VISA: Versatile Impulse Structure Approximation for Time-Domain Linear Macromodeling

Chi-Un Lei and Ngai Wong
Dept. of Electrical and Electronic Engineering
The University of Hong Kong

Presented by C.U. Lei

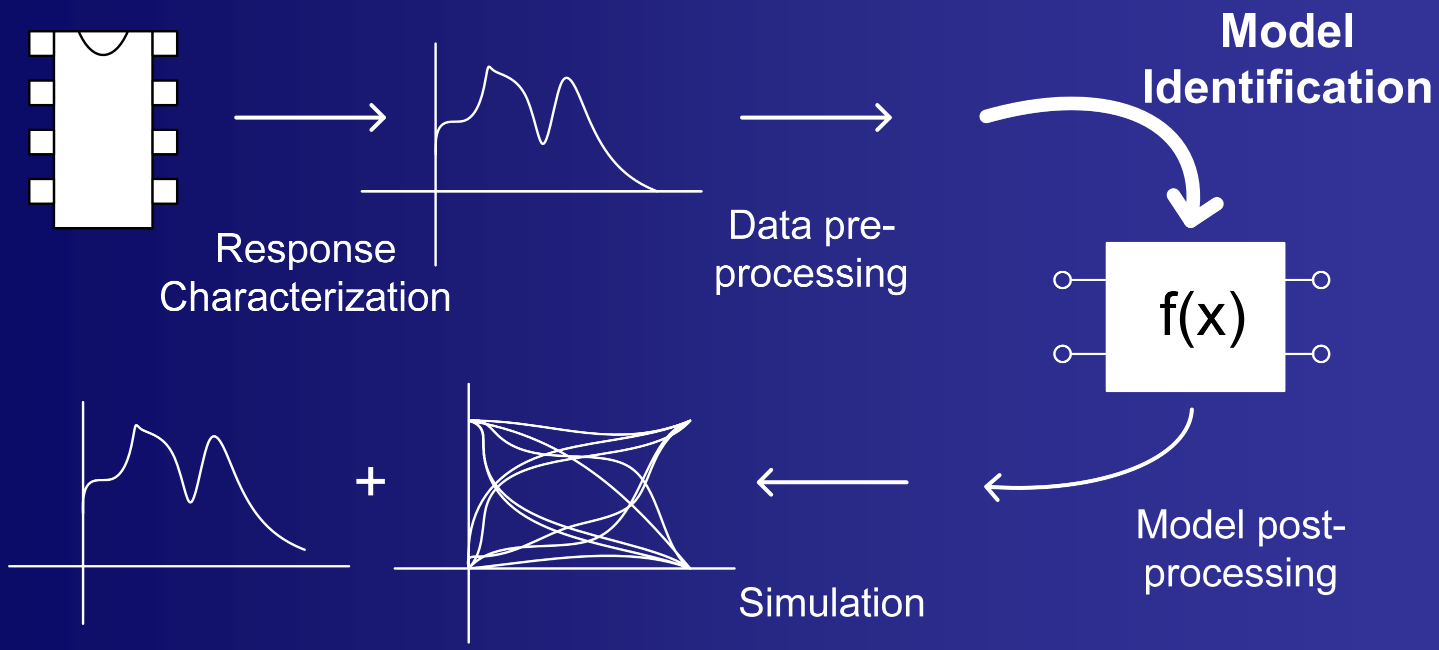
ASP-DAC 2010

Outline

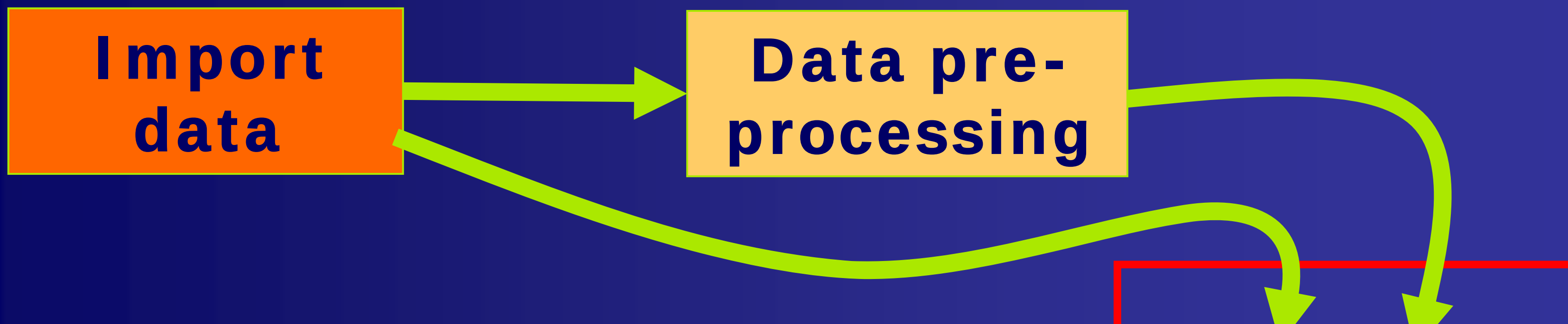
- Digital IIR filter approximation technique → Eigenvalue-free, initial-guess-free, linear macromodeling technique
- Introduction
  - Macromodeling Process
  - Time Domain Macromodeling
- Algorithm (VISA)
- Numerical Results
- Conclusion

