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Qubit Placement to Minimize Communication Overhead in 2D Quantum Architectures

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Outline

- Introduction
  - Quantum Computing Technologies
- Geometric Constraints
  - Nearest Neighbor Architectures
- Proposed Solution
  - MIP-based Qubit Placement
  - Force-directed Qubit Placement
- Results
- Conclusion

Quantum Computing

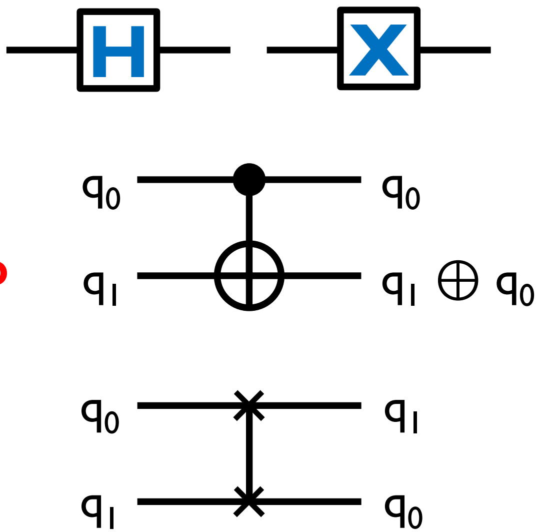
- Motivation: Faster Algorithms http://math.nist.gov/quantum/zoo/
  - Shor's factoring algorithm (Superpolynomial)
  - Grover's search algorithm (Polynomial)
  - Quantum walk on binary welded trees (Superpolynomial)
  - Pell's equation (Superpolynomial)
  - Formula evaluation (Polynomial)
- Representation

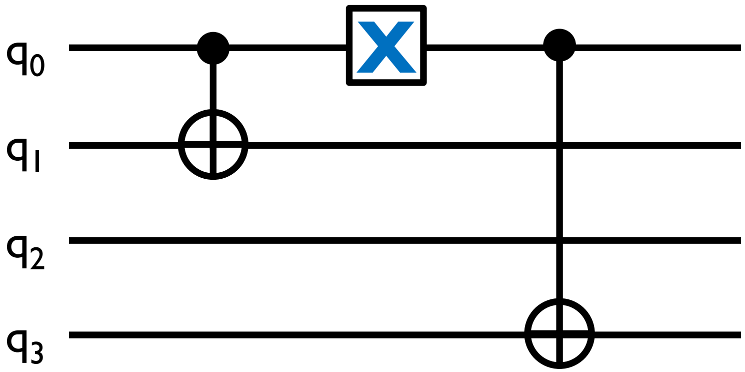
Quantum Algorithm → Quantum Circuit → Physical Realization (PMD)

PMD: Physical Machine Description

Quantum Circuits

- Qubits
  - Data is carried by quantum bits or qubits
  - Physical objects are ions, photons, etc.
- Quantum Gates
  - Single-qubit: H (Hadamard), X (NOT)
  - Two-qubit: CNOT (Controlled NOT), SWAP
- Quantum Circuit





Quantum PMDs

- Move-based PMDs
  - Explicit move instruction
    - There are routing channels for qubit routing
  - Examples: Ion-Trap, Photonics, Neutral Atoms
- SWAP-based PMDs
  - No move instruction
    - There are no routing channels
  - Qubit routing via SWAP gate insertion
  - Examples: Quantum Dot, Superconducting

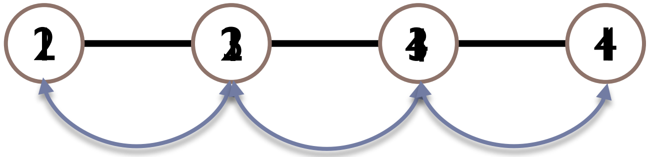
- Focus of this presentation is on SWAP-based PMDs

Geometric Constraints

- Limited Interaction Distance
  - Adjacent qubits can be involved in a two-qubit gate
  - Nearest neighbor architectures

