

# An Adaptive Parallel Flow for Power Distribution Network Simulation Using Discrete Fourier Transform

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# Outline

- Motivation
- Adaptive simulation flow using discrete Fourier transform (DFT)
  - Basic DFT flow
  - Problems with the basic DFT flow
  - Adaptive DFT flow
- Experimental results
  - Error analysis
  - Time efficiency of the adaptive flow
  - Time efficiency of parallel processing
- Conclusions

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# PDN Simulation: Why Frequency Domain?

- Huge PDN netlists
  - Time-domain simulation: serial - slow
  - Frequency-domain simulation: parallel – fast
- Frequency dependent parasitics
- Simulation results
  - Time-domain: voltage drops, simultaneous switching noise (SSN) – input dependent
  - Frequency-domain: impedance, anti-resonance peaks – input independent

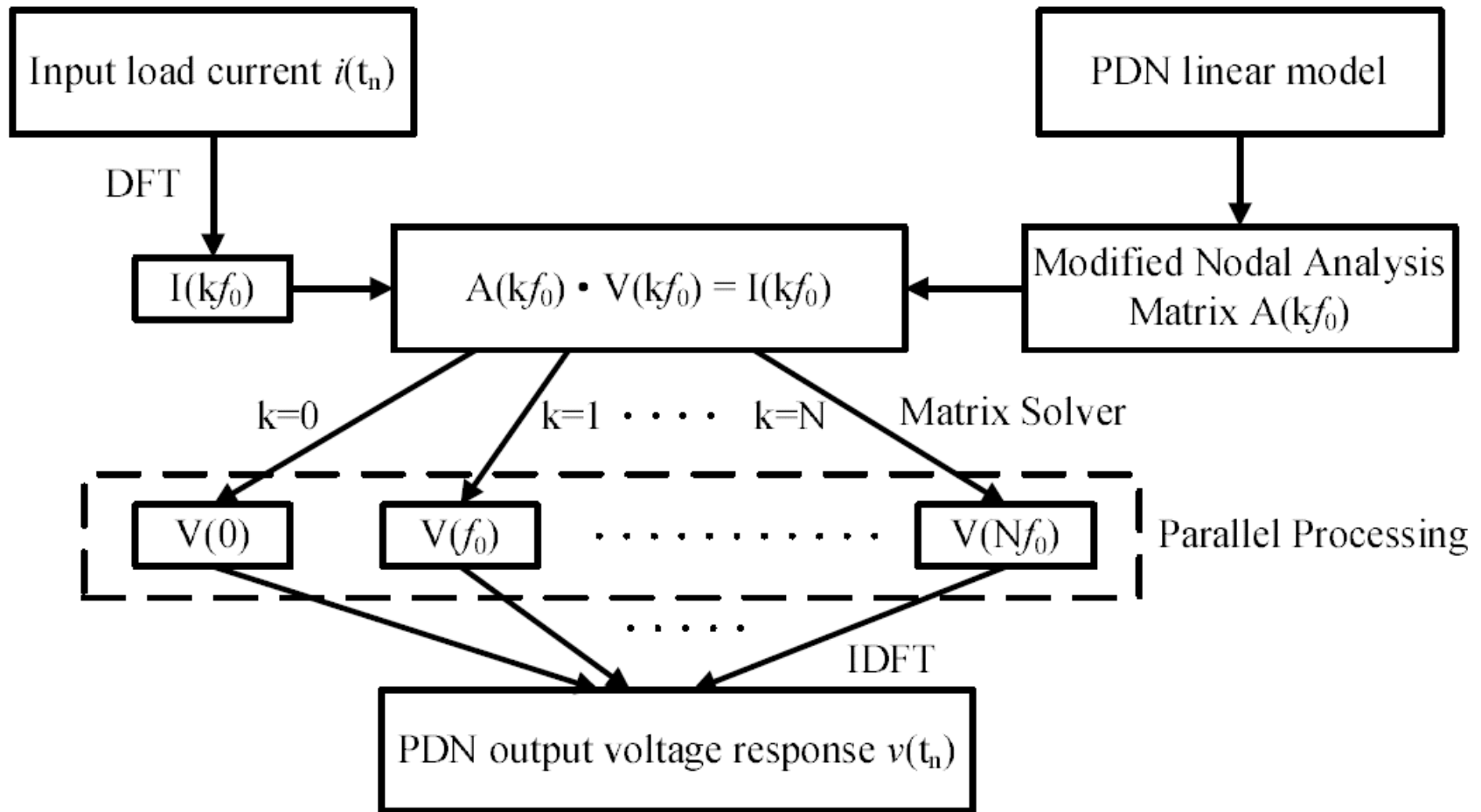
# Transform Operations: Why Discrete Fourier Transform

- Laplace Transform [Wanping '07]
  - Input: Series of ramp functions
  - Output: Rational expressing via vector fitting
    - Vector fitting may introduce large errors
  - Choice of frequency samples is case dependent
- Discrete Fourier Transform (DFT)
  - Input: periodic signal
  - Inverse DFT is straightforward: vector fitting is not needed
  - Frequency sample points with uniform steps

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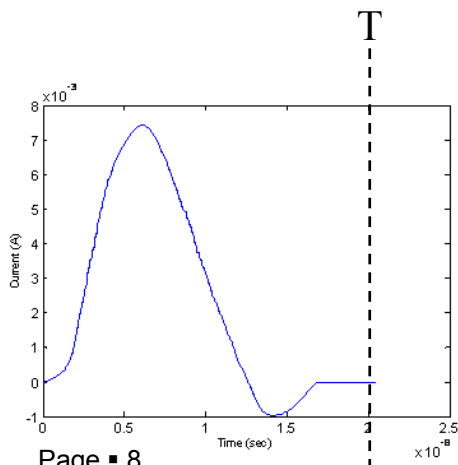
# Basic DFT Simulation Flow