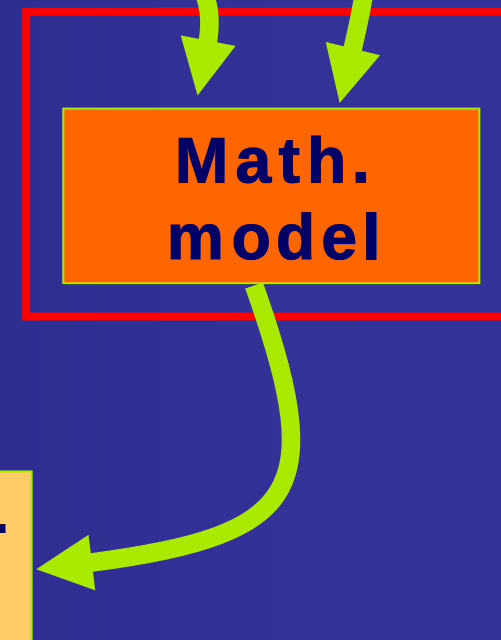
Introduction

- Accurate interconnect/via/package models esp. in high frequency simulation to capture EM effects
- Compact & efficient macromodels for simulation



Introduction





- Tabulated data of input-output response
  - Electromagnetic data
  - Time / frequency
  - Calculation / measurement

- Macromodel requirement: Accurate, causal, stable, passive

Existing algorithms

- Frequency-domain Vector Fitting (VF) [99]
  - Iterative continuous frequency domain approximation (*p* input ports, *q* output ports)

$$f_{u,v}(s) \approx \hat{f}_{u,v}(s) = \frac{P_{u,v}(s)}{Q(s)} = \sum_{m=0}^{M} \frac{p_{u,v}(m)\,s^m}{q(m)\,s^m}$$

- Time-domain Macromodeling Techniques
  - Approximation using (truncated) time-sampled system response ← Easier to capture
  - e.g. 4SID [99], GPOF [99], TD-VF(z) [03,08]

Problems Issues and Alternatives

- Numerical-sensitive calculation
  - Affected by the initial poles, noisy signal
- Require eigenvalues calculation
- Unstable pole flipping → Not an accurate approx.
- High (multiport) computation complexity
- Other techniques has much higher computation complexity (Expensive SVD computation)

- Digital IIR filter approximation → Macromodeling SISO LS [ISCAS 08] → VISA [ASPDAC 10]
- Implication: finite-length discrete response sequences ... like FIR filter responses!!!

Outline

- Introduction
- Algorithm (VISA)
  - "Walsh's theorem" & "Complementary signal"
  - Formulation
  - Convergence: SM iteration, model order reduction, *P*-norm approximation
- Numerical Results
- Conclusion

Least-Squares Approximation

Macromodeling of a time-sampled response = IIR Approx. of FIR filter

Problem: Approximation → Interpolation problem → Input / Output description of an allpass filtering operation → Designing an allpass operator

$$F(z) = f_0 + f_1 z^{-1} + \cdots + f_L z^{-L}$$

$$H(z) = \frac{P(z)}{Q(z)} = \frac{p_0 + p_1 z^{-1} + \cdots + p_N z^{-N}}{1 + q_1 z^{-1} + \cdots + q_N z^{-N}}, N < L$$

$$\Delta(z) = F(z) - H(z)$$

$$l_2\text{norm:} \quad \left\|\Delta(z)\right\|_2 = \sqrt{\frac{1}{2\pi j} \oint_{|z|=1} \Delta(z)\Delta^*(z)\frac{dz}{z}}$$

Find P(z) Suppose Q(z) Known

Walsh's Theorem:

$$z_0 = \infty,\ z_1 = 1/\alpha_1^*,\ z_2 = 1/\alpha_2^*, \cdots,\ z_N = 1/\alpha_N^*$$

$$\frac{P(z_i)}{Q(z_i)} = \frac{p_0 + p_1 z_i^{-1} + \cdots + p_N z_i^{-N}}{1 + q_1 z_i^{-1} + \cdots + q_N z_i^{-N}} = F(z_i),\ i = 0, 1, \cdots, N$$

$$\Delta(z_i) = F(z_i) - \frac{P(z_i)}{Q(z_i)} = 0 \Rightarrow \left\|\Delta(z)\right\|_2 \text{ minimized!}$$