- Route distant qubits to make them adjacent
  - Move-based: MOVE instruction

  - SWAP-based: insert SWAP gates



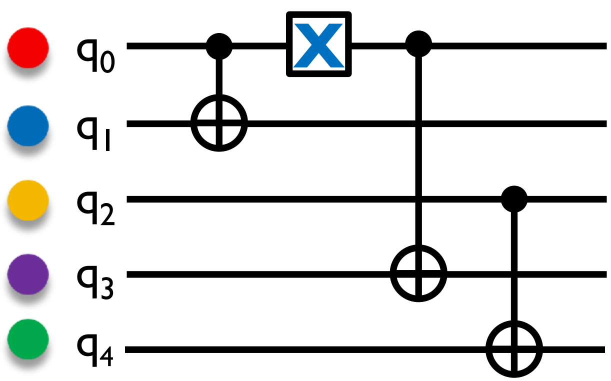
SWAP-based PMDs

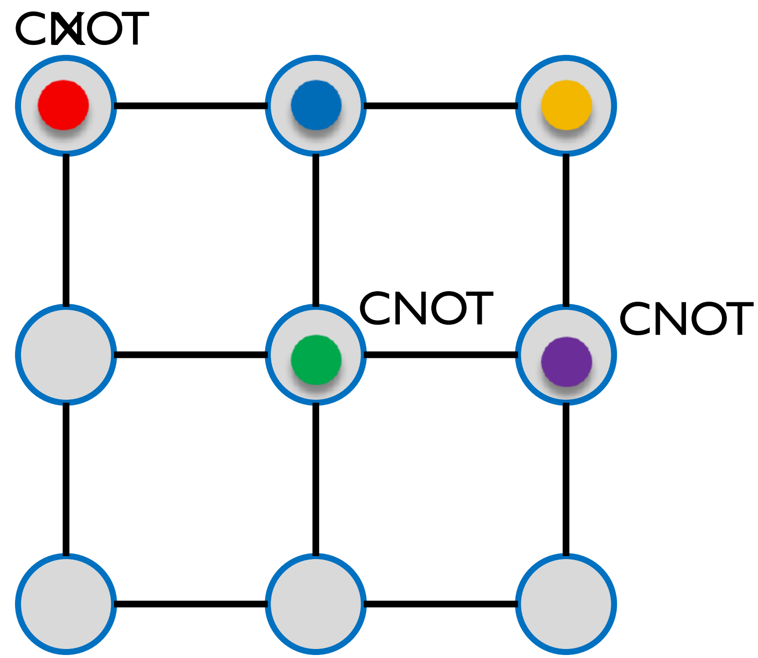
- SWAP insertion
  - Objective
    - Ensure that all two-qubit gates perform local operations (on adjacent qubits)
  - Side effects
    - More gates, and hence more area
    - Higher logic depth, and thus higher latency and higher error rate
  - Minimize the number of SWAP gates by placing frequently interacting qubits as close as possible on the fabric
    - This paper: MIP-based qubit placement
    - Future work: Force-directed qubit placement (a more scalable solution)

MIP: Mixed Integer Programming

Example on Quantum Dot

- Simple qubit placement: place qubits considering only their immediate interactions and ignoring their future interactions

