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A Moment-Matching Scheme for the Passivity-Preserving Model Order Reduction of Indefinite Descriptor Systems with Possible Polynomial Parts

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Need for Reduced Order Models (ROMs)

$$\hat{E}\frac{d\hat{x}}{dt} = \hat{A}\,\hat{x}(t) + \hat{B}\,u(t)$$

Reduced-order models (tens of Eqns), easy to simulate...

Accuracy, stability, passivity

Step 2. Model Order Reduction

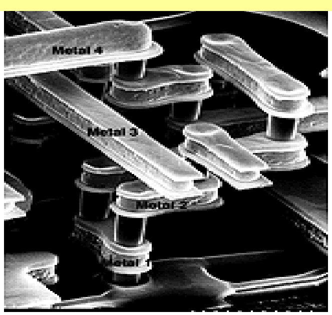
$$E\frac{dx}{dt} = A\,x(t) + B\,u(t)$$

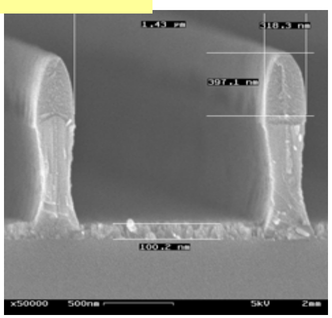
Large LTI descriptor systems (Thousands to Millions Eqns)

Very expensive to simulate...

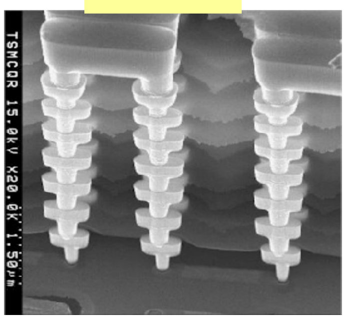
Step 1. PDE Field Solvers, SPICE Netlist, Measured Data

interconnect

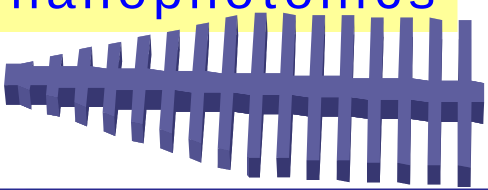




vias



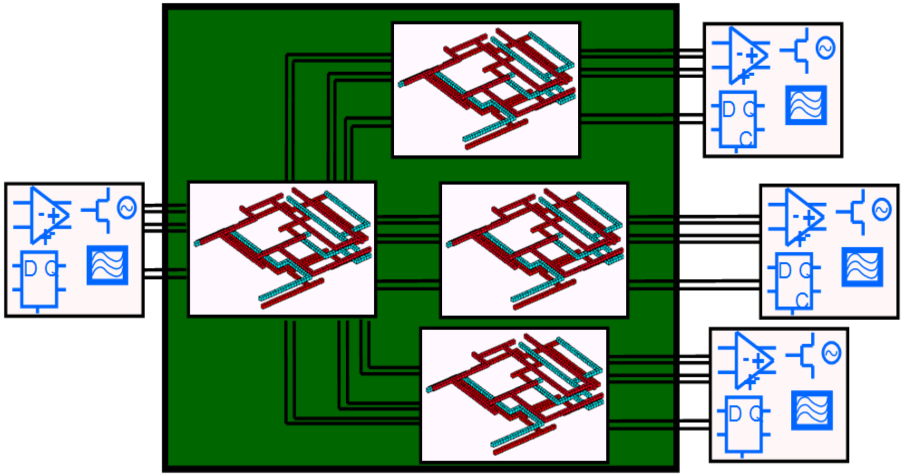
nanophotonics



carbon nanotubes

Need for Passivity Preservation

- The composition of "stable" models can be ***unstable***
- But the interconnection of "passive models" is stable.



If the models are not ***passive*** they can generate energy, and the time-domain simulation may explode in the connected systems-level simulation!

