Efficient Sensitivity-Based Capacitance Modeling for Systematic and Random Geometric Variations

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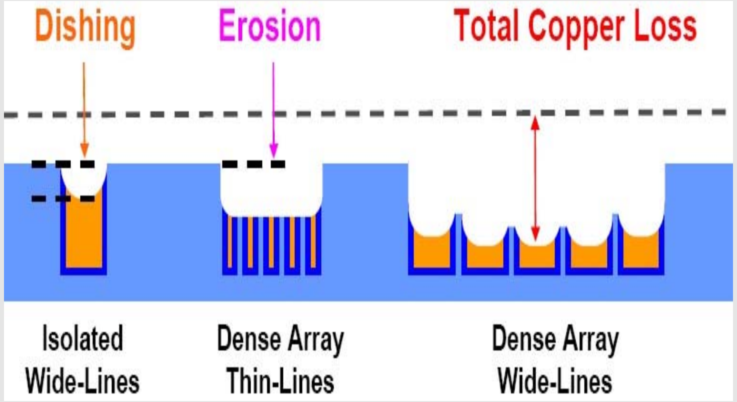
Outline

- Introduction
  - Systematic & random variations
  - Modeling method (panel sensitivity)
- Modeling for systematic variations
- Modeling for random variations
  - Panel sensitivity based statistical modeling method
  - Experiments and results
  - Case study: 8-bit binary-scaled charge-redistribution DAC
- Modeling for both variations
  - Diagram
  - Experiment and result
- Conclusion

Process Variability

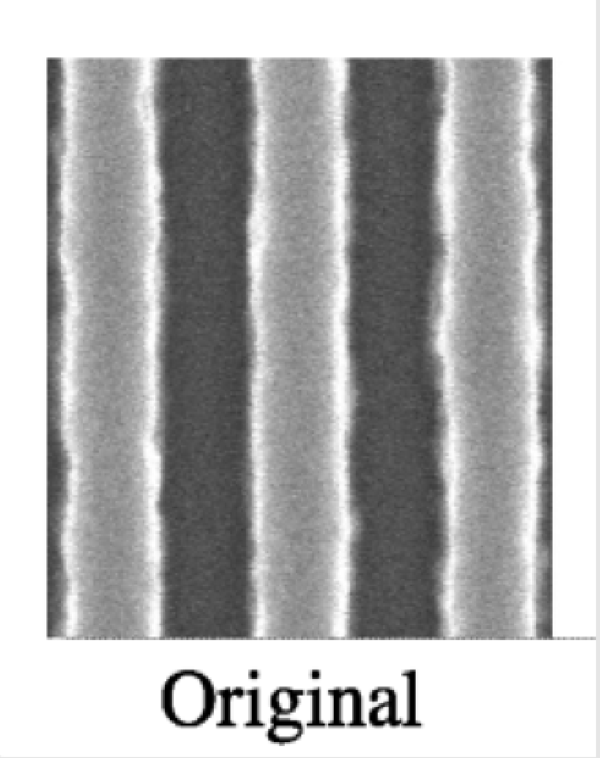
- Systematic Variation
  - Lithography, Etching, CMP
  - Layout dependent
- Random Variation



[Scheffer - 2006]

Process Variability

- Systematic Variation
  - Lithography, Etching, CMP
  - Layout dependent
- Random Variation
  - Line-edge roughness
  - Spatial correlation



[Asenov - 2003]

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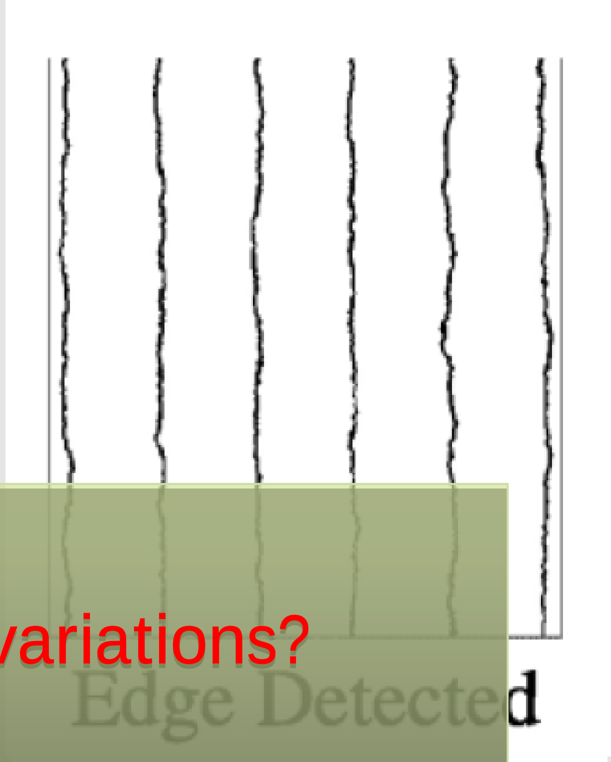


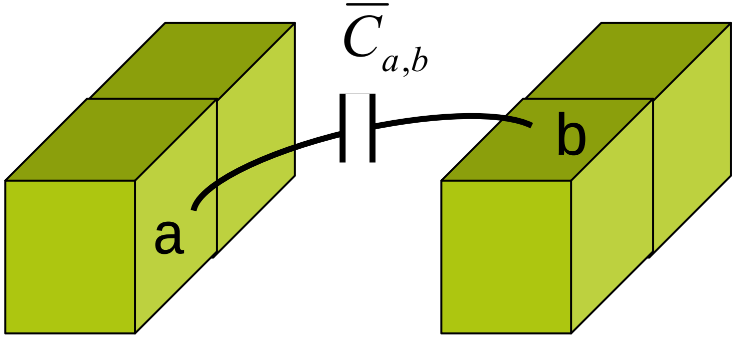
[Asenov - 2003]

Variabilities of R & C !