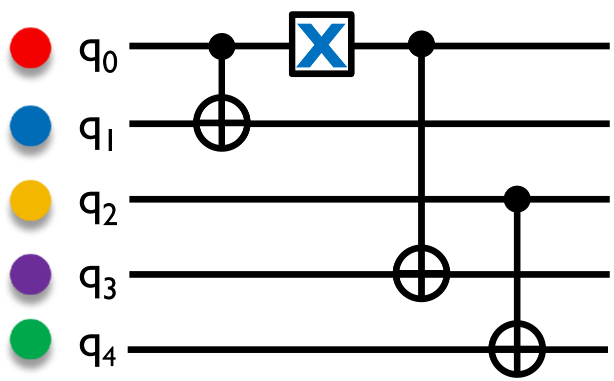


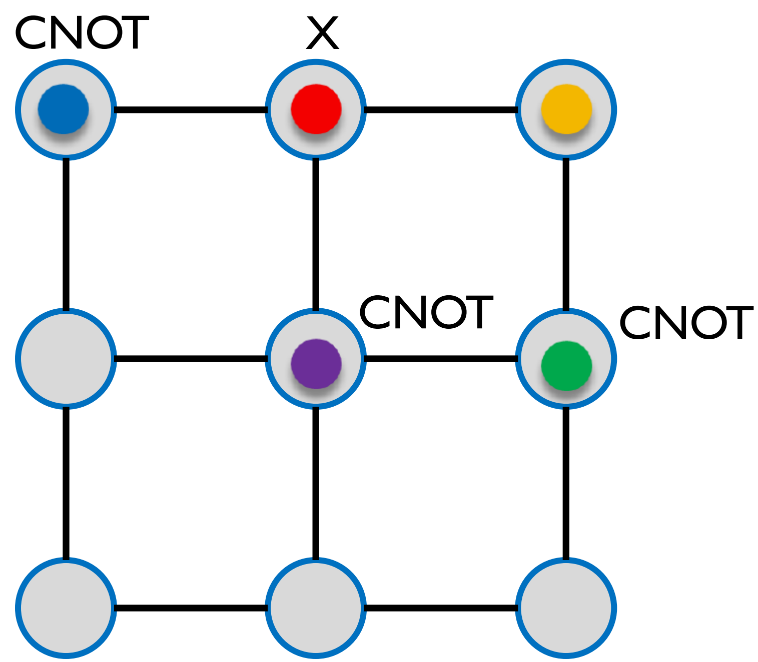


Two SWAP gates

Example on Quantum Dot (cont'd)

- Improved qubit placement: place qubits by considering their future interactions





No SWAP gate

Qubit Placement

- Assign each qubit to a location on the 2D grid such that frequently interacting qubits are placed close to one another

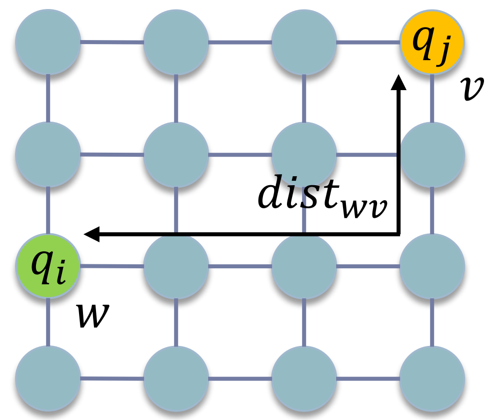
x_{iw}: assignment of q_i to location w

x_{jv}: assignment of q_j to location v

m_{ij}: number of 2-qubit gates working on q_i and q_j

$dist_{wv}$: Manhattan distance between locations w and v

$c_{iwjv} = m_{ij} \times dist_{wv}$



Min $\sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{iw} x_{jv}$

subject to

$$\sum_{w=1}^{n} x_{iw} = 1, \quad i = 1, \ldots, n,$$
$$\sum_{i=1}^{n} x_{iw} = 1, \quad w = 1, \ldots, n,$$
$$x_{iw} \in \{0, 1\}, \quad i, w = 1, \ldots, n. \qquad (1)$$

Kaufmann and Broeckx's Linearization

$$\alpha_{iw} = \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv}, \quad i, w = 1, \dots, n$$

$$z_{iw} = x_{iw} \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{jv}, \quad i, w = 1, \dots, n$$

$$\text{Min} \quad \sum_{i=1}^{n} \sum_{w=1}^{n} z_{iw}$$

subject to

$$\sum_{w=1}^{n} x_{iw} = 1, \quad i = 1, \dots, n,$$

$$\sum_{i=1}^{n} x_{iw} = 1, \quad w = 1, \dots, n,$$

$$\alpha_{iw} x_{iw} + \sum_{j=1}^{n} \sum_{v=1}^{n} c_{iwjv} x_{jv} - z_{iw} \le \alpha_{iw}, \quad i, w = 1, .., n,$$