- A LTI system is passive if *H(s)* is positive real:

1. $H(s)$ has no poles in $\mathrm{Re}(s) > 0$
2. $\overline{H(s)} = H(\overline{s})$ for all $s \in C$
3. $H(s) + H^*(s) \geq 0$ for all $\mathrm{Re}(s) > 0$

Passive MORs: Moment Matching vs PRBT

Basic MOR Procedures:

$$\begin{bmatrix} E \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x(t) \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} u(t), \quad y = Cx + Du$$

Large system, n eqns

Find U and V (nxq), and perform projection

$$\underbrace{E_r}_{U^T E V} \frac{dx_r}{dt} = \underbrace{A_r}_{U^T A V} x_r + \underbrace{B_r}_{U^T B} u(t), \quad y_r = \underbrace{C_r}_{C V} x_r + Du$$

Small system, q eqns, q<<n

How to construct U and V to preserve passivity?

1. For positive semi-definite systems, moment matching (e.g., PRIMA) $O(n^2)$

$$E + E^T \geq 0, \; A + A^T \leq 0, \; B^T = C$$

$U = V$ (congruence transform)

$$E_r + E_r^T \geq 0, \; A_r + A_r^T \leq 0, \; B_r^T = C_r$$

2. For indefinite systems, use positive-real balanced truncation, cost $O(n^3)$
need solve two generalized algebraic Riccati equations (coming later...)

Moment matching is more efficient thus preferred ...

Limitations of the Conventional Moment Matching

1. Indefinite models: passivity is not guaranteed.

2. If E is singular, the ROM may be very inaccurate.

Why??

Consider the following passive descriptor system model

$$E\frac{dx}{dt} = Ax + Bu, \quad y = Cx$$

Transfer function $H(s) = C(sE - A)^{-1}B = H_{sp}(s) + P(s)$

Strictly proper

Polynomial part

$$H_{sp}(s) = \sum_{i=1}^{m_1} \frac{F_i}{s - p_i}, \quad H_{sp}(j\infty) = 0$$

$$P(s) = M_0 + \sum_{k=1}^{m_2} s^k M_k$$

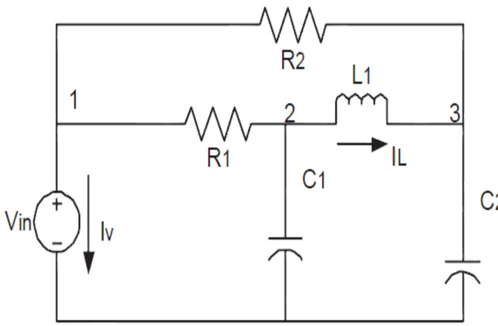
$P(s)=0$: *H(s)* is strictly proper.

The Krylov-subspace based moment matching generates *nonsingular* E_r, so $H_r(s)$ is *strictly proper*, and it is inaccurate when $H(s)$ is not strictly proper!!

Practical Examples:

RLC circuits with L-I cutsets or C-V loops; descretized PDEs, linearized MOS circuits, MEMs devices

Polynomial Parts: Simple RLC Examples



$$E = diag\begin{bmatrix}0 & C_1 & C_2 & L_1 & 0\end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{1}{R_1}-\frac{1}{R_2} & \frac{1}{R_1} & \frac{1}{R_2} & 0 & -1 \\ \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 1 & 0 \\ \frac{1}{R_2} & 0 & -\frac{1}{R_2} & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

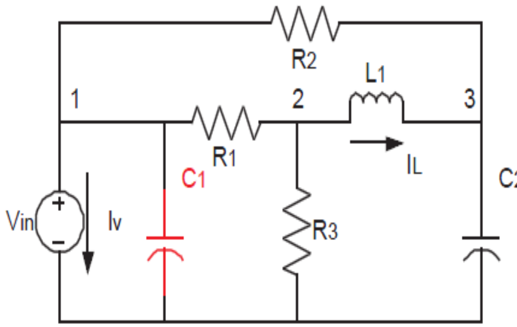
$$B = C^T = \begin{bmatrix}0 & 0 & 0 & 0 & 1\end{bmatrix}^T$$

$$P(s) = \frac{1}{R_1} + \frac{1}{R_2} \neq 0$$

MNA (modified nodal analysis)