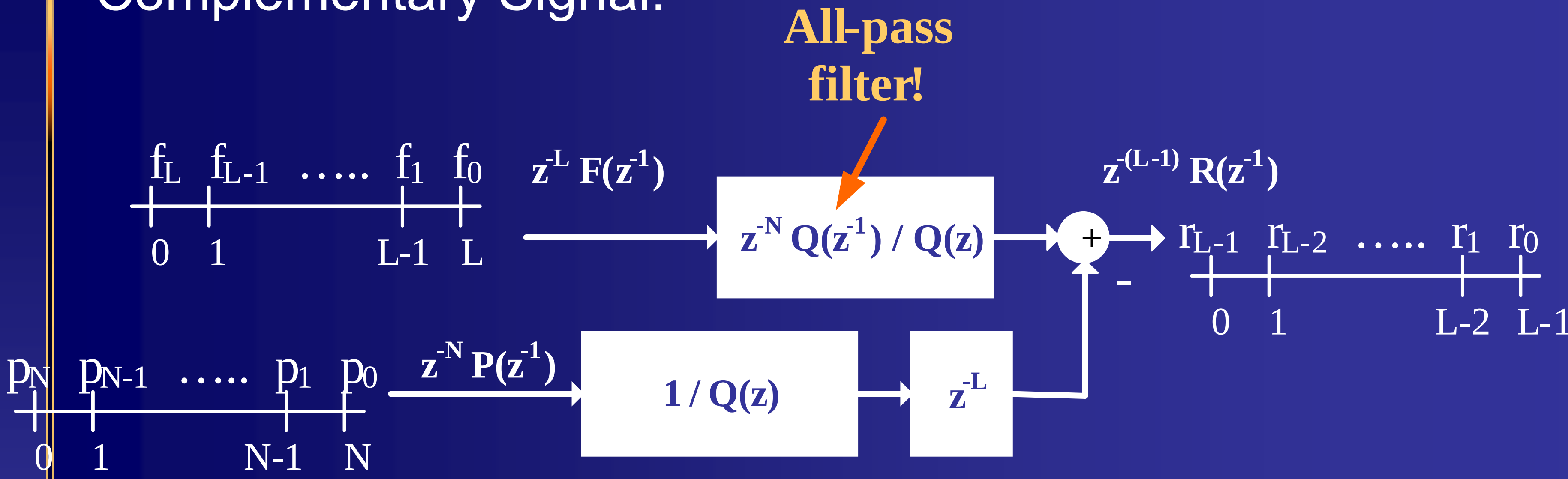
Finding z_i's can be numerically ill. Smarter way making use of the FIR nature of F(z), we can rewrite Δ(z) as

$$\Delta(z) = \frac{z^{-(N+1)} Q(z^{-1})}{Q(z)} R(z) \quad \text{where} \quad R(z) = r_0 + r_1 z^{-1} + \cdots + r_{L-1} z^{-(L-1)}$$

$$\frac{z^{-(N+1)} Q(z^{-1})}{Q(z)} R(z) = F(z) - \frac{P(z)}{Q(z)}$$

Equivalent Filtering Problem

Complementary Signal:



For the first L time instances

$$r_{L-1} \; r_{L-2} \; \ldots\ldots \; r_1 \; r_0 = z^{-N}\,Q(z^{-1})\,/\,Q(z) \otimes f_L \; f_{L-1} \; \ldots\ldots \; f_1 \; f_0$$

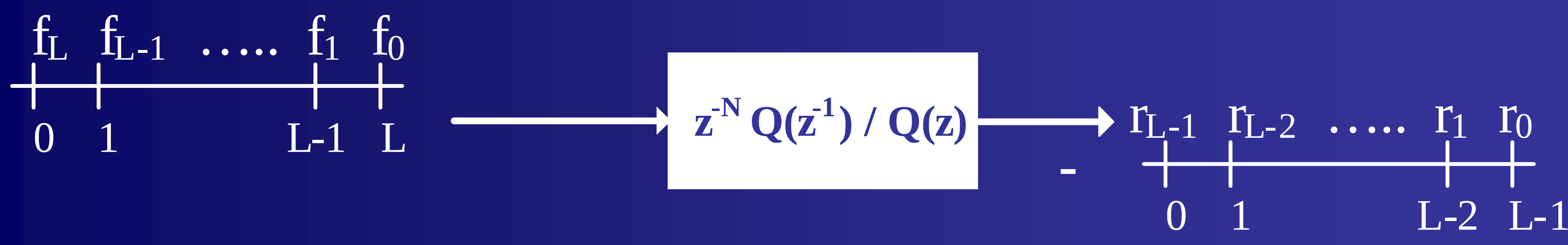
Finding Numerator P(z)

$$\Delta(z) = \frac{z^{-(N+1)}Q(z^{-1})}{Q(z)}R(z) = F(z) - \frac{P(z)}{Q(z)}$$

P(z) easily found!!

