Generating Stable and Sparse Reluctance/Inductance Matrix under Insufficient Conditions

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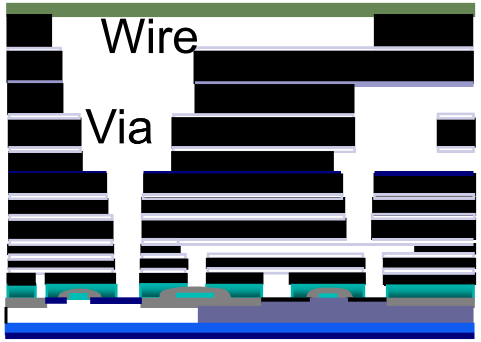
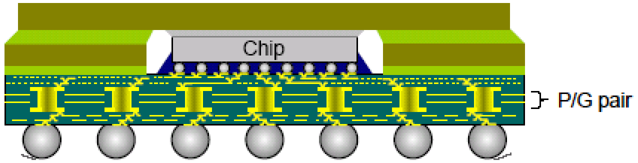
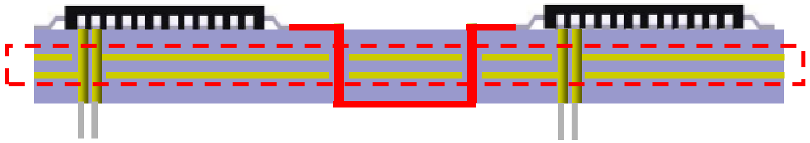
Background



VLSI's



Packages



PCB's

RLC Network

MNA Eqn.

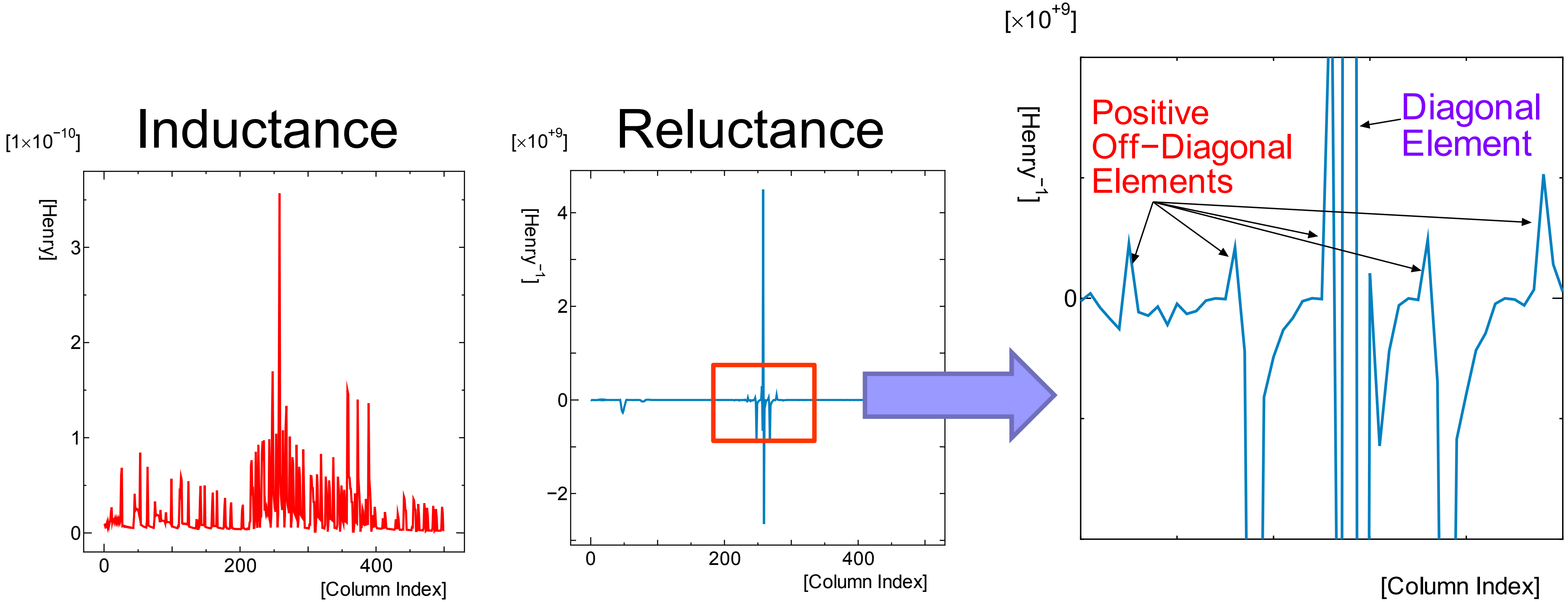
$$\begin{bmatrix} G & A^T \\ -A & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} = b(t)$$

Fast Simulation → Using Sparse Inductance/Reluctance Matrices

- To speed up the transient simulation, we should make the inductance or reluctance matrix sparse.
- Positive definiteness of the inductance or reluctance matrix must be guaranteed for stable circuit simulation.

Method		Fault
Shift & Truncate	ICCAD1995	Inaccurate
Double Inverse	DAC'01	Inaccurate
INDUCTWISE	TCAD'03	No guaranteed Stable
Wire Duplication	TCAD'04	Zero initial Condition Only
Block K	ASP-DAC'04	No guaranteed Stable
Band Matching	ASP-DAC'05	Restricted Structure

- To get a stable and sparse reluctance matrix, off-diagonal elements of reluctance matrix must be negative.
- Most previous works with guaranteed stability are based on a fine discretization of conductors.



- Even if the off-diagonal elements are all negative, this does not mean positive definite.

Negative Eigen Value

$$\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^T \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

- To get the stable and sparse reluctance matrix using the previous methods, the conductors must be more finely discretized to make the reluctance matrix diagonally dominant.

we present how to obtain the stable and sparse reluctance/inductance matrix under insufficient discretization.

