QF_BV Property Directed Reachability with Mixed Type Atomic Reasoning Units

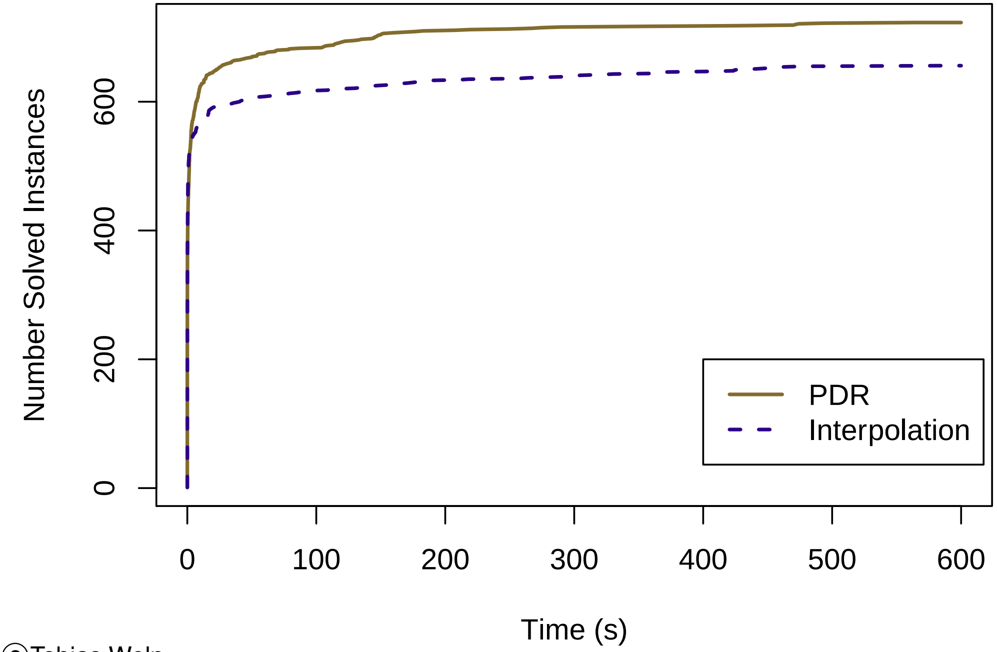
Tobias Welp and Andreas Kuehlmann

Outline

1. **Introduction**
2. QF_BV Property Directed Reachability
3. Mixed Type Atomic Reasoning Units
4. Experimental Results
5. Summary

Motivation for Property Directed Reachability

- In 2011, Bradley proposed *Property Directed Reachability* (a.k.a IC^3) for model checking [Brad11].
- Experiments indicate that PDR outperforms model checking based on *Interpolation* [McMi03] on representative benchmark sets.

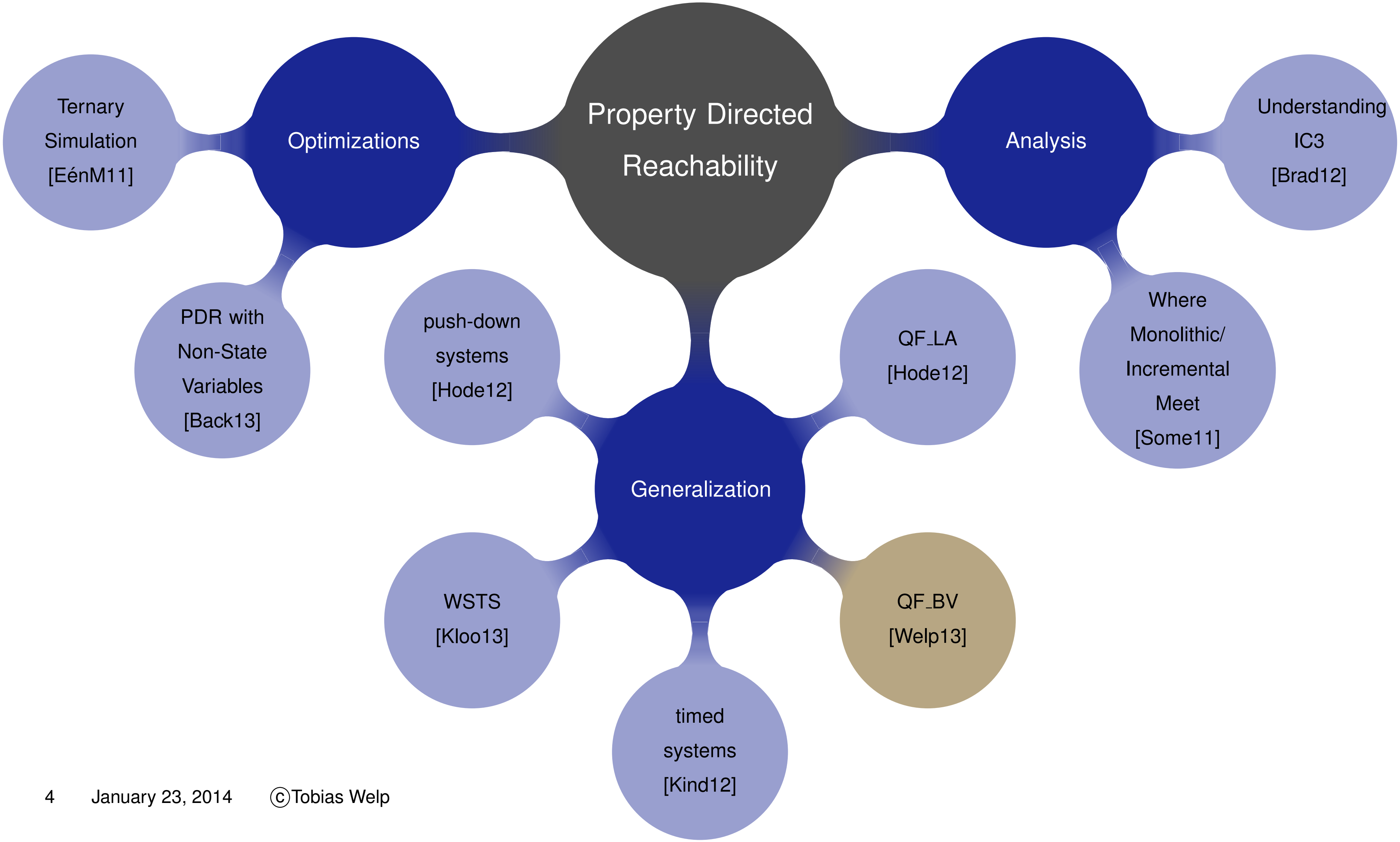


[EénM11]

Other Favorable Properties of PDR

- ☺ No unrolling of transition relation.
- ☺ Parallizable.
- ☺ Allows for initialization with known invariants.
- ☺ Good for finding counterexamples and proving that none exists.

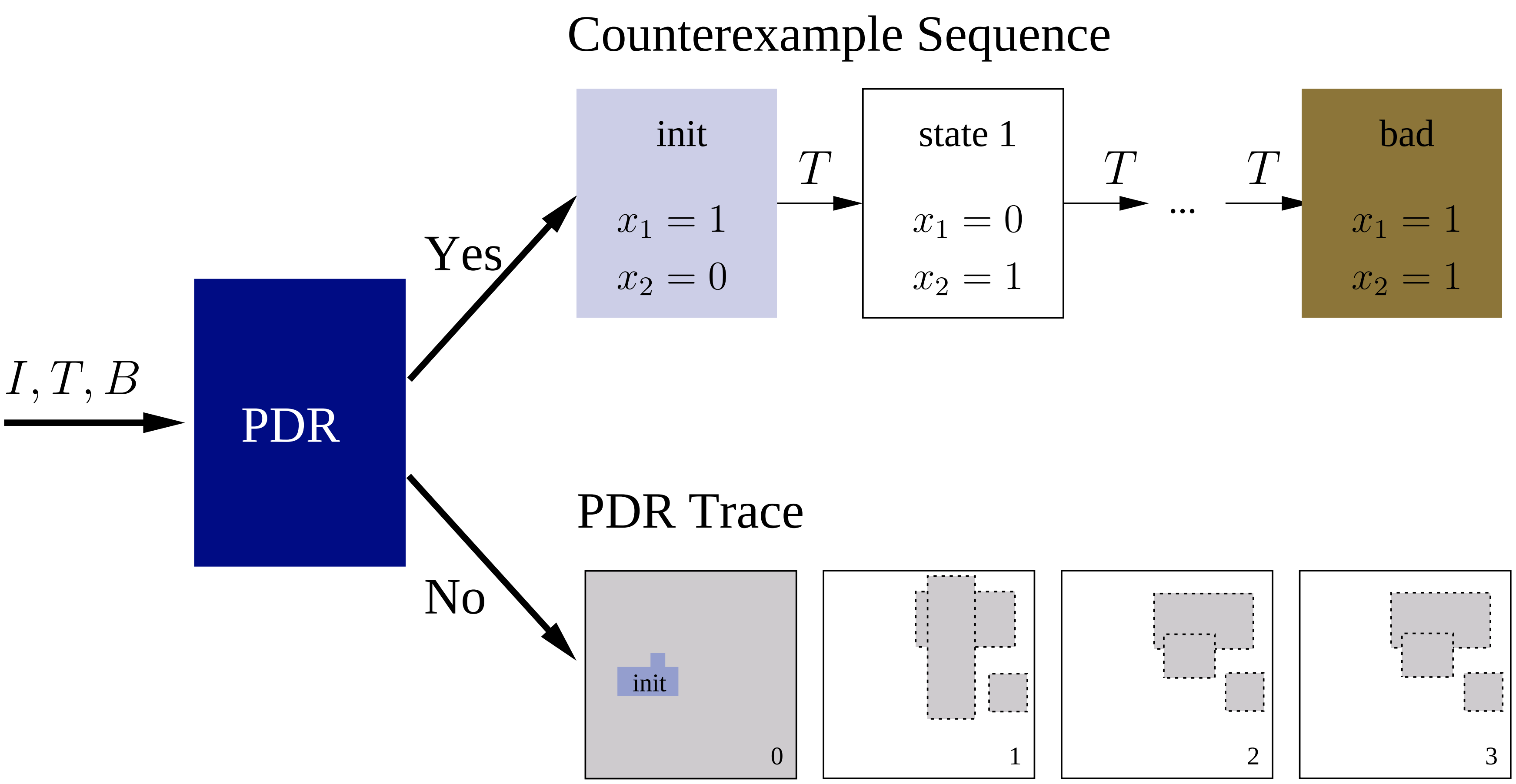
Research Pertaining PDR



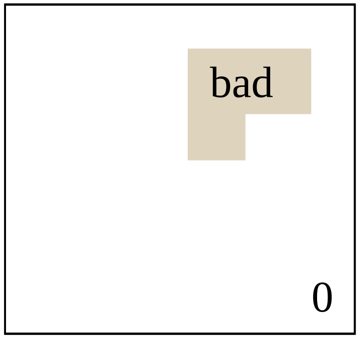
Model Checking

- Given are
  - A set of initial states: $I(\mathbf{x})$
  - A set of bad states: $B(\mathbf{x})$
  - A transition relation: $T(\mathbf{x}, \mathbf{x}')$
- Question: Is a bad state reachable from an initial state using valid transitions?

Model Checking with PDR



Proving a Safety Property with PDR



Legend:

- Initial set I
- Bad set B
- Proof oblig.
- Cover

- Can bad be reached within zero steps?