$$x_{iw} \in \{0, 1\}, \quad i, w = 1, \dots, n,$$

$$z_{iw} \ge 0, \quad i, w = 1, \dots, n. \qquad (2)$$

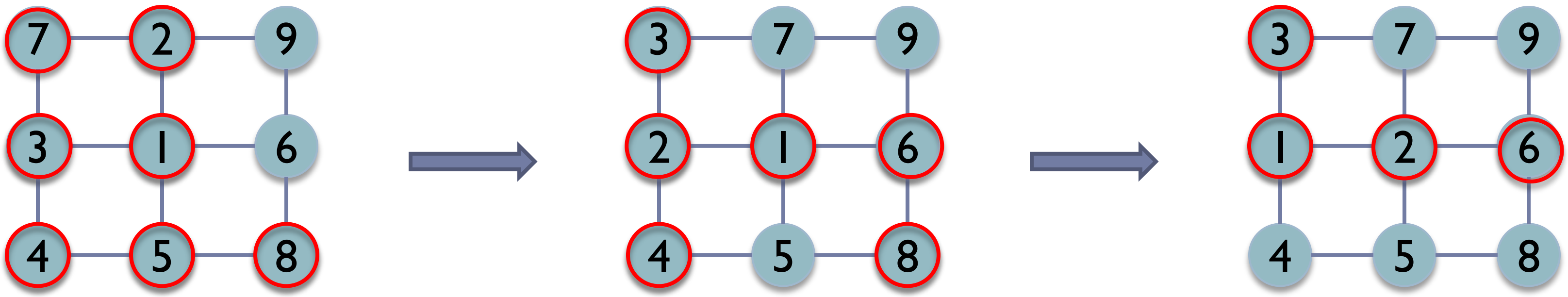
n^2 binary variables (x_{iw}), n^2 real variables (z_{iw}), and $n^2 + 2n$ constraints

R. E. Burkard, E. ela, P. M. Pardalos, and L. S. Pitsoulis. The Quadratic Assignment Problem. Handbook of Combinatorial Optimization, Kluwer Academic Publishers, pp. 241-338, 1998.

MIP Optimization Framework

- GUROBI Optimizer 5.5 (http://www.gurobi.com)
  - Commercial solver with parallel algorithms for large-scale linear, quadratic, and mixed-integer programs (free for academic use)
  - Uses linear-programming relaxation techniques along with other heuristics in order to quickly solve large-scale MIP problems
- Qubit placement (the MIP formulation) does not guarantee that all two-qubit gates become localized; Instead, it ensures the placement of qubits such that the frequently interact qubits are as close as possible to one another
  - SWAP insertion

SWAP Insertion



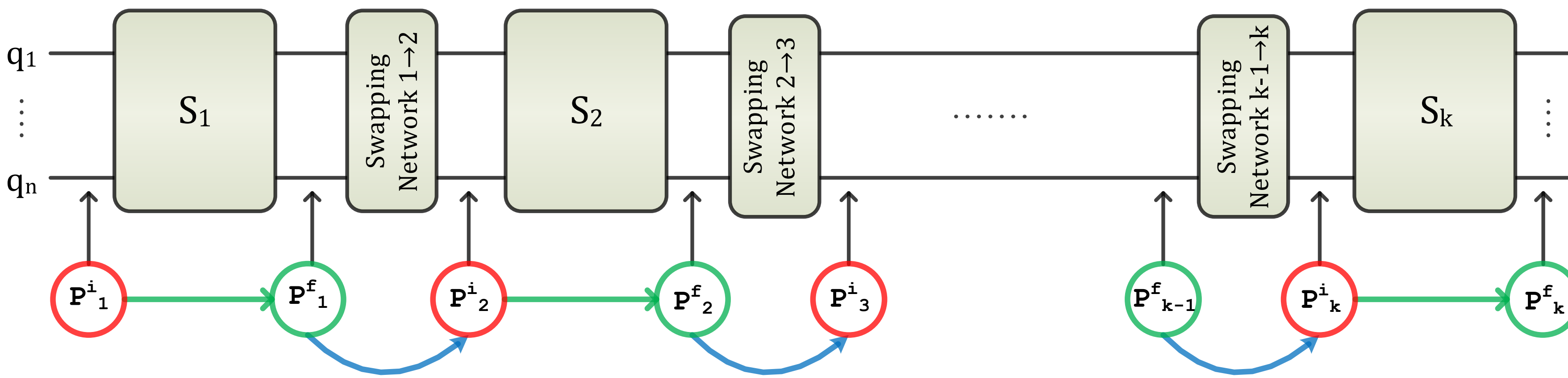

```
CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
CNOT 2, 4
CNOT 6, 8
CNOT 1, 3
CNOT 2, 6
```

```
CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
SWAP 2, 7
SWAP 2, 3
CNOT 2, 4
CNOT 6, 8
CNOT 1, 3
CNOT 2, 6
```

```
CNOT 1, 2
CNOT 5, 8
CNOT 3, 7
SWAP 2, 7
SWAP 2, 3
CNOT 2, 4
CNOT 6, 8
SWAP 1, 2
CNOT 1, 3
CNOT 2, 6
```