? How to model BOTH variations?

Modeling Method

- BEM-based capacitance extraction
  - Partial short-circuit capacitances $\overline{\mathbf{C}}$
  - Nominal capacitance $\mathbf{C_0}$: sum of associated partial short-circuit capacitances $\overline{\mathbf{C}}$



Modeling Method

- Panel sensitivity

have been produced in calculation of the nominal C_0 using BEM

$$S_{k,ij} = \frac{\partial C_{ij}}{\partial \rho_k} = -\frac{1}{\varepsilon A_k} \sum_{a \in N_i} \sum_{b \in N_j} \overline{C}_{k,a} \overline{C}_{k,b}$$

[Bi-CICC-2009]

- ρ_k : a small displacement of a panel *k*
- ε : material permittivity around panel *k*
- A_k : the area of panel *k*
- $\overline{C}_{k,a}$: an entry in the partial short-circuit capacitance matrix
- $\sum_{a \in N_i} \overline{C}_{k,a}$: capacitance between a panel *k* and a node *i*

Modeling Method

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$$S_{k,ij} = \frac{\partial C_{ij}}{\partial \rho_k} = -\frac{1}{\varepsilon A_k} \sum_{a \in N_i} \sum_{b \in N_j} \overline{C}_{k,a} \overline{C}_{k,b}$$

[Bi-CICC-2009]

- no extra costly computation
- partial capacitances
  (data for standard capacitance extraction)
- BEM
- **FAST!**

Modeling for Systematic Variation

- Linear / sensitivity model

$$\mathbf{C} = \mathbf{C_0} + \sum_i \frac{\partial \mathbf{C}}{\partial \mathbf{p}_i} \cdot \Delta \mathbf{p}_i$$

$$\frac{\partial \mathbf{C_{ij}}}{\partial \mathbf{p}} = \sum_{\mathbf{k} \in \mathbf{s_p}} \mathbf{S_{k,ij}}$$

  - s_p : the set of panels incident to the geometric parameter p

Modeling for Systematic Variation

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$$\frac{\partial \mathbf{C}_{ij}}{\partial \mathbf{p}} = \sum_{k \in s_p} \mathbf{S}_{k,ij}$$

  - s_p : the set of panels incident to the geometric parameter p

- Variance of capacitance due to the dimensional variation:

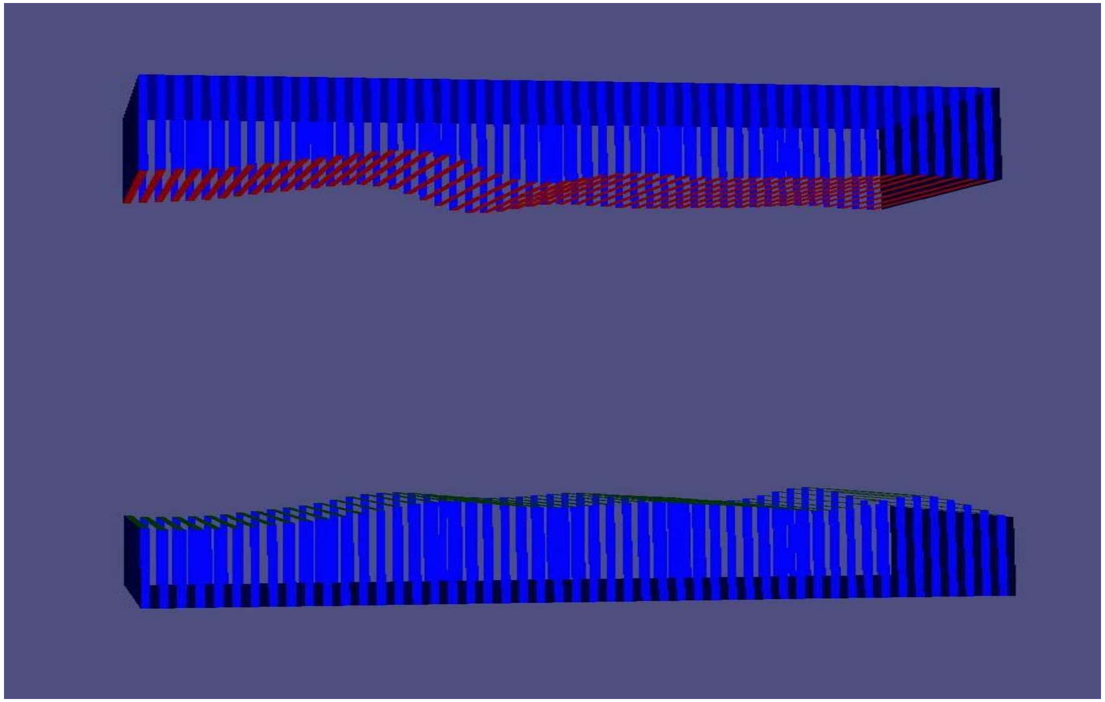
$$\mathrm{var}(\mathbf{C}_{ij})_{sys} = \Big(\sum_{k \in s_p} \mathbf{S}_{k,ij}\Big)^2 \boldsymbol{\sigma}_p^2$$