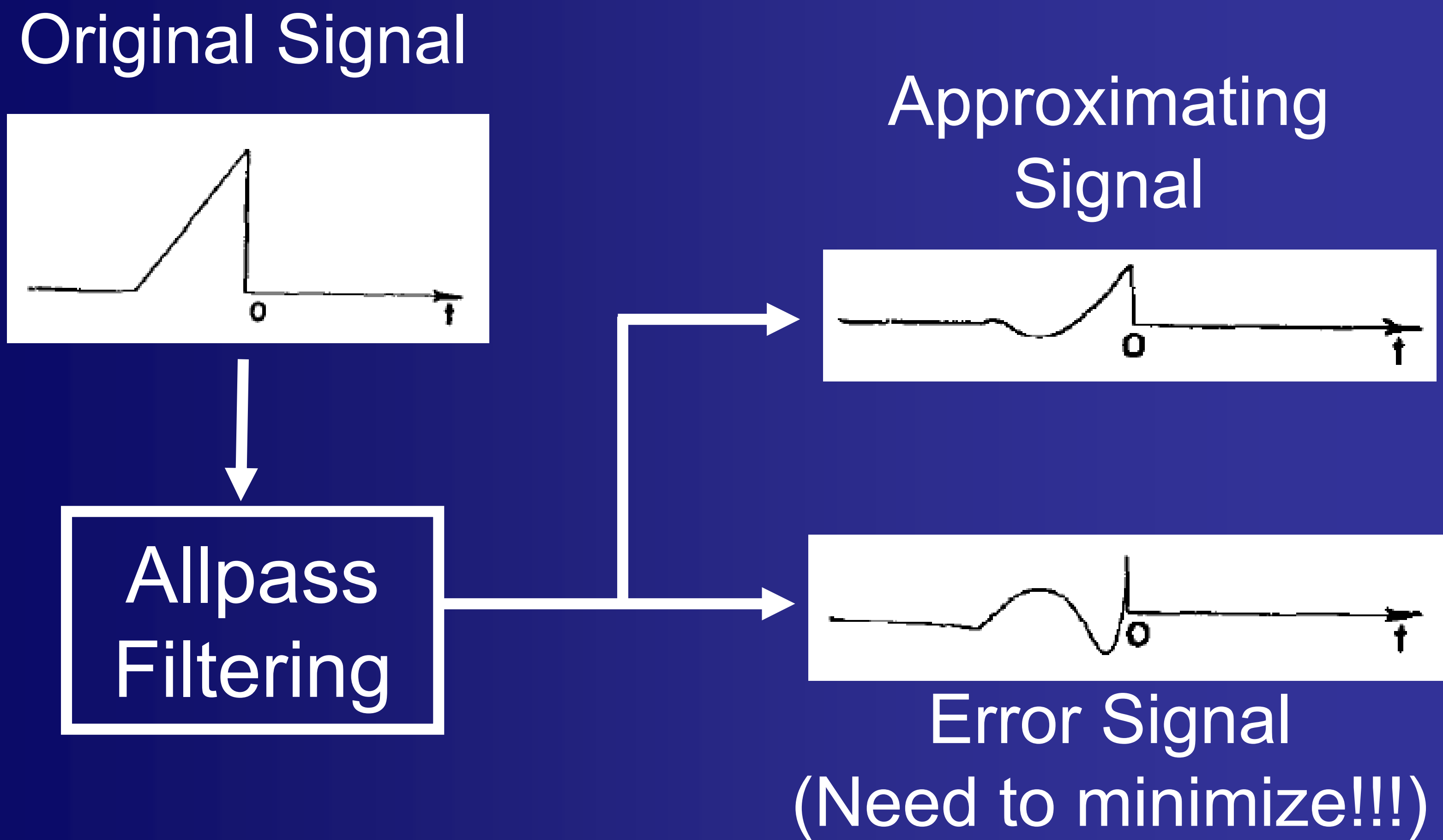
Easily prove that $$\left\|\Delta(z)\right\|_2 = \sum_{i=0}^{L-1} r_i^2$$

For the first time L instances:



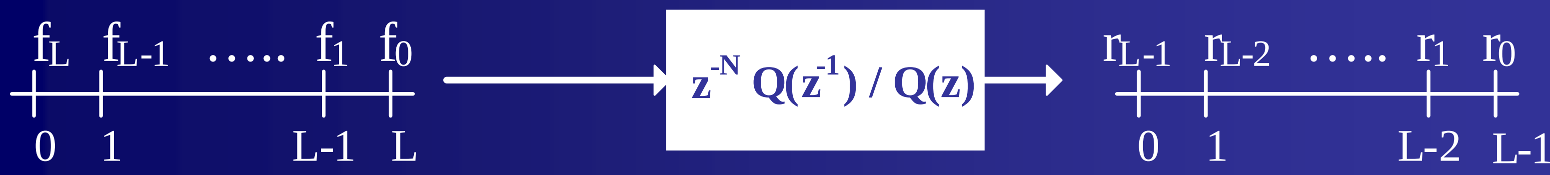
Complementary Signal

- Adopted from [YH62]



Finding Denominator Q(z)

For the first L time instances



*Find Q(z) such that energy output of the all-pass filter $A(z)=z^{-N}Q(z^{-1})/Q(z)$ is concentrated in time ≥ L,

Recall $$Q(z) = 1 + q_1 z^{-1} + \cdots + q_N z^{-N}$$

Define $$Q^{(0)}(z) = 1$$

$$Q^{(k)}(z) = 1 + q_1^{(k)} z^{-1} + q_2^{(k)} z^{-2} + \cdots + q_N^{(k)} z^{-N}$$

