16th Asian Pacific Design Automation Conference

Analog Circuit Verification by Statistical Model Checking

Author Ying-Chih Wang (Presenter)

Anvesh Komuravelli

Paolo Zuliani

Edmund M. Clarke

Date 2011/1/26

This work is sponsored by the GigaScale Systems Research Center, Semiconductor Research Corporation, National Science Foundation, General Motors, Air Force and the Office of Naval Research

Overview

- **Introduction**
- **Background**
  - Assumptions
  - Bayesian Statistics
- **Algorithm**
- **Application to Analog Circuits**
  - Bounded Linear Temporal Logic
- **Experimental Results**
- **Discussion**
- **Conclusion and Future Work**

Introduction – Motivation

- **Analog circuits' behaviors vary dramatically with changes in prameters**
  - MOSFET parameters, temperature, etc.
  - Performance, correctness
- **Circuits' resiliency under uncertainties**
  - Process variation, noise, etc.
- **How to deal with uncertainties?**

Introduction – Problem

- **“Does the design satisfy the property with probability $p \geq$ threshold θ?”**
  - Monte Carlo: estimate the probability p
    - How good is the estimate?
  - Property : Bounded Linear Temporal Logic (BLTL)
- **Statistical Model Checking (SMC)**
  - Recently applied to Stateflow/Simulink models[1]
  - Probabilistic Guarantees
  - Bayesian Hypothesis Testing
  - Bayesian Estimation

[1] P. Zuliani, A. Platzer and E. M. Clarke, “Bayesian Statistical Model Checking with Application to Stateflow/Simulink Verification,” in HSCC 2010.

Background – Assumptions

Monte Carlo Simulation

- Draw independent and identically distributed (i.i.d.) sample traces
- Characterize each simulation trace as Pass/Fail

Property Φ

$+$

*Stochastic system **M***

$=$

Biased coin

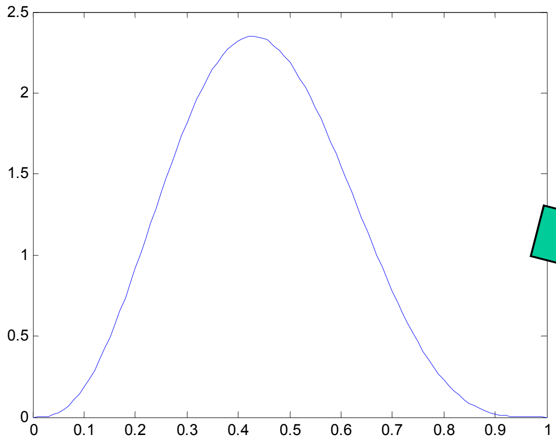
Bernoulli Random Variable Y

$P(Y=1) = p$

$P(Y=0) = 1-p$

$\mathbf{M} \models \Phi$ *with probability p*

Background – Bayesian Statistics



Prior density of p
(Beta Distribution)