[Bi-CICC-2009]

  - $\boldsymbol{\sigma}_p$: standard deviation of parameter p

Modeling for Random Variations

- Modeling the effects of Line-Edge Roughness (LER) on capacitances



Statistical Model of Capacitances

- Capacitance Modeling

$$\Delta C = \sum_{l=1}^{L} \sum_{i=1}^{n_l} S_i \rho_i$$

- ΔC : capacitance variation induced by the LER
- ρ : a sequence of random variables (panel displacements)
- S_i : panel sensitivity associated with panel displacement ρ_i
- n_l : the number of deviation panels for rough line l
- L : the number of rough lines

Statistical Model of Capacitances

- Capacitance Modeling

$$\Delta C = \sum_{l=1}^{L} \sum_{i=1}^{n_l} S_i \rho_i$$

- Variance of ΔC

$$\sigma_{\Delta C}^2 = var\left(\sum_{l=1}^{L} \sum_{i=1}^{n_l} S_i \rho_i \right)$$

Some ρ_i s are NOT independent !!

Statistical Model of Capacitances

- Capacitance Modeling

$$\Delta C = \sum_{l=1}^{L} \sum_{i=1}^{n_l} S_i \rho_i$$

- Variance of ΔC

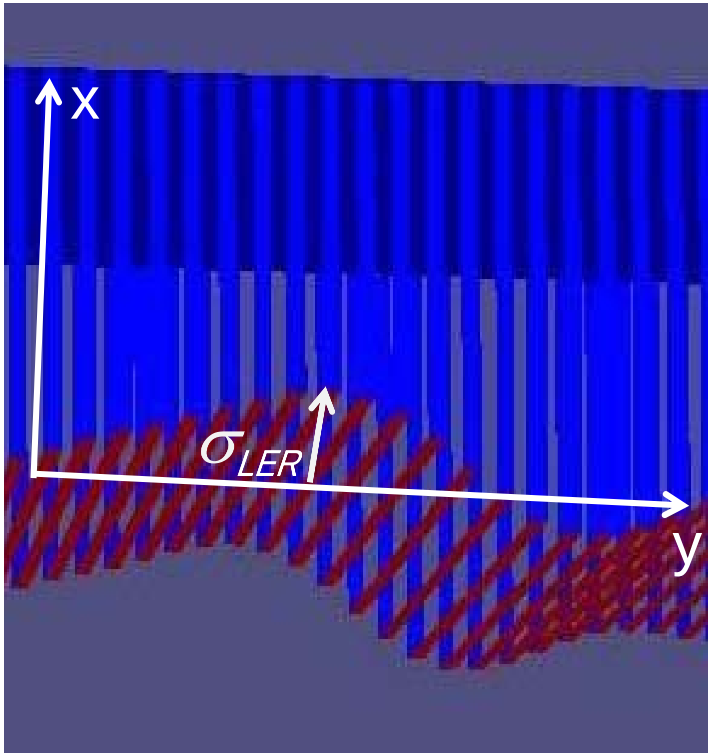
$$\sigma^2_{\Delta C} = \mathrm{var}\left(\sum_{l=1}^{L} \sum_{i=1}^{n_l} S_i \rho_i \right)$$

$$\sigma^2_{\Delta C} = \sum_{l=1}^{L} \left[\sum_{i=1}^{n_l} s_i^2 \underline{\mathrm{var}(\rho_i)} + 2 \sum_{i,j:i<j} s_i s_j \underline{\mathrm{cov}(\rho_i, \rho_j)} \right]$$

Random Variation

$$\boldsymbol{\sigma}^2_{\Delta C} = \sum_{l=1}^{L}\left[\sum_{i=1}^{n_l} s_i^2 \underline{\mathrm{var}(\rho_i)} + 2\sum_{i,j:i<j} s_i s_j \mathrm{cov}(\rho_i,\rho_j)\right]$$