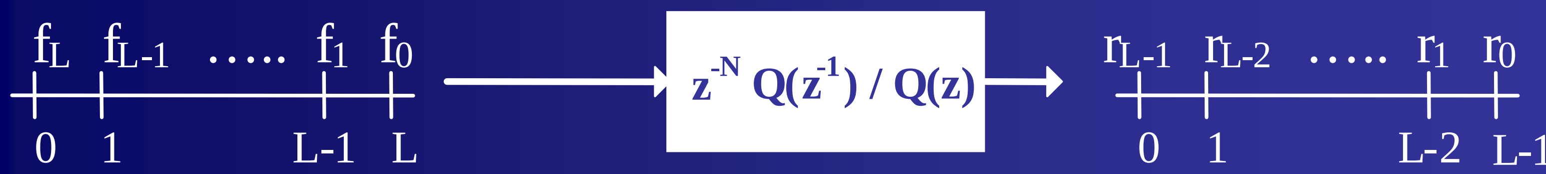
$$= 1 + z^{-1}\left(q_1^{(k)} + q_2^{(k)} z^{-1} + \cdots + q_N^{(k)} z^{-(N-1)}\right)$$

$$= 1 + z^{-1} Q_1^{(k)}(z)$$

Finding Q(z) by Iterations

$$Q^{(0)}(z)=1$$

$$\begin{aligned}Q^{(k)}(z)&=1+q_1^{(k)}z^{-1}+q_2^{(k)}z^{-2}+\cdots+q_N^{(k)}z^{-N}\\&=1+z^{-1}\left(q_1^{(k)}+q_2^{(k)}z^{-1}+\cdots+q_N^{(k)}z^{-(N-1)}\right)\\&=1+z^{-1}Q_1^{(k)}(z)\end{aligned}$$



Define $$A^{(k)}(z)=\frac{z^{-N}Q^{(k)}(z^{-1})}{Q^{(k-1)}(z)}$$

$$U^{(k)}(z)=\frac{z^{-N}Q^{(k)}(z^{-1})}{Q^{(k-1)}(z)}X(z)=z^{-N}Q^{(k)}(z^{-1})X^{(k)}(z)$$

$$X^{(k)}(z)\left[z^{-(N-1)}Q_1^{(k)}(z^{-1})\right]=U^{(k)}(z)-z^{-N}X^{(k)}(z)$$

Finding Q(z) by Iterations

$$X^{(k)}(z)\left[z^{-(N-1)}Q_1^{(k)}(z^{-1})\right]=U^{(k)}(z)-z^{-N}X^{(k)}(z)$$

$$\begin{bmatrix} x^{(k)}(0) & 0 & \cdots & 0 \\ x^{(k)}(1) & x^{(k)}(0) & \cdots & \vdots \\ \vdots & & \ddots & 0 \\ x^{(k)}(N-1) & \cdots & & x^{(k)}(0) \\ \vdots & & & \\ x^{(k)}(L-1) & \cdots & & x^{(k)}(L-N) \end{bmatrix} \begin{bmatrix} q_N^{(k)} \\ q_{N-1}^{(k)} \\ \vdots \\ q_1^{(k)} \end{bmatrix} = \begin{bmatrix} u^{(k)}(0) \\ u^{(k)}(1) \\ \\ \vdots \\ \\ u^{(k)}(L-1) \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ x^{(k)}(0) \\ \vdots \\ x^{(k)}(L-N-1) \end{bmatrix}$$

$$A^{(k)}q^{(k)}=u^{(k)}+b^{(k)}$$

Automatically $u(k):=A^{(k)}q^{(k)}-b^{(k)}$ has minimum norm

By iterations, $Q^{(k)}(z)$ converges to $Q(z)$

What's amazing: $Q^{(k)}(z)$ is always stable!

Finding Q(z) by Iterations

$$X^{(k)}(z)\left[z^{-(N-1)}Q_1^{(k)}(z^{-1})\right] = U^{(k)}(z) - z^{-N}X^{(k)}(z)$$

$$\begin{bmatrix} x^{(k)}(0) & 0 & \cdots & 0 \\ x^{(k)}(1) & x^{(k)}(0) & \cdots & \vdots \\ \vdots & & \ddots & 0 \\ x^{(k)}(N-1) & \cdots & & x^{(k)}(0) \\ \vdots & & & \\ x^{(k)}(L-1) & \cdots & & x^{(k)}(L-N) \end{bmatrix} \begin{bmatrix} q_N^{(k)} \\ q_{N-1}^{(k)} \\ \vdots \\ q_1^{(k)} \end{bmatrix} = \begin{bmatrix} u^{(k)}(0) \\ u^{(k)}(1) \\ \\ \vdots \\ \\ u^{(k)}(L-1) \end{bmatrix} - \begin{bmatrix} 0 \\ \vdots \\ 0 \\ x^{(k)}(0) \\ \vdots \\ x^{(k)}(L-N-1) \end{bmatrix}$$

$$A^{(k)}q^{(k)} = u^{(k)} + b^{(k)}$$

Automatically $u(k) := A^{(k)}q^{(k)} - b^{(k)}$ has minimum norm

By iterations, $Q^{(k)}(z)$ converges to $Q(z)$

What's amazing: $Q^{(k)}(z)$ is always stable!