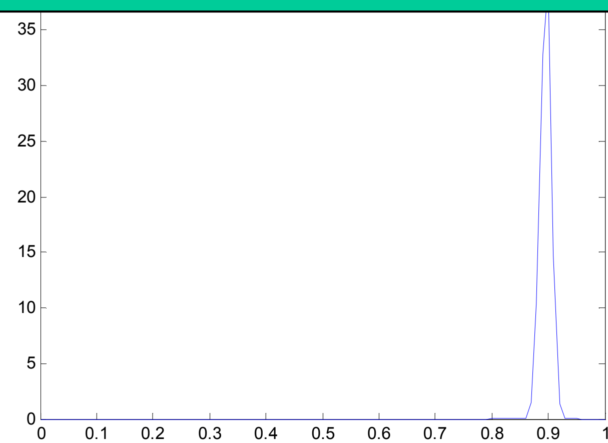
Perform Experiments Sequentially

Experimental Results
{Pass,Fail,Pass,Pass,…}

Bayesian Estimation

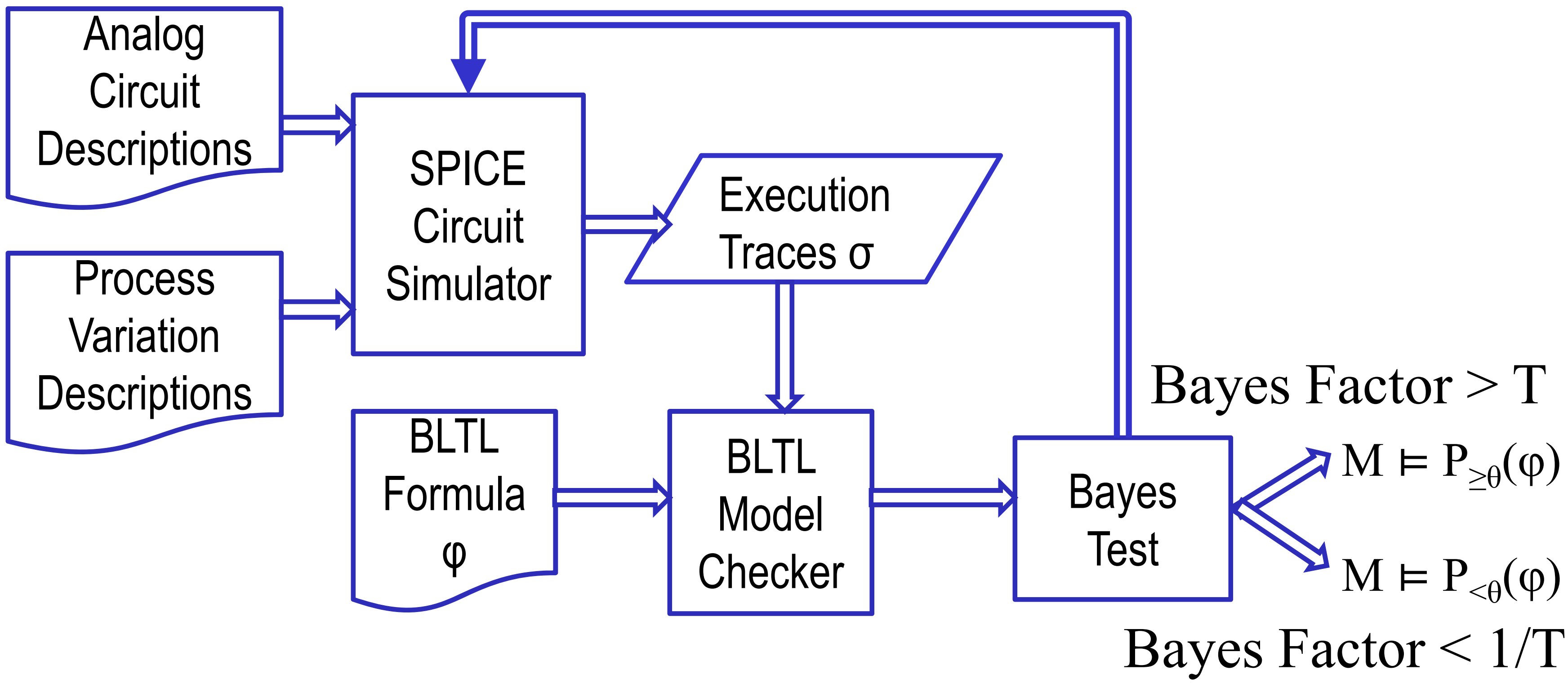
Bayesian
Hypothesis Testing

Bayes Factor



Posterior density of p

Overview –Analog Circuit Verification



Background – Bayesian Statistics

- **Null hypothesis $H_0(p \geq \theta)$**
- **Alternative hypothesis $H_1(p < \theta)$**
  - $P(H_0)$, $P(H_1)$ sum to 1
- **X: experimental results**
- **Bayes Factor** $B = \frac{P(X|H_0)}{P(X|H_1)}$
- **Jefferys' Test [1960s]**
  - Fixed Sample Size
  - Bayes Factor > 100 : Strongly supports H_0
  - Sequential Version (Fixed Bayes Factor Threshold)

Statistical Model Checking – Algorithm

Require: ***Property $P_{\geq\theta}(\Phi)$, Threshold $T \geq 1$, Prior density g***

n := 0 ***{number of traces drawn so far}***

x := 0 ***{number of traces satisfying Φ so far}***

repeat

  σ := draw a sample trace of the system (iid)

  n := n + 1

  if $\sigma \models \Phi$ **then**

    x := x + 1

  endif

  B := *BayesFactor(n, x, θ, g)*

until (B > *T* v B < *1/T*)

if (B > *T*) then

  return "*H_0 accepted*"

else

  return "*H_0 rejected*"

endif

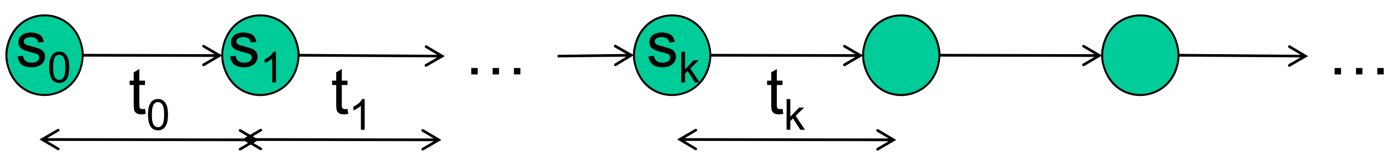
Theorem (Error bounds). When the Bayesian algorithm – using threshold *T* – stops, the following holds:

Prob ("accept H_0" | H_1) ≤ 1/*T*

Prob ("reject H_0" | H_0) ≤ 1/*T*

Bounded Linear Temporal Logic

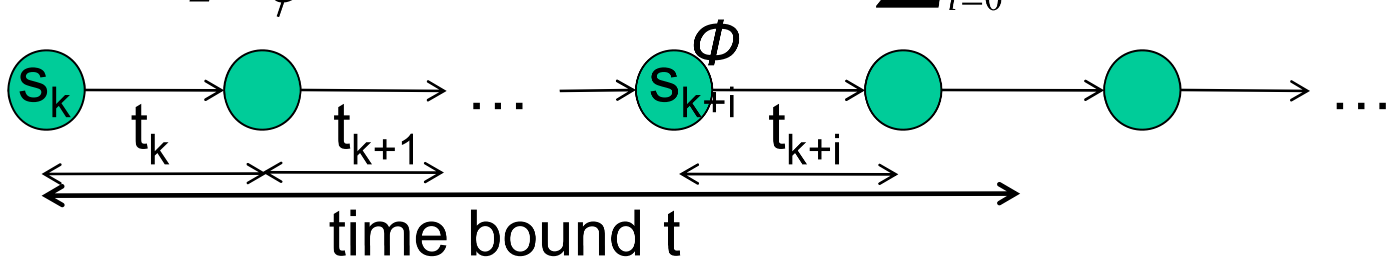
- **States: State Variables**
- **Atomic Propositions (AP): $e_1 \sim e_2$**
  - e_1, e_2: arithmetic expressions over state variables, $\sim \in \{<, \leq, >, \geq, =\}$
- **Syntax** $\phi ::= AP \mid \phi_1 \vee \phi_2 \mid \neg \phi \mid F^t \phi \mid G^t \phi$
- **Let $\sigma = (s_0,t_0), (s_1,t_1), \ldots$ be a simulation trace of the model**
  - stays in s_i for duration t_i.