$$\mathrm{var}(\rho_i) = \sigma^2_{LER}$$



Random Variation

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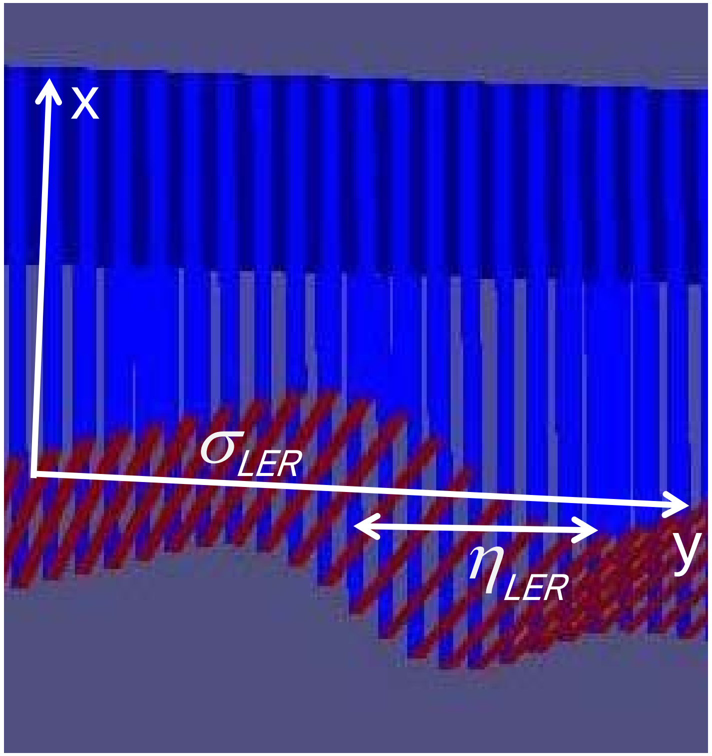
$$var(\rho_i) = \sigma^2_{LER}$$

Any correlation function!



Random Variation

$$\sigma^2_{\Delta C} = \sum_{l=1}^{L}\left[\sum_{i=1}^{n_l} s_i^2 \underline{var(\rho_i)} + 2\sum_{i,j:i<j} s_i s_j \underline{cov(\rho_i,\rho_j)}\right]$$

$$var(\rho_i) = \sigma^2_{LER}$$

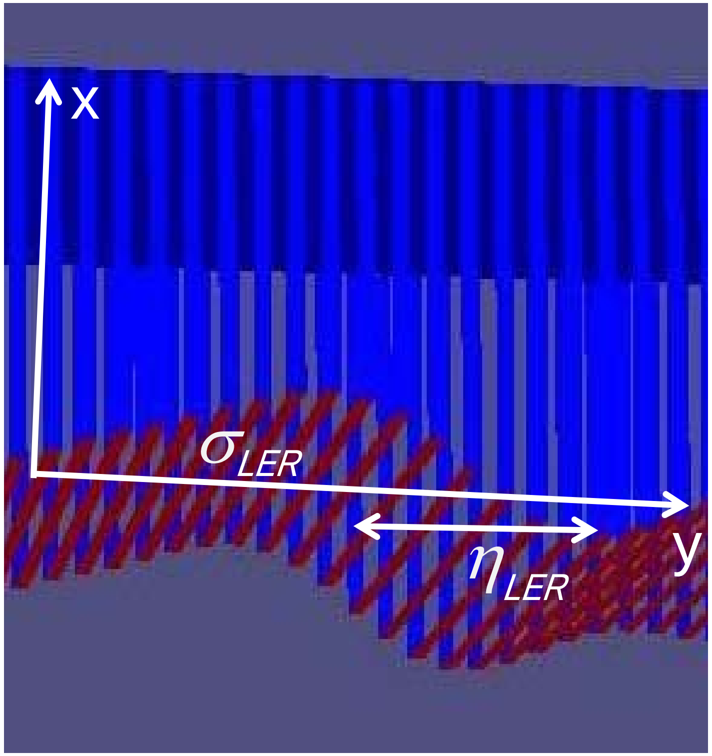
- Gaussian correlation function:

$$cov(\rho_i,\rho_j) = \sigma^2_{LER}\exp\left(-\frac{|r_{i,y}-r_{j,y}|^2}{\eta^2_{LER}}\right)$$

$r_{i,y}$ is the y-coordinate of the position associated with ρ_i

η_{LER} is the correlation length in y-direction

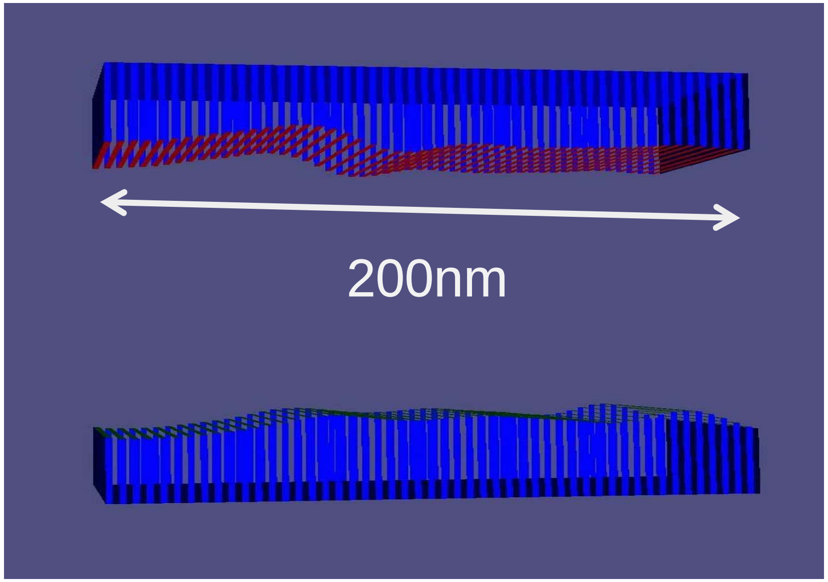
Two characterization parameters



Experiment - I

$$\sigma_{LER} = 3.5nm \quad \eta_{LER} = 16nm$$

- Measurement data from IMEC [Stucchi - 2007]