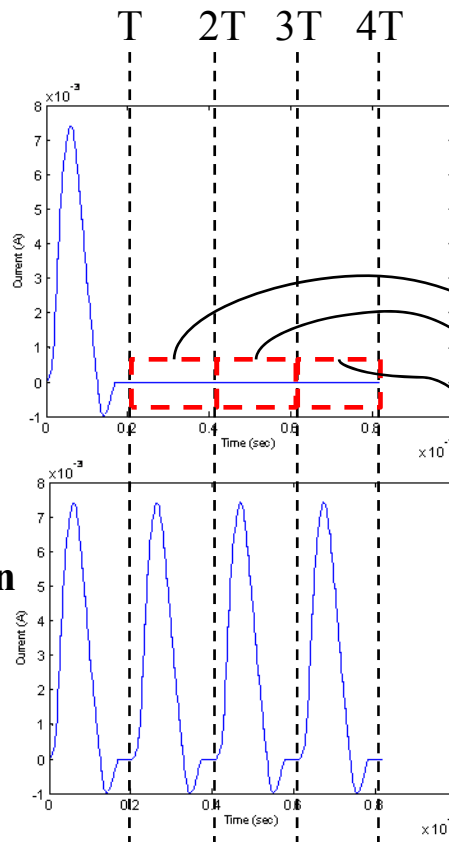
# Problem with Basic DFT Flow

- “Wrap-around effect” requires long padding zeros at the end of the input
  - Periodicity nature of DFT
- Small uniform time steps are needed to cover the input frequency range

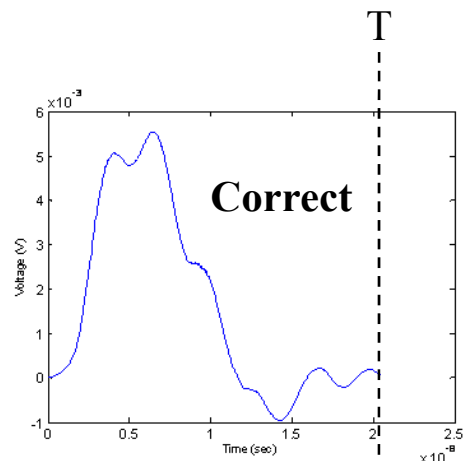
Large number of simulation points!