- **σ^k: suffix trace of σ starting at state *k***
  - $\sigma^k = (s_k,t_k), (s_{k+1},t_{k+1}), \ldots$

Bounded Linear Temporal Logic

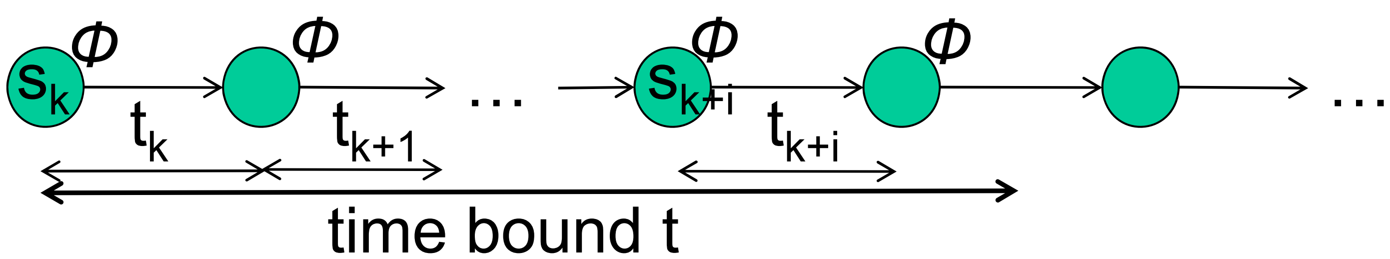
- **The semantics of BLTL for suffix trace σ^k *(trace starting at state k)*:**
  - $\sigma^k \models AP$ iff atomic proposition *AP is* true in state s_k
  - $\sigma^k \models \phi_1 \vee \phi_2$ iff $\sigma^k \models \Phi_1$ or $\sigma^k \models \Phi_2$
  - $\sigma^k \models \neg\phi$ iff $\sigma^k \models \Phi$ does not hold
  - $\sigma^k \models F^t\phi$ iff exists i ≥0 s.t. $\sum_{l=0}^{i-1} t_{k+l} \leq t$ and $\sigma^{k+i} \models \Phi$



  - **F**[10ns] (Vout>1)
  - Within 10ns, Vout should eventually be larger than 1 volt.

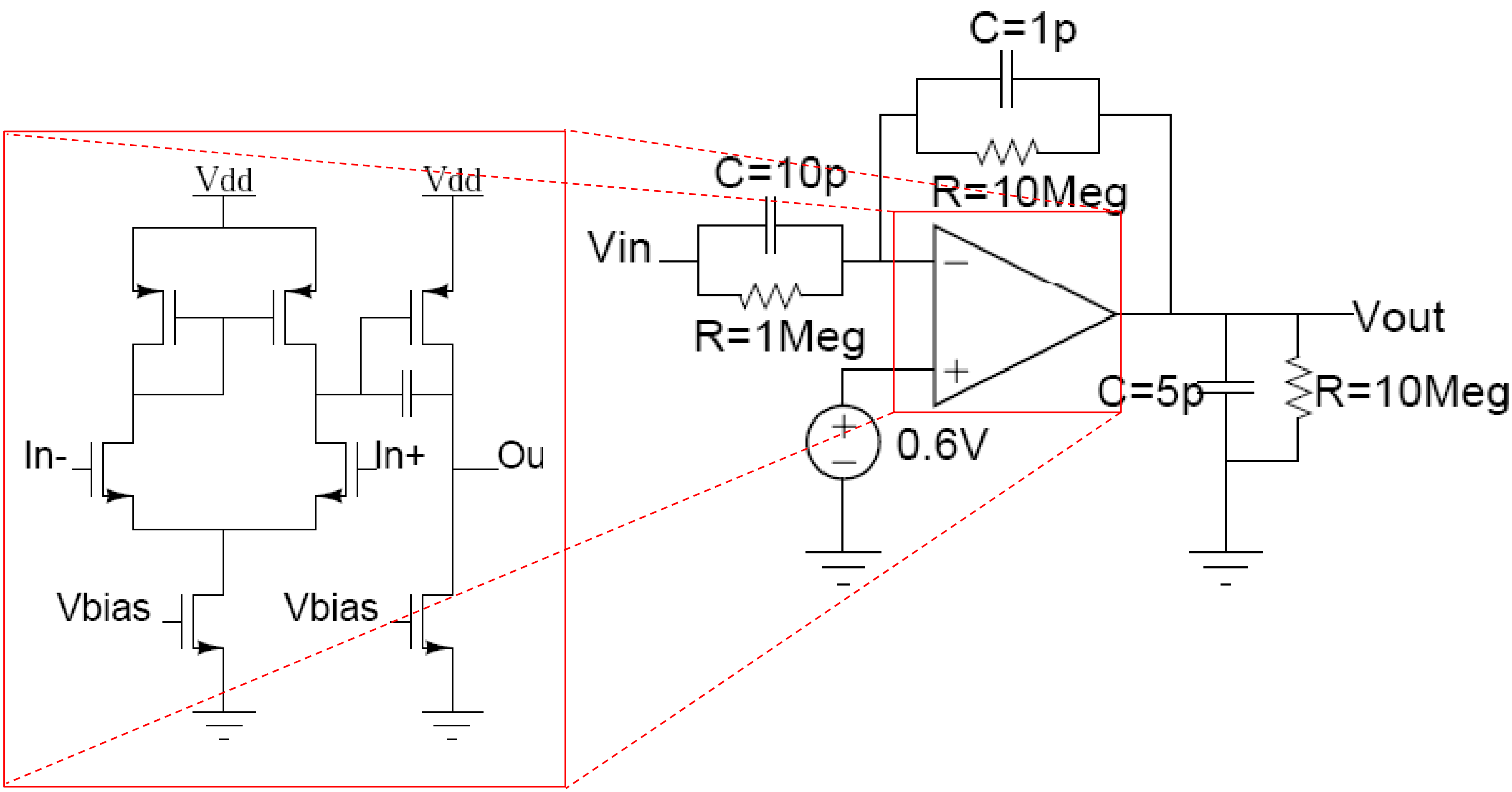
Bounded Linear Temporal Logic

- The **semantics** of BLTL for suffix trace σ^k *(trace starting at state k):*
  - $\sigma^k \vDash G^t \phi$ iff for all i ≥0 s.t. $\sum_{l=0}^{i-1} t_{k+l} \leq t$, $\sigma^{k+i} \vDash \Phi$