- Relative std. deviation: $\frac{\sigma_C}{C}$
- Monte-Carlo simulation
  - 1000 samples

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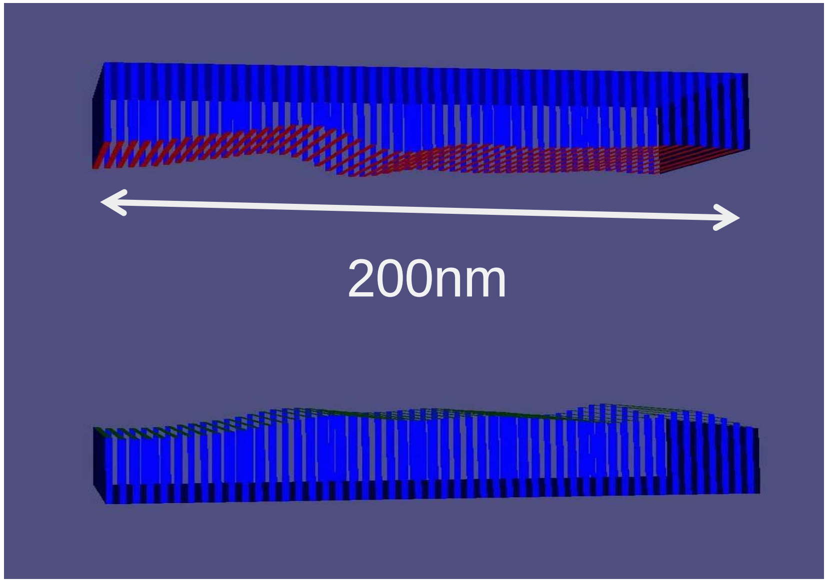


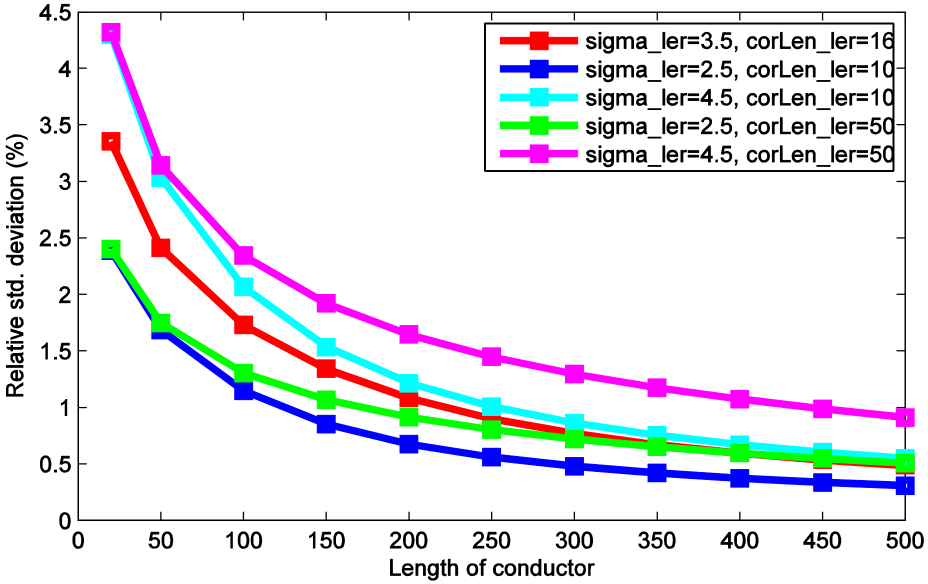
- Relative std. deviation: $\frac{\sigma_C}{C}$
- Monte-Carlo simulation
  - 1000 samples

	σ_C / C	Error	CPU Time
MC simulation	0.681%	-	48653''
Proposed model	0.603%	11.5%	50''

Experiment - II

Using the proposed model, one can easily study:

1. The relationship between $\frac{\sigma_C}{C}$ and the conductor length;
2. The impact of parameters σ_{LER} and η_{LER} on $\frac{\sigma_C}{C}$.