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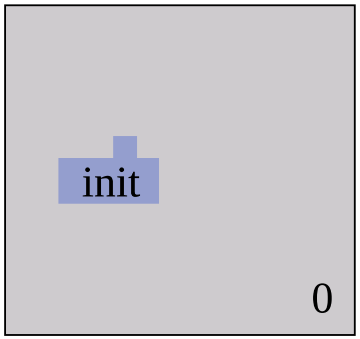

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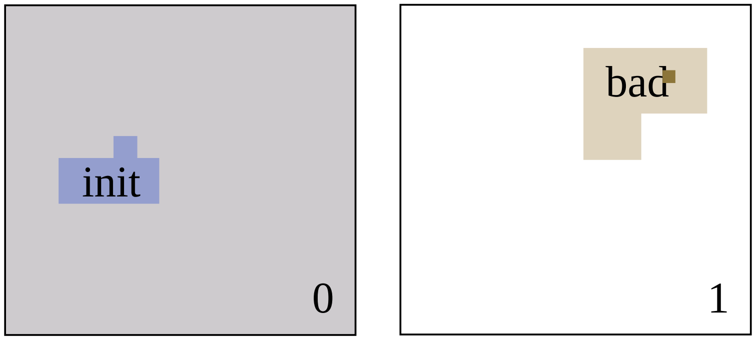
- No, only the initial set is reachable within zero steps.
- Everything else is *covered*, i.e. not reachable.

Proving a Safety Property with PDR

- Can bad be reached within one step?
- Conservatively, we initially assume that everything is *reachable*.

Proving a Safety Property with PDR



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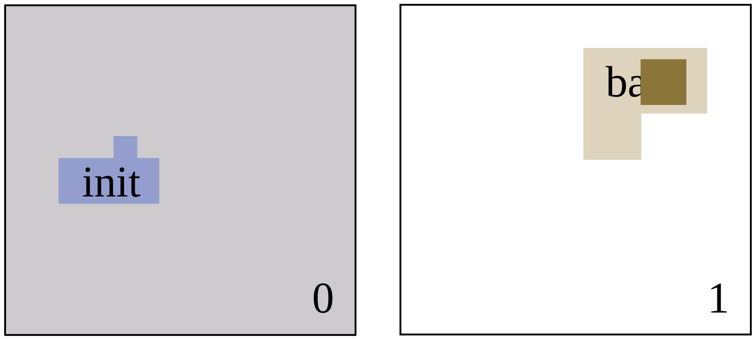
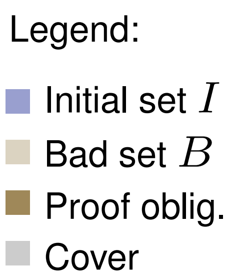
- Find a point in bad that is not yet covered.

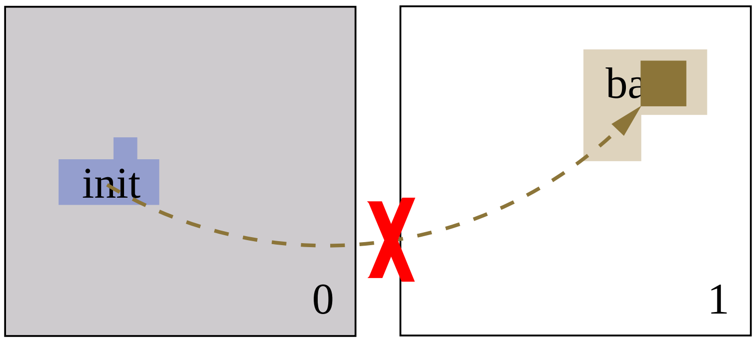
Proving a Safety Property with PDR





- Expand proof obligation using simulation.

Proving a Safety Property with PDR



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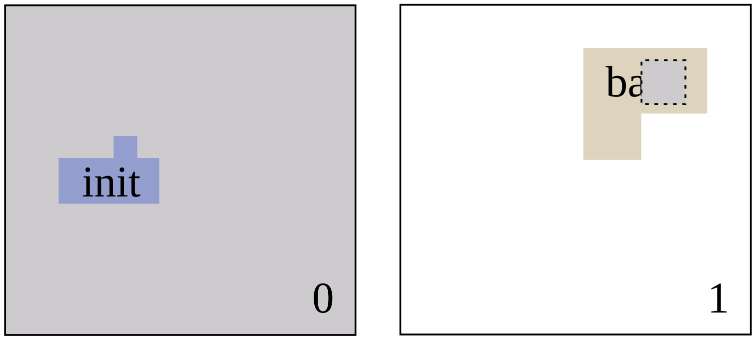
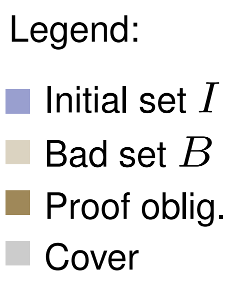
- The cube cannot be reached from the reachable area in frame 0.

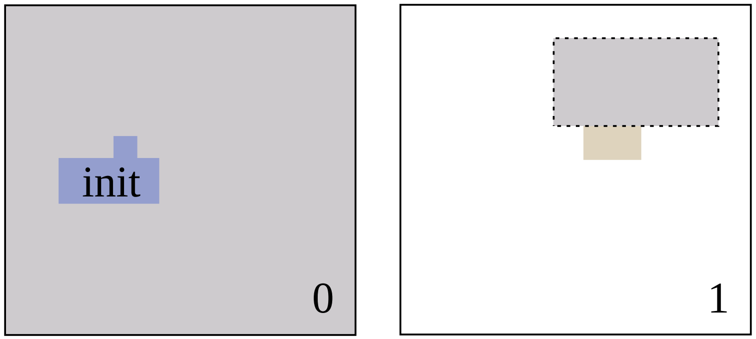
Proving a Safety Property with PDR





- Hence, we can consider the proof obligation covered.

Proving a Safety Property with PDR



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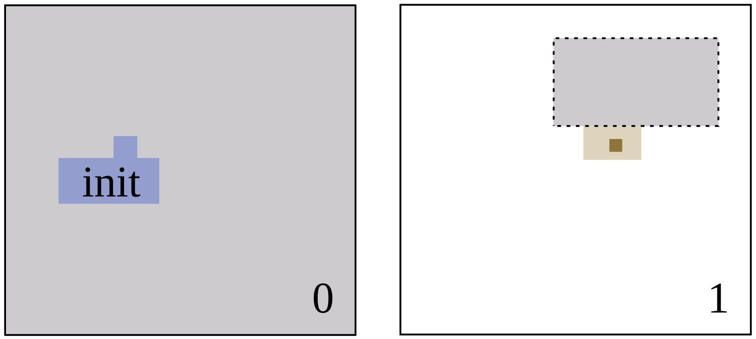
- Expand the covered cube as much as possible.

Proving a Safety Property with PDR



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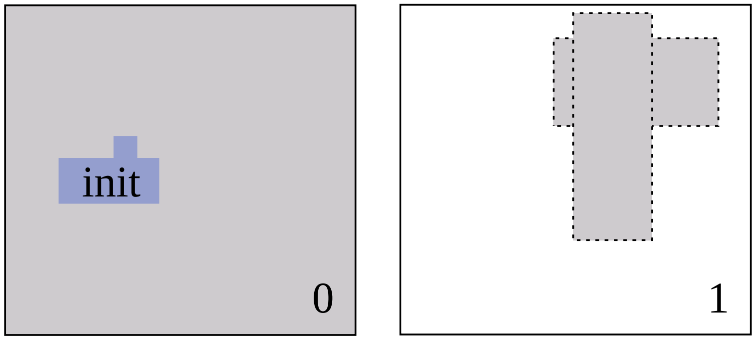
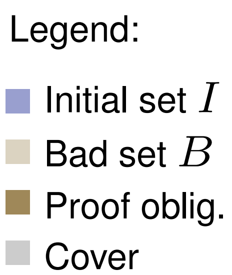
- Repeat with finding a new point in bad that is not covered.

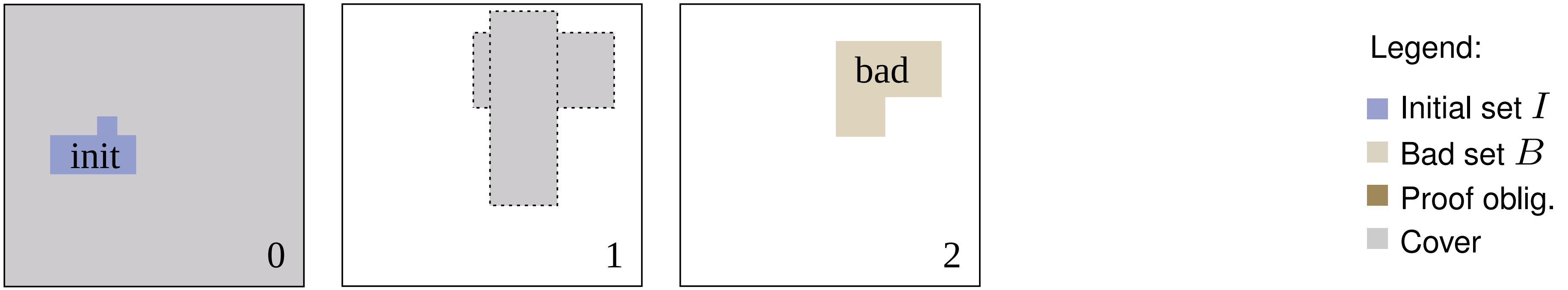
Proving a Safety Property with PDR





- Again, the point cannot be reached from the reachable area in the previous frame.
- Expand the covered cube.
- Now, bad is completely covered.

Proving a Safety Property with PDR

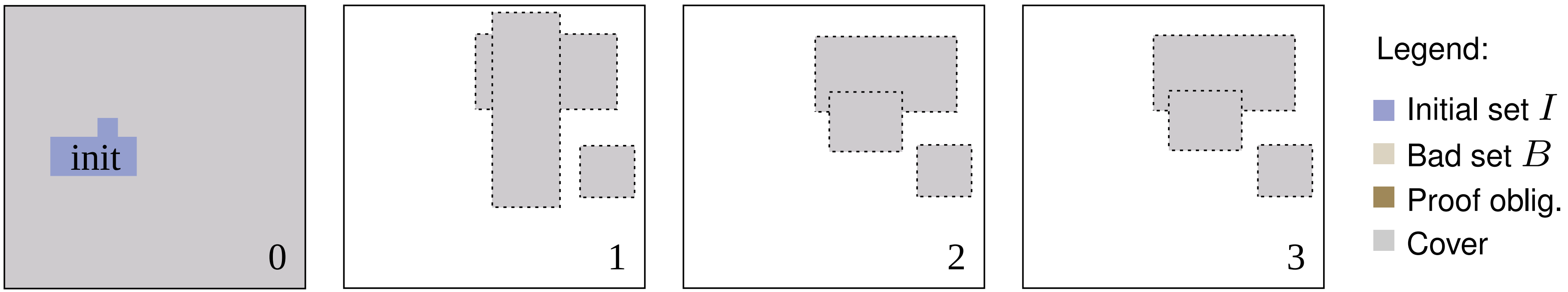


- Can bad be reached within two steps?

Proving a Safety Property with PDR

. . .

Proving a Safety Property with PDR



- Identified an inductive invariant disjoint from bad.
- This proves the property.