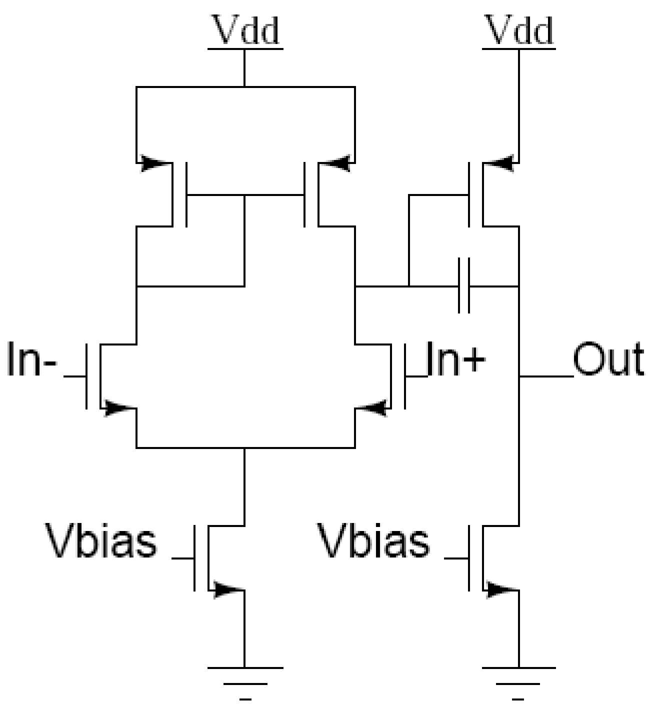


  - $G^{[10ns]}$ (Vout>1)
  - In the next 10ns, Vout should always be greater than 1 volt.
- **Nesting Operators**
  - $F^{[100ns]}G^{[10ns]}$ (Vout>1)
  - Within 100ns, Vout will be greater than 1 volt and stay greater than 1 volt for at least 10 ns.

An Example : OP amp



An Example : OP amp – Specifications



Specifications		
1	Input Offset Voltage	< 1 mV
2	Output Swing Range	0.2 V to 1 V
3	Slew Rate	> 25 V/µSec
4	Open-Loop Voltage Gain	> 8000 V/V
5	Loop-Gain Unit-gain Frequency	> 10 MHz
6	Phase Margin	> 60°

Example : OP amp – Specifications

- **Specifications : Quantities computed from simulation traces**
  - in most cases, can be translated to BLTL directly from definitions
- **e.g. Swing Range:**
  - Max(Vout) > 1V and Min(Vout) < 0.2V
  - $\mathbf{F}^{[100\mu s]}(V_{out} < 0.2) \wedge \mathbf{F}^{[100\mu s]}(V_{out} > 1)$
  - "Within entire trace, Vout will eventually be greater than 1V and smaller than 0.2V"
  - 100μs : the end time of transient simulation

Example : OP amp – Specifications

- **Frequency-domain properties?**
  - *Transient response* $\sigma = (s_0,t_0), (s_1,t_1),(s_2,t_2),\ldots$
  - *Frequency response* $\sigma = (s_0,f_0), (s_1,f_1),(s_2,f_2),\ldots$
- **Substitute frequency-stamps for time-stamps**
  - Specify frequency domain properties in BLTL
  - Check with the same BLTL checker
  - Semantics Changed
    - $G^{[1GHz]}\varphi$:φ holds for all frequencies from current to 1GHz larger
    - $F^{[1GHz]}\varphi$:φ holds for some frequency from current to 1GHz larger

An Example : OP amp – Specifications

Specifications (Transient)			BLTL Specifications
1	Input Offset Voltage	< 1 mV	$\mathbf{F}^{[100\mu s]}(V_{out} = 0.6) \wedge \mathbf{G}^{[100\mu s]}((V_{out} = 0.6) \rightarrow (\lvert V_{in+} - V_{in-}\rvert < 0.001))$
2	Output Swing Range	0.2 V to 1 V	$\mathbf{F}^{[100\mu s]}(V_{out} < 0.2) \wedge \mathbf{F}^{[100\mu s]}(V_{out} > 1)$
3	Slew Rate	> 25 V/μSec	
	$\mathbf{G}^{[100\mu s]}(\ ((V_{out} = 1 \wedge V_{in} > 0.65) \rightarrow \mathbf{F}^{[0.008\mu s]}(V_{out} = 0.8)) \wedge ((V_{out} = 0.2 \wedge V_{in} < 0.55) \rightarrow \mathbf{F}^{[0.008\mu s]}(V_{out} = 0.4))\ )$		

Specifications (Frequency-domain)			BLTL Specifications
4	Open-Loop Voltage Gain	> 8000 V/V	$\mathbf{G}^{[1KHz]}(V\ mag_{out} > 8000)$
5	Loop-Gain Unit-gain Frequency	> 10 MHz	$\mathbf{G}^{[10MHz]}(V\ mag_{out} > 1)$
6	Phase Margin	> 60°	$\mathbf{F}^{[10GHz]}(V\ mag_{out} = 1) \wedge \mathbf{G}^{[10GHz]}(\ (V\ mag_{out} = 1) \rightarrow (Vphase_{out} > 60^{\circ})\ )$

Experimental Setup

- **Platform: Linux virtual machine running on a 2.26GHz i3-350M, 4GB RAM computer**
- **We model-check the BLTL formulae on previous slides**
  - e.g. (Swing Range)
    $H_0{:}\mathcal{M} \models P_{\geq\theta}[\mathbf{F}^{[100\mu s]}(V_{out} < 0.2) \wedge \mathbf{F}^{[100\mu s]}(V_{out} > 1)]$