Generating Sparse Reluctance Matrix

Inductance (INDUCTWISE)

$$\begin{bmatrix} 1.04 & 0.34 & 0.37 & 0.24 & 0.51 \\ 0.34 & 0.45 & 0.09 & 0.06 & 0.27 \\ 0.37 & 0.09 & 1.04 & 0.34 & 0.41 \\ 0.24 & 0.06 & 0.34 & 0.45 & 0.11 \\ 0.51 & 0.27 & 0.41 & 0.11 & 1.69 \end{bmatrix}$$

reluctance

$$\begin{bmatrix} 1.57 & -0.94 & -0.22 & -0.47 & -0.25 \\ -0.94 & 3.02 & 0.15 & 0.01 & -0.23 \\ -0.22 & 0.15 & 1.42 & -0.93 & -0.24 \\ -0.47 & 0.01 & -0.93 & 3.12 & 0.16 \\ -0.25 & -0.23 & -0.24 & 0.16 & 0.75 \end{bmatrix}$$

Eigen Value $[0.4, 1.0, 1.4, 3.5, 3.7]$

$$\begin{bmatrix} 1.57 & -0.94 & -0.22 & -0.47 & -0.25 \\ -0.94 & 3.02 & 0.00 & 0.00 & -0.23 \\ -0.22 & 0.00 & 1.42 & -0.93 & -0.24 \\ -0.47 & 0.00 & -0.93 & 3.12 & 0.16 \\ -0.25 & -0.23 & -0.24 & 0.16 & 0.75 \end{bmatrix}$$

Eigen Value $[0.4, 1.0, 1.4, 3.4, 3.7]$

Negative off-diagonal elements are found relatively small.

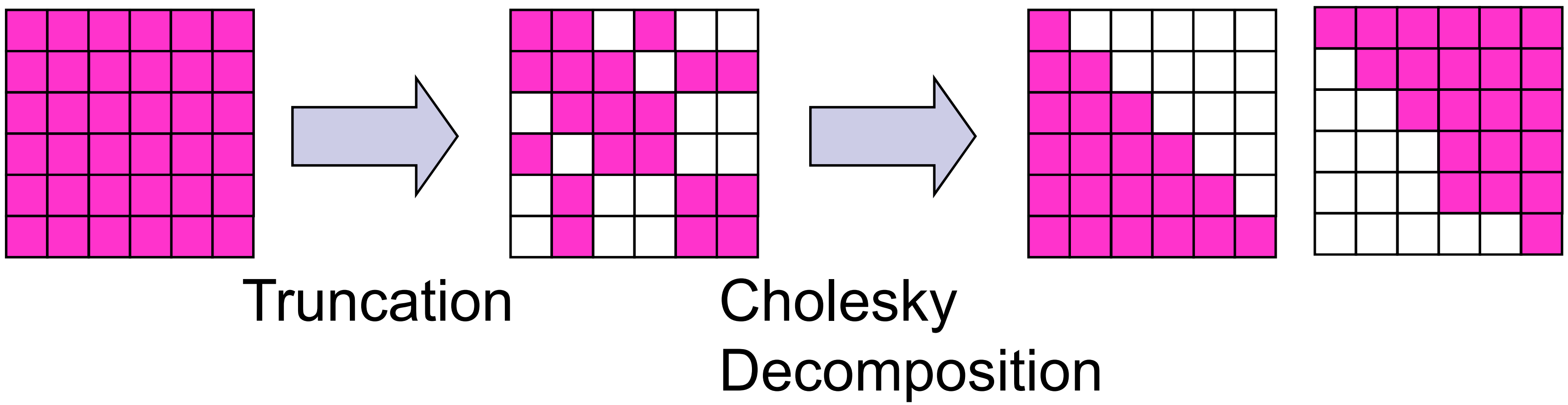
Truncation affects a small change to the eigen values.

- Three methods for generating stable and sparse reluctance matrix are proposed.
  - Direct Truncation

    is accurate but check of positive definiteness is necessary.
  - Enforcing Positive Definiteness
  - Enforcing Diagonally Dominance

    provide a provably positive definite reluctance matrix but are not accurate.

Direct Truncation



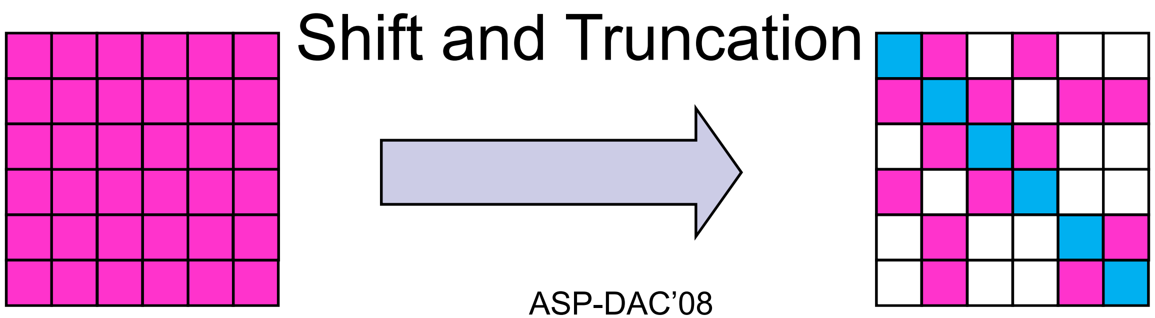
- An off-diagonal element is truncated, if the absolute value is less than a threshold.
- If the Cholesky decomposition is successfully done, the truncated matrix is positive definite.

Enforcing Positive Definiteness

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{\text{Positive}} - \underbrace{\begin{bmatrix} -|a_{13}| & 0 & a_{13} \\ 0 & 0 & 0 \\ a_{31} & 0 & -|a_{31}| \end{bmatrix}}_{\text{Negative}} = \underbrace{\begin{bmatrix} \tilde{a}_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & \tilde{a}_{33} \end{bmatrix}}_{\text{Positive Definite}}$$

Positive — Negative — Positive Definite

- The off-diagonal element is shifted and truncated and the absolute value is added to the diagonal.



Enforcing Diagonally Dominance

- A diagonally dominant matrix can be extracted from the original matrix by removing the negative definite matrices.