DFT repetition

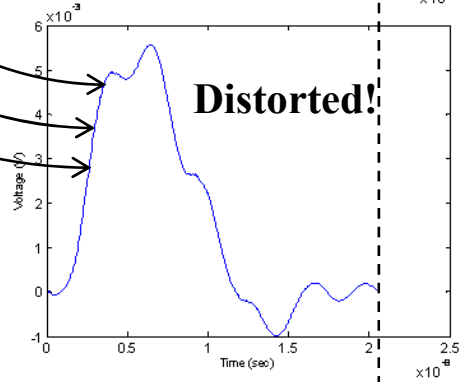


output



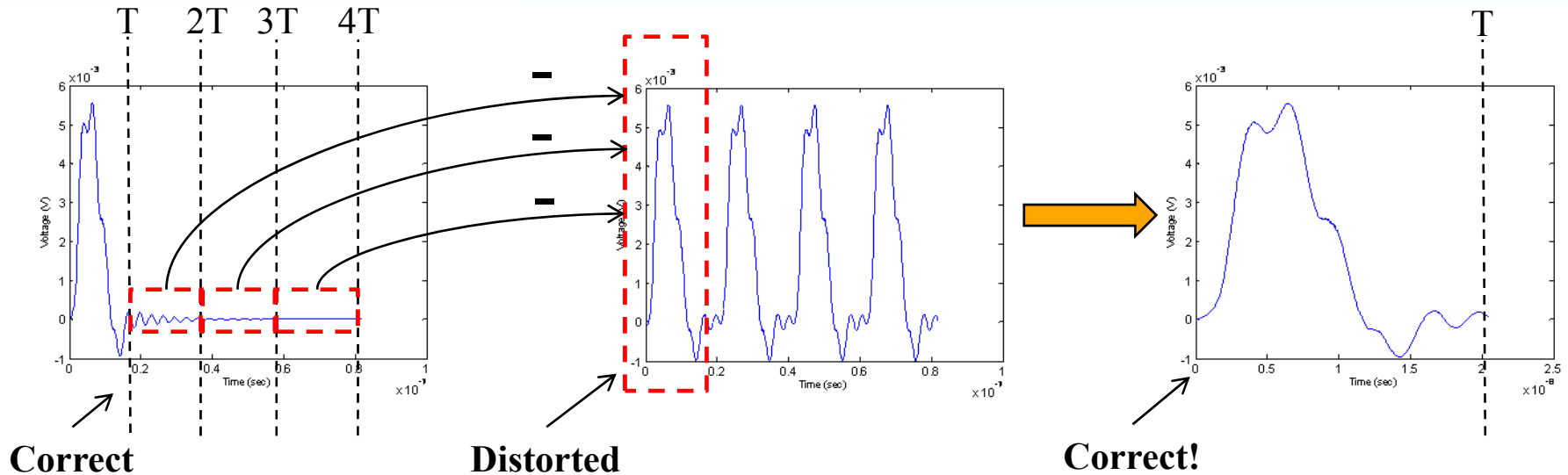
Wrap-around

output





# Adaptive DFT Simulation

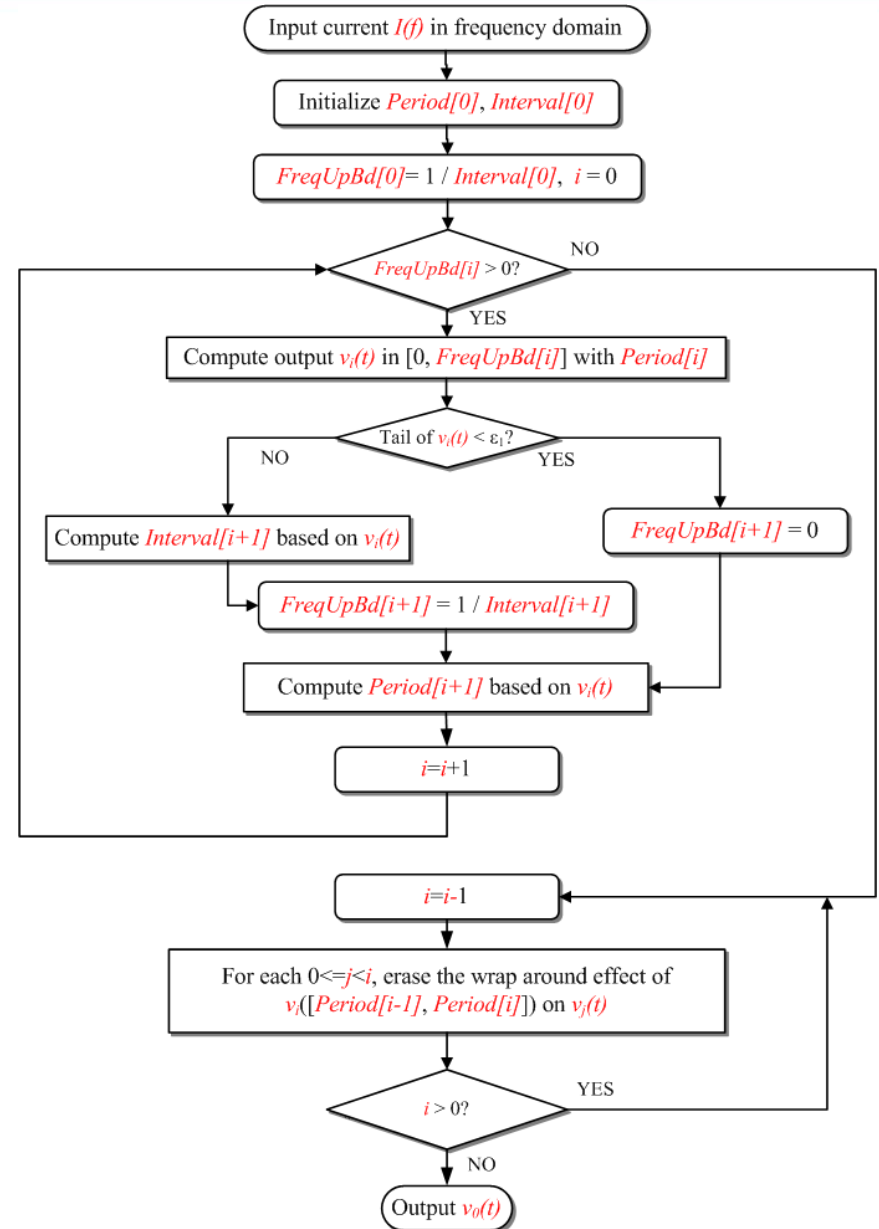


- Basic ideas of the adaptive DFT flow: cancel out the wrap-around effect by subtracting the tail from the main part of the output
  - Main part of the output: obtained with small time step and small period; distorted by the wrap-around effect
  - Tail of the output: low frequency oscillation; can be captured with large time steps

**Total number of simulation points is reduced significantly!**

# Adaptive DFT Flow

- $Period[i]$ : the input period at each iteration
- $Interval[i]$ : the simulation time step at each iteration
- $FreqUpBd[i]$ : the upper bound of the input frequency range at each iteration
