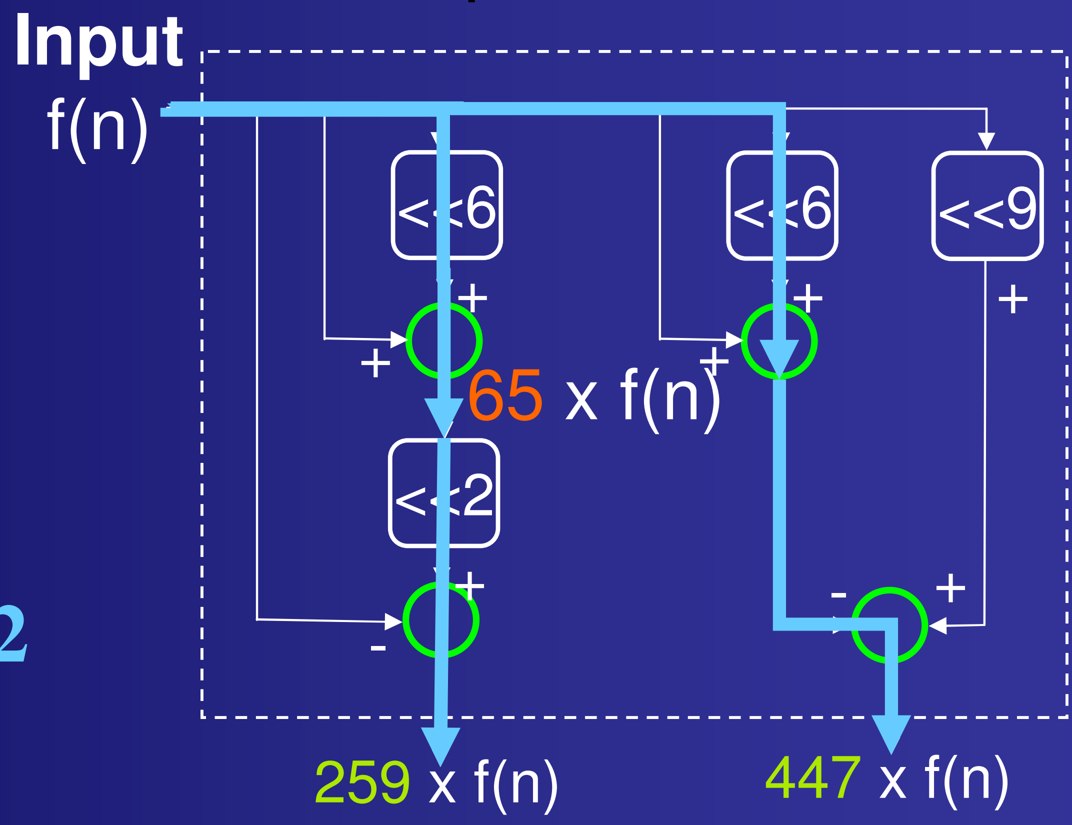
- Delay of the MCM block:
  - determined by the max. No. of adders that the signal goes through from input to output
- Logic depth :
  - the No. of adders from MCM input to the coefficients

logic depth:

$D_{65} = 1$

logic depth:

$D_{259} = D_{447} = 2$



Logic Depth Constraint in MILP

- Set max. logic depth, LD_{max}
- Define a continuous variable D_n for each sub-expression or output coefficients n
  - If $C_n = 1$, D_n = logic depth of coefficient n
  - If $C_n = 0$, D_n is made to equal to LD_{max} (for disabling the constraints)
  - No. of non-zero bit of $n = 2$

$$D_n = LD_{max}(1 - C_n) + C_n$$

  - No. of non-zero bit of $n \geq 2$

$$LD_{max}(1 - C_n) + 2C_n \leq D_n \leq LD_{max}$$

Logic Depth Constraint in MILP

- Example : 673

LT_{673}

(1)	673 =	2^9 + 161
(2)	673 =	2^7 + 545
(3)	673 =	2^5 + 641
(4)	673 =	21 x 2^5 + 1
(5)	673 =	5 x 2^7 + 33
(6)	673 =	17 x 2^5 + 129
(7)	673 =	5 x 2^5 + 513
(8)	673 =	145 x 2 + 383
(9)	673 =	33 x 2^5 – 383
(10)	673 =	7 x 2^5 + 449
(11)	673 =	-255 + 449 x 2
(12)	673 =	145 + 33 x 2^4