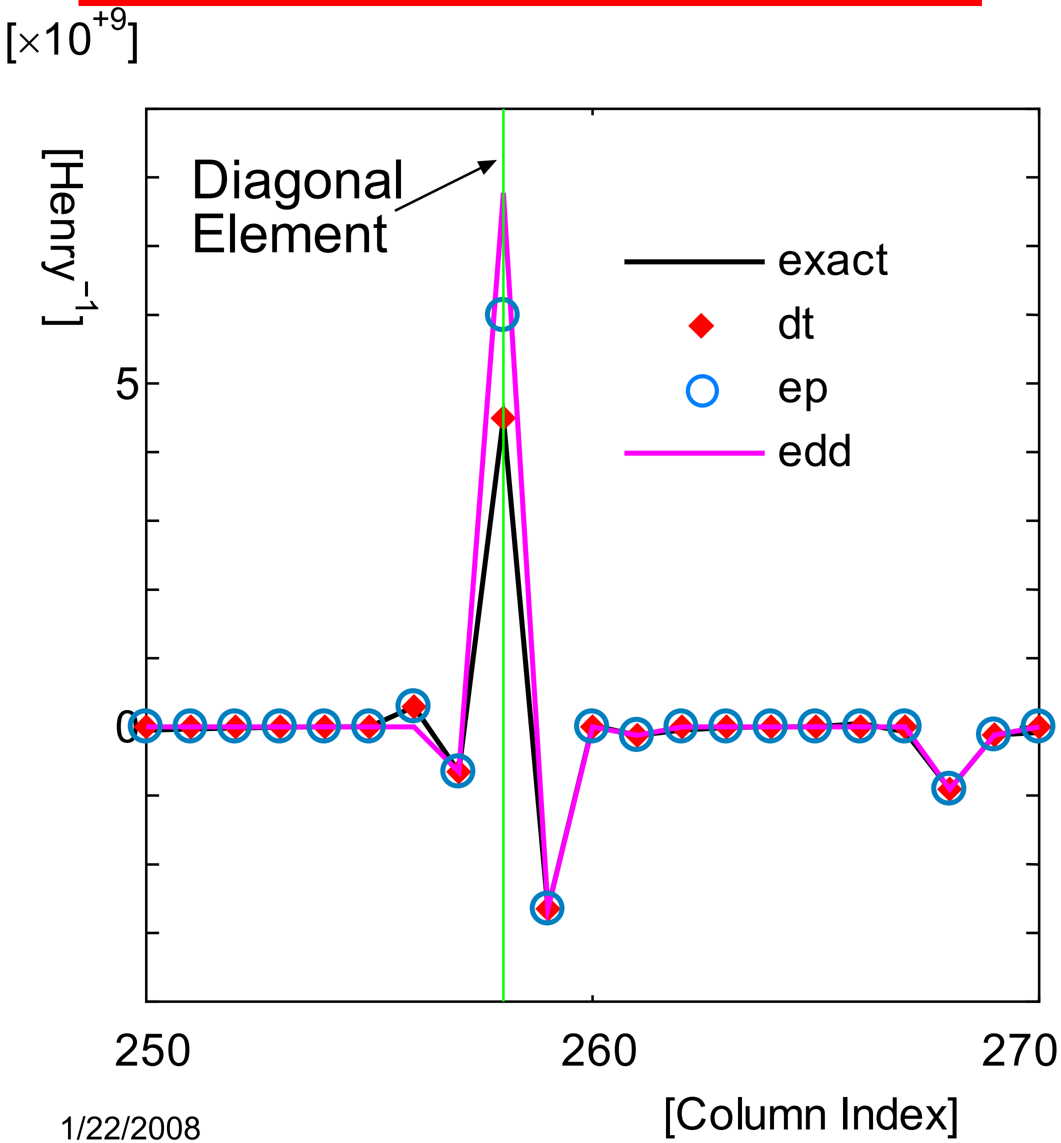
Positive Definite and Diagonally Dominant

$$\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -4 \\ 1 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2+4 & -4 \\ 0 & -4 & 4 \end{bmatrix} + diag\,(2,0,0)$$