- $v_i(t)$ : tentative time-domain output within the frequency range  $[0, FreqUpBd]$  at each iteration
- Iteration #1: obtain the main part of the output
- Iteration #2~k: capture the oscillations in the tail of the output (high, middle, and low resonant frequencies)
- For each iteration #i,  $i=k, k-1, \dots, 2$ , subtract the captured tail from the outputs at iteration #j,  $j < i$  to eliminate the wrap-around effect



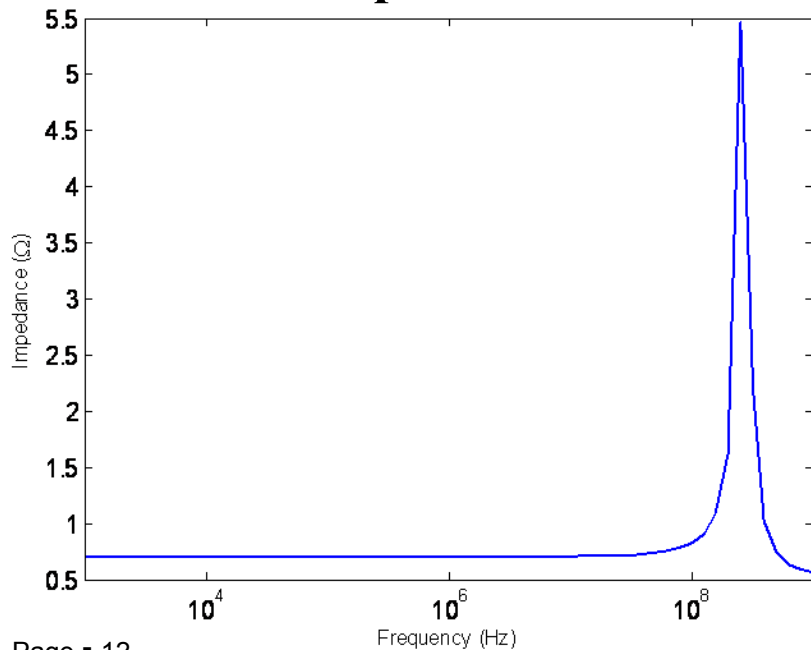
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- **Experimental results**
  - Error analysis
  - Time efficiency of the adaptive flow
  - Time efficiency of parallel processing
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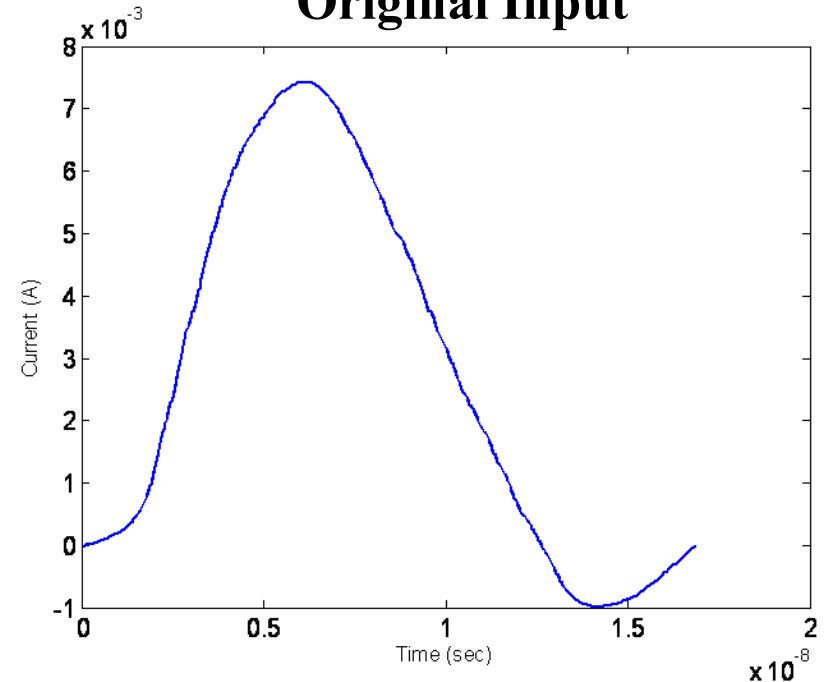
# Experimental Results: Test Case & Input