Moment matching

$$P(s) = 0$$



$$E = diag\begin{bmatrix}C_1 & 0 & C_2 & L_1 & 0\end{bmatrix}$$

$$A = \begin{bmatrix} -(\frac{1}{R_1}+\frac{1}{R_2}) & \frac{1}{R_1} & \frac{1}{R_2} & 0 & -1 \\ \frac{1}{R_1} & -(\frac{1}{R_1}+\frac{1}{R_3}) & 0 & -1 & 0 \\ \frac{1}{R_2} & 0 & -\frac{1}{R_2} & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P(s) = \frac{1}{R_1 + R_3} + \frac{1}{R_2} + sC_1 \neq 0$$

$$P(s) = \infty, \text{ as } s \to \infty$$

$$B = C^T = \begin{bmatrix}0 & 0 & 0 & 0 & 1\end{bmatrix}^T$$

Conventional Krylov-subspace moment matching cannot preserve ***P(s)*!!**

Proposed Model Order Reduction Scheme

1. Passive Moment Matching for Nonsingular Indefinite Systems
   - How to guarantee accuracy?
     - use moment matching to guarantee accuracy.
   - How to preserve passivity?
     - Relate the MOR method to positive real lemma.

2. Model Reduction for Singular Descriptor Systems
   - How to preserve the possible polynomial part?
   - How to preserve passivity?

MOR for Indefinite LTI Systems

Given the passive indefinite model $E\dot{x} = Ax + Bu, \quad y = Cx + D$

Assume that *E* is nonsingular and $D + D^T > 0$

Proposed MOR Flow (motivated by Bond ICCAD2008)

- **Step 1:** Construct the right projection matrix *V*, by *any* existing *q-th* order Krylov-subspace *moment matching, e.g.*

$$\mathbf{V} = \left\{ (\mathbf{s_0E - A})^{-1}\mathbf{B},\ (\mathbf{s_0E - A})^{-1}\mathbf{E}(\mathbf{s_0E - A})^{-1}\mathbf{B}, \mathbf{L}, \left[(\mathbf{s_0E - A})^{-1}\mathbf{E} \right]^{\mathbf{q-1}} (\mathbf{s_0E - A})^{-1}\mathbf{B} \right\}$$

- **Step 2:** Compute the positive real observability Gramian *Qo>=0,* the stabilizing solution to the GARE *:*

$$\tilde{A}^T Q_o E + E^T Q_o \tilde{A} + E^T Q_o \tilde{B}\tilde{B}^T Q_o E + \tilde{C}\tilde{C} = 0,\ \tilde{B} = B\left(D + D^T\right)^{-\frac{1}{2}},\ \tilde{C} = \left(D + D^T\right)^{-\frac{1}{2}} C,\ \tilde{A} = A - \tilde{B}\tilde{C}$$

- **Step 3:** Compute the left projection matrix *U:* $U = Q_o EV\left(V^T E^T Q_o EV\right)^{-1}$

- **Step 4:** Construct the ROM: $E_r = U^T EV,\ A_r = U^T AV,\ B_r = U^T B,\ C_r = CV$

Accuracy and Complexity

Accuracy

- The first q moments of $H(s)$ are preserved, due to the (block) Krylov-subspace used for constructing V

$$V = \left\{(s_0E-A)^{-1}B,\ (s_0E-A)^{-1}E(s_0E-A)^{-1}B,\cdots,\left[(s_0E-A)^{-1}E\right]^{q-1}(s_0E-A)^{-1}B\right\}$$

This guarantees a similar accuracy with PRIMA (using U=V).