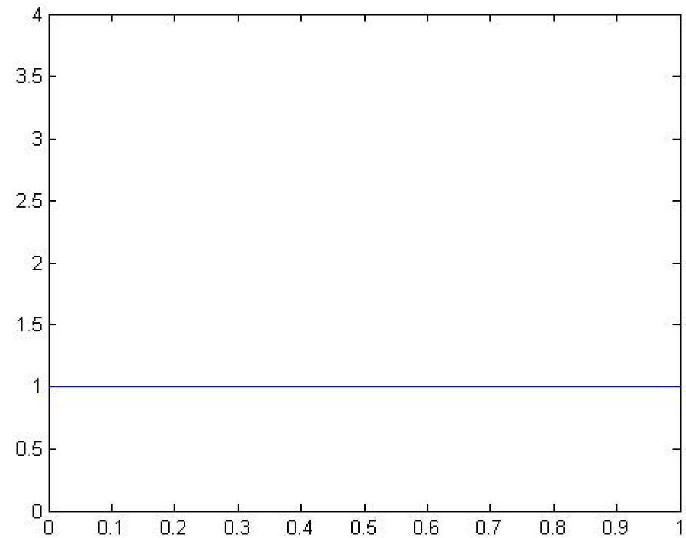
Experimental Results

- **Target yield: 0.95**
- **Monte Carlo analysis:**
  - do not satisfy /satisfy yield threshold

Monte Carlo (1000 samples) — Measured Value				
Specifications		Mean	Stddev	Yield
1	Input Offset Voltage (V)	.436	.597	.826
2	Swing Range Min (V)	.104	.006	1.00
	Swing Range Max (V)	1.08	.005	1.00
3	Negative Slew Rate (V/μSec)	-40.2	1.17	1.00
	Positive Slew Rate (V/μSec)	56.4	2.54	1.00
4	Open-Loop Voltage Gain (V/V)	8768.	448	.975
5	Loop-Gain UGF (MHz)	19.9	.30	1.00
6	Phase Margin (°)	64.1	.44	1.00

Experimental Results

- **Statistical Model Checking**
  - *Null Hypothesis (H_0):* From Specs,
    - *e.g.* $H_0 : \mathcal{M} \models P_{\geq\theta}[\mathbf{F}^{[100\mu s]}(V_{out} < 0.2) \wedge \mathbf{F}^{[100\mu s]}(V_{out} > 1)]$
  - *Probability Threshold θ*: Set to Target Yield
  - *Bayes Factor Threshold T:* Test Strength Needed
    - T = 1000 : Probability of error is less than 0.001
  - *Prior Distribution of p*
    - Uniform : No idea



Experimental Results

- **Monte Carlo : do not satisfy /satisfy yield threshold 0.95**
- **SMC testing result : Prior: Uniform, T = 1000, θ = 0.95**
  reject / accept null hypothesis

Monte Carlo (1000 samples) — Measured Value					SMC(θ = 0.95)
Specifications		Mean	Stddev	Yield	Samples/Runtime
1	Input Offset Voltage (V)	.436	.597	.826	31/39s
2	Swing Range Min (V)	.104	.006	1.00	77/98s
	Swing Range Max (V)	1.08	.005	1.00	77/98s
3	Negative Slew Rate (V/μSec)	-40.2	1.17	1.00	77/98s
	Positive Slew Rate (V/μSec)	56.4	2.54	1.00	77/98s
4	Open-Loop Voltage Gain (V/V)	8768.	448	.975	239/303s
5	Loop-Gain UGF (MHz)	19.9	.30	1.00	77/98s
6	Phase Margin (°)	64.1	.44	1.00	77/98s

Experimental Results

- ***Prior: uniform, T = 1000, θ in [0.7, 0.999]***
  - SMC testing result : reject / accept null hypothesis

Spec	Null Hypothesis
1	$\mathbf{M} \models P_{\geq\theta}[\mathbf{F}^{[100\mu s]}(V_{out} = 0.6) \wedge \mathbf{G}^{[100\mu s]}((V_{out} = 0.6) \rightarrow (\lvert V_{in+} - V_{in-}\rvert < 0.001))]$
2	$\mathbf{M} \models P_{\geq\theta}[\mathbf{F}^{[100\mu s]}(V_{out} < 0.2) \wedge \mathbf{F}^{[100\mu s]}(V_{out} > 1)]$
3	$\mathbf{M} \models P_{\geq\theta}[\mathbf{G}^{[100\mu s]}(((V_{out} = 1 \wedge V_{in} > 0.65) \rightarrow \mathbf{F}^{[0.008\mu s]}(V_{out} < 0.8)) \wedge ((V_{out} < 0.2 \wedge V_{in} < 0.55) \rightarrow \mathbf{F}^{[0.008\mu s]}(V_{out} < 0.4)))]$

Samples/Runtime

Probability threshold θ					
Spec	0.7	0.8	0.9	0.99	0.999
1	77/105s	9933/12161s	201/275s	10/13s	7/9s
2	16/18s	24/27s	44/51s	239/280s	693/813s
3	16/23s	24/31s	44/57s	239/316s	693/916s

Experimental Results

- ***Prior: uniform, $T = 1000$, θ in [0.7, 0.999]***
  - SMC testing result : reject / accept null hypothesis

Spec	Null Hypothesis
4	$\mathbf{M} \models P_{\geq\theta}[\mathbf{G}^{[1KHz]}(Vmag_{out} > 8000)]$
5	$\mathbf{M} \models P_{\geq\theta}[\mathbf{G}^{[10MHz]}(Vmag_{out} > 1)]$
6	$\mathbf{M} \models P_{\geq\theta}[\mathbf{F}^{[10GHz]}(Vmag_{out} = 1) \wedge \mathbf{G}^{[10GHz]}((Vmag_{out}=1) \rightarrow (Vphase_{out}>60^{\circ}))]$