- Test case: 3D PDN
  - One resonant peak in the impedance profile
- Input current
  - Time step:  $\Delta t = 20\text{ps}$
  - Duration:  $T_0 = 16.88\text{ns}$

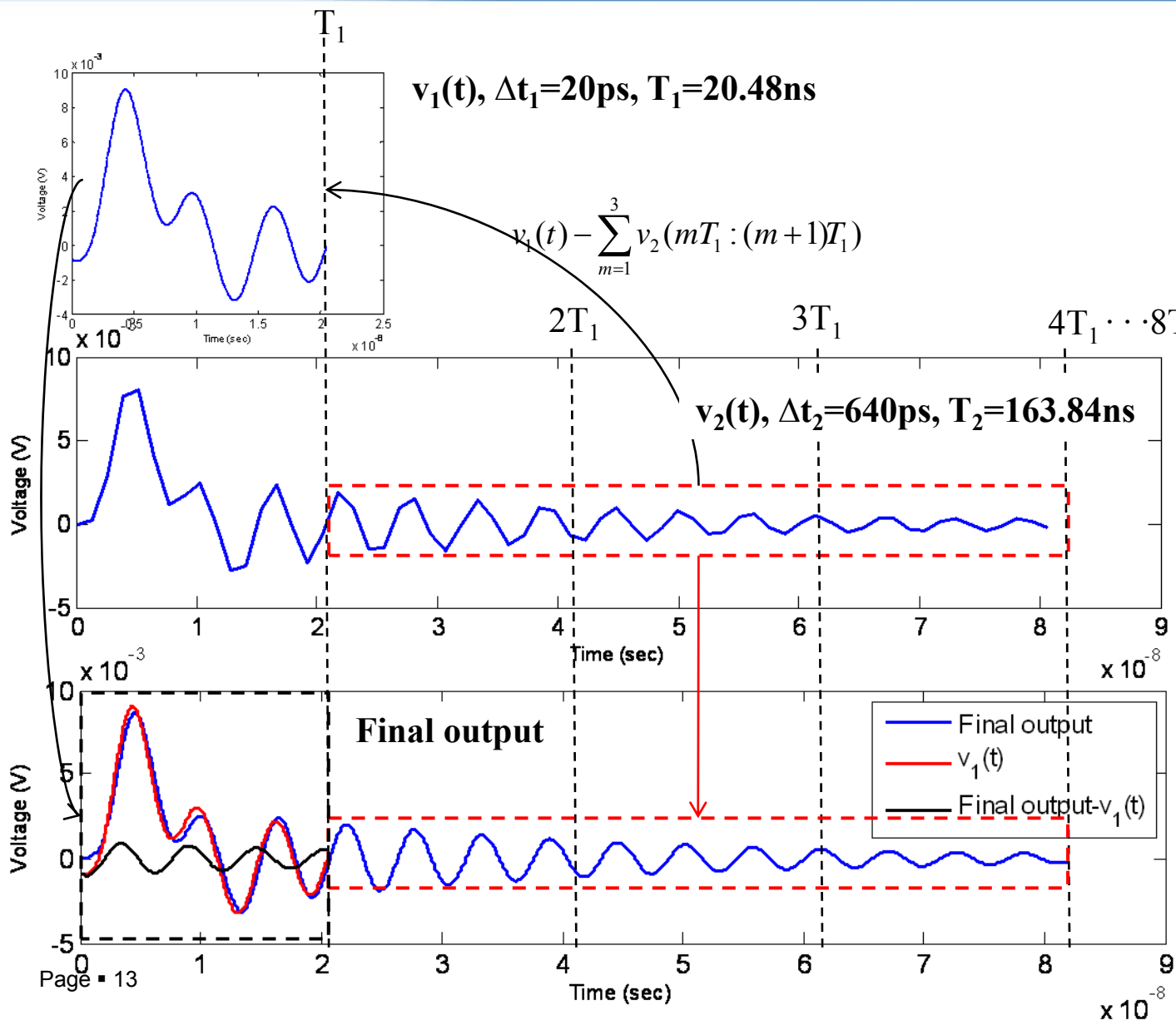
## Impedance



## Original Input

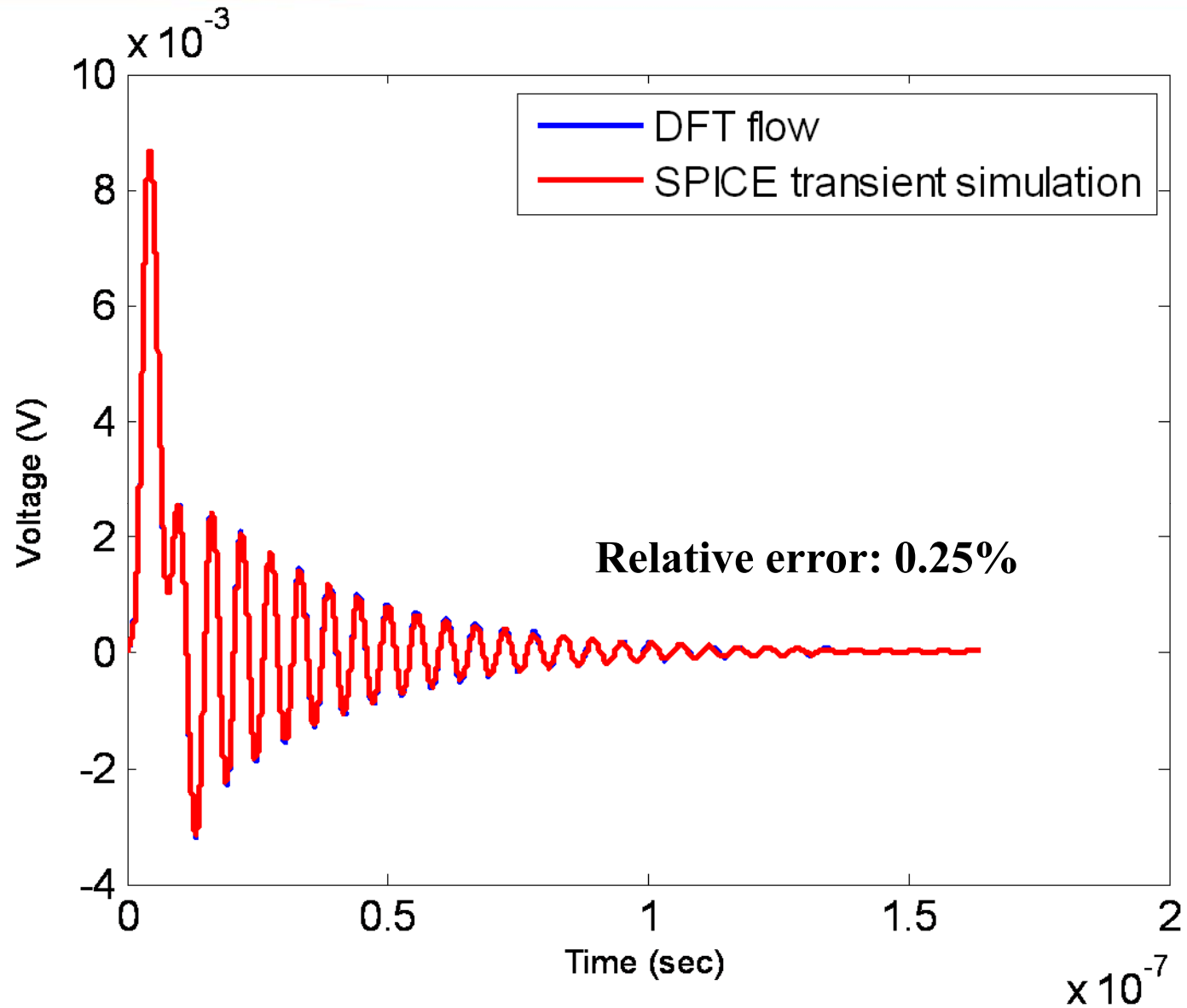


# Experimental Results: Adaptive Flow Process



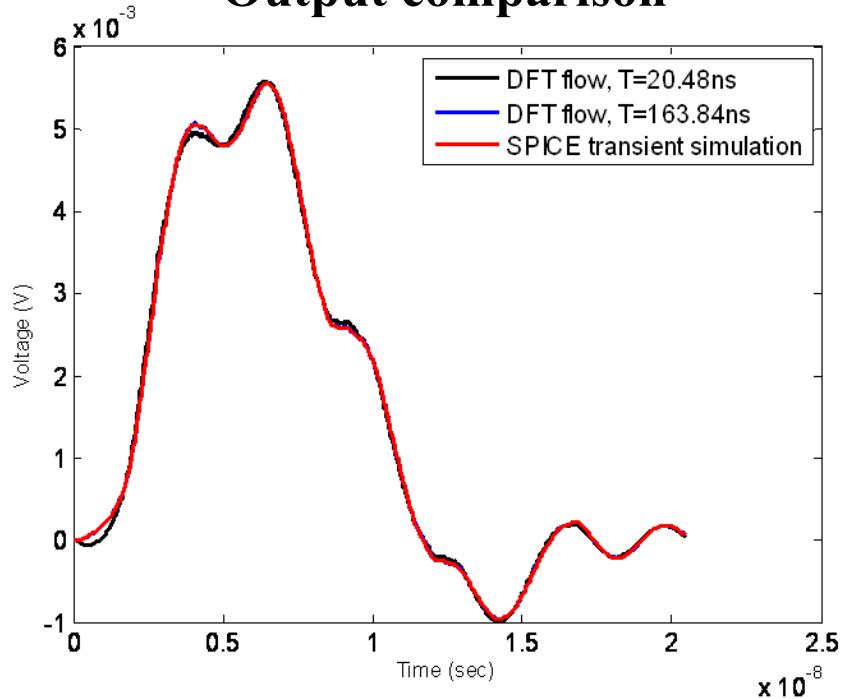
- Iteration #1:  $v_1(t)$ 
  - $\Delta t_1=20\text{ps}$
  - $T_1=20.48\text{ns}$
- Iteration #2:  $v_2(t)$ 
  - $\Delta t_2 = 64\Delta t_1$   
= 640ps
  - $T_2 = 8T_1$   
= 163.84ns
- Final output:
  - Main part:  
 $v_1(t) - \sum_{m=1}^3 v_2(mT_1 : (m+1)T_1)$
  - Tail:  
 $v_2(T_1 : 8T_1)$