$$+\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} + diag\,(0,-2,-3)$$

Negative Definite

Sparse Reluctance Matrix



exact: no truncation

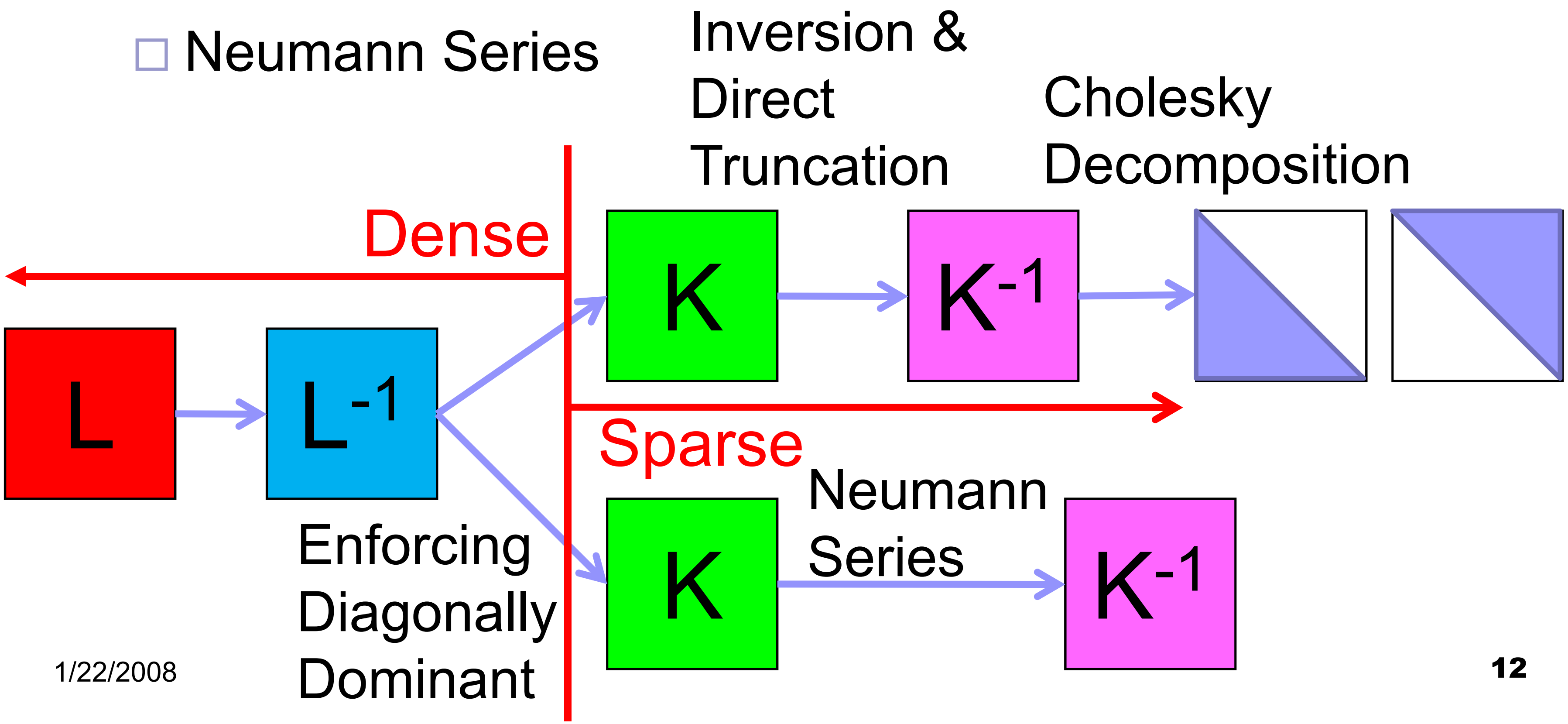
dt: direct truncation

ep: enforcing positive definiteness

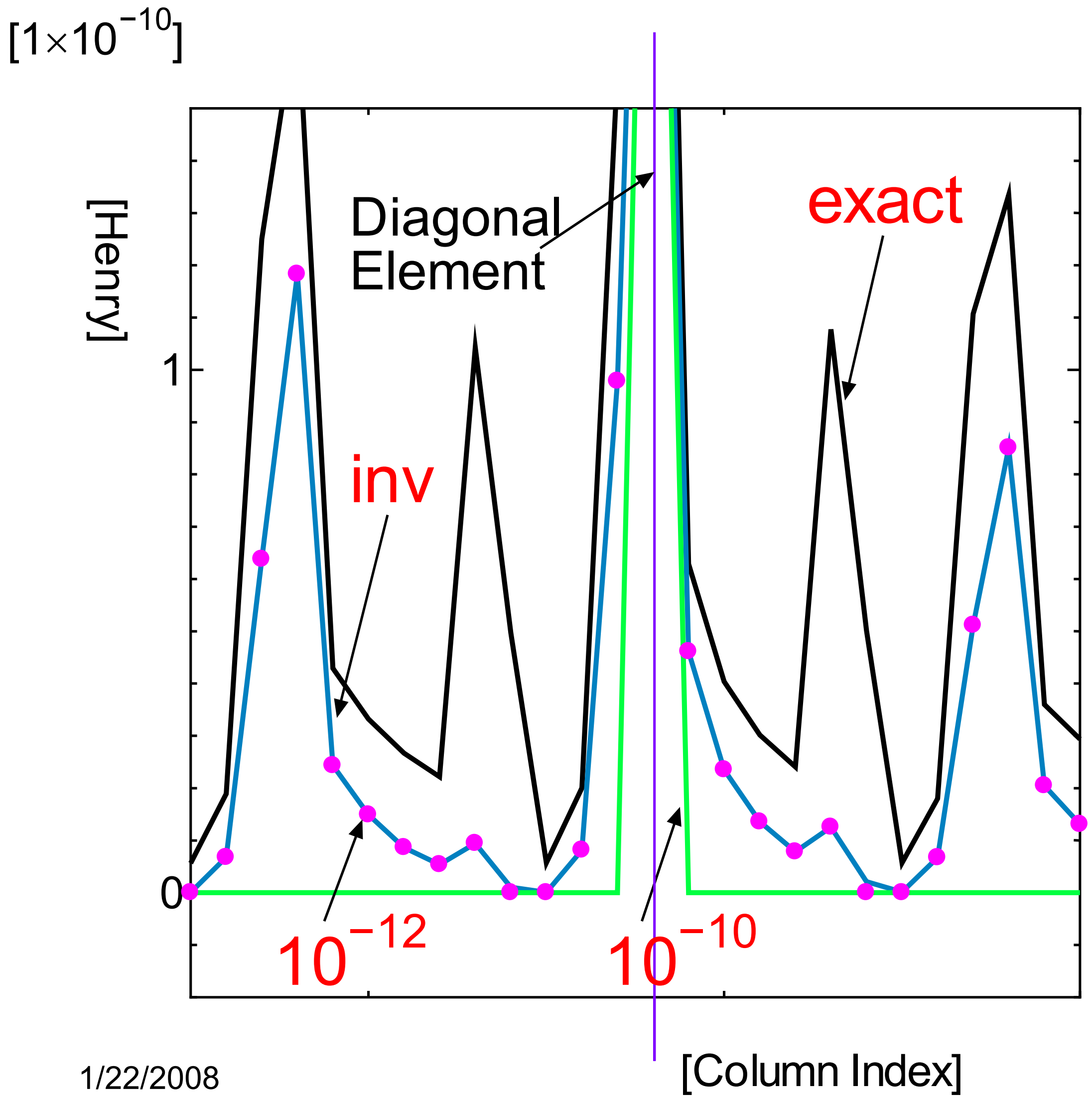
edd: enforcing diagonally dominance

Generating Sparse Inductance Matrix

- After getting the reluctance matrix, the matrix is again inverted. Then, we can obtain the sparse inductance matrix.
  - Direct Truncation
  - Neumann Series