Convergence of VISA

- Calculation of Q(z) is a simplification of Steiglitz-McBride (SM) iteration without initial guess
  - P(z) is not required for the calculation of Q(z)

$$\min \sum_{k=1}^{N_s} \left| \frac{z_k^{-N} Q^{(i)}\left(z_k^{-1}\right)}{Q^{(i-1)}\left(z_k\right)} z^{-L} F\left(z_k^{-1}\right) \right|^2$$

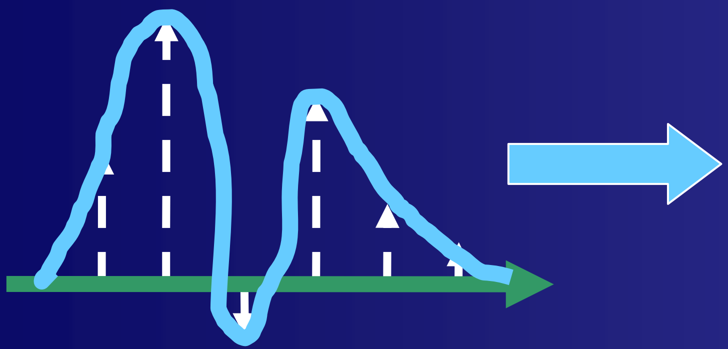
$$= \min \sum_{k=0}^{N_s} \frac{1}{\left|Q^{(i-1)}\left(z_k\right)\right|^2} \left| z_k^{-N} Q^{(i)}\left(z_k^{-1}\right) z^{-L} F\left(z_k^{-1}\right) \right|^2$$

$$= \min \sum_{k=0}^{N_s} \frac{1}{\left|Q^{(i-1)}\left(z_k\right)\right|^2} \left| Q^{(i)}\left(z_k\right) F\left(z_k\right) - P^{(i)}\left(z_k\right) \right|^2$$

SM iteration

Model order selection

- Appropriate model order (N) for efficient and accurate simulation
- Similar pattern between approximant error and Hankel singular value (HSV) of the impulse response
- HSV distribution and the ratio of first and last HSV



$$H = eig\begin{bmatrix} f_1 & f_2 & \cdots & f_L \\ f_2 & f_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_L & 0 & \cdots & 0 \end{bmatrix}$$

P-norm approximation

- Modify the minimization criteria to suit different macromodeling requirements
  - *P*-norm approximation
  - User-defined norm approximation
  - Norm-constrained approximation

$$\min \sum_{n=0}^{L-1} \left\| \Delta^{(k)}\left(z_n\right) \right\|_p = \min \left\| \mathbf{B}^{(k)} \mathbf{q}^{(k)} - \mathbf{d}^{(k)} \right\|_p$$

$$= \min \sum_{n=0}^{L-1} \frac{1}{\left\| Q^{(k-1)}\left(z_n\right) \right\|_p} \left\| Q^{(k)}\left(z_n\right) H\left(z_n\right) - P^{(k)}\left(z_n\right) \right\|_p$$

- Applying convex programming for solving

MIMO VISA Extension

- Walsh's theorem can be applied to MIMO situation
- Stacking the responses into a column for LS solving

$$\begin{bmatrix} \mathbf{B}_{1,1}^{(k)} \\ \mathbf{B}_{1,2}^{(k)} \\ \vdots \\ \mathbf{B}_{p,q}^{(k)} \end{bmatrix} \left[q^{(k)} \right] = \begin{bmatrix} \mathbf{d}_{1,1}^{(k)} \\ \mathbf{d}_{1,2}^{(k)} \\ \vdots \\ \mathbf{d}_{p,q}^{(k)} \end{bmatrix}$$

Single-port response

- Less computation complexity
  - TD-VF: O((pq+1)N^2Lpq) ← Much higher!
  - VISA: O(N^2Lpq)