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Property Directed Reachability for QF_BV

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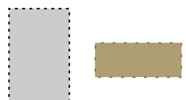
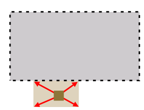
Atomic Reasoning Unit 	Boolean Cubes		
Expansion of Proof Obligations 	Ternary Simulation		
Strengths			
Weaknesses			

Example Hybrid Invariant

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\begin{aligned}
I &:= (2 \times y \equiv x) \wedge (x + y \leq 3) \\
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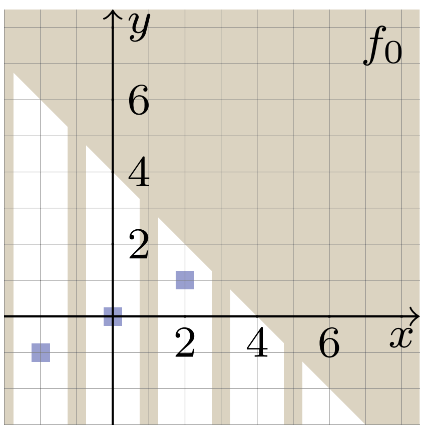

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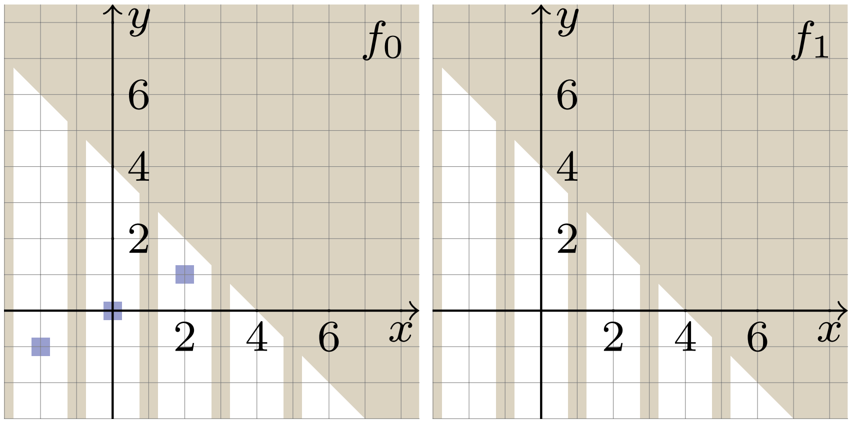

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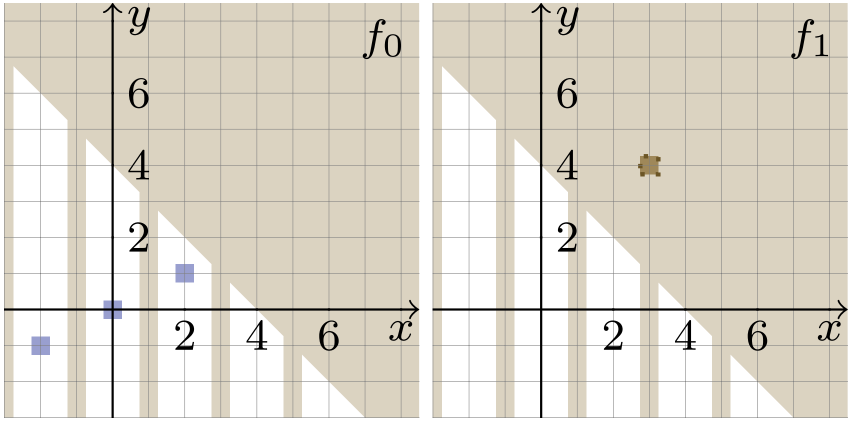

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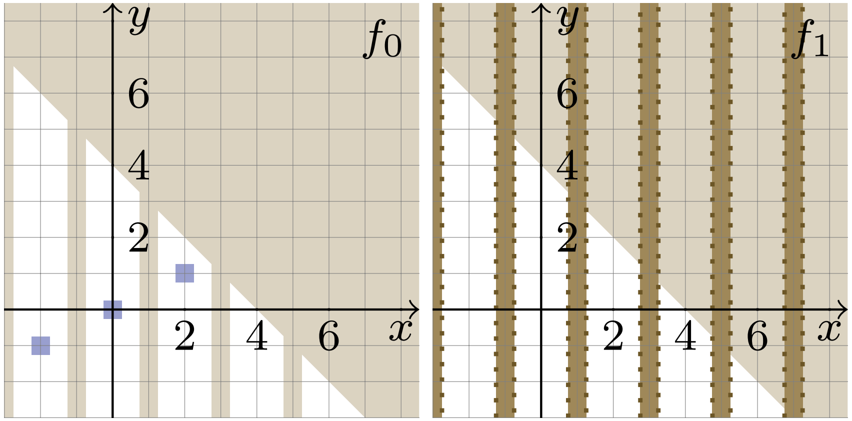

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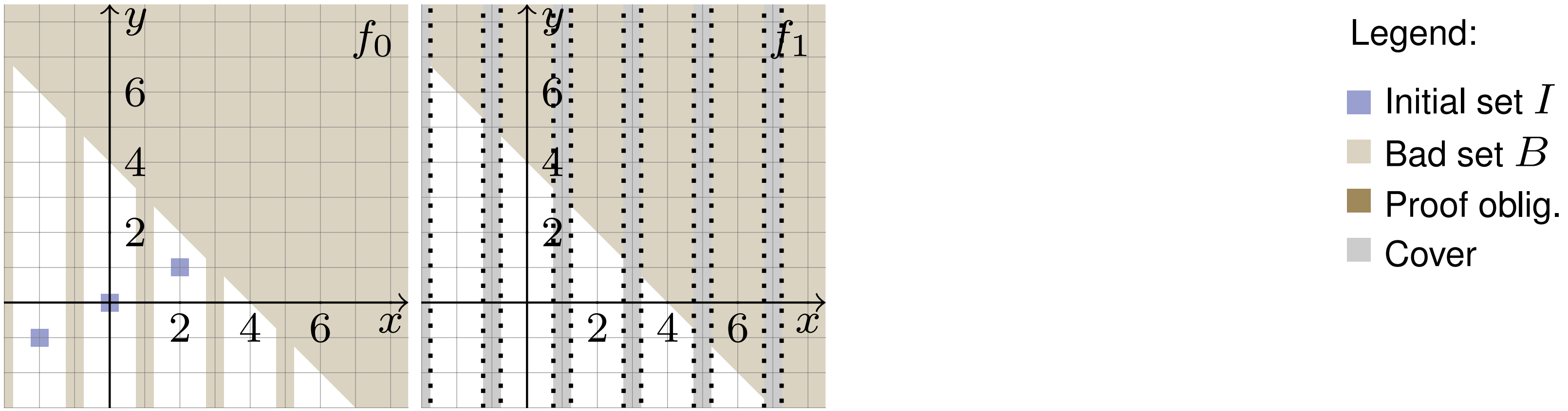

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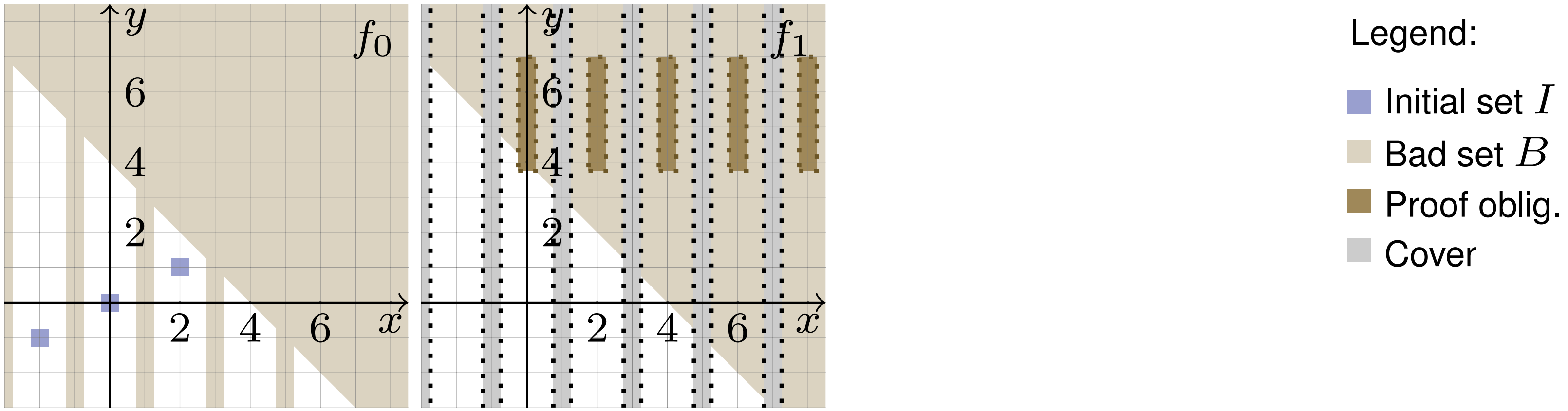

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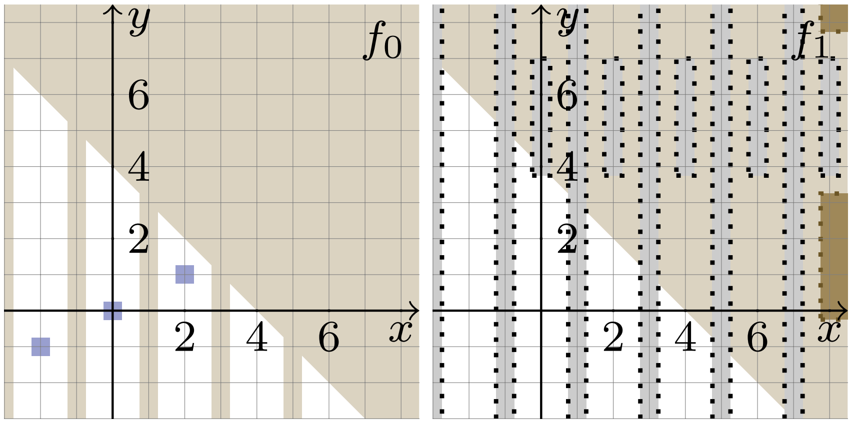

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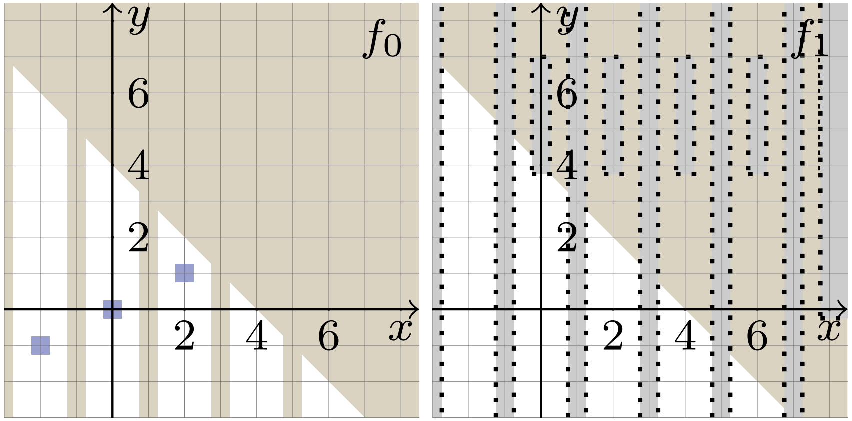

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Property Directed Reachability for QF_BV

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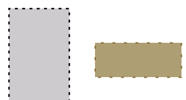
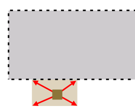
Atomic Reasoning Unit 	Boolean Cubes		
Expansion of Proof Obligations 	Ternary Simulation		
Strengths	logic		
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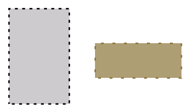
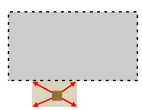
Property Directed Reachability for QF_BV

	Original Formulation	Polytopes [Welp13]	
Atomic Reasoning Unit 	Boolean Cubes	Polytopes	
Expansion of Proof Obligations 	Ternary Simulation	Interval Simulation	
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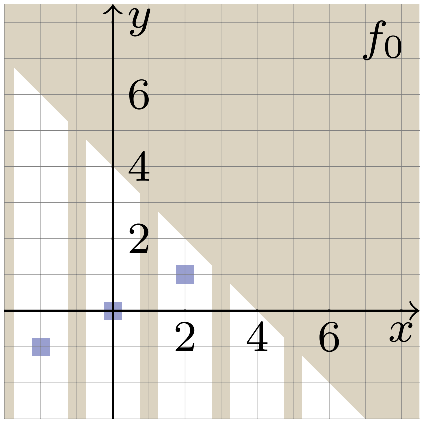