Logic Depth Constraint in MILP

- Example : 673

$$C_{673} \le (LD_{max} - D_{161})$$
$$+ (LD_{max} - D_{545})$$
$$+ (LD_{max} - D_{641})$$
$$+ (LD_{max} - D_{21})$$
$$+ (LD_{max} - Max\{D_5, D_{33}\})$$
$$+ (LD_{max} - Max\{D_{17}, D_{129}\})$$
$$+ (LD_{max} - Max\{D_5, D_{513}\})$$
$$+ (LD_{max} - Max\{D_{145}, D_{383}\})$$
$$+ (LD_{max} - Max\{D_{33}, D_{383}\})$$
$$+ (LD_{max} - Max\{D_7, D_{449}\})$$
$$+ (LD_{max} - Max\{D_{225}, D_{449}\})$$
$$+ (LD_{max} - Max\{D_{33}, D_{145}\})$$

LT_{673}

(1)	673 =	$2^9 + 161$
(2)	673 =	$2^7 + 545$
(3)	673 =	$2^5 + 641$
(4)	673 =	$21 \times 2^5 + 1$
(5)	673 =	$5 \times 2^7 + 33$
(6)	673 =	$17 \times 2^5 + 129$
(7)	673 =	$5 \times 2^5 + 513$
(8)	673 =	$145 \times 2 + 383$
(9)	673 =	$33 \times 2^5 - 383$
(10)	673 =	$7 \times 2^5 + 449$
(11)	673 =	$-255 + 449 \times 2$
(12)	673 =	$145 + 33 \times 2^4$

Logic Depth Constraint in MILP

- Example : 673

$$
\begin{aligned}
C_{673} \le\ & (LD_{\max} - D_{161}) \\
&+ (LD_{\max} - D_{545}) \\
&+ (LD_{\max} - D_{641}) \\
&+ (LD_{\max} - D_{21}) \\
&+ (LD_{\max} - Max\,\{D_5, D_{33}\}) \\
&+ (LD_{\max} - Max\,\{D_{17}, D_{129}\}) \\
&+ (LD_{\max} - Max\,\{D_5, D_{513}\}) \\
&+ (LD_{\max} - Max\,\{D_{145}, D_{383}\}) \\
&+ (LD_{\max} - Max\,\{D_{33}, D_{383}\}) \\
&+ (LD_{\max} - Max\,\{D_7, D_{449}\}) \\
&+ (LD_{\max} - Max\,\{D_{225}, D_{449}\}) \\
&+ (LD_{\max} - Max\,\{D_{33}, D_{145}\})
\end{aligned}
$$

→

If 673 is synthesized, i.e. $C_{673}=1$

$$
\begin{aligned}
& D_{161} \le LD_{max} - 1;\ or \\
& D_{545} \le LD_{max} - 1;\ or \\
& D_{641} \le LD_{max} - 1;\ or \\
& D_{21} \le LD_{max} - 1;\ or \\
& Max\,\{D_5, D_{33}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_{17}, D_{129}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_5, D_{513}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_{145}, D_{383}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_{33}, D_{383}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_7, D_{449}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_{225}, D_{449}\} \le LD_{max} - 1;\ or \\
& Max\,\{D_{33}, D_{145}\} \le LD_{max} - 1;
\end{aligned}
$$