Solution Improvement (1)

- Two qubits may interact with one another at different times
  - Not satisfactorily captured by a global qubit placer
  - Solution: Partition the circuit into k sub-circuits $(S_1, \cdots, S_k)$



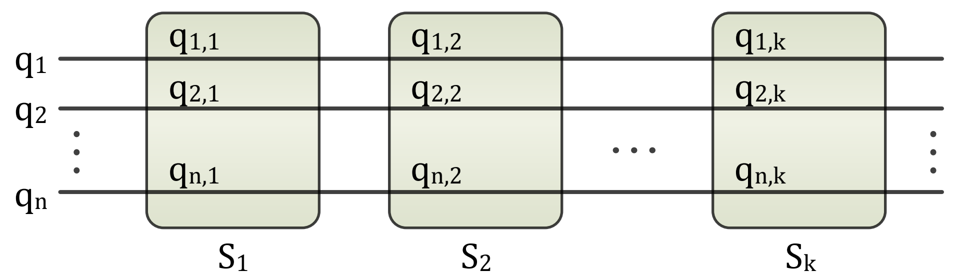
(1) The placement tool finds initial qubit placements (P_j^i).

(2) A SWAP insertion block generates final qubit placements (P_j^f) by inserting <u>*intra-set* SWAP</u> gates.

(3) A swapping network inserts <u>*inter-set* SWAP</u> gates to change the final placement of S_j to the initial placement of S_{j+1} as generated by the qubit placer

Solution Improvement (2)

- In the previous solution, P_j^f is obtained without considering P_{j+1}^i, for $j \geq 2$
  - Large swapping networks
  - Objective function of (1) only minimizes the intra-set communication distances
  - Solution: Add a new term to the objective function in order to capture inter-set communication distances



$q_{i,s}$: qubit i in sub-circuit s

x_{iw}^s: assignment of $q_{i,s}$ to location w

x_{jv}^s: assignment of $q_{j,s}$ to location v

m_{ij}^s: number of 2-qubit gates working on $q_{i,s}$ and $q_{j,s}$

Improved Qubit Placement

Intra-set communication distance

Inter-set communication distance

Min $\sum_{s=1}^{k} \sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{n} m_{ij}^{s} dist_{wv} x_{iw}^{s} x_{jv}^{s} +$

$\sum_{s=1}^{k} \sum_{i=1}^{n} \sum_{w=1}^{n} \sum_{v=1}^{n} dist_{wv} x_{iw}^{s} x_{jv}^{s+1}$

subject to

$$\sum_{w=1}^{n} x_{iw} = 1, \quad i = 1, \dots, n,$$
$$\sum_{i=1}^{n} x_{iw} = 1, \quad w = 1, \dots, n,$$
$$x_{iw} \in \{0, 1\}, \quad i, w = 1, \dots, n. \qquad (3)$$

Force-directed Qubit Placement

- Attractive forces
  - A force proportional to m_{ij}^{s} between $q_{i,s}$ and $q_{j,s}$.
  - A (unit) force between between $q_{i,s}$ and $q_{i,s+1}$.
- Can be solved by quadratic programming

Results (1)