Samples/Runtime

Probability threshold θ					
Spec	0.7	0.8	0.9	0.99	0.999
4	23/26s	43/49s	98/114s	1103/1309s	50/57s
5	16/18s	24/28s	44/51s	239/279s	693/807s
6	16/20s	24/30s	44/55s	239/303s	693/882s

Discussion

- **Statistical Model Checking is faster when**
  - The threshold probability θ is away from the unknown probability p,
    - e.g. p ≈ 1

θ	0.7	0.8	0.9	0.99	0.999
Samples	16	24	44	239	693

- **Easily integrated in the validation flow**
  - Relies only on Monte Carlo sampling and SPICE
    - Add-ons: Online BLTL monitoring, Bayes factor calculation
  - Low computation overhead
    - Runtime dominated by SPICE simulation

Conclusion

- **Introduced SMC to analog circuit verification**
  - Avoid Monte Carlo simulation drawing unnecessary samples when evidence is enough.
- **Demonstrated the feasibility of using BLTL specifications for a simple analog circuit**
  - BLTL can specify complex interactions between signals
- **Future Works**
  - More experiments
    - larger examples with more complicated specifications
  - Introducing the technique of importance sampling

- **Thank you.**

Bayesian Statistical Model Checking

- Suppose $\mathcal{M}$ satisfies ϕ with (unknown) probability p
  - *p* is given by a random variable (defined on [0,1]) with density *g*
  - *g* represents the prior belief that $\mathcal{M}$ satisfies ϕ
- Generate independent and identically distributed (iid) sample traces $\sigma_1,\ldots,\sigma_n$
- $X = \{x_1,\ldots,x_n\}$, x_i: the i^{th} sample trace σ_i satisfies ϕ
  - $x_i = 1$ iff $\sigma_i \models \phi$ $x_i = 0$ iff $\sigma_i \not\models \phi$
- Then, x_i will be a Bernoulli trial with conditional density (likelihood function) : $f(x_i|u) = u^{x_i}(1 - u)^{1-x_i}$

Statistical Model Checking

- **Calculate Bayes Factor using posterior and prior probability**

$$\frac{P(X|H_0)}{P(X|H_1)} = \frac{P(H_0|X)}{P(H_1|X)} \cdot \frac{P(H_1)}{P(H_0)}$$

- **Posterior density (Bayes Theorem) (cond. iid Bernoulli's)**

$$f(u \mid x_1, \ldots, x_n) = \frac{f(x_1 \mid u) \cdots f(x_n \mid u) \cdot g(u)}{\int_0^1 f(x_1 \mid v) \cdots f(x_n \mid v) \cdot g(v)\ dv}$$

Likelihood function

Statistical Model Checking

- H_0: $p \geq \theta$, H_1: $p < \theta$,
- Prior Probability (g: prior density of p)

$$P(H_0) = \pi_0 = \int_\theta^1 g(u)du \qquad P(H_1) = \int_0^\theta g(u)du = 1 - \pi_0$$

- Bayes Factor of sample $X=\{x_1,\ldots,x_n\}$, and hypotheses H_0, H_1 *is*

$$\frac{P(H_0 \mid X)}{P(H_1 \mid X)} \cdot \frac{P(H_1)}{P(H_0)} = \frac{\int_\theta^1 f(x_1 \mid u) \cdots f(x_n \mid u) \cdot g(u)du}{\int_0^\theta f(x_1 \mid u) \cdots f(x_n \mid u) \cdot g(u)du} \cdot \frac{1-\pi_0}{\pi_0}$$

Beta Prior

- Prior g is Beta of parameters $\alpha>0, \beta>0$

$$\forall u \in [0,1] \quad g(u, \alpha, \beta) = \frac{1}{B(\alpha, \beta)} u^{\alpha-1}(1-u)^{\beta-1}$$

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$$

- $F_{(\cdot,\cdot)}(\cdot)$ is the Beta distribution function (i.e., Prob(X ≤ u))

$$F_{(\alpha,\beta)}(u) = \int_0^u g(t, \alpha, \beta)\, dt$$

Why Beta priors?

- Defined over [0,1]
- Beta distributions are *conjugate* to Binomial distributions:
  - If prior *g* is Beta and likelihood function is Binomial then *posterior is Beta*
- Suppose likelihood Binomial(n,x), prior Beta(α,β): posterior

$$f(u \mid x_1, \ldots, x_n) \approx f(x_1|u) \cdot \cdot \cdot f(x_n|u) \cdot g(\mathrm{u})$$

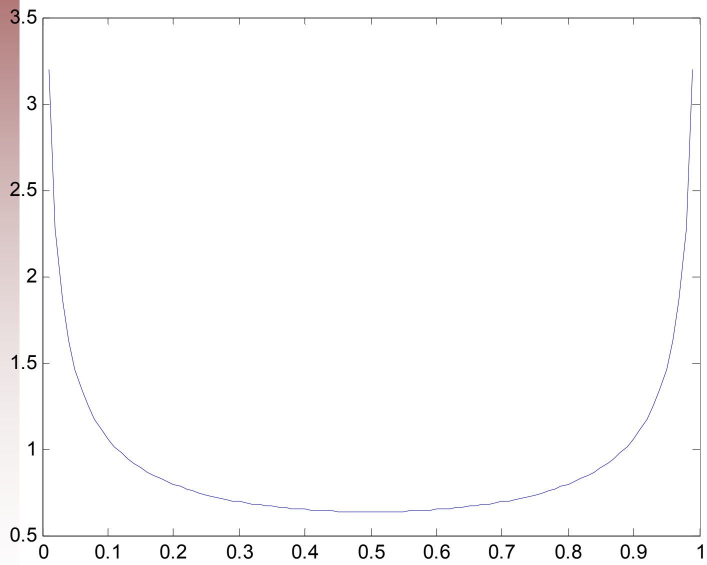
$$= u^{x}(1-u)^{n-x} \cdot u^{\alpha-1}(1-u)^{\beta-1}$$