# Experimental Results: DFT Flow vs. SPICE

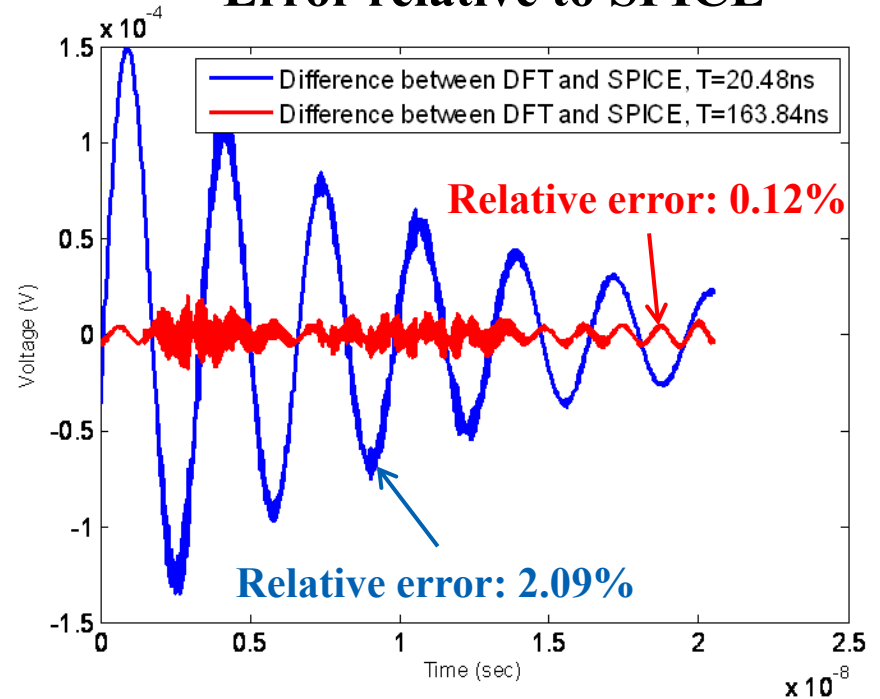


# Error Analysis: Error Caused by Wrap-around Effect

## Output comparison



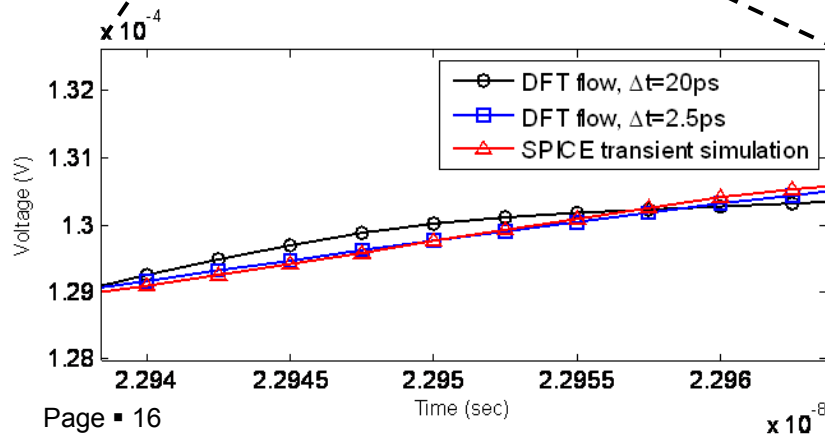
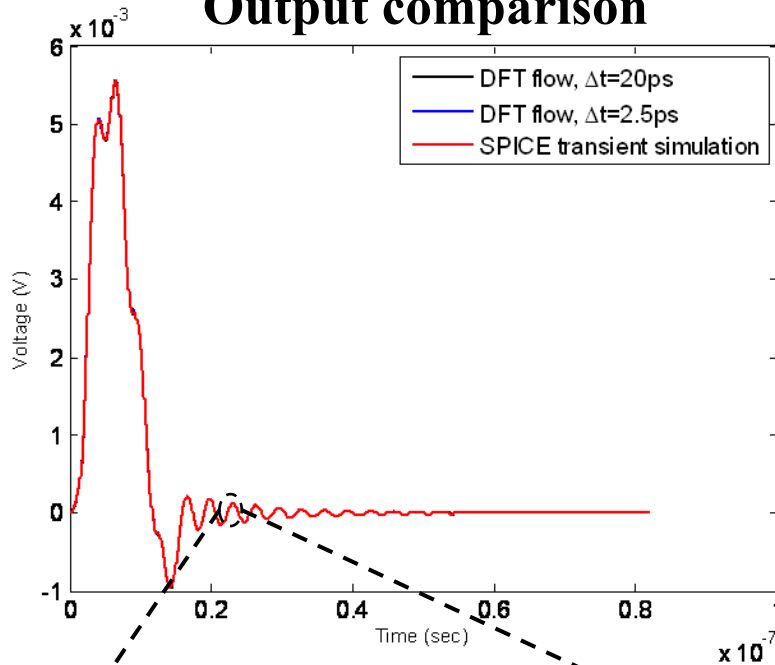
## Error relative to SPICE



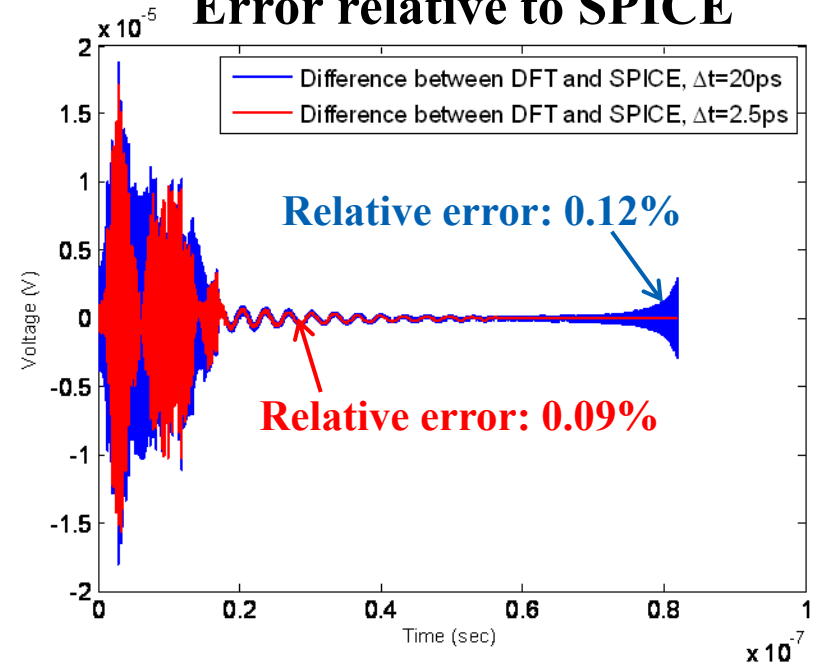
**Theorem 1:** Let  $v_0$  be the initial value of the output voltage. Suppose  $\{\int_{T-T'}^T (v(t)-v_0)^2 dt\}^{1/2} \leq \epsilon$  for some  $T' > 0$ , then the mean square error, i.e.,  $\|\Delta v(t)\|$  is bounded by  $\epsilon\sqrt{T/T'}$ .