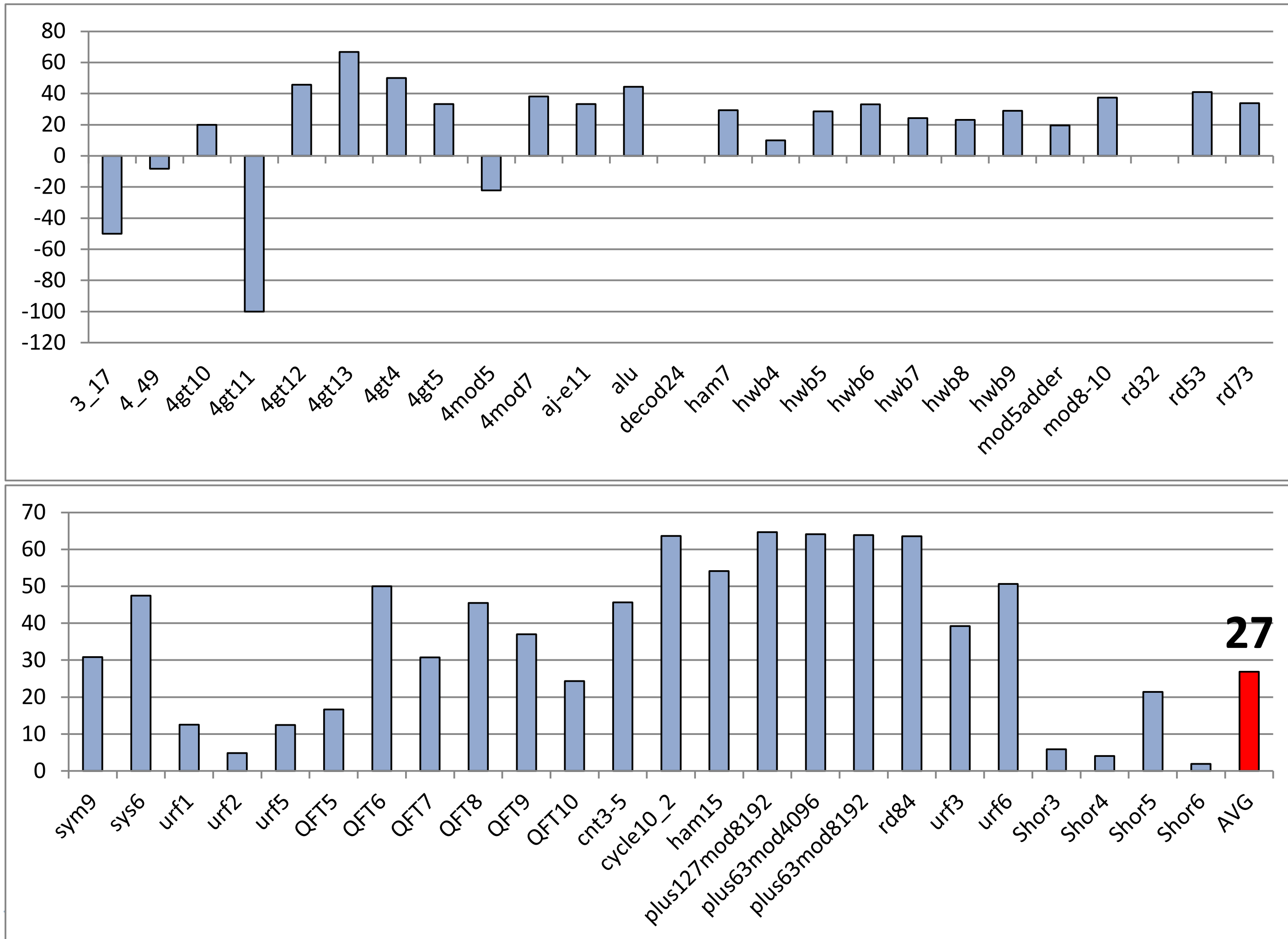
	# of qubits	# of gates	Our Method: Grid Size	Our Method: #SWAPs	Best 1D: #SWAPs	Imp. (%)	Ref.
3_17	3	13	2x2	6	4	-50	[1]
4_49	4	30	2x2	13	12	-8	[1]
4gt10	5	36	3x2	16	20	20	[1]
4gt11	5	7	2x3	2	1	-100	[1]
4gt12	5	52	3x2	19	35	46	[1]
4gt13	5	16	3x3	2	6	67	[1]
4gt4	5	43	2x3	17	34	50	[1]
4gt5	5	22	3x3	8	12	33	[1]
4mod5	5	24	2x3	11	9	-22	[1]
4mod7	5	40	3x3	13	21	38	[1]
aj-e11	4	59	2x3	24	36	33	[1]
alu	5	31	2x3	10	18	44	[1]
decod24	4	9	2x2	3	3	0	[1]
ham7	7	87	3x3	48	68	29	[1]
hwb4	4	23	3x3	9	10	10	[1]
hwb5	5	106	3x2	45	63	29	[1]
hwb6	6	146	2x3	79	118	33	[1]
hwb7	7	2659	3x3	1688	2228	24	[1]
hwb8	8	16608	3x3	11027	14361	23	[1]
hwb9	9	20405	4x3	15022	21166	29	[1]
mod5adder	6	81	3x2	41	51	20	[1]
mod8-10	5	108	3x3	45	72	38	[1]
rd32	4	8	2x3	2	2	0	[1]
rd53	7	78	5x2	39	66	41	[1]
rd73	10	76	4x4	37	56	34	[1]