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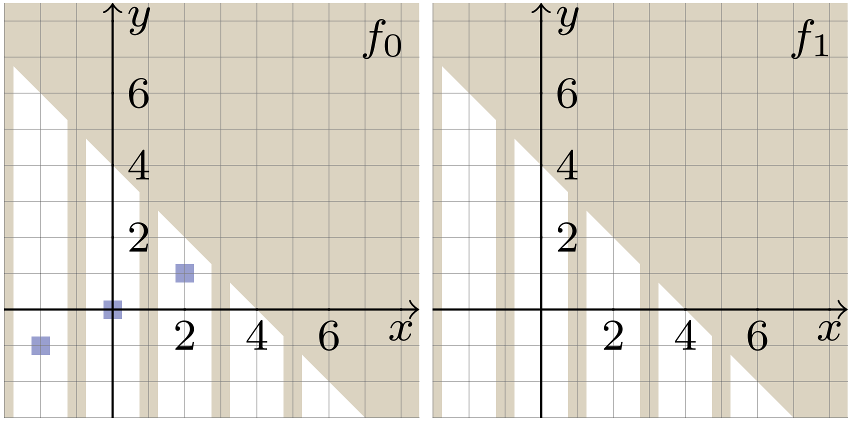

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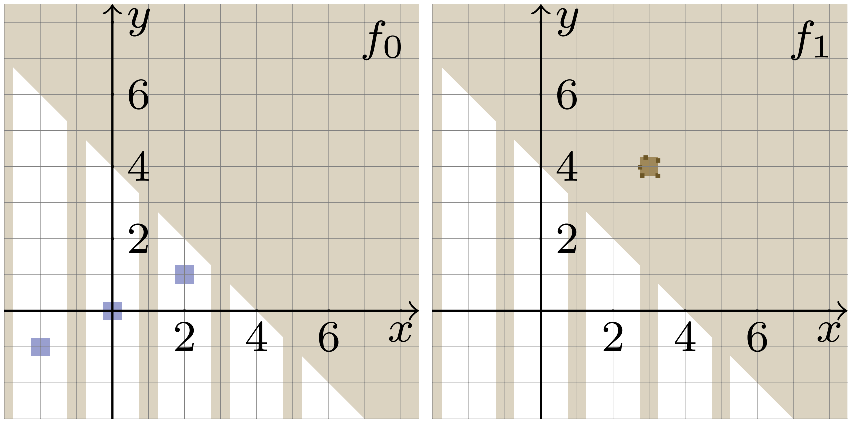

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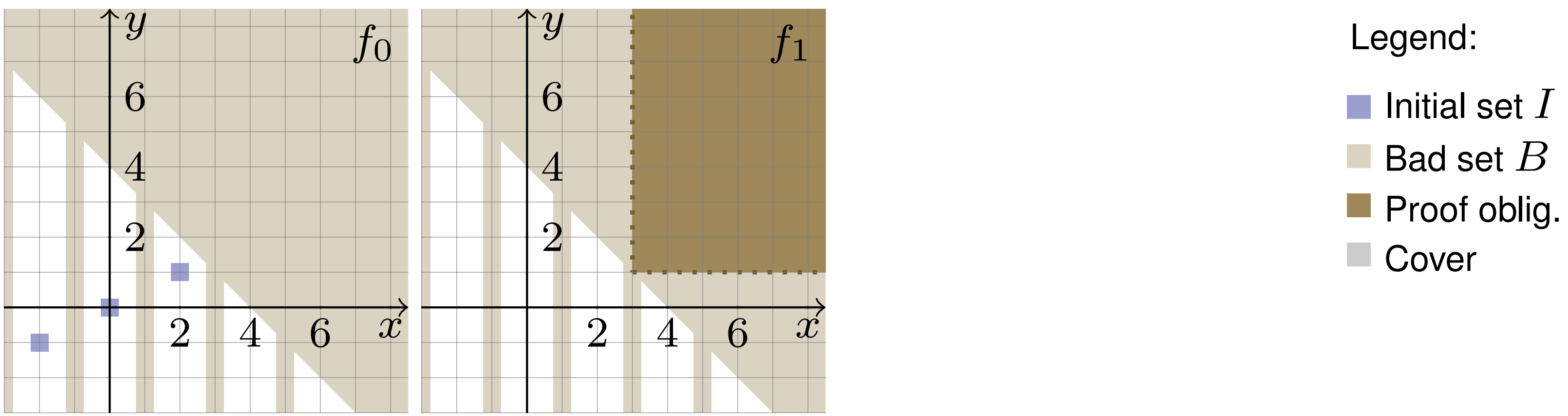

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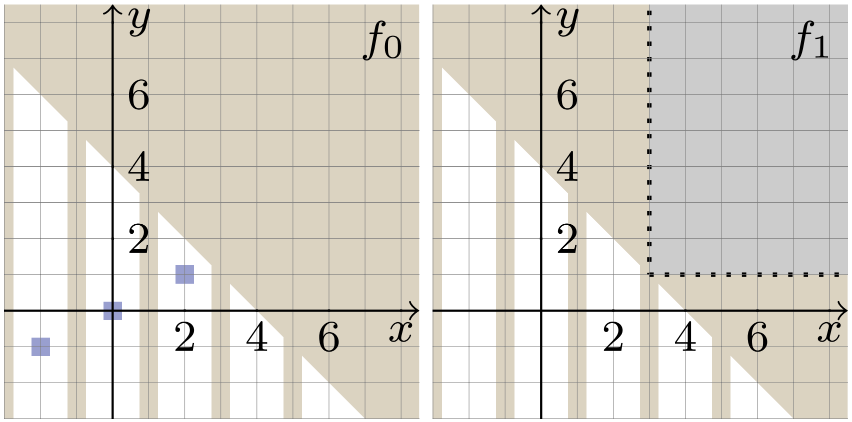

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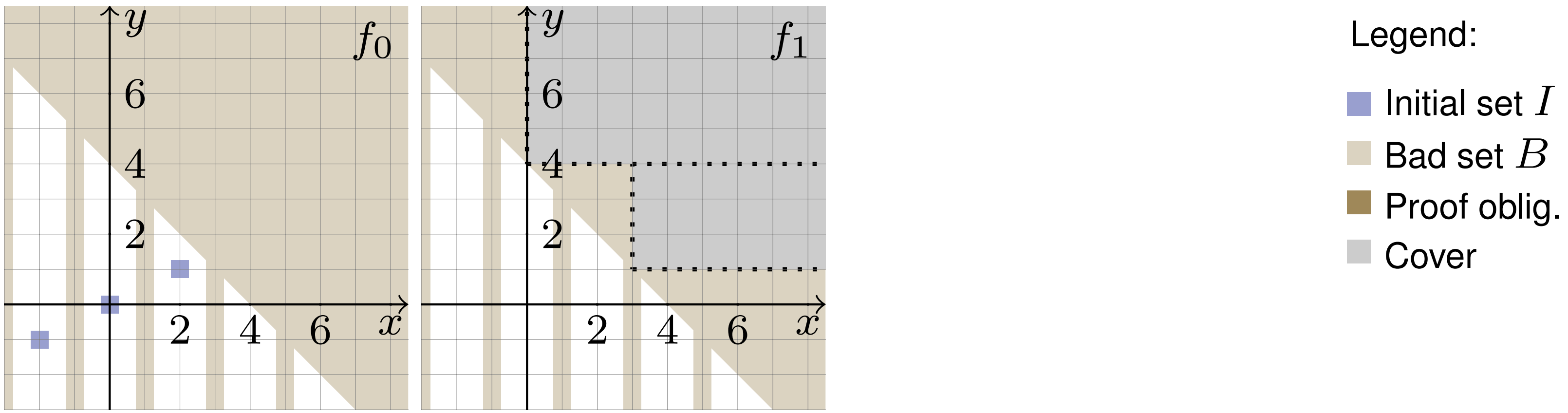

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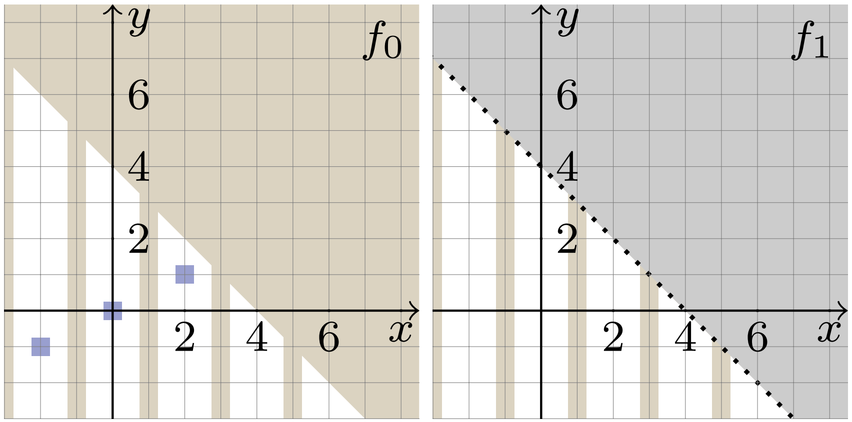

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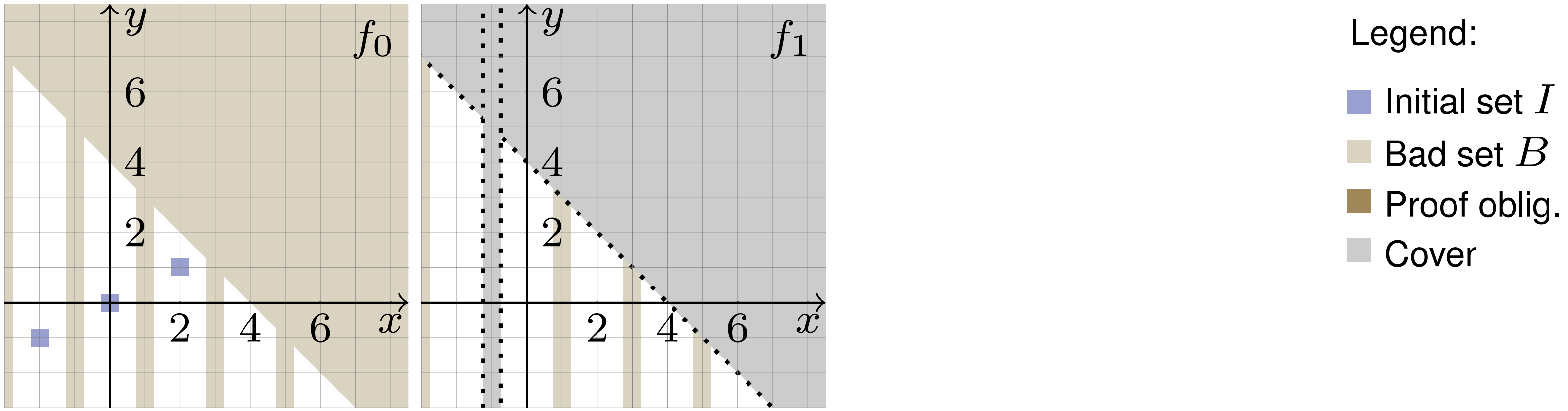

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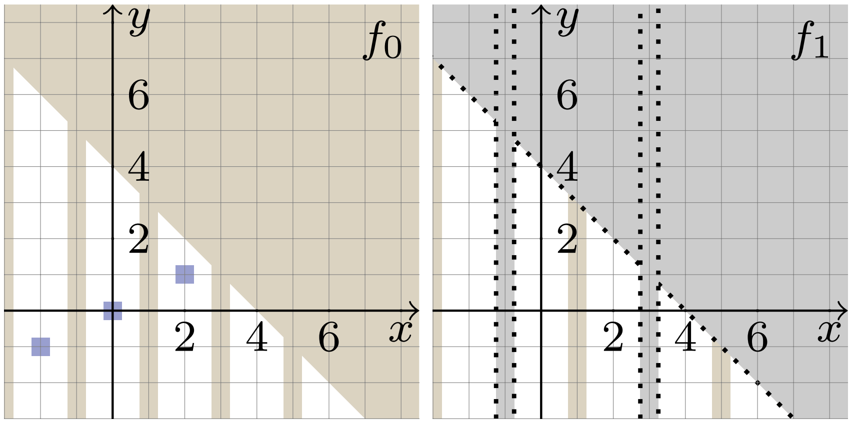