Sparse Inductance Matrix

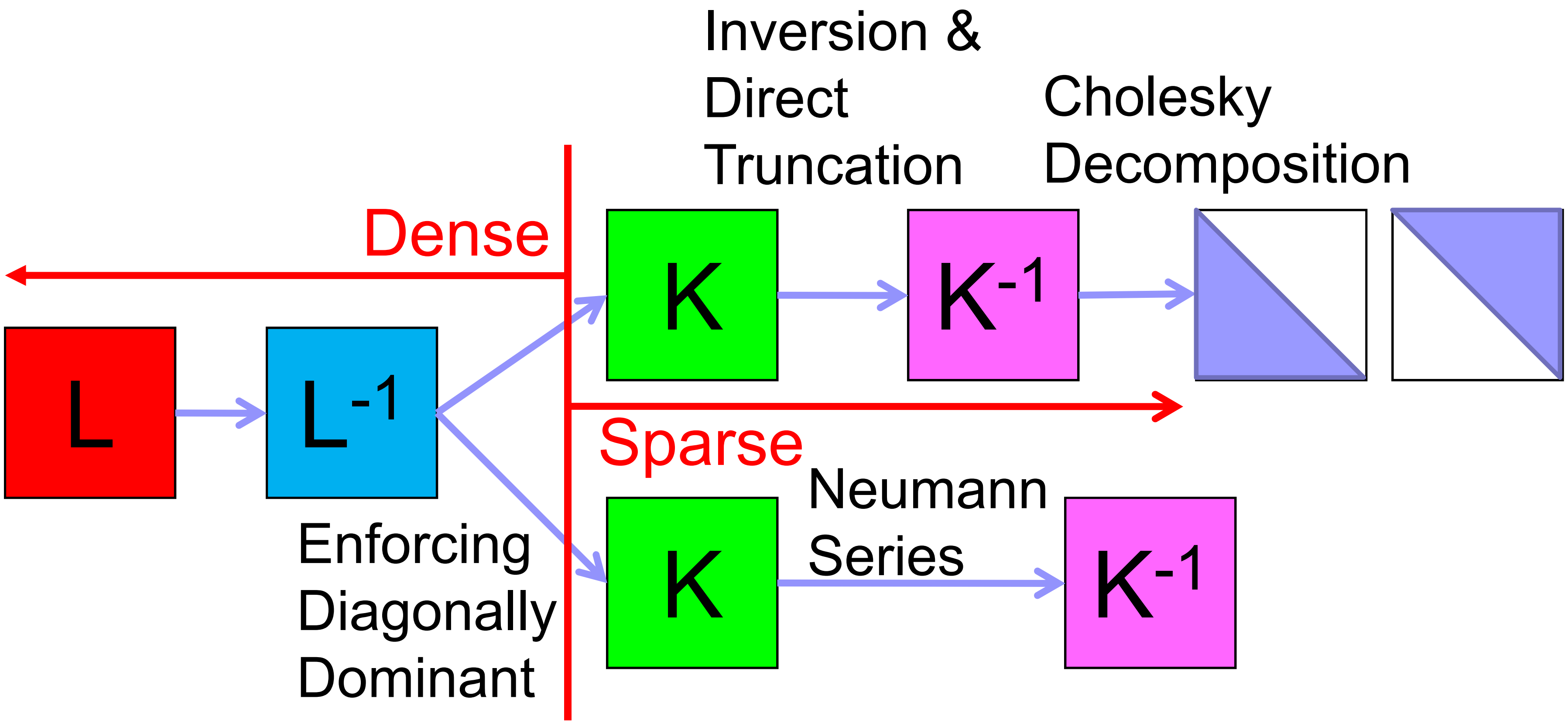


exact: no truncation

inv: inverse of truncated reluctance matrix

10^{-10},10^{-12}: threshold of direct truncation

Neumann Series (1/2)

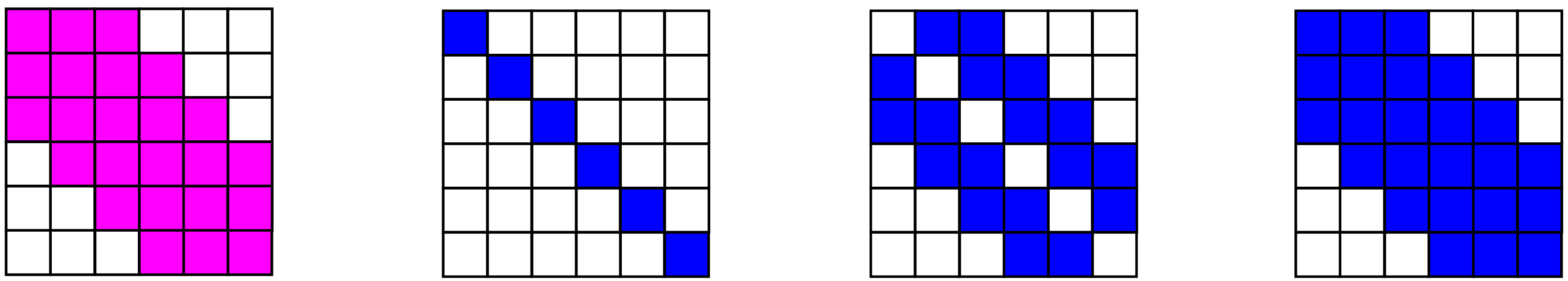


Neumann Series (2/2)

- Inversion of the reluctance matrix is done using the Neumann series.

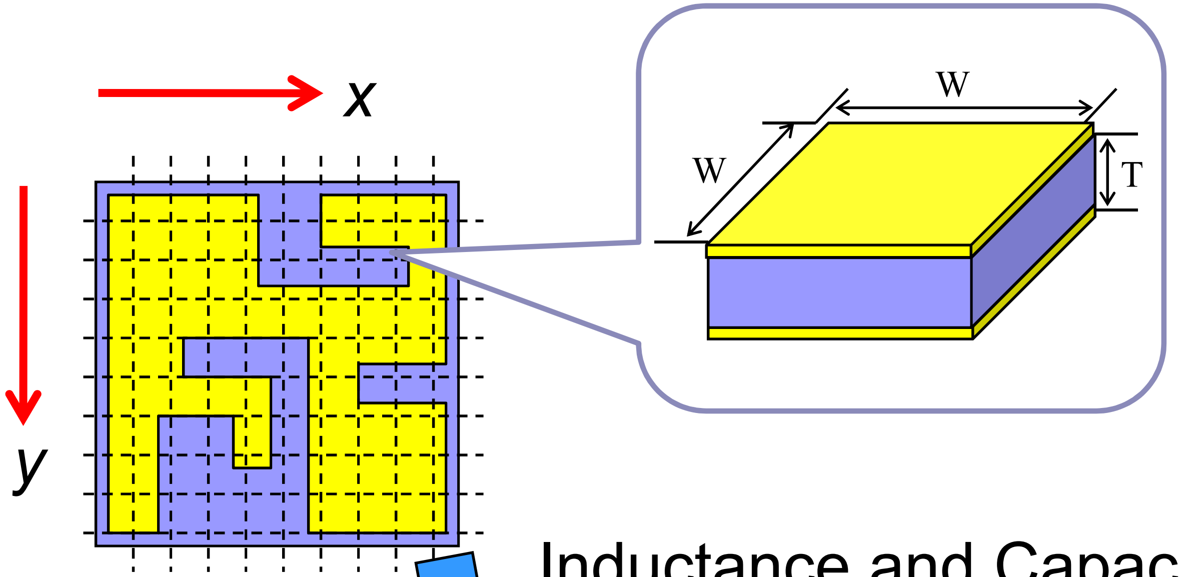
Reluctance Matrix: $\mathbf{K} = \mathbf{D} - \mathbf{N}$ $\quad \mathbf{D}$: Diagonal Matrix

Neumann Series: $\mathbf{K}^{-1} \approx \mathbf{L}_p = \left(\sum_{i=0}^{p} \left(\mathbf{D}^{-1}\mathbf{N} \right)^i \right) \mathbf{D}^{-1}$



$\mathbf{K}$ $\quad \mathbf{D}^{-1}$ $\quad \mathbf{D}^{-1}\mathbf{N}\mathbf{D}^{-1}$ $\quad \mathbf{L}_1$