- Same structure as Experiment-I
- 5 examples of mismatches
- Combination of various σ_{LER} and η_{LER}
- Application:
  - High accuracy vs. low power consumption

Experiment - II

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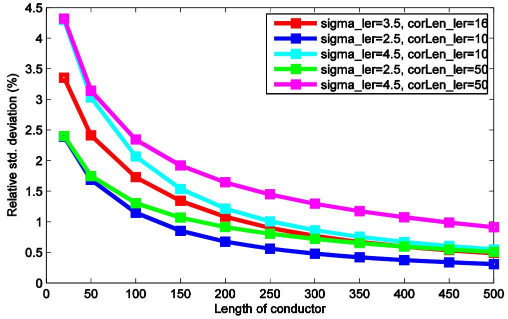

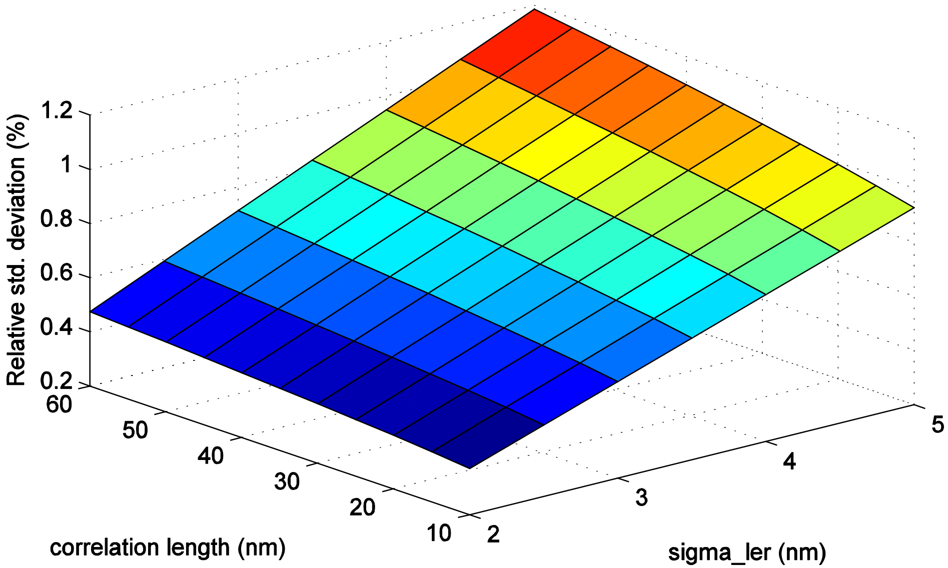

- Two sweeping parameters
- Proposed method: an hour
- MC approach: 43 days

Experiment - II

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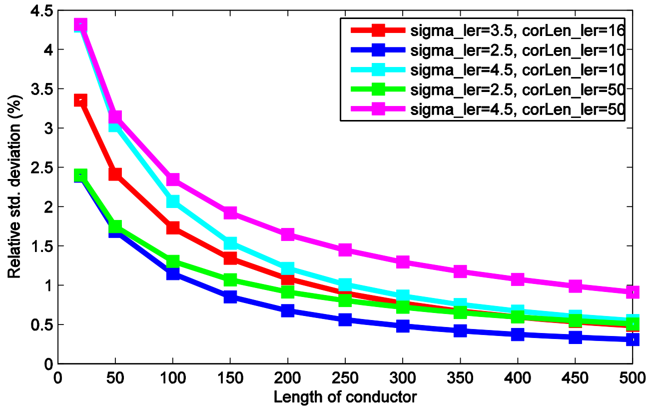
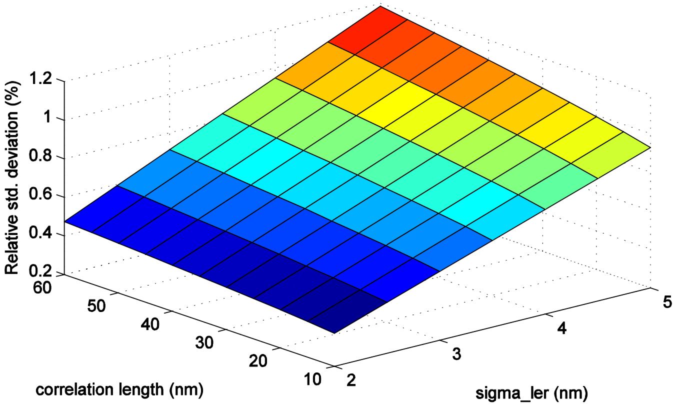
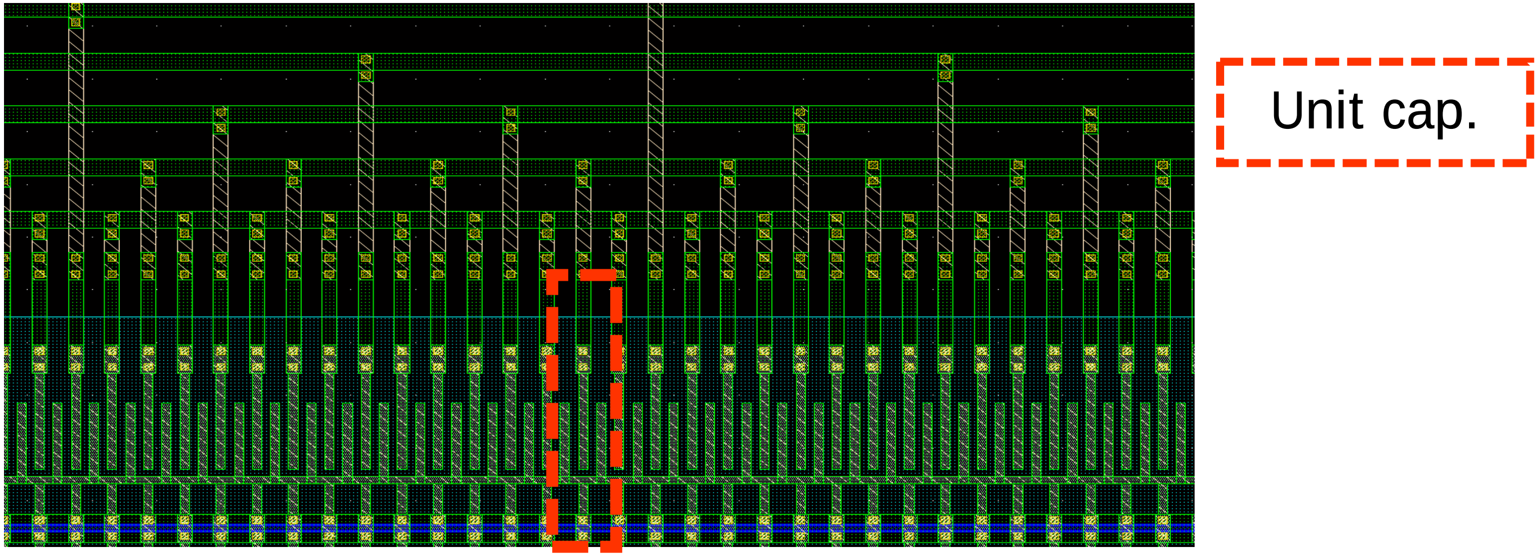