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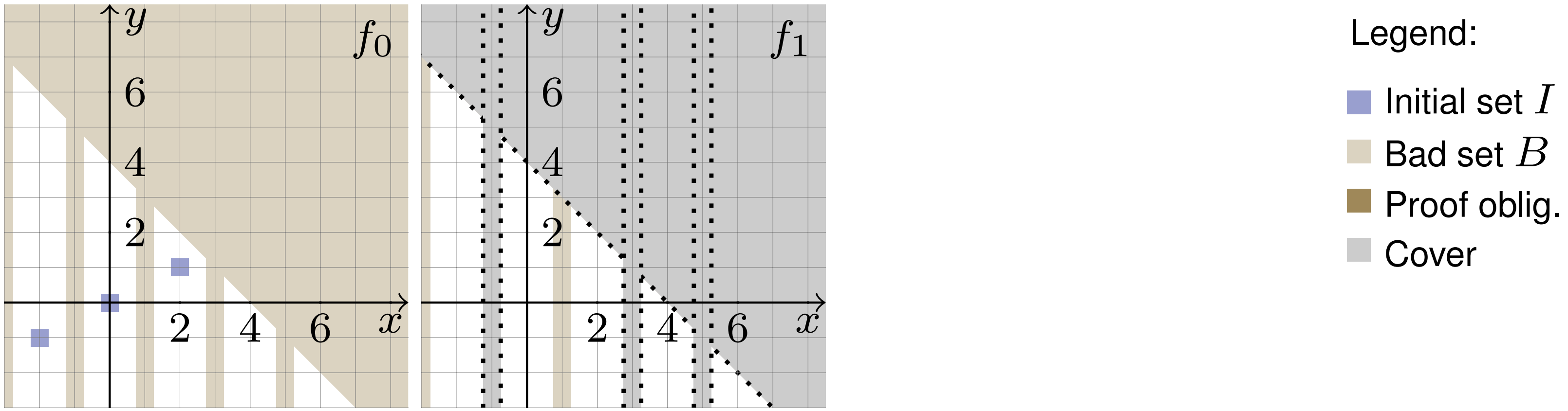
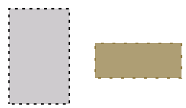
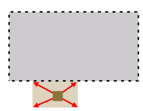


Property Directed Reachability for QF_BV

	Original Formulation	Polytopes [Welp13]	
Atomic Reasoning Unit 	Boolean Cubes	Polytopes	
Expansion of Proof Obligations 	Ternary Simulation	Interval Simulation	
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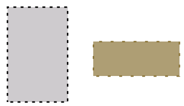
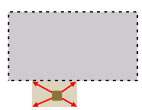
Atomic Reasoning Unit 	Boolean Cubes	Polytopes	Boolean Cubes and Polytopes
Expansion of Proof Obligations 	Ternary Simulation	Interval Simulation	Hybrid Simulation
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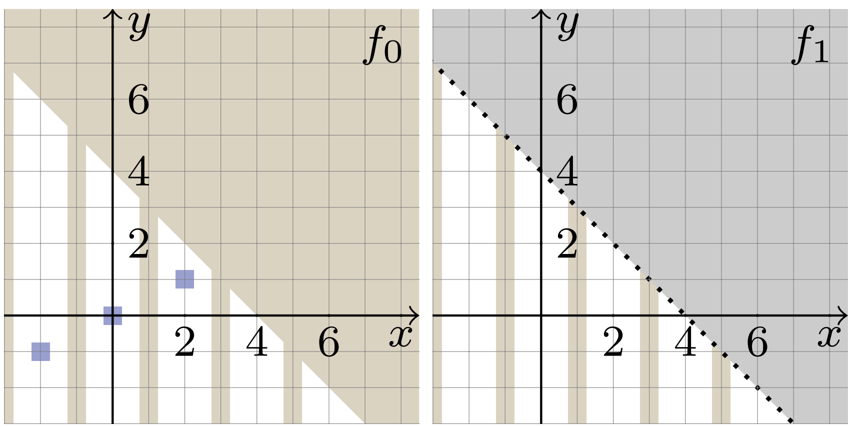
B &:= (x + y \geq 4) \vee (x \bmod 2 \equiv 1)
\end{aligned}
$$



Legend:

- Initial set I
- Bad set B
- Proof oblig.
- Cover

Example Hybrid Invariant

$$I := (2 \times y \equiv x) \wedge (x + y \leq 3)$$
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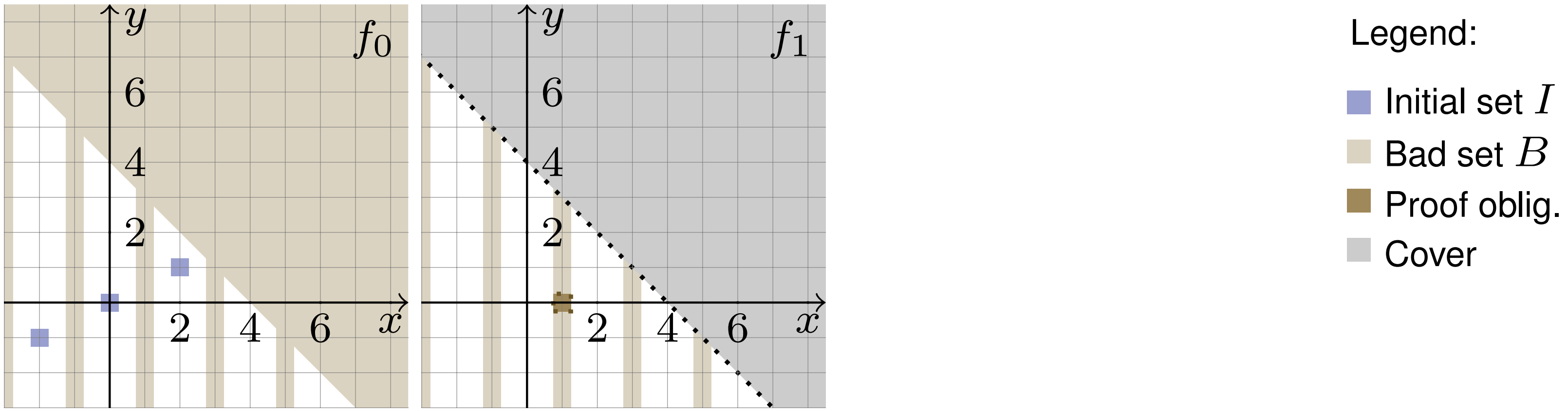
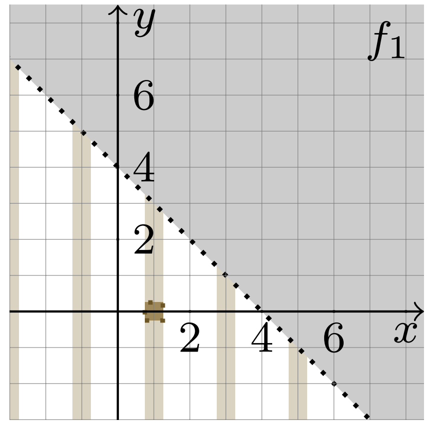
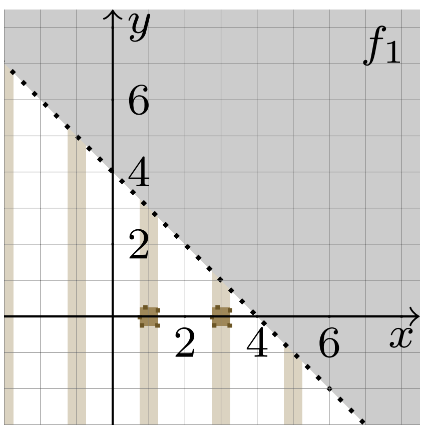


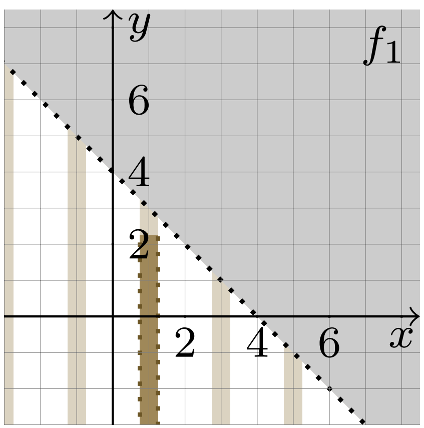
Probabilistic Specialization



specialize to Boolean cube with probability c

specialize to polytope with probability $1 - c$





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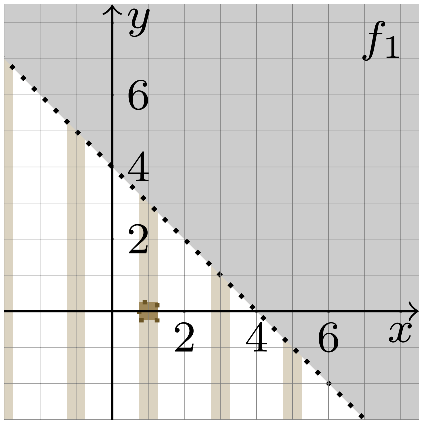

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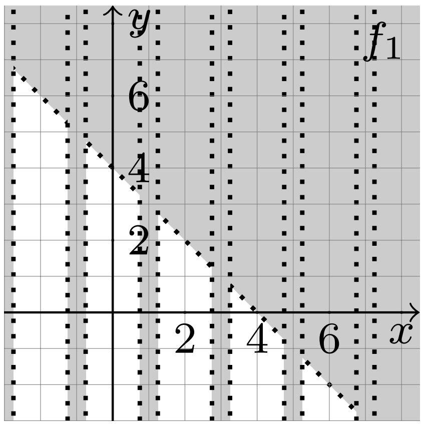

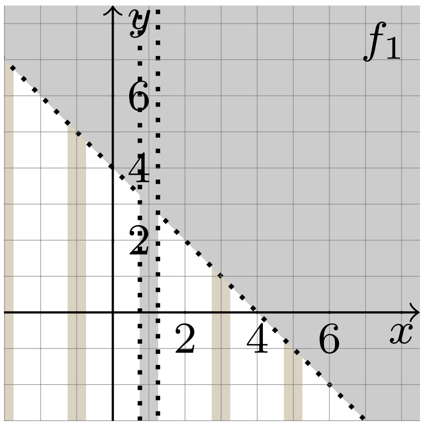

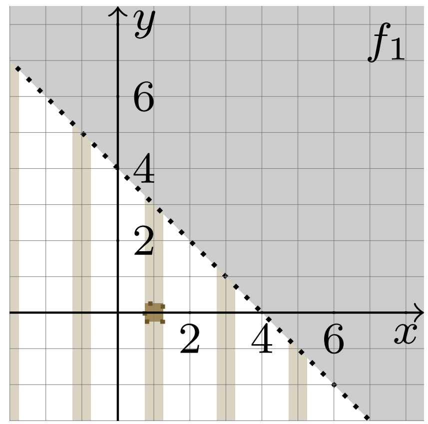

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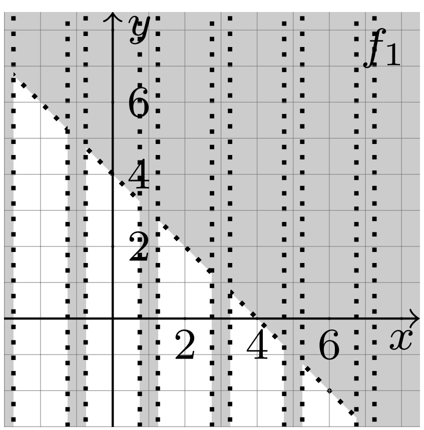
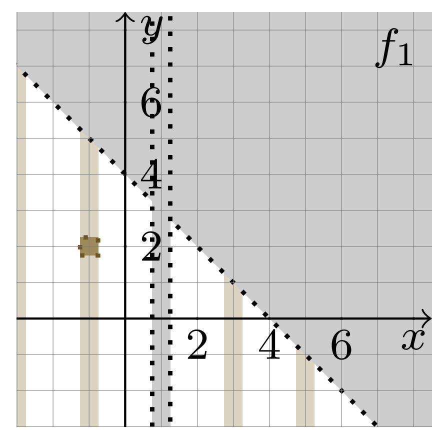




Probabilistic Specialization

- The favorable event can be expected to happen in a constant number of steps.