- Furthermore, U provides extra accuracy!

$$U = Q_oEV\left(V^TE^TQ_oEV\right)^{-1} \Rightarrow \text{range}(U)\subset\text{range}(Q_o)$$

Provides system oberservability information !!

Complexity

- Compared with PRBT, our method is 2X faster.

Our method (one GARE):

$$\tilde{A}^TQ_oE+E^TQ_o\tilde{A}+E^TQ_o\tilde{B}\tilde{B}^TQ_oE+\tilde{C}\tilde{C}=0$$

PRBT (two GAREs):

$$\begin{cases}\tilde{A}^TQ_oE+E^TQ_o\tilde{A}+E^TQ_o\tilde{B}\tilde{B}^TQ_oE+\tilde{C}^T\tilde{C}=0\\ \tilde{A}Q_CE^T+EQ_C\tilde{A}^T+EQ_C\tilde{C}^T\tilde{C}Q_CE^T+\tilde{B}\tilde{B}^T=0\end{cases}$$

Passivity Preservation

$$U = Q_o EV\left(V^T E^T Q_o EV\right)^{-1},\ E_r = U^T EV,\ A_r = U^T AV,\ B_r = U^T B,\ C_r = CV$$

For the obtained ROM, we can prove that the following LMIs

$$\begin{cases} A_r^T X E_r + E_r^T X A_r = -L_r L_r^T, \\ E_r^T X B_r - C_r^T = -L_r W_r, \\ D + D^T \ge W^T W \end{cases}$$

has a positive semi-definite solution:

$$X = V^T E^T Q_o EV \ge 0$$

Sufficient condition for passivity!!

According to the extended positive real lemma (*R. W. Freund and F. Jarre 2004*), the constructed ROM is passive.

MOR for Singular Systems

Singular Descriptor systems

$$E\frac{dx}{dt} = Ax + Bu, \quad y = Cx + Du$$

E is not invertible

Canonical Form of the Matrix Pencil

$$E = T_l \begin{bmatrix} I & \\ & N \end{bmatrix} T_r, \quad A = T_l \begin{bmatrix} J & \\ & I \end{bmatrix} T_r$$

T_l and T_r are invertible, J corresponds to finite system poles, N is a nilpotent matrix that leads to infinite system poles.

Spectral Projectors $$P_l = T_l \begin{bmatrix} I & \\ & 0 \end{bmatrix} T_l^{-1}, \quad P_r = T_r^{-1} \begin{bmatrix} I & \\ & 0 \end{bmatrix} T_r$$

Note: in practice, we do not use the canonical form to compute the spectral projectors; instead, we use the sparse-LU based canonical projector technique to compute the two projector matrices.

Details: Z. Zhang and N. Wong, "*An Efficient Projector-Based Passivity Test for Descriptor Systems*", IEEE TCAD, Aug. 2010

MOR for Singular Systems

Singular Descriptor systems

$$E\frac{dx}{dt} = Ax + Bu, \quad y = Cx + Du$$

E is not invertible

The transfer function of a passive descriptor system:

$$H(s) = C(sE - A)^{-1}B + D = H_{sp}(s) + \underbrace{M_0 + sM_1}_{P(s)}$$

Passivity Condition for Descriptor Systems:
The proper part $H_{sp}(s)+M_0$ is positive real, and M_1 is positive semi-definite.