The proposed modeling method provides a fast and practical tool for circuit designers to estimate mismatches and optimize dimensions of critical structures accordingly.

A Case Study

- Novel passive devices with high-precision structures
- 8-bit charge-redistribution DAC
  - 255 identical unit capacitors
  - Min. value of a unit capacitor for high power efficiency (0.5fF)
  - Main consideration: mismatch of capacitors ($\frac{\sigma_{C_0}}{C_0}$)



[Harpe-ESSCIRC-2010]

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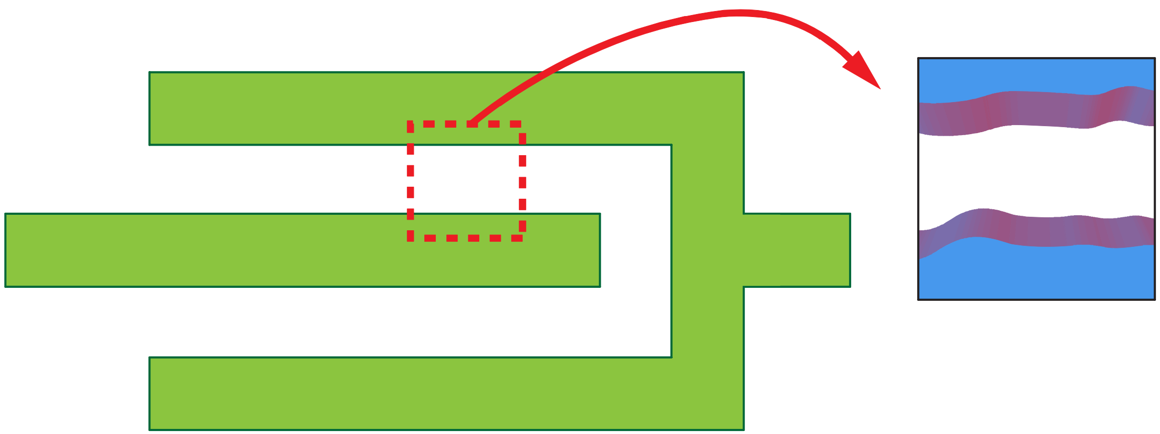