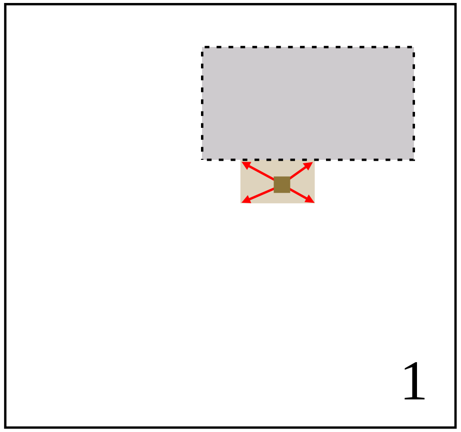
$$E\{\text{trials until Boolean cube specialization}\} = c\sum_{i=1}^{\infty} i(1-c)^{i-1} = \frac{1}{c}$$

- Analogously, one calculates

$$E\{\text{trials until polytope specialization}\} = (1-c)\sum_{i=1}^{\infty} ic^{i-1} = \frac{1}{1-c}$$

Simulation-based Expansion

Simulation-based expansion of proof obligations is e.g. used to expand a bad ARU that is not yet covered:



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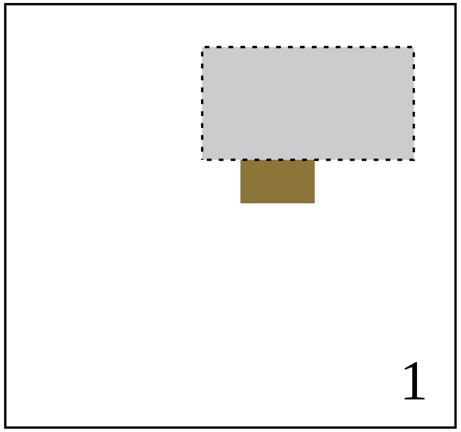
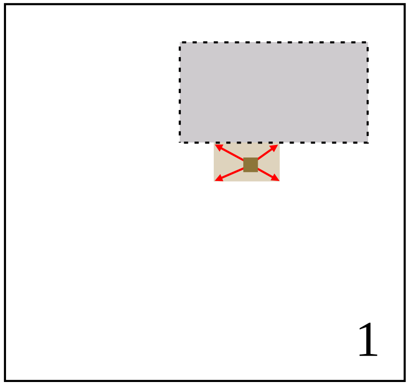


Simulation-based Expansion

The check whether or not an expansion is valid can be reduced to simulation on sets of points [EénM11].

Assume bad is defined as $e_1 < 2$ and we already covered $e_1 < -1$ with

$$e_1 := (x_1 - x_2 + 2) \wedge (y_1 \vee y_2)$$

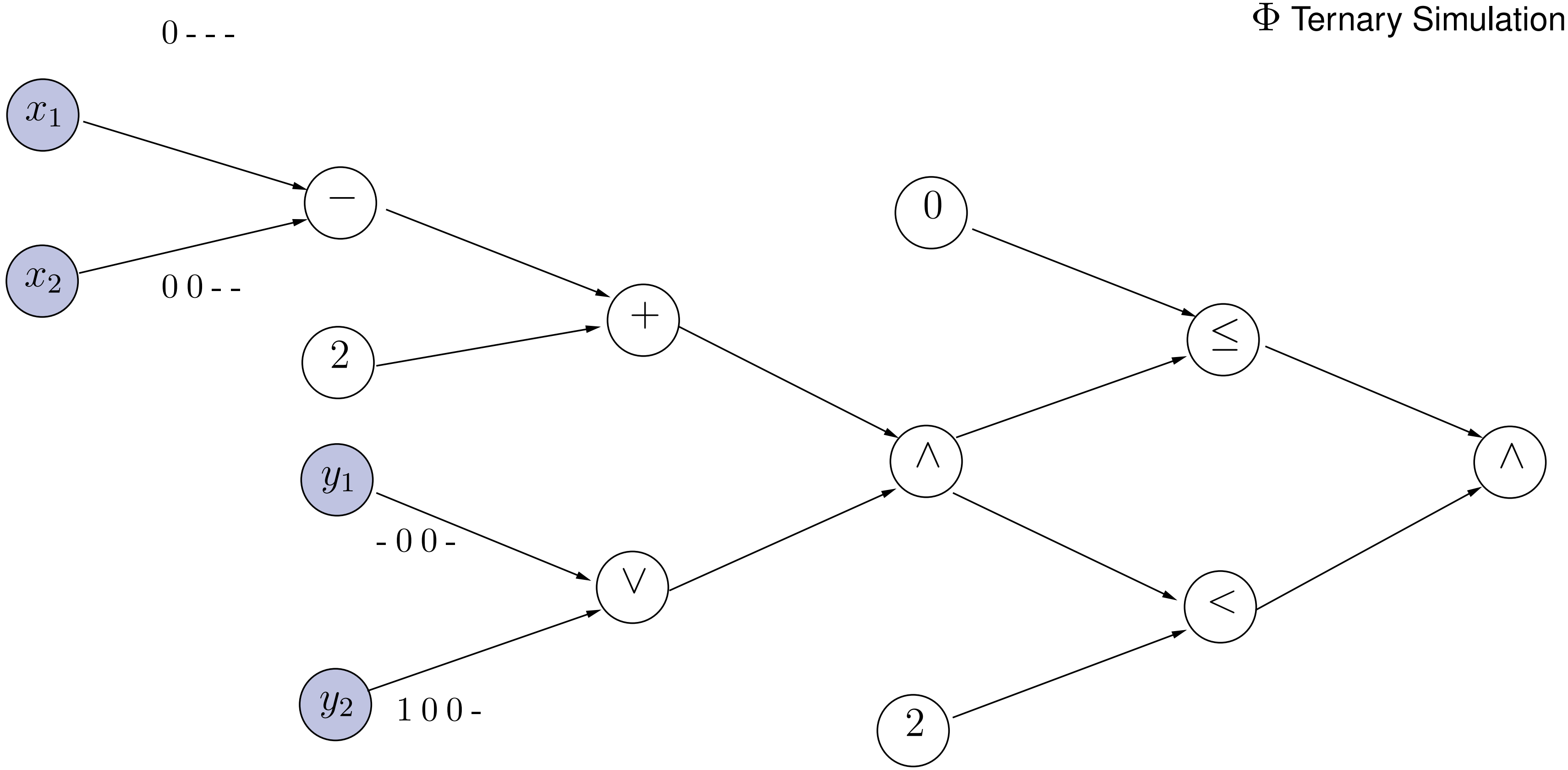


Then we may expand an ARU to a larger ARU $\hat{a}$ if $e := \underbrace{(e_1 < 2)}_{\text{bad}} \wedge \neg \underbrace{(e_1 < -1)}_{\text{covered}}$ evaluates to **true** for all values in $\hat{a}$.

Ternary Simulation

Let $\hat{a} := (1 \leq x_1 \leq 5) \wedge (0 \leq x_2 \leq 3) \wedge (y_1 \in \text{-00-}) \wedge (y_2 \in 100\text{-})$

Φ Ternary Simulation



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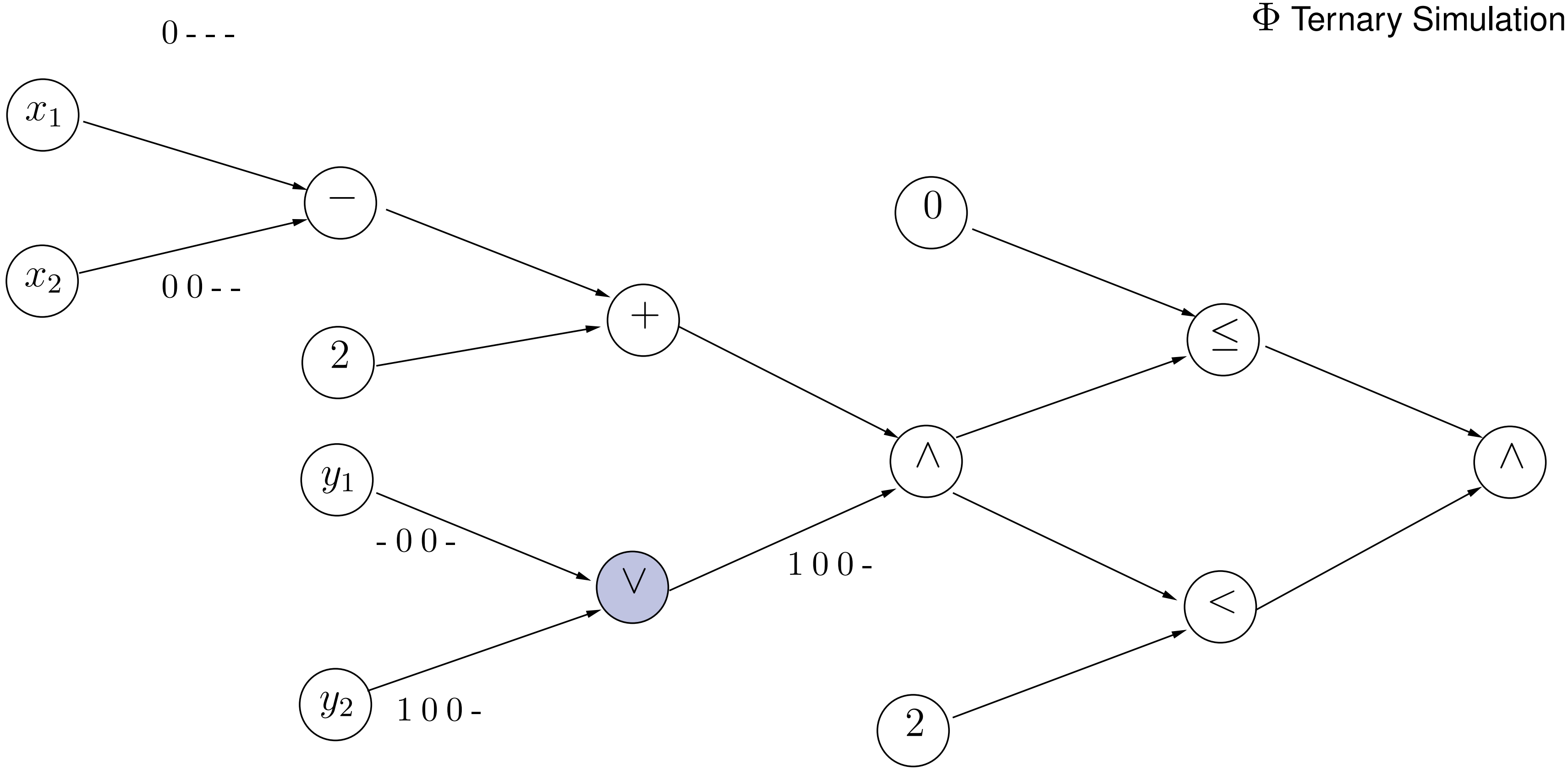