Logic Depth Constraint in MILP

If 673 is synthesized, i.e. C_{673}=1

$D_{161} \leq LD_{max} - 1;$ or

$D_{545} \leq LD_{max} - 1;$ or

$D_{641} \leq LD_{max} - 1;$ or

$D_{21} \leq LD_{max} - 1;$ or

$Max\ \{D_5, D_{33}\} \leq LD_{max} - 1;$ or

$Max\ \{D_{17}, D_{129}\} \leq LD_{max} - 1;$ or

$Max\ \{D_5, D_{513}\} \leq LD_{max} - 1;$ or

$Max\ \{D_{145}, D_{383}\} \leq LD_{max} - 1;$ or

$Max\ \{D_{33}, D_{383}\} \leq LD_{max} - 1;$ or

$Max\ \{D_7, D_{449}\} \leq LD_{max} - 1;$ or

$Max\ \{D_{225}, D_{449}\} \leq LD_{max} - 1;$ or

$Max\ \{D_{33}, D_{145}\} \leq LD_{max} - 1;$

Logic Depth Constraint in MILP

If 673 is synthesized, i.e. $C_{673}=1$

$D_{161} \leq LD_{max} - 1;$ *or*

……

$Max\ \{D_{33}, D_{145}\} \leq LD_{max} - 1;$

Logic Depth Constraint in MILP

If 673 is synthesized, i.e. $C_{673}=1$

$$D_{161} \leq LD_{max} - 1; \ \ or$$

……

$$Max\ \{D_{33}, D_{145}\} \leq LD_{max} - 1;$$

LT_{673}

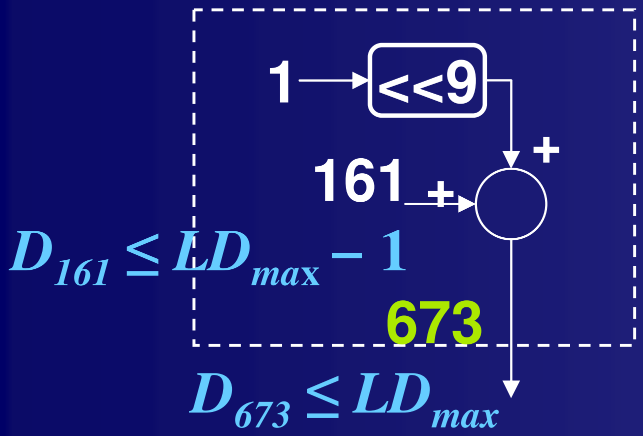
(1) 673 = 2^9 + 161

……

(12) 673 = 145 + 33 x 2^4

Logic Depth Constraint in MILP

(1) 673 = 1 x 2^9 + 161



If 673 is synthesized, i.e. $C_{673}=1$

$D_{161} \le LD_{max} - 1;$ or

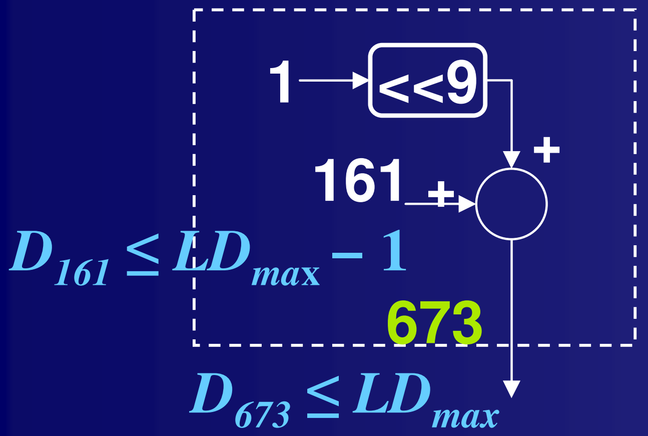
……

$Max\ \{D_{33}, D_{145}\} \le LD_{max} - 1;$

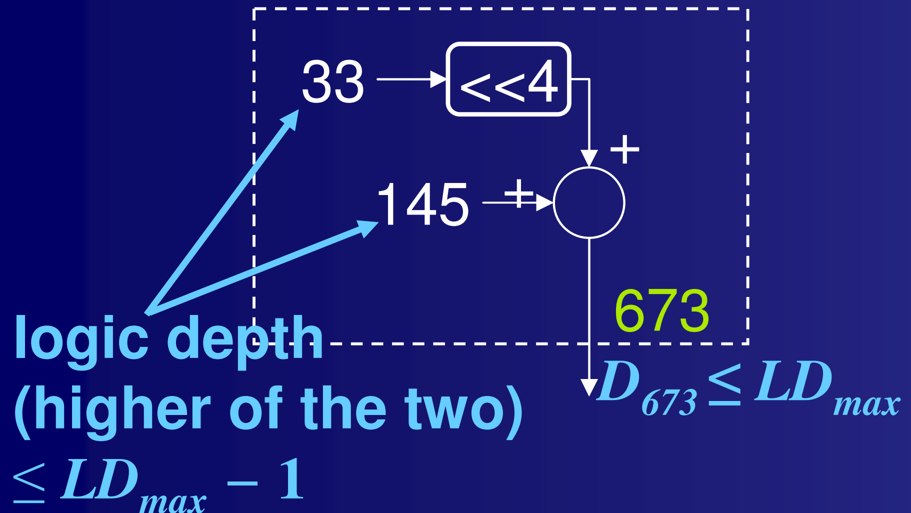
LT_{673}			
(1) 673 =	2^9	+ 161	
……			
(12) 673 =	145	+	33 x 2^4