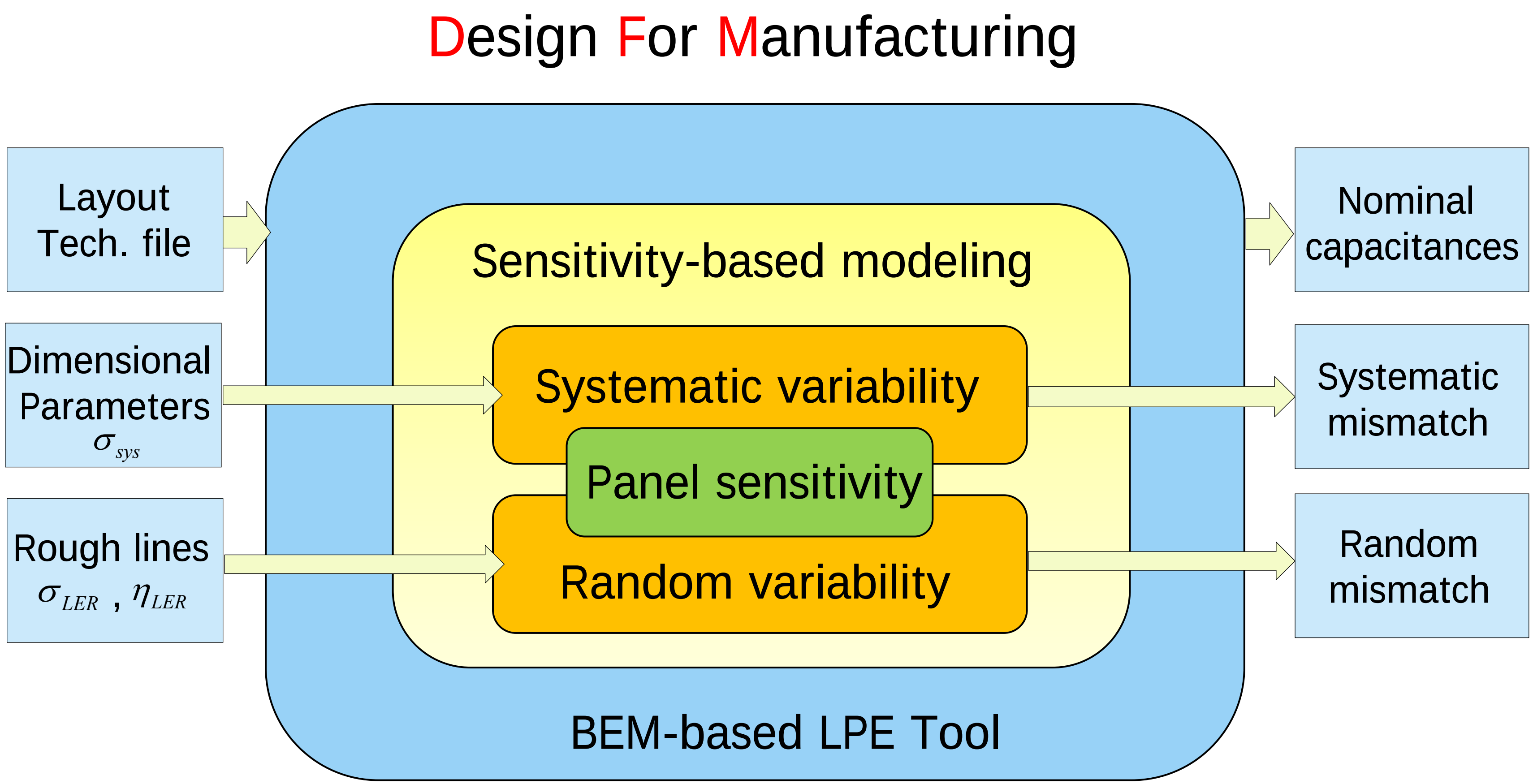
  - Design requirement: mismatch of the unit capacitor < 1%

A Case Study

- Design requires:
  - mismatch of the unit capacitor **< 1%**
- Simulation shows:
- The mismatch of the unit capacitor caused by the LER is around **0.25%**;
- Measurement indicates:
- A random mismatch of the unit capacitor being **better than 0.6%**;
- Simulation and measurement together conclude:
- Simulation results are very reasonable;
- The structure can be used for more accurate designs: e.g. 10-bit DAC (designer's plan!);

Design tool enables a new design,
based on proper modeling but NOT guessing

Sensitivity Based Modeling for Both Systematic and Random Variations



Experiment - III

- Two parallel conductors
- width/space = 2μm/2μm; thickness = 2μm; length = 8μm
- LER on four edges of two conductors:

$$\sigma_{LER} = 0.03\mu m, \quad \eta_{LER} = 2.00\,\mu m$$
$$\sigma_{LER} = 0.04\mu m, \quad \eta_{LER} = 2.88\,\mu m$$

- Systematic variation of the two conductors:

$$\sigma_{sys} = 0.03\mu m, \quad \sigma_{sys} = 0.04\,\mu m$$

- Parameters are chosen based on pure assumption

Experiment - III

- Two parallel conductors
- width/space = 2μm/2μm; thickness = 2μm; length = 8μm
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- Systematic variation of the two conductors:

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- 3 Monte Carlo simulations with 1000 samples each
  - Systematic variation
  - Random variation
  - Superposition of the above two

Experiment - III

	MC simulation	Proposed method
$\sigma_{C_{sys}} / C$	2.22%	2.05% (7.72% error)
$\sigma_{C_{LER}} / C$	0.21%	0.24% (14.23% error)
$\sigma_{C_{sNr}} / C$	2.22%	2.06% (7.18% error)
CPU Time	38h52’	58’’

- The systematic variation is the dominant one
- Some designs are sensitive to both variations and some (e.g. 8-bit DAC) are only vulnerable to random variations
- Being able to apply the appropriate modeling techniques is essential

Extremely high efficiency + good enough accuracy
= a fast and convenient tool for DFM !

Conclusion

- **Sensitivity based** method for statistical property of capacitances due to **both** systematic and random variations
- Modeling method for the effect of LER on capacitance
  - Simulations & **measurement on** chips
  - Good enough accuracy & **high efficiency**
  - Useful and convenient tool for **mismatch estimation** & **circuit optimization**
- Overall picture of the sensitivity based method for both variations
  - Extension of BEM LPE tools
  - Serving **DFM**!