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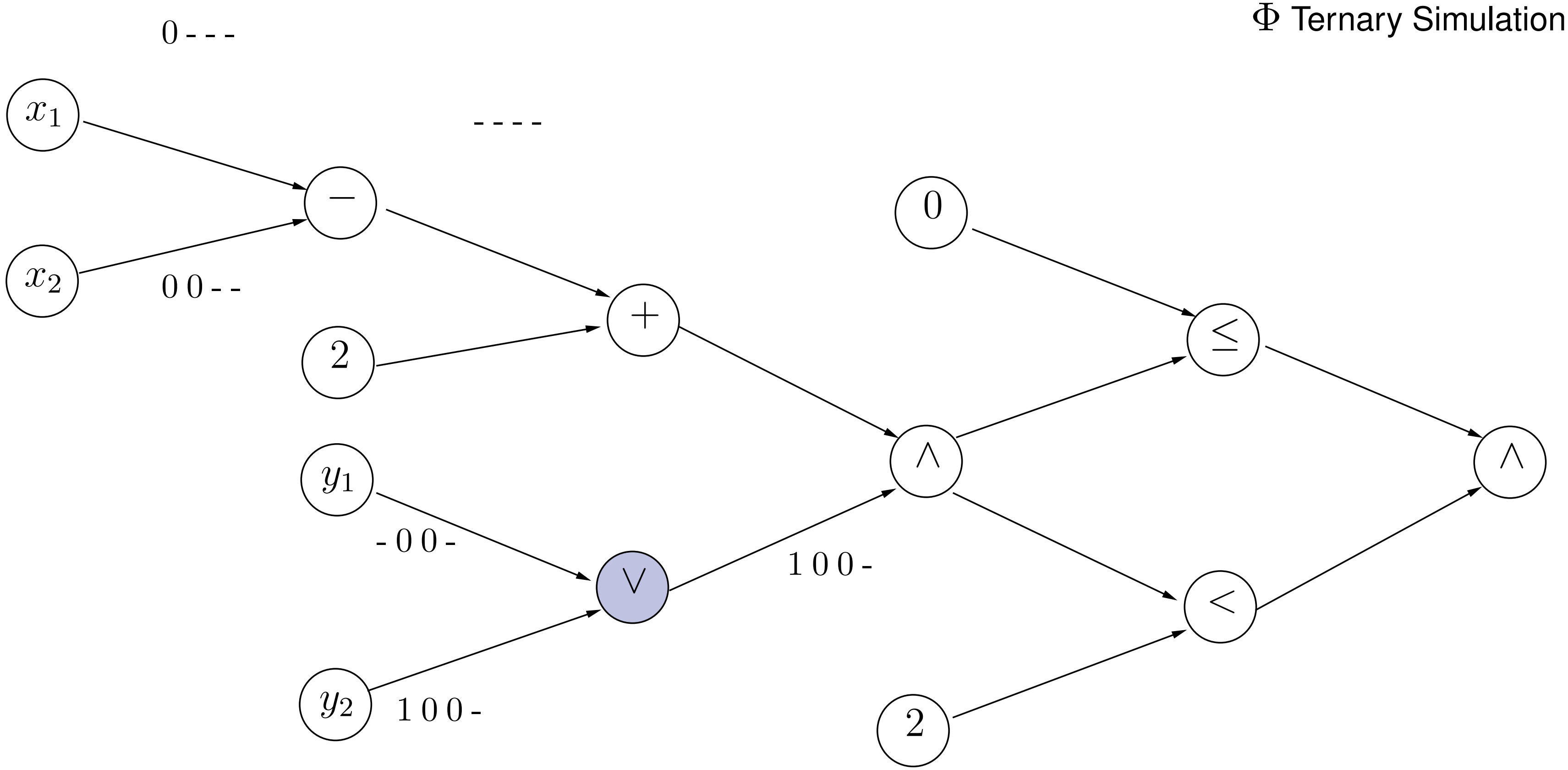

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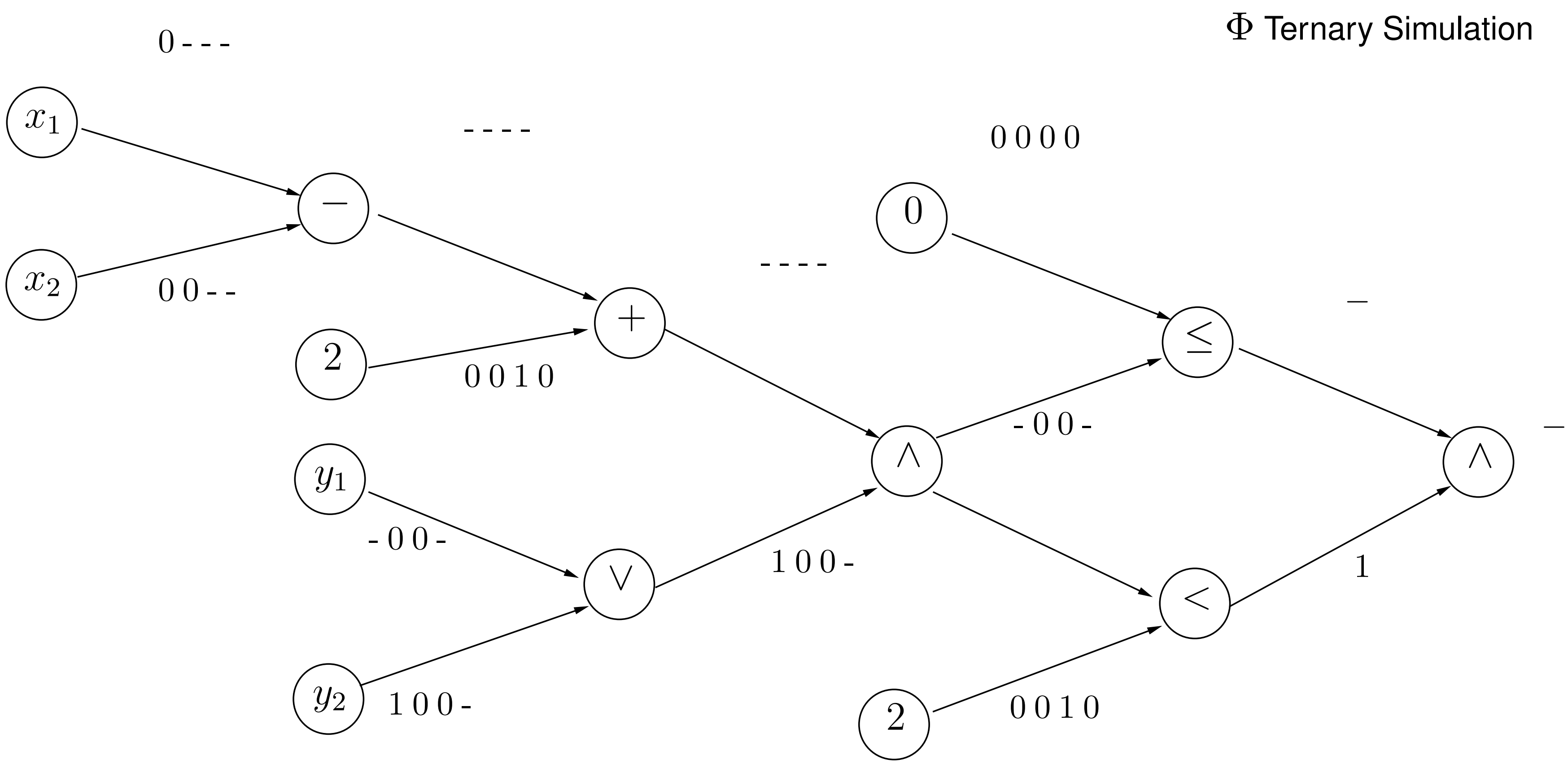

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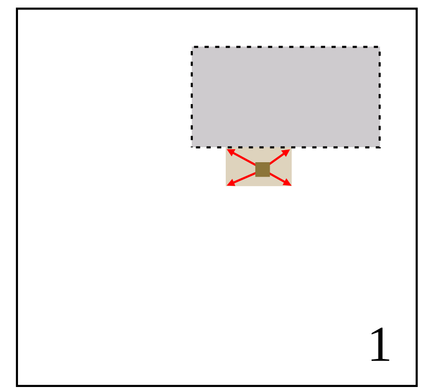

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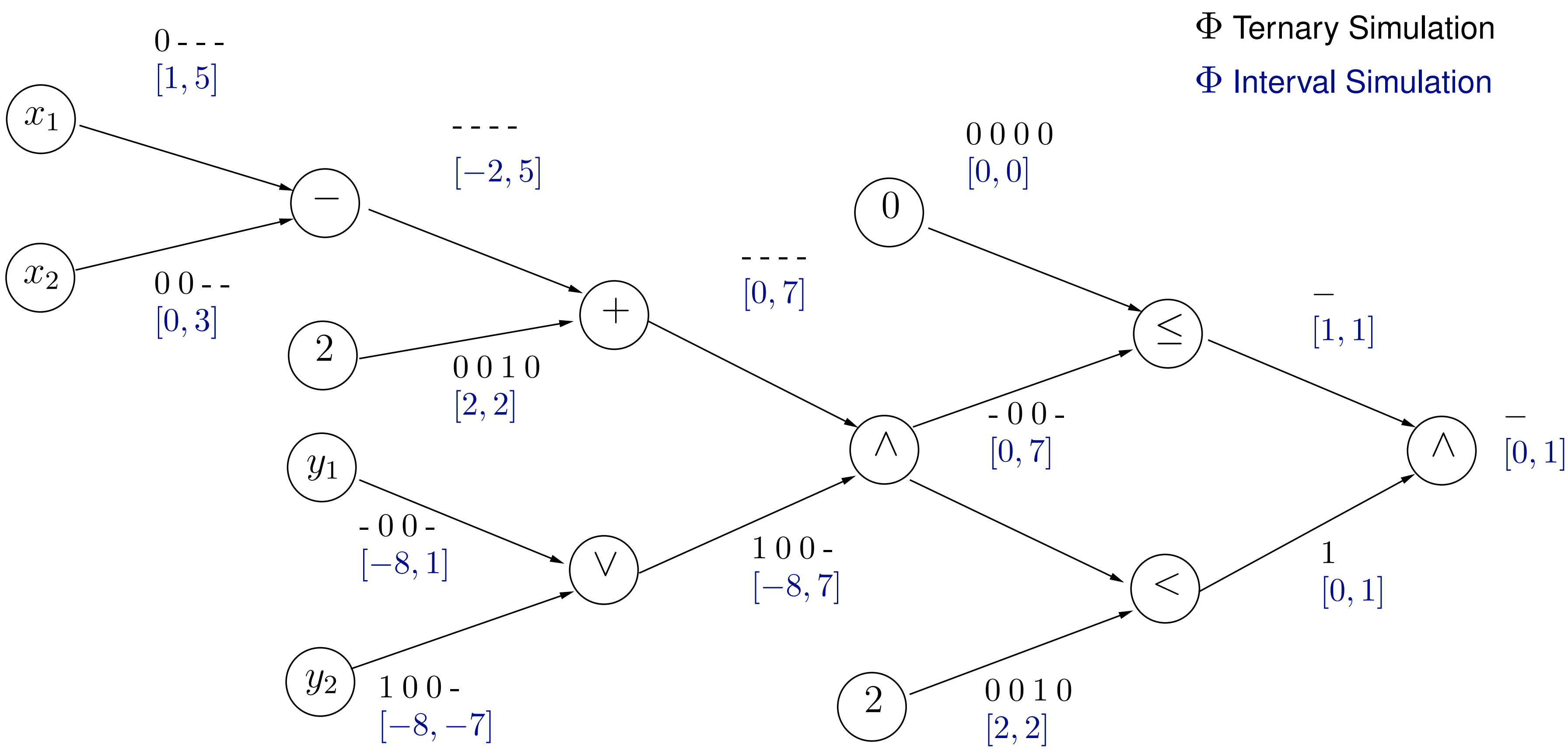
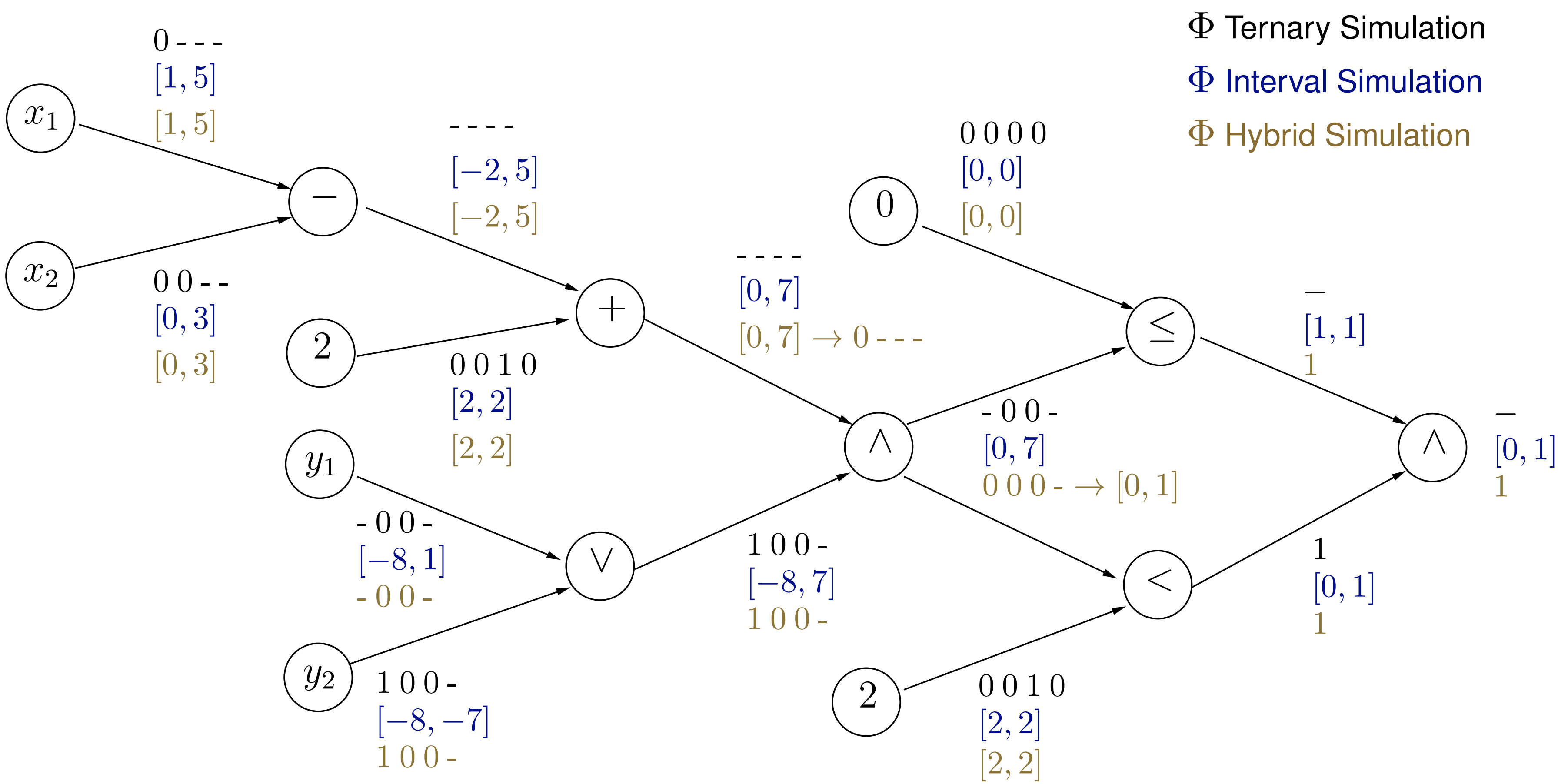
Interval Simulation