# Error Analysis: Error Caused by Different Interpolation Methods

## Output comparison



## Error relative to SPICE



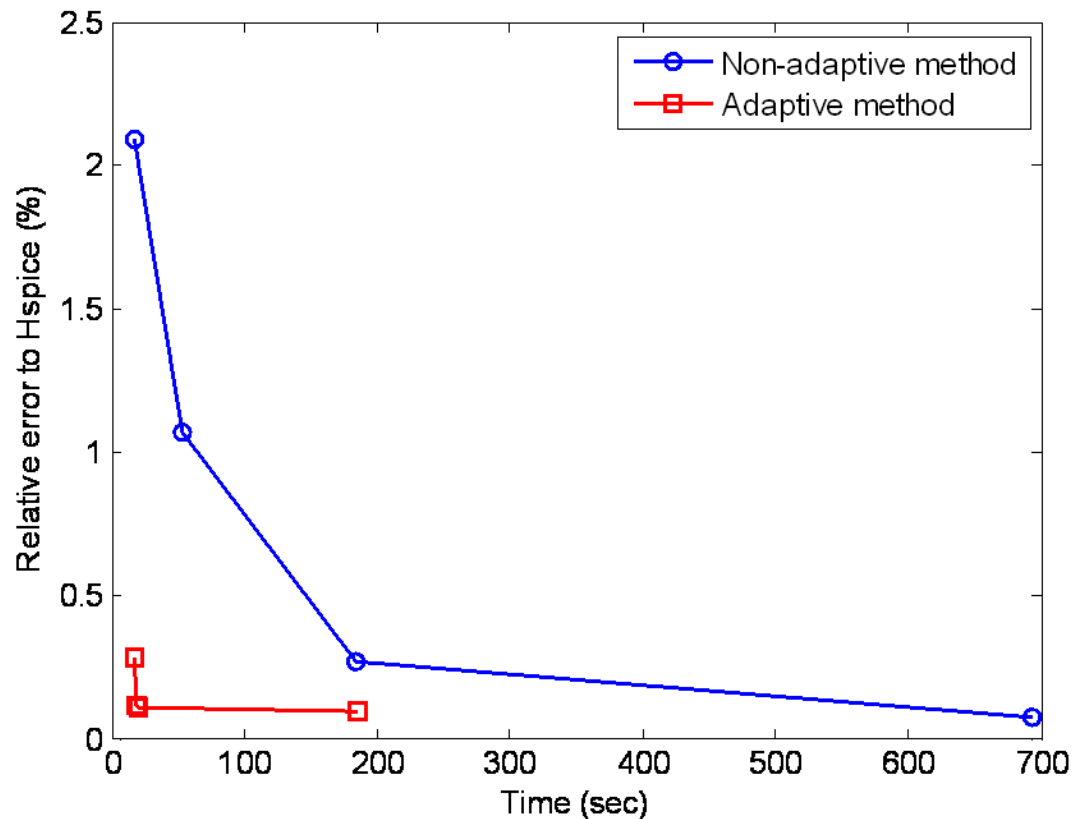
- SPICE: PWL interpolation
- DFT: sinusoidal interpolation

$$v(t) = \sum_{k=-N/2+1}^{N/2} V(f_k) e^{j2\pi tk/T}$$



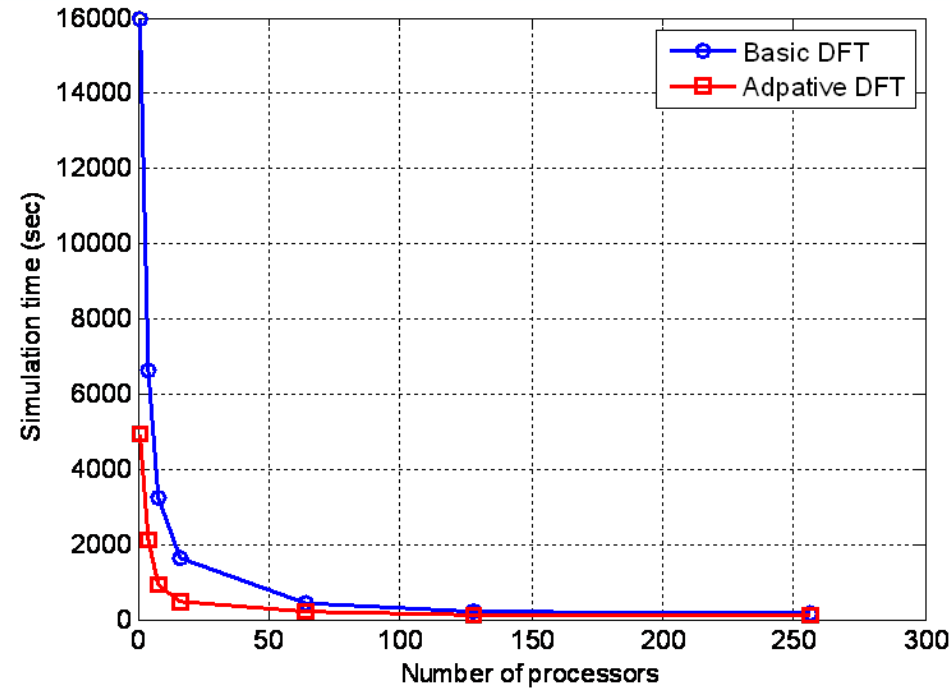
# Time Complexity Analysis: Adaptive vs. Non-adaptive