Requirements for our moment-matching flow:

1. Preserve the possible polynomial part $P(s)$
2. Reduce *Hsp(s),* and ensure the passivity of $H_{sp}(s)+M_0$

Our Solution:

System Decomposition	+	Passive moment matching for indefinite systems

MOR for Singular Systems

$$E\frac{dx}{dt} = Ax + Bu, \quad y = Cx + Du$$ E is not invertible

Step 1: System Decomposition by P_r

1. Define $G(s) = C(I - P_r)(sE - A)^{-1}B + D$, then $H_{sp}(s)+M_0$ can be realized by a nonsingular indefinite system (E_p,A_p,B_p,C_p,D_p):

$$E_p = EP_r - \alpha A(I - P_r),\ A_p = A,\ C_p = CP_r,\ B_p = B,\ D_p = G(0),\ \text{with}\ \alpha > 0$$

2. Extract M_1 $M_1 = \frac{G(s_1) - G(s_2)}{s_1 - s_2}$, with $s_1, s_2 > 0$

Step 2: Reduce the indefinite nonsingular model (E_p,A_p,B_p,C_p,D_p) by our proposed moment matching method, get the ROM

$$E_{pr}\frac{d\tilde{x}}{dt} = A_{pr}\tilde{x} + B_{pr}u, \quad y = C_{pr}\tilde{x} + D_p u$$

Step 3: Reconstruct the singular ROM:

$$Er = \begin{bmatrix} E_{pr} & & \\ & 0 & I_m \\ & & 0 \end{bmatrix},\ A_r = \begin{bmatrix} A_{pr} & & \\ & I_m & \\ & & I_m \end{bmatrix},\ B_r = \begin{bmatrix} B_{pr} \\ 0 \\ M_1 \end{bmatrix},\ C_r^T = \begin{bmatrix} C_{pr}^T \\ -I_m \\ 0 \end{bmatrix},\ D_r = D_p$$

Note: The final ROM is passive and the polynomial part $P(s)$ is preserved.

Numerical Results

Benchmark: an order-1505 RLC MNA model, 5 ports

Verification flow:

Use sparse LU-based spectral projector → **System Decomposition**

↓

Compare with PRIMA and PRBT

Use single-point moment matching to construct V

Reduce the proper part ($H_{sp}(s)+M_0$) by the proposed passive moment matching

Use Matlab built-in function "gcare" to solve the GARE

↓

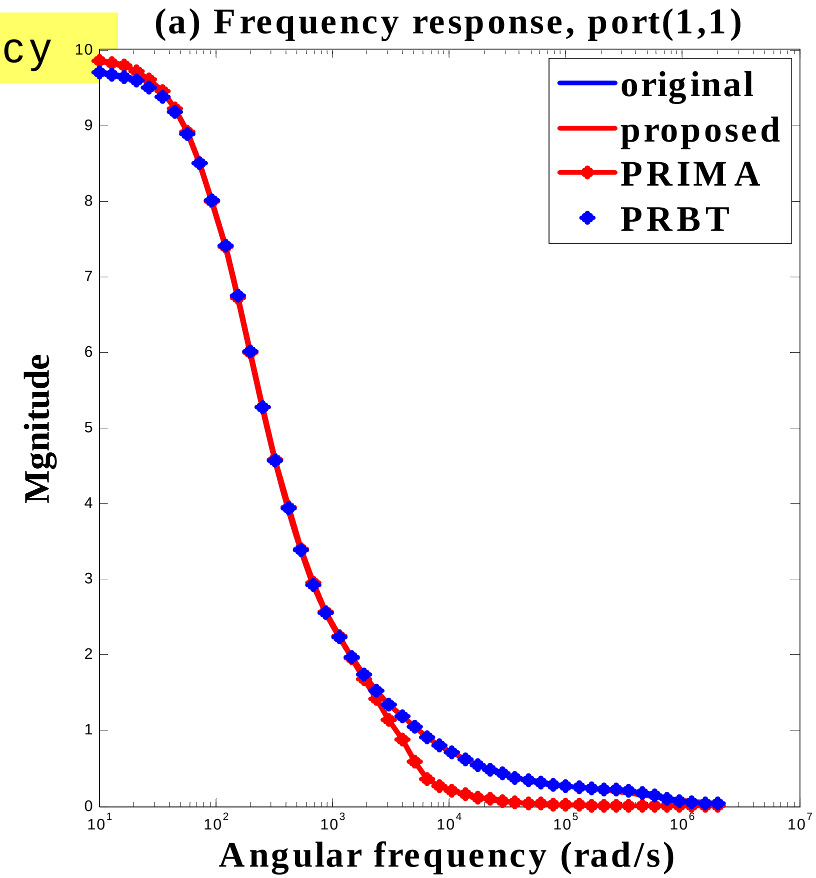
Reconstruction to a singular ROM

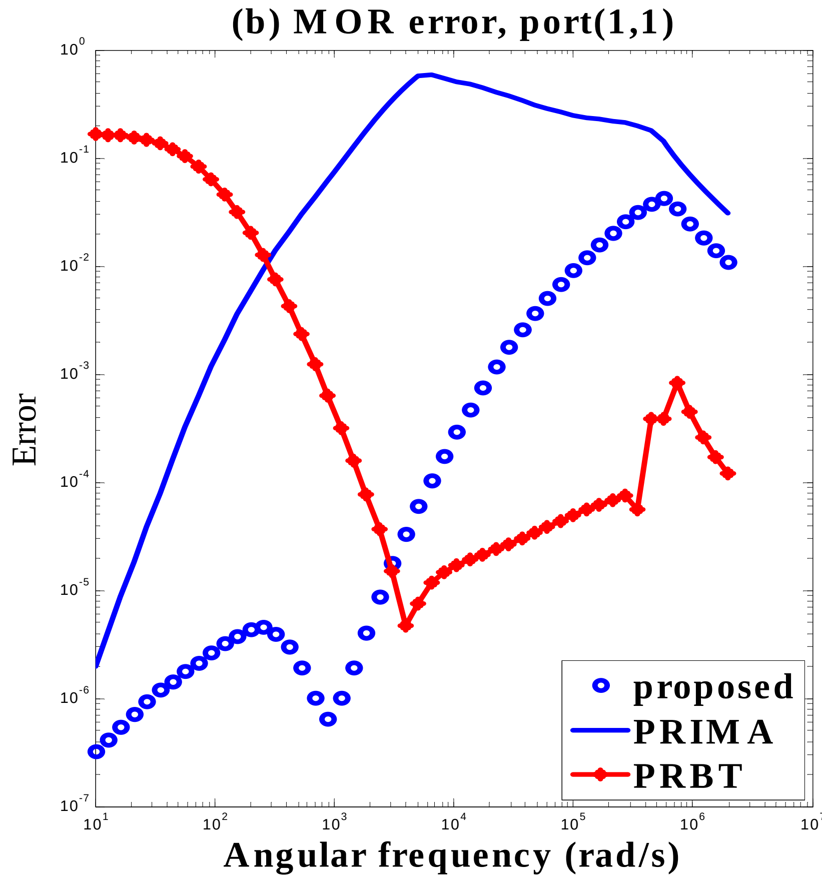
Compare PRIMA (directly performed on the original MNA model)

Numerical Results

MOR of the Indefinite Proper Part $H_{sp}(s)+M_0$

Accuracy





Low-freq. band: both moment matching methods are better than PRBT
The proposed method is better than PRIMA in the whole frequency band