$$= u^{x+\alpha-1}(1-u)^{n-x+\beta-1}$$

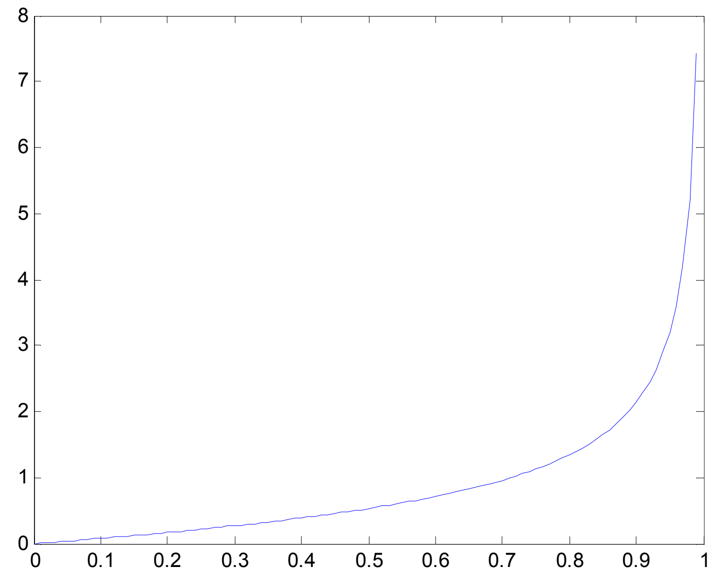
where $x = \Sigma_i x_i$

- Posterior is Beta of parameters $x+\alpha$ and n-x+β

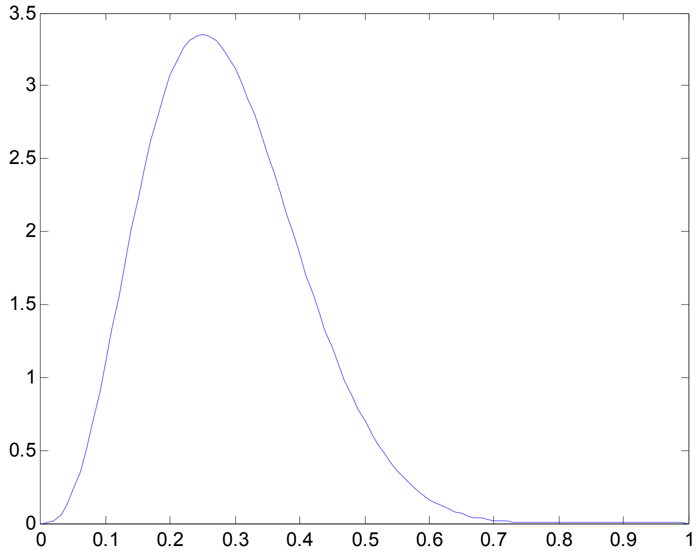
Beta Density Shapes



α=.5 β=.5



α=2 β=.5



α=4 β=10

Computing the Bayes Factor

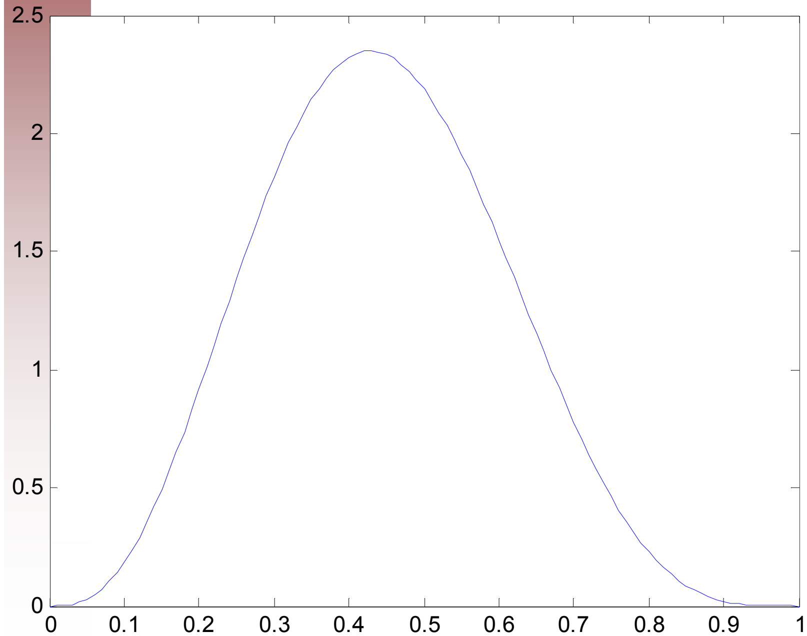
- ***Proposition***
- **The Bayes factor of $H_0 : \mathcal{M} \models P_{\geq\theta}(\Phi)$ vs $H_1 : \mathcal{M} \models P_{<\theta}(\Phi)$ for n Bernoulli samples (with $x \leq n$ successes) and prior Beta(α, β)**

$$B = \frac{1-\pi_0}{\pi_0} \cdot \left(\frac{1}{F_{(x+\alpha, n-x+\beta)}(\theta)} - 1 \right)$$