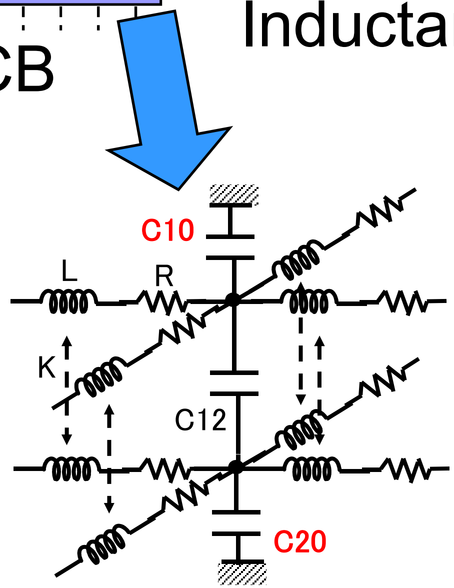
Example



PCB

Division into Unit Cells

Inductance and Capacitance Extraction



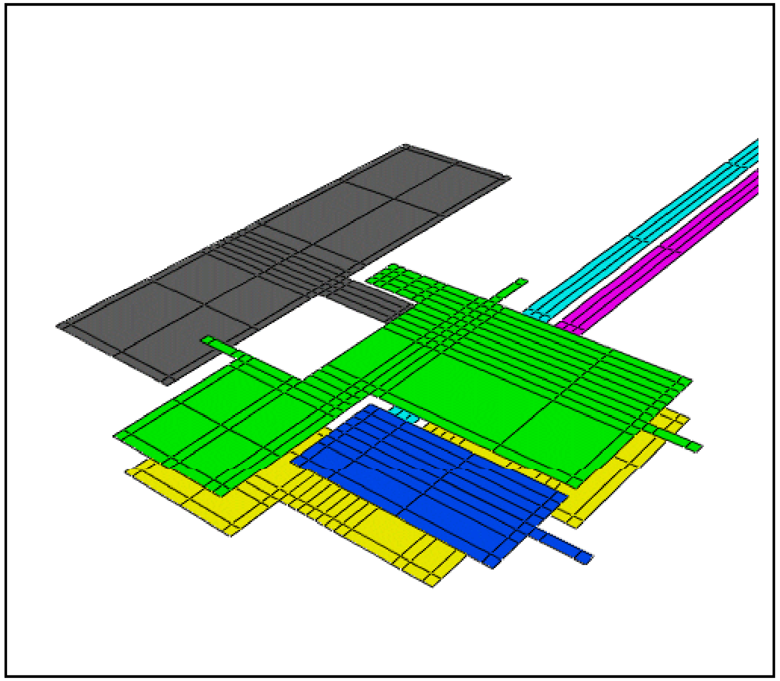
Reluctance:

500 × 500: 7.8% positive

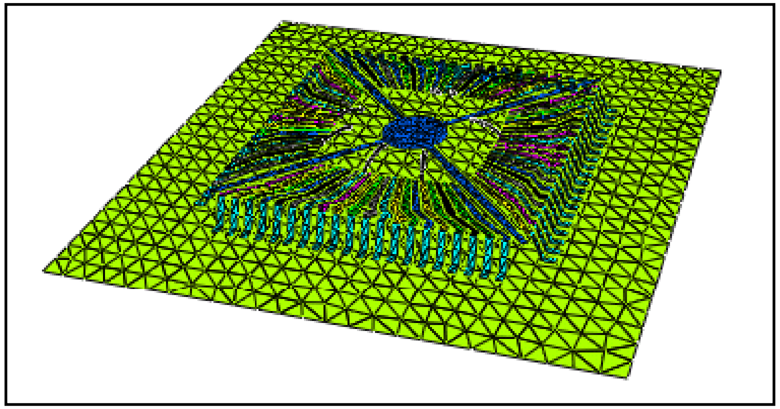
493 × 493: 7.0% positive

Capacitance:

568 × 568: diagonally dominant



PCB Model (L or K, C, R Matrices)

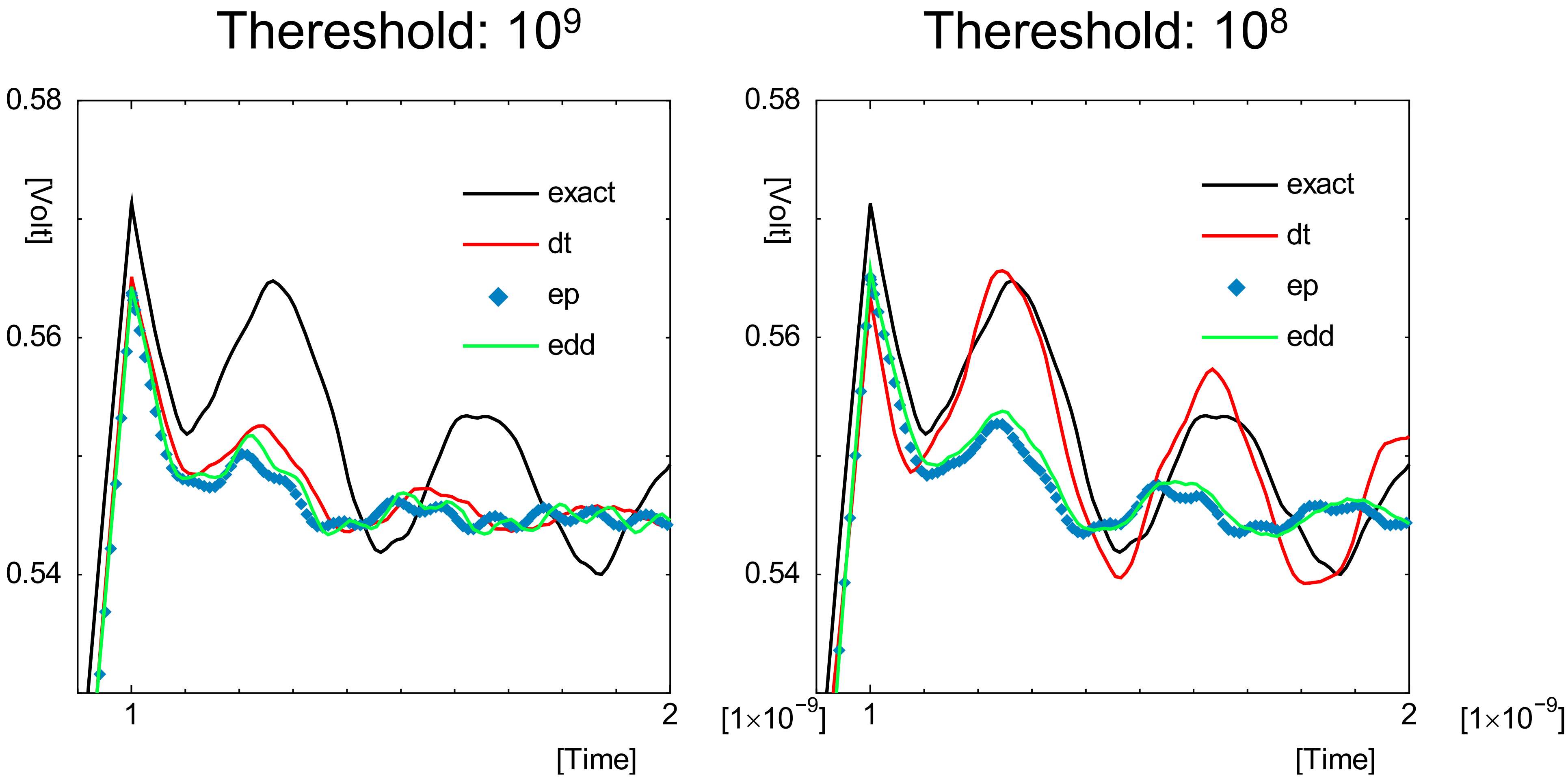


Package Model (R, L Matrices)