Results (2)

	# of qubits	# of gates	Grid Size (Our Method)	#SWAPs (Our Method)	#SWAPs (Best 1D)	Imp. (%)	Ref.
sym9	10	4452	4x4	2363	3415	31	[1]
sys6	10	62	4x4	31	59	47	[1]
urf1	9	57770	3x3	38555	44072	13	[1]
urf2	8	25150	2x4	16822	17670	5	[1]
urf5	9	51380	3x3	34406	39309	12	[1]
QFT5	5	10	3x2	5	6	17	[1]
QFT6	6	15	2x3	6	12	50	[1]
QFT7	7	21	5x2	18	26	31	[1]
QFT8	8	28	4x2	18	33	45	[1]
QFT9	9	36	3x3	34	54	37	[1]
QFT10	10	45	5x3	53	70	24	[1]
cnt3-5	16	125	3x6	69	127	46	[2]
cycle10_2	12	1212	3x4	839	2304	64	[2]
ham15	15	458	5x3	328	715	54	[2]
plus127mod8192	13	65455	5x4	53598	151794	65	[2]
plus63mod4096	12	29019	5x3	22118	61556	64	[2]
plus63mod8192	13	37101	5x3	29835	82492	64	[2]
rd84	15	112	5x3	54	148	64	[2]
urf3	10	132340	4x3	94017	154672	39	[2]
urf6	15	53700	5x3	43909	88900	51	[2]
Shor3	10	2076	4x3	1710	1816	6	[3]
Shor4	12	5002	3x6	4264	4339	4	[3]
Shor5	14	10265	5x4	8456	10760	21	[3]
Shor6	16	18885	4x6	20386	20778	2	[3]
					On average	**27**	

Results (3)

Improvement over best 1D solution



Conclusion

- Qubit placement methods for 2D quantum architectures
  - Directly applicable to Quantum Dot PMD
- 27% improvement over best 1D results

- Future work: force-directed qubit placement
  - Better results by considering both intra- and inter-set SWAP gates in the optimization problem

References

[1] A. Shafaei, M. Saeedi, and M. Pedram, "Optimization of quantum circuits for interaction distance in linear nearest neighbor architectures," *Design Automation Conference (DAC)*, 2013.

[2] M. Saeedi, R. Wille, R. Drechsler, "Synthesis of quantum circuits for linear nearest neighbor architectures," *Quantum Information Processing*, 10(3):355–377, 2011.

[3] Y. Hirata, M. Nakanishi, S. Yamashita, Y. Nakashima, "An efficient conversion of quantum circuits to a linear nearest neighbor architecture," *Quantum Information & Computation*, 11(1–2):0142–0166, 2011.

Thank you!