CPU Times

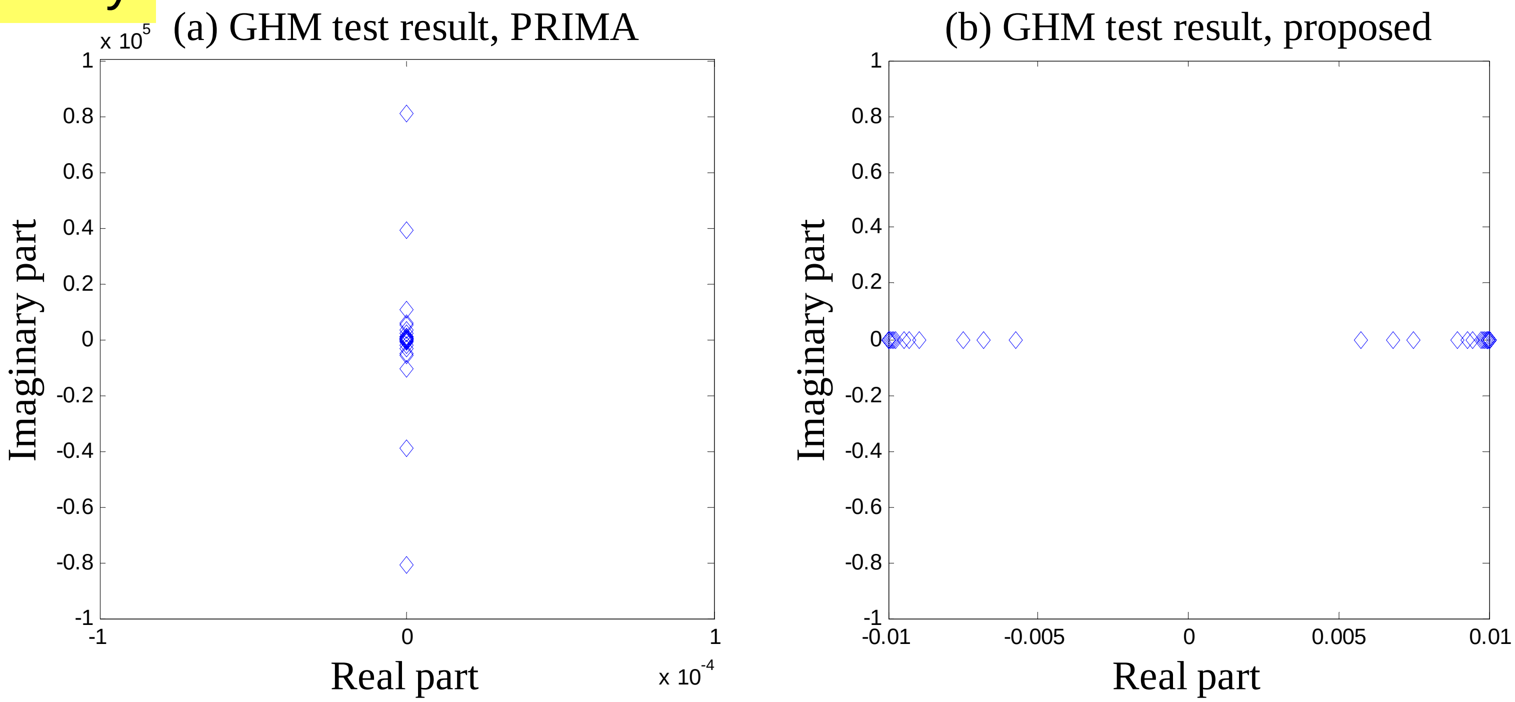
Model Size	Number of Port	PRIMA (sec)	PRBT (sec)	Proposed (sec)
1505	5	1.76	507.8	243.1

MOR of the Indefinite Proper Part $H_{sp}(s)+M_0$

The passivity is tested by the generalized Hamiltonian method (GHM

Z. Zhang and N. Wong, *"Passivity test of immitance descriptor systems based on generalized Hamiltonian methods"*, IEEE TCAS2, Jan.2010