Sparse Approximation

SPICE Transient Analysis

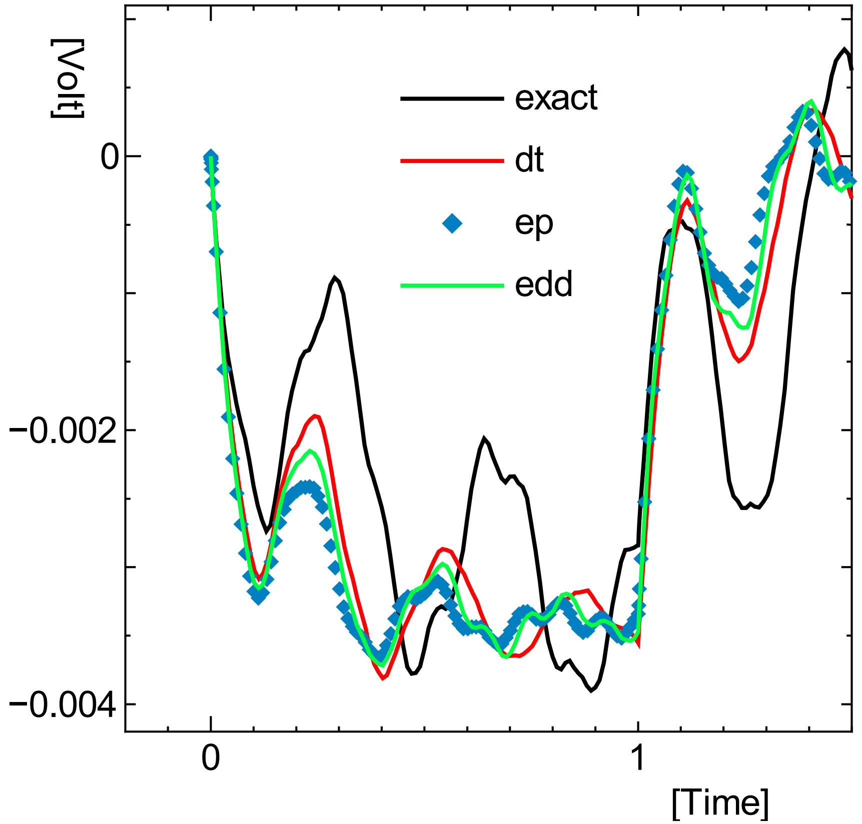
Transient Response (Reluctance) 1/2



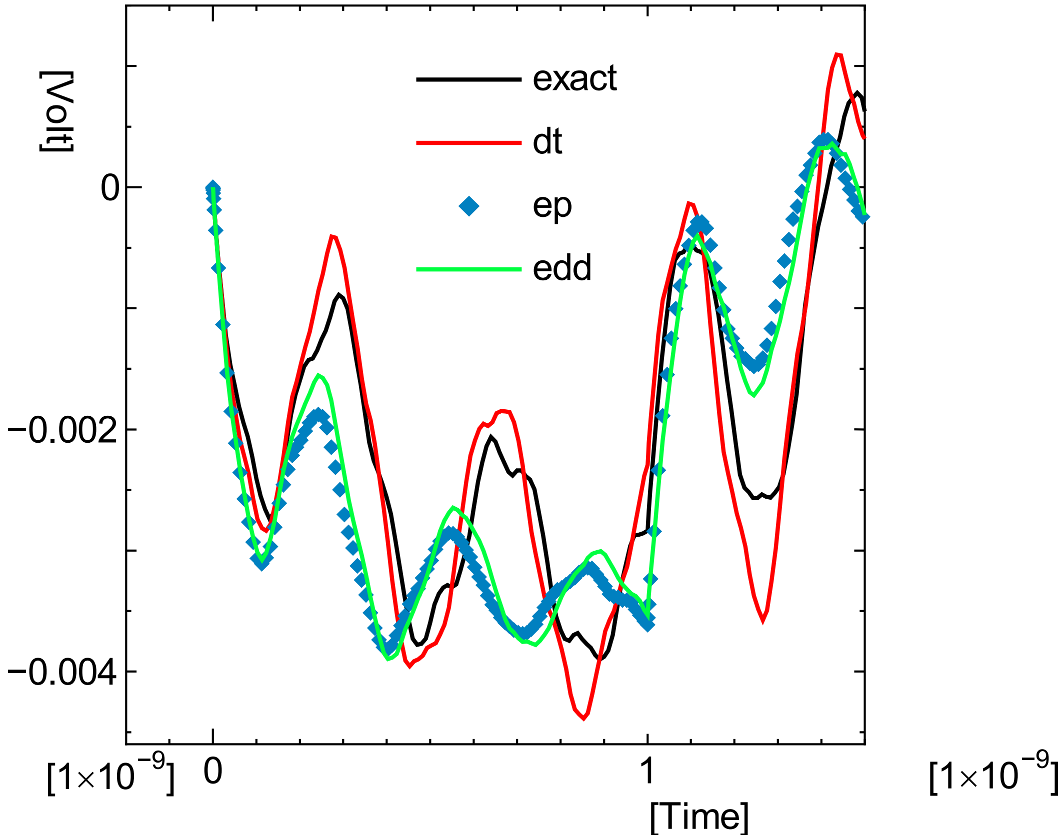
dt: direct truncation, ep: enforcing positive definiteness,
edd: enforcing diagonally dominance