Logic Depth Constraint in MILP

(1) 673 = 1 x 2^9 + 161



(12) 673 = 145 + 33 x 2^4



If 673 is synthesized, i.e. C_{673}=1

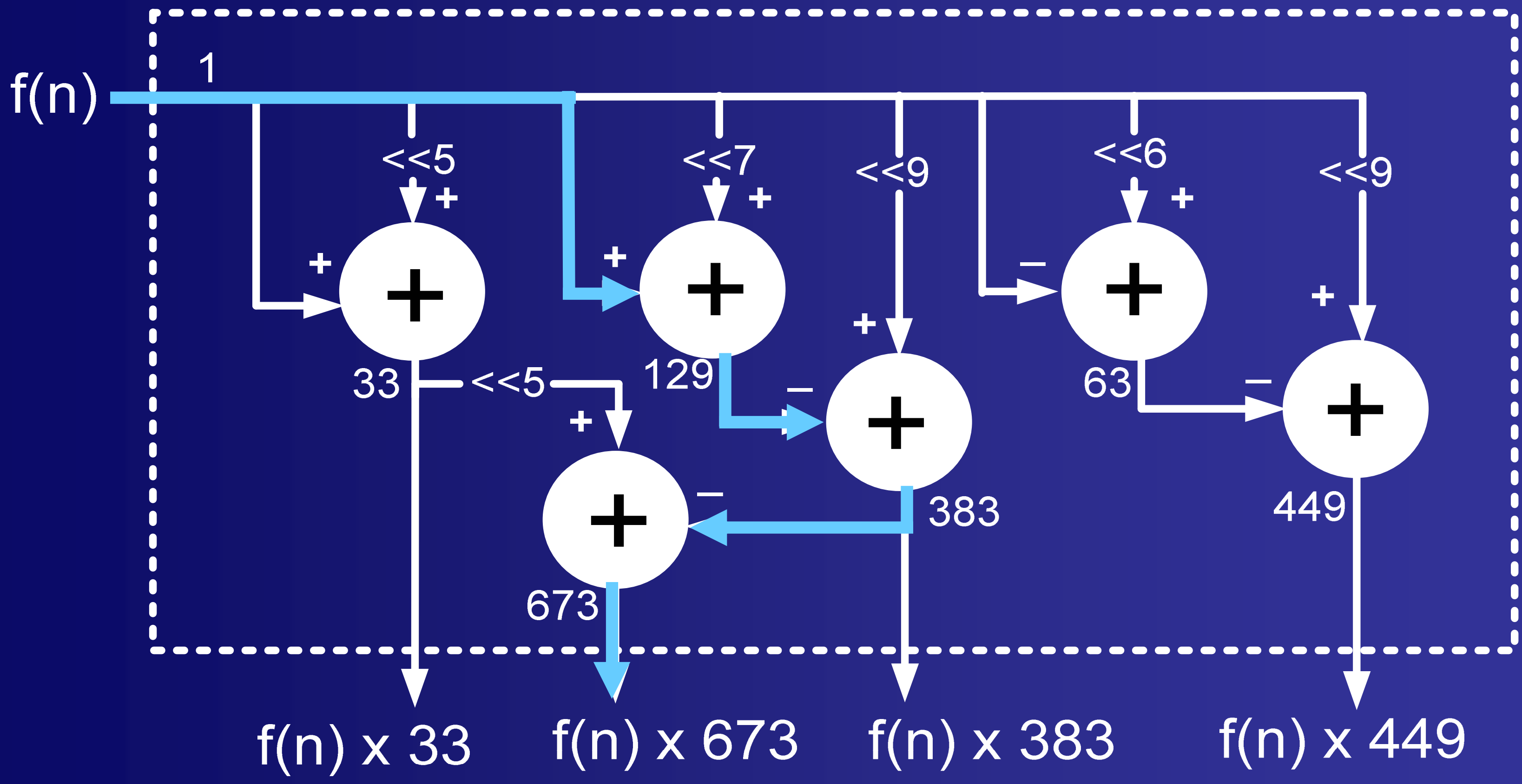
$D_{161} \le LD_{max} - 1$; or

……

$Max\ \{D_{33}, D_{145}\} \le LD_{max} - 1$;