- Adaptive flow time complexity:  $O(\sum_{i=1}^k T_i / \Delta t_i)$ 
  - $T_i$ : simulation period at iteration #i,  $T_1 < T_2 < \dots < T_k$
  - $\Delta t_i$ : simulation time step at iteration #i,  $t_1 < t_2 < \dots < t_k$
- Non-adaptive flow time complexity:  $O(T_k / \Delta t_1)$



# Parallel Processing

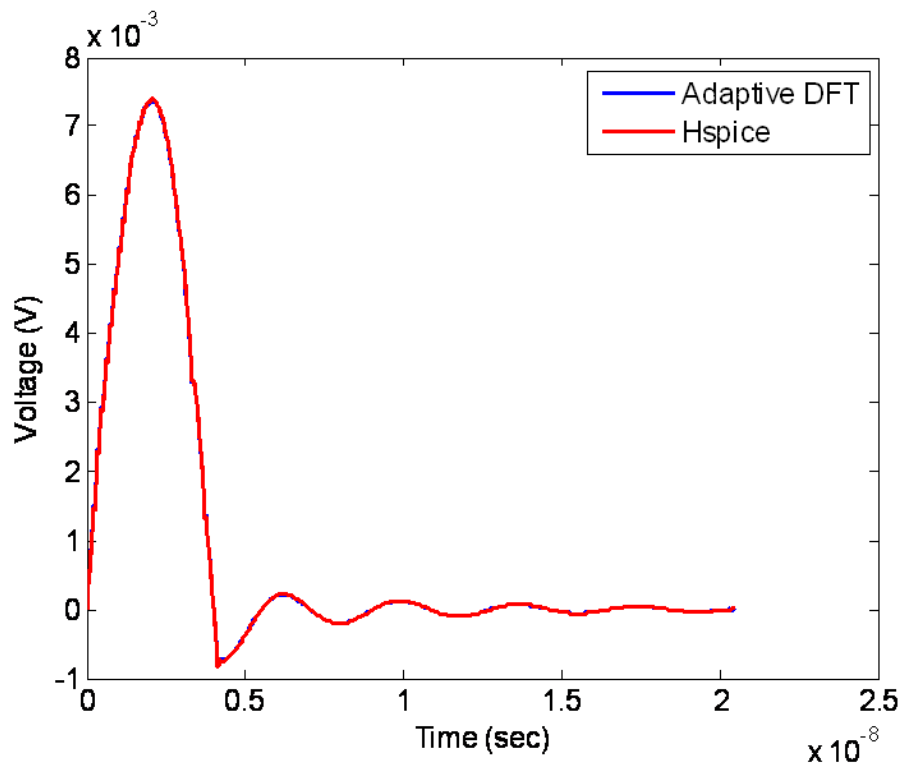
- Test case: 3D PDN
  - ~ 0.17 million nodes
- The adaptive DFT flow has more advantage when the number of available processors is limited.
- Simulations between each iteration of the adaptive flow need to be processed serially



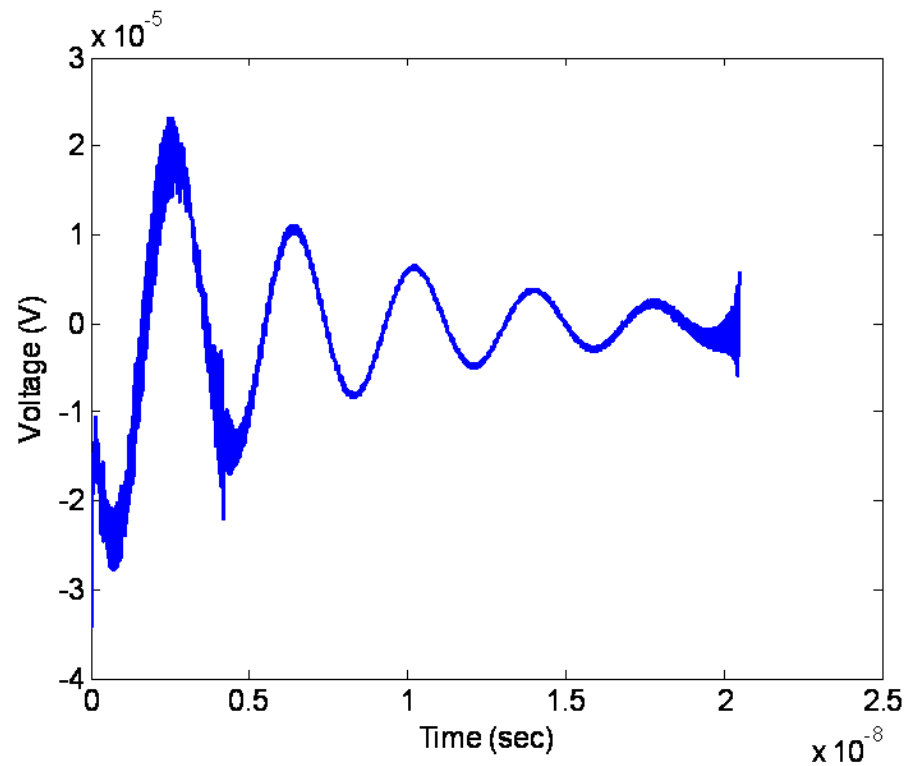
	Simulation time with different number of processors (sec)						
	1 prc	4 prcs	8 prcs	16 prcs	64 prcs	128 prcs	256 prcs
Hspice	21374	NA	NA	NA	NA	NA	NA
Basic DFT	15976	6635	3225	1647	425	218	143
Adaptive DFT	4947	2096	904	490	180	120	115

# Parallel Processing: DFT Flow vs. SPICE

## Simulation result



## DFT error compared to Hspice



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# Conclusions

- Implemented an adaptive flow for large PDN simulation using DFT
- Total number of simulation points is reduced significantly compared to the basic DFT flow
- Achieved a relative error of the order of 0.1% compared to SPICE
- 10x speed up with a single processor compared to SPICE.
- Parallel processing is incorporated to reduce the simulation time even more significantly

**Thank You !**