Transient Response (Reluctance) 2/2

Thereshold: 10^9



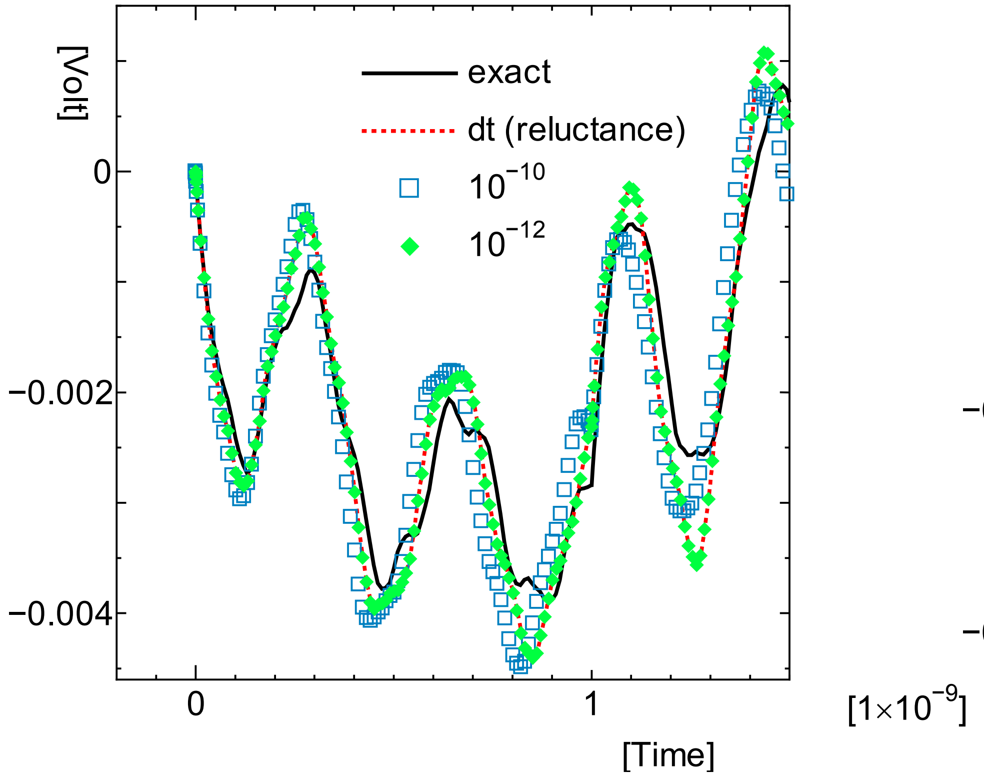
Thereshold: 10^8



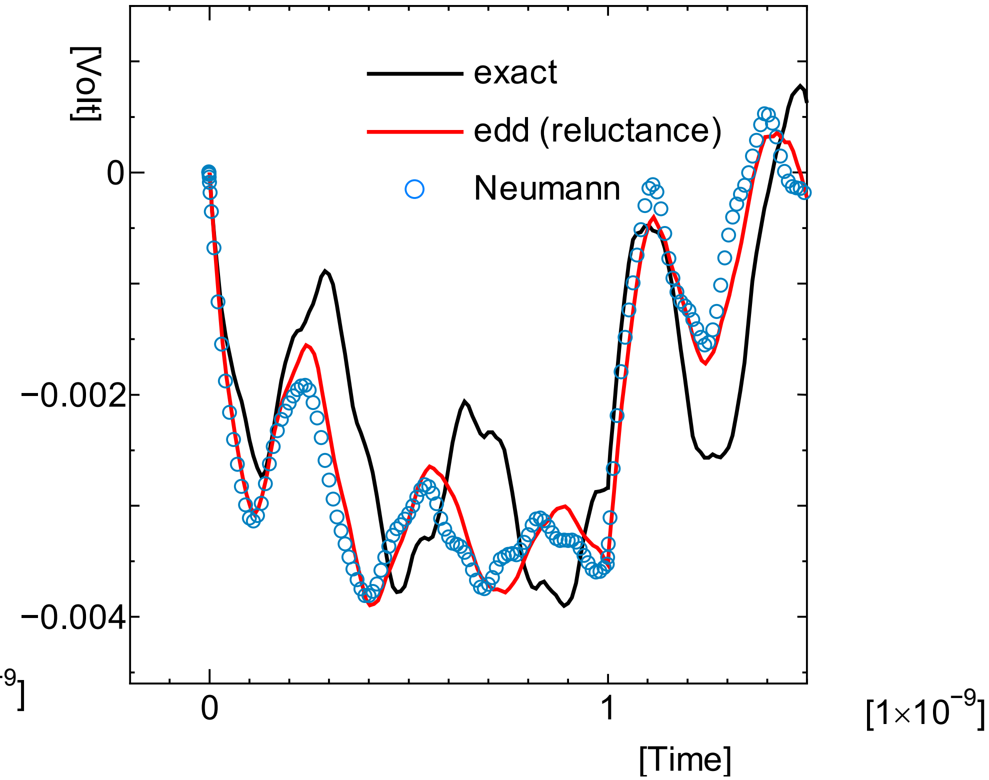
dt: direct truncation, ep: enforcing positive definiteness,
edd: enforcing diagonally dominance

Transient Response (Inductance)

Direct Truncation



Neumann Series



dt: direct truncation, edd: enforcing diagonally dominance

CPU Time Comparison

Reluctance

Method	Sparsity [%]	CPU times [sec.]
Direct Truncation	0.70	61
Direct Truncation	2.10	1,090
Enforcing Positive Definiteness	0.70	51
Enforcing Positive Definiteness	2.10	1,185
Enforcing Diagonally Dominance	0.68	53
Enforcing Diagonally Dominance	1.82	867

Inductance

Method	Sparsity [%]	CPU times [sec.]
Exact	100	21,108
Direct Truncation	0.38	10
Direct Truncation	6.87	847
Neumann Series	2.01	175

Conclusions

- Generating the sparse and stable reluctance/inductance matrices with insufficient conditions is presented.
- When the discretiztion of conductors is not fine, we find positive-off diagonal elements.
- Even if the positive-off diagonal elements were found, the positive definiteness was guaranteed after truncation.
- The key techniques are as follows:

  1)check of positive definiteness by Cholesky decomposition

  2)enforcing positive definiteness

  3)enforcing diagonally dominance

  4)Neumann series

SPICE Model of Reluctance (Block K Method)

- A row of reluctance matrix is represented with a inductance and voltage control voltage sources.

Relation between current and voltage $\mathbf{V} = s\mathbf{LI} \rightarrow s\mathbf{I} = \mathbf{KV}$

$$sI_i = \sum_{j=1}^{N} K_{ij} V_j \rightarrow V_i = s\boxed{\frac{1}{K_{ii}}} I_i - \sum_{j=1, j \neq i}^{N} \boxed{\frac{K_{ij}}{K_{ii}}} V_j$$

inductance — voltage control voltage sources