Passivity

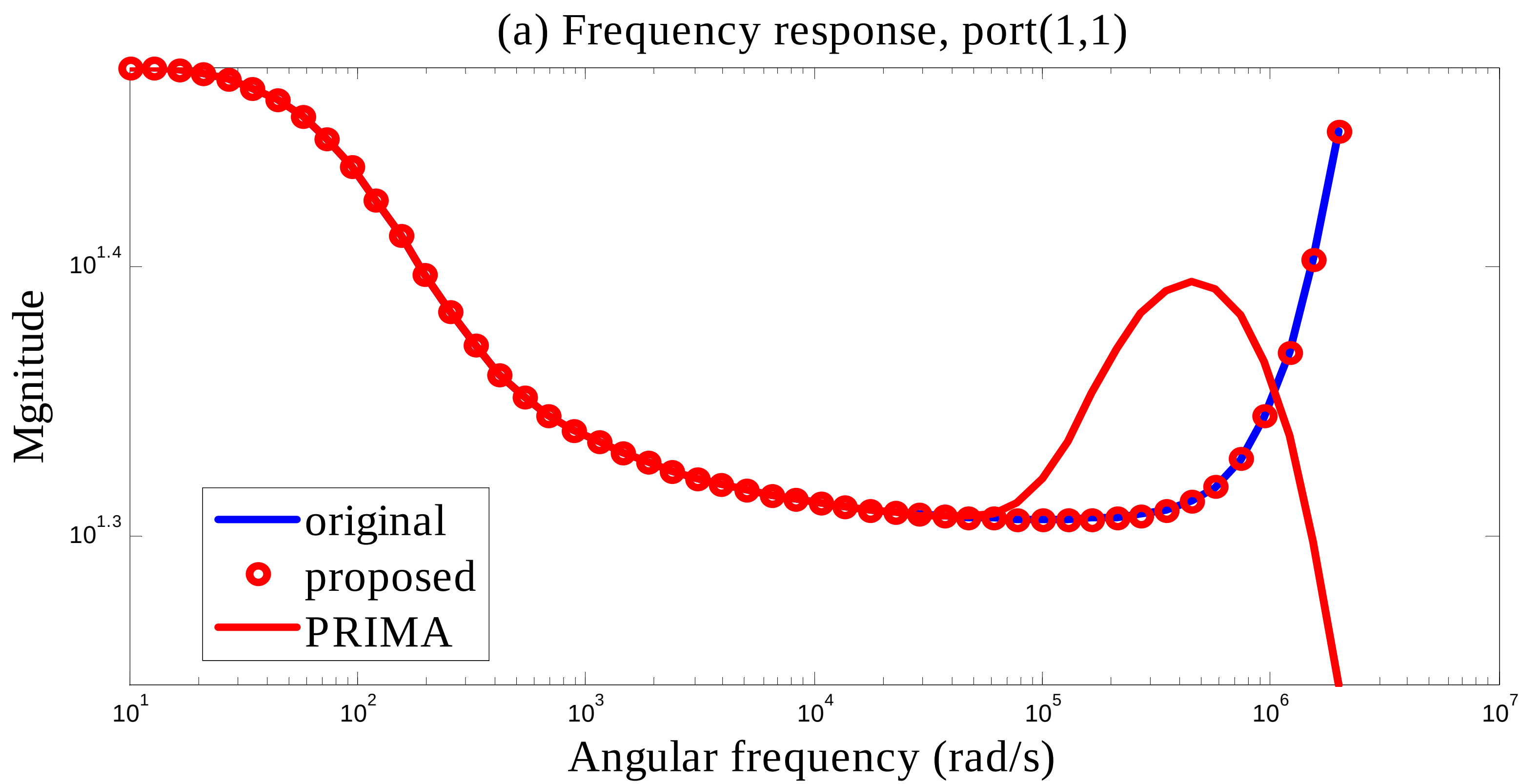


PRIMA: Nonpassive ROM   Proposed: Passive ROM

Numerical Results

MOR of whole MNA descriptor-system model

To preserve passivity, PRIMA is directly performed on the original positive semi-definite descriptor system



The proposed moment-matching method produces a ROM that overlaps the original model in the whole frequency band; but the conventional moment matching (e.g. PRIMA) can't preserve the polynomial part ***P(s).***

Thanks for your attention!

Q&A

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