VISA Features

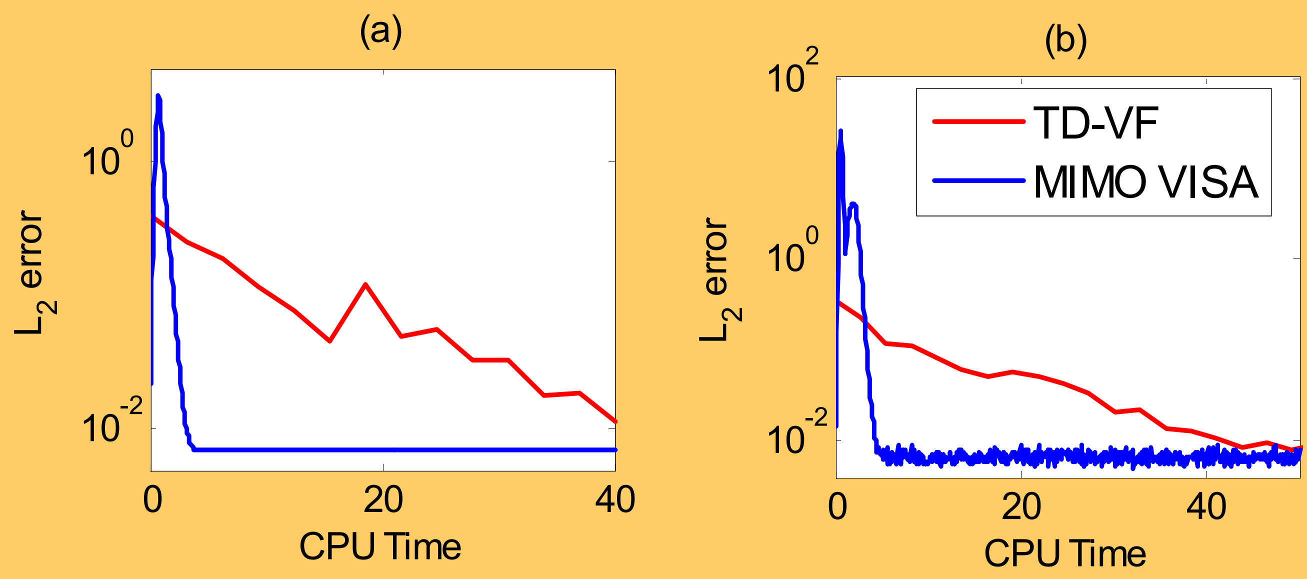
- No eigenvalue computations
- No initial guess required
  - Less initial-pole-sensitive calculation
- Guaranteed stable pole calculation
- Converge to the near-L_2 optimal solution
- Low (multi-port) computation complexity
- *P*-norm approximation for different modeling requirement
- Quasi-error bound for model order selection