LT_{673}				
(1) 673 =		2^9	+ 161	
……				
(12) 673 =	145		+	33 x 2^4

Example



Logic depth = 3

$33 = 1 + 2^5$

$129 = 1 + 2^7$

$63 = -1 + 2^6$

$383 = -129 + 2^9$

$673 = 33 \times 2^5 - 383$

$449 = 2^9 - 63$

Outline

- Introduction
- Algorithm
  - Literature Review
  - Extended search space
  - Mixed Linear Integer Programming
- Results and Comparisons
- Conclusion

Design examples

Filter	Bit Width (Word-length)	No. of taps
1	8	120
2	10	100
3	12	40
4	12	80
5	12	120
6	14	60
7	14	60
8	14	60

Result comparison (Adder Cost)

Filter	Number of adders			
	Flores et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	10	10	10	10
2	18	18	17	17
3	16	16	15	16
4	29	29	28	28
5	34	34	34	34
6	22	22	21	22
7	34	34	30	30
8	33	33	30	31

$^*LD_{max}$ = 3

Result comparison (Adder Cost)

MSD

- As optimal as literature result

Filter	Number of adders			
	Flores et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	10	10	10	10
2	18	18	17	17
3	16	16	15	16
4	29	29	28	28
5	34	34	34	34
6	22	22	21	22
7	34	34	30	30
8	33	33	30	31

$*LD_{max} = 3$

Result comparison (Adder Cost)

MSD

- As optimal as literature result

SSD

- -10% adder cost

Filter	Number of adders			
	Flores et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	10	10	10	10
2	18	18	17	17
3	16	16	15	16
4	29	29	28	28
5	34	34	34	34
6	22	22	21	22
7	34	34	30	30
8	33	33	30	31

$^*LD_{max} = 3$

Result comparison (Adder Cost)

MSD

- As optimal as literature result

SSD

- -10% adder cost

SSD vs SSD*

- Tradeoff
- +3.3% to +6.7% adder cost
- -25% logic depth

Filter	Number of adders			
	Flores et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	10	10	10	10
2	18	18	17	17
3	16	16	15	16
4	29	29	28	28
5	34	34	34	34
6	22	22	21	22
7	34	34	30	30
8	33	33	30	31

*LD_{max} = 3

Computation Complexity Comparison

	Flores. et. al. 05' (MSD)			Proposed formulation (MSD)		
Filter	#bvar	#cvar	#const	#bvar	#cvar	#const
1	383	0	732	36	19	119
2	818	0	1683	19	69	430
3	1443	0	3172	112	153	949
4	2028	0	4694	143	227	1639
5	1433	0	3104	116	135	934
6	3768	0	9522	222	710	4437
7	2254	0	4979	175	230	1540
8	3568	0	8426	243	419	3268

Computation Complexity Comparison

	Flores. et. al. 05' (MSD)			Proposed formulation (MSD)		
Filter	#bvar	#cvar	#const	#bvar	#cvar	#const
1	383	0	732	36	19	119
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8	3568	0	8426	243	419	3268

Computation Complexity Comparison

Avg: -82% total variables

	Flores. et. al. 05' (MSD)			Proposed formulation (MSD)		
Filter	#bvar	#cvar	#const	#bvar	#cvar	#const
1	383	0	732	36	19	119
2	818	0	1683	19	69	430
3	1443	0	3172	112	153	949
4	2028	0	4694	143	227	1639
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6	3768	0	9522	222	710	4437
7	2254	0	4979	175	230	1540
8	3568	0	8426	243	419	3268

Computation Complexity Comparison

Avg: -82% total variables -34% constraints

	Flores. et. al. 05' (MSD)			Proposed formulation (MSD)		
Filter	#bvar	#cvar	#const	#bvar	#cvar	#const
1	383	0	732	36	19	119
2	818	0	1683	19	69	430
3	1443	0	3172	112	153	949
4	2028	0	4694	143	227	1639
5	1433	0	3104	116	135	934
6	3768	0	9522	222	710	4437
7	2254	0	4979	175	230	1540
8	3568	0	8426	243	419	3268