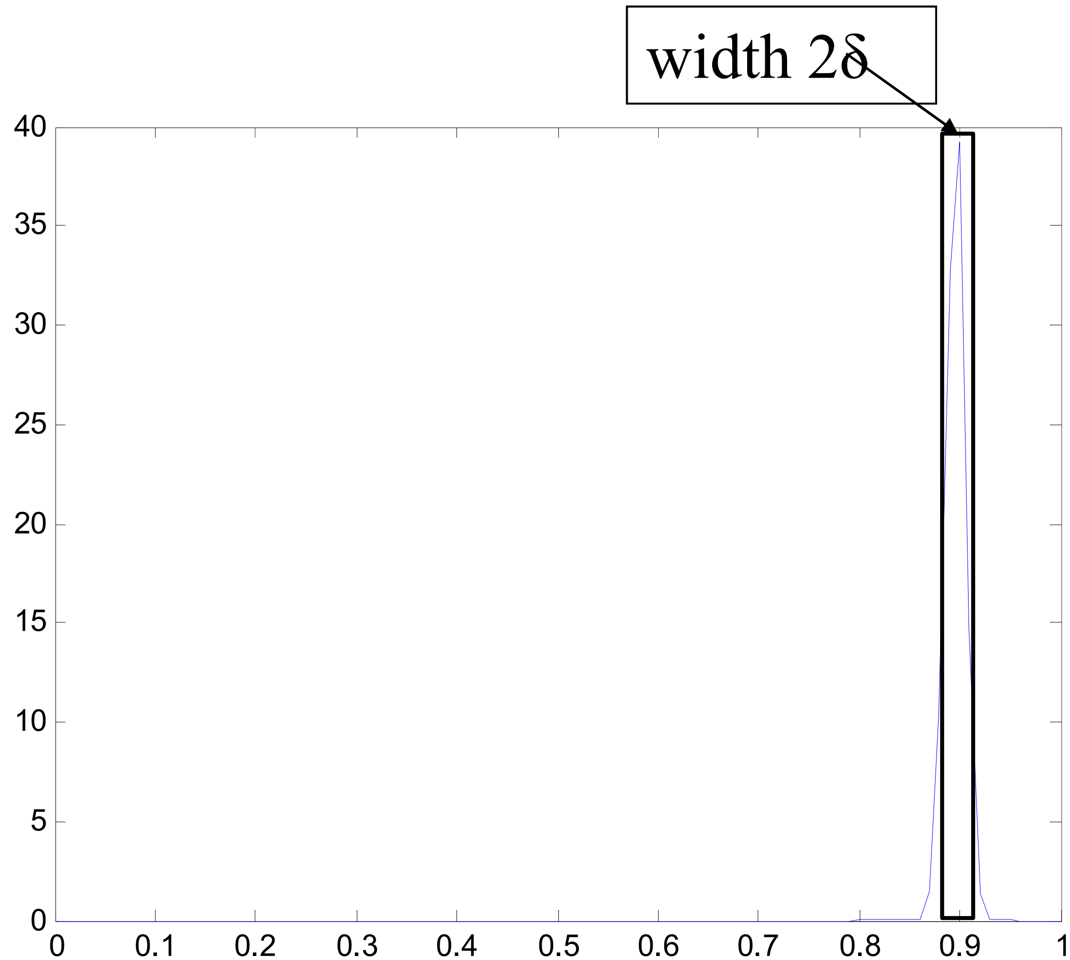
- **where $F_{(\cdot,\cdot)}(\cdot)$ is the Beta distribution function.**

$$F_{(x+\alpha,n-x+\beta)}(\theta) = \frac{1}{B(x+\alpha,n-x+\beta)} \int_0^\theta u^{x+\alpha-1}(1-u)^{n-x+\beta-1}\, du$$

Bayesian Interval Estimation - IV



prior is beta(α=4,β=5)



posterior density after 1000 samples and 900 “successes” is beta(α=904,β=105) posterior mean = 0.8959

Related Work – Monte Carlo Simulation

- A random variable X with unknown distribution f(x) $p = P(X > t)$
- Monte Carlo is a statistical method to estimate probability of an event (such as X > t) under unknown distribution.
- The goal is to find $p = E\{I(X > t)\}$

$$\hat{p}_{MC} = \frac{1}{N}\sum_{i=1}^{N} I(X_i > t) \rightarrow p \qquad I(x > t) = \begin{cases} 1, x > t \\ 0, otherwise \end{cases}$$

$$N \rightarrow \infty$$

- The Monte Carlo method generates N independent samples of X ($X_1, \ldots, X_N$) to form a estimate of p

Related Work – Importance Sampling

- **Importance Sampling samples from a biased distribution that the rare event happens more often.** Let f(x) is the density of X, f*(x) is the density of X*

$$
\begin{aligned}
p &= E\{I(X > t)\} \\
&= \int I(x > t) f(x)dx \\
&= \int I(x > t)\frac{f(x)}{f^*(x)} f^*(x)dx \\
&= E\{I(X^* > t)W(X^*)\}
\end{aligned}
$$

$$W(x) = \frac{f(x)}{f^*(x)}$$

- **Sampling from a biased random variable X* , IS estimator:**

$$\hat{p}_{IS} = \frac{1}{N}\sum_{i=1}^{N} W(X_i^*)I(X_i^* > t)$$

Bayesian Interval Estimation - V

Require: BLTL *property* Φ, *interval-width* δ, *coverage* c,
***prior* *beta parameters* α,β**

```
n := 0                {number of traces drawn so far}
x := 0                {number of traces satisfying  so far}
repeat
        σ := draw a sample trace of the system (iid)
        n := n + 1
        if  σ ⊨ Φ  then
                x := x + 1
        endif
        mean = (x+α)/(n+α+β)
        (t0,t1) = (mean-δ, mean+δ)
        I := PosteriorProbability (t0,t1,n,x,α,β)
until (I > c)
return (t0,t1), mean
```

Bayesian Interval Estimation - VI

- **Recall the algorithm outputs the interval (t_0,t_1)**
- **Define the null hypothesis**
- **H_0: $t_0 < p < t_1$**
- ***Theorem (Error bound).* When the Bayesian estimation algorithm (using coverage ½< c < 1) stops – we have**

  Prob (“accept H_0” | H_1) ≤ $(1/c - 1)\pi_0/(1-\pi_0)$

  Prob (“reject H_0” | H_0) ≤ $(1/c - 1)\pi_0/(1-\pi_0)$
- **π_0 is the prior probability of H_0**