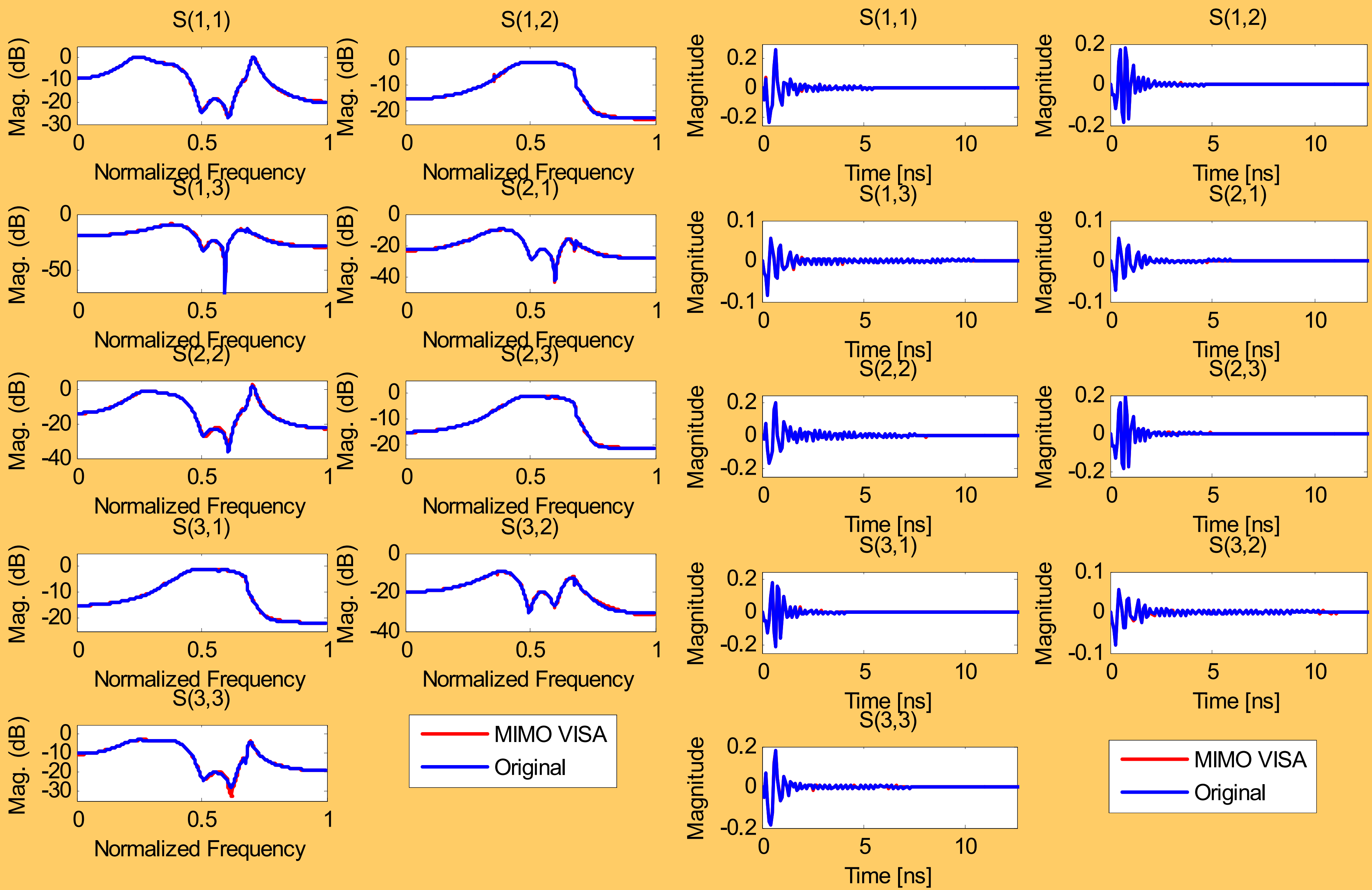
Outline

- Introduction
- Algorithm (VISA)
- Numerical Results
  - Three-port counterwise RF circulator
  - Benchmark examples
- Conclusion

Numerical Examples

- Three-port counterwise RF circulator
- More accurate : 18% less avgerage R.M.S. error after convergence
- Faster : >15X faster for convergence and >17X faster to achieve a -40dB accuracy

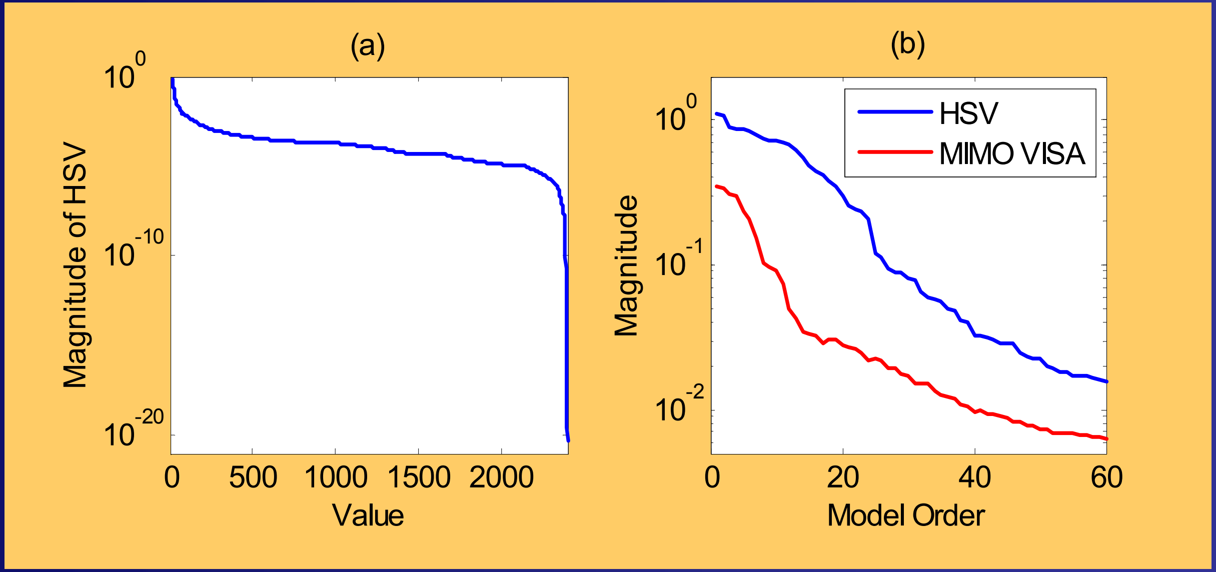




S(1,1)
S(1,2)
S(1,3)
S(2,1)
S(2,2)
S(2,3)
S(3,1)
S(3,2)
S(3,3)
Mag. (dB)
Normalized Frequency
Magnitude
Time [ns]
MIMO VISA
Original

Numerical Examples

- HSV (error bound) computation: 58 sec. CPU time
- *P*-norm approximation
  - 3.5% L_2 error, 29.5% L_{inf} error and 24.5% CPU time reduction
- Four benchmark examples [IG08]
  - 51% less L_2 error, >21 X faster, comparing to TD-VF



Conclusion

- VISA: Linear Macromodeling using time-sampled response
  - Simplified Steiglitz-McBride algorithm
  - No initial-pole assignment
  - No eigenvalues calculation
  - Robust and efficient computation
  - *P*-norm approximation

Thank you!

Thank you!

Questions are welcome

- Authors:

  C. U. Lei * culei@eee.hku.hk

  Dr. N. Wong nwong@eee.hku.hk

Department of Electrical and Electronic Engineering,

The University of Hong Kong

Back-Up Slides

Walsh Theorem

- Among the set of rational functions $H(z)=P(z)/Q(z)$, with prescribed poles $a_1, a_2, \ldots, a_n$ that are fixed and located in $|z| < 1$, the best approximation in the LS sense to $F(z)$ (analytic in $|z| > 1$ and continuous in $|z| \geq 1$) is the unique function that interpolates to $F(z)$ in all the points. $z = \infty, 1/a^*_1, 1/a^*_2, \ldots, 1/a^*_n$, where * denotes complex conjugate
- Approximation

→ Interpolation problem

→ Input / Output description of an allpass filtering operation

→ Design of an allpass operator

Complementary Signal

- Adopted from [YH62]