Computation Time Comparison

Filter	Flores. et. al. 05′ (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	2.38	0.89	1.39	2.38
2	4.69	1.49	4.89	9.21
3	8.80	2.60	5.50	13.83
4	12.80	3.90	12.92	23.51
5	8.30	2.50	13.80	26.90
6	33.03	9.72	25.30	86.00
7	20.8	4.18	15.80	37.49
8	156.82	9.38	17.74	92.13

$^*LD_{max}$ = 3

Computation Time Comparison

MSD:

- -70% CPU time

Filter	Flores. et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	2.38	0.89	1.39	2.38
2	4.69	1.49	4.89	9.21
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6	33.03	9.72	25.30	86.00
7	20.8	4.18	15.80	37.49
8	156.82	9.38	17.74	92.13

$*LD_{max} = 3$

Computation Time Comparison

MSD:

- -70% CPU time

SSD:

- -18% CPU time
- More optimal result (less adders)

Filter	Flores. et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	2.38	0.89	1.39	2.38
2	4.69	1.49	4.89	9.21
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4	12.80	3.90	12.92	23.51
5	8.30	2.50	13.80	26.90
6	33.03	9.72	25.30	86.00
7	20.8	4.18	15.80	37.49
8	156.82	9.38	17.74	92.13

*LD_{max} = 3

Computation Time Comparison

MSD:

- -70% CPU time

SSD:

- -18% CPU time
- More optimal result (less adders)

SSD*

- Longer computation time
- More optimal result
  - Less adders
  - lower logic depth

Filter	Flores. et. al. 05' (MSD)	Proposed MILP formulation		
		MSD	SSD	SSD*
1	2.38	0.89	1.39	2.38
2	4.69	1.49	4.89	9.21
3	8.80	2.60	5.50	13.83
4	12.80	3.90	12.92	23.51
5	8.30	2.50	13.80	26.90
6	33.03	9.72	25.30	86.00
7	20.8	4.18	15.80	37.49
8	156.82	9.38	17.74	92.13

$*LD_{max} = 3$

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Conclusion

- Multiplierless Synthesis of multiple constant multiplications block
  - Extended MSD-based search space
  - MILP framework
  - Logic depth constraint
- Improvement in hardware cost and propagation delay of the synthesis designs
- Significant improvement in computation time

Thank you!

Questions are welcome.

Thank you!

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Back-Up Slides

Generating expanded search space

$n = \pm S1 \times 2^{p} \pm S2 \times 2^{q},$

- $S1 = (n + S2) \times 2^{-p};$

  or

- $S1 = (n - S2) \times 2^{-p};$

  or

- $S1 = (-n + S2) \times 2^{-p};$

  or

- $S1 = n + S2 \times 2^{q};$

  or

- $S1 = n - S2 \times 2^{q};$

  or

- $S1 = -n + S2 \times 2^{q}$

Generating expanded search space

$n = \pm S1 \times 2^p \pm S2 \times 2^q,$

- $S1 = (n + S2) \times 2^{-p};$

  or
- $S1 = (n - S2) \times 2^{-p};$

  or
- $S1 = (-n + S2) \times 2^{-p};$

  or
- $S1 = n + S2 \times 2^q;$

  or
- $S1 = n - S2 \times 2^q;$

  or
- $S1 = -n + S2 \times 2^q$

- $S2$ must be a coefficient in $O_{set};$
- No. of non-zero bit of n $\geq$ No. of non-zero bit of $S1$ $+1;$
- No. of non-zero bit of n $\geq$ No. of non-zero bit of $S2$ $+1.$

Constraints forcing minimum logic depth on top of minimum adder cost

- Minimum logic depth is also desired
- For 673,
  - when $LD_{max} = 3$
  - and No. of non-zero bit of $n > 2$, set
    - $D_{673} \le LD_{max}(2 - C_{673}) - Min\{C_5, C_{33}\}$
    - $D_{673} \le LD_{max}(2 - C_{673}) - Min\{C_{17}, C_{129}\}$
    - $D_{673} \le LD_{max}(2 - C_{673}) - Min\{C_5, C_{513}\}$
    - $D_{673} \ge LD_{max}(1 - Min\{C_5, C_{33}\} - Min\{C_{17}, C_{129}\} - Min\{C_5, C_{513}\})$
    - No of non-zero bit of {5, 17, 33, 129, 513} is two only.
- Force $D_{673} = 2$ if 673 can be synthesized from coefficients that have their logic depth equal to one