Let $\hat{a} := (1 \leq x_1 \leq 5) \wedge (0 \leq x_2 \leq 3) \wedge (y_1 \in \text{-00-}) \wedge (y_2 \in \text{100-})$

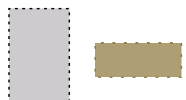
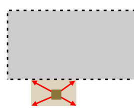


Hybrid Simulation

Let $\hat{a} := (1 \leq x_1 \leq 5) \wedge (0 \leq x_2 \leq 3) \wedge (y_1 \in \text{-00-}) \wedge (y_2 \in 100\text{-})$



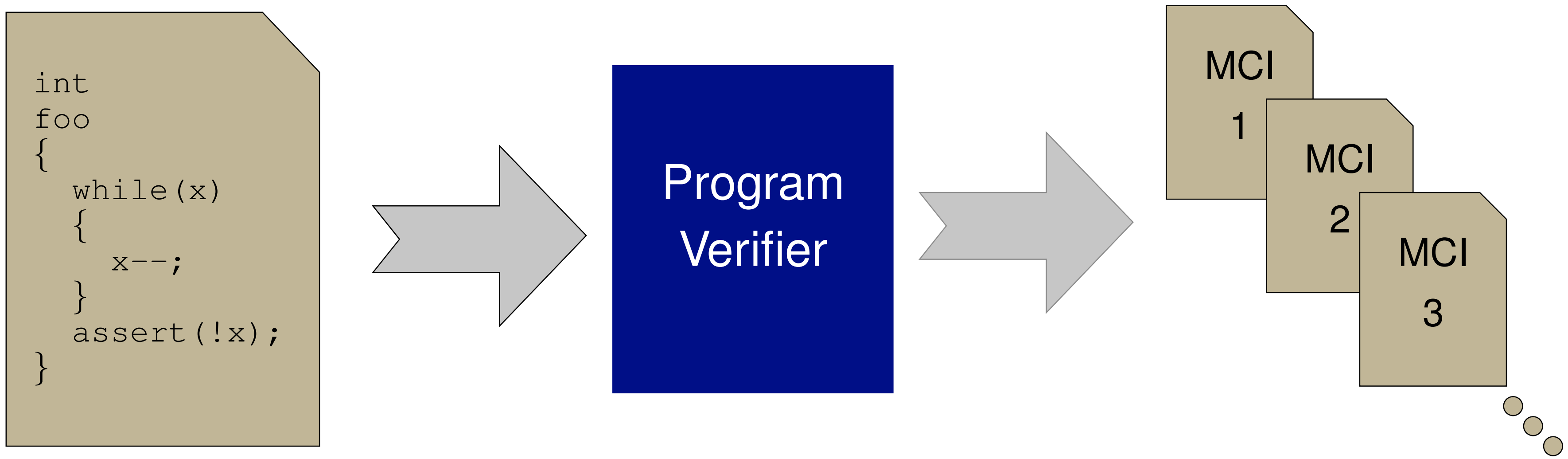
Property Directed Reachability for QF_BV

	Original Formulation	Polytopes [Welp13]	Hybrid
Atomic Reasoning Unit 	Boolean Cubes	Polytopes	Boolean Cubes and Polytopes
Expansion of Proof Obligations 	Ternary Simulation	Interval Simulation	Hybrid Simulation
Strengths	logic	arithmetic	arithmetic logic
Weaknesses	arithmetic	logic	-

Outline

1. Introduction
2. QF_BV Property Directed Reachability
3. Mixed Type Atomic Reasoning Units
4. **Experimental Results**
5. Summary

Experimental Setup



Benchmark Sets

Bitvector set of SV-Comp [Beye12]

```
int
foo(int n)
{
  int x = 1;
  while(1)
  {
    x += 2*n;
    assert(x);
  }
}
```

Mostly Logic Invariants

InvGen-Benchmarks [Gupt09]

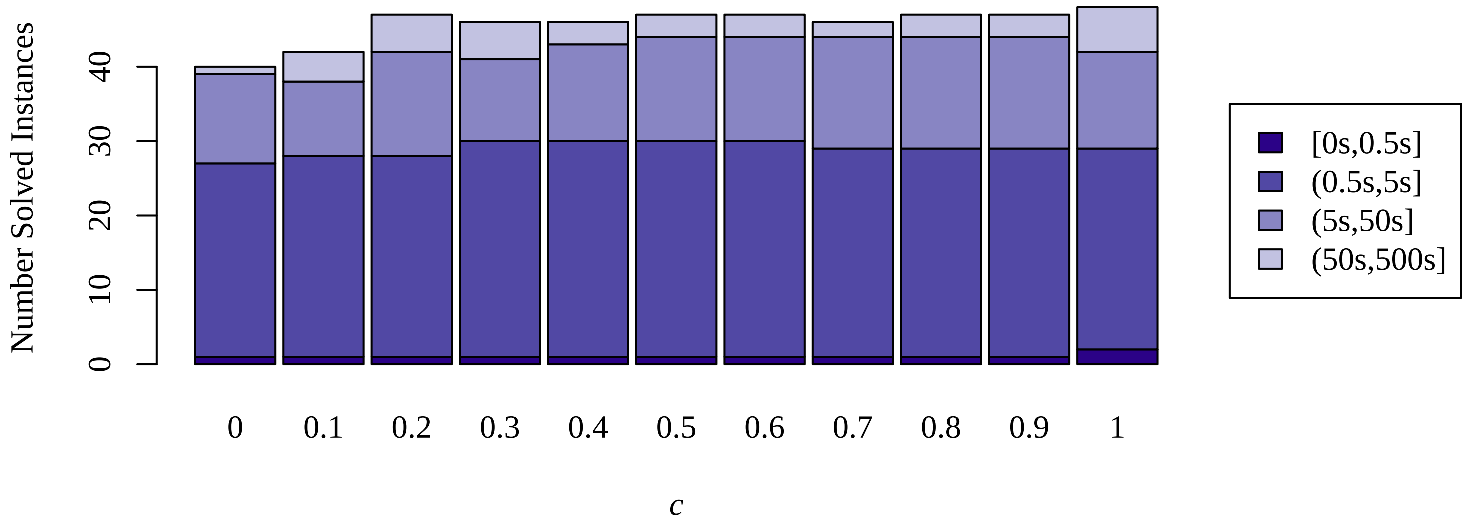
```
int
foo(int n)
{
  int x = 0;
  assume(n>=0);
  while(x < n)
  {
    x--;
  }
  assert(x <= n);
}
```

Mostly Arithmetic Invariants

Overall Performance

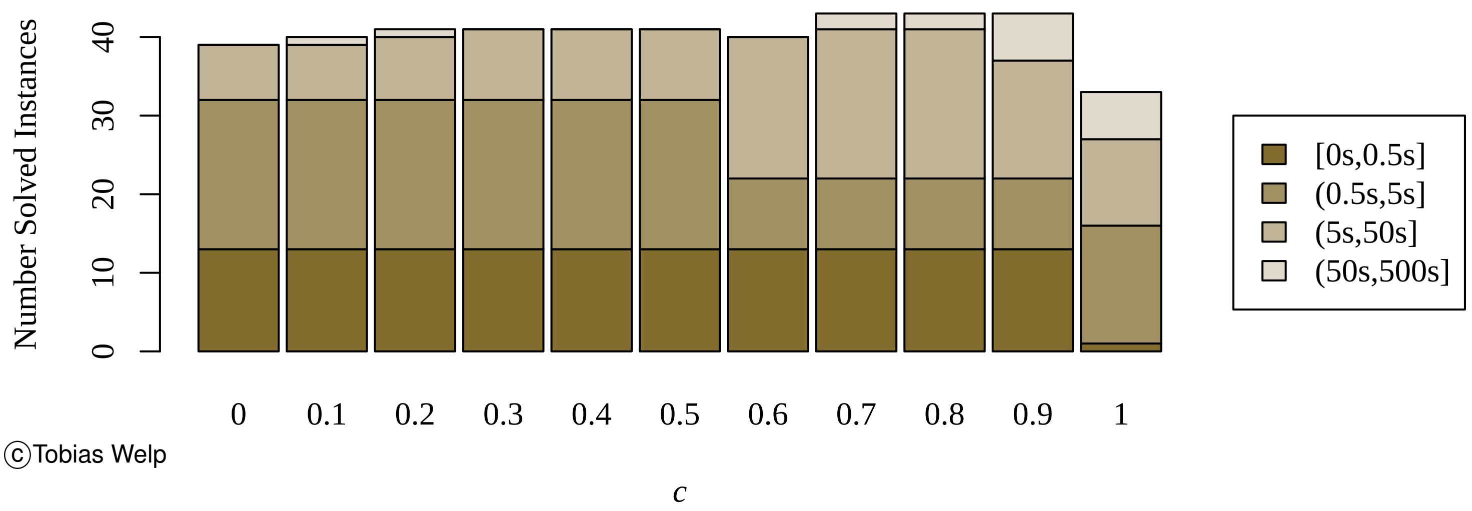
SV-COMP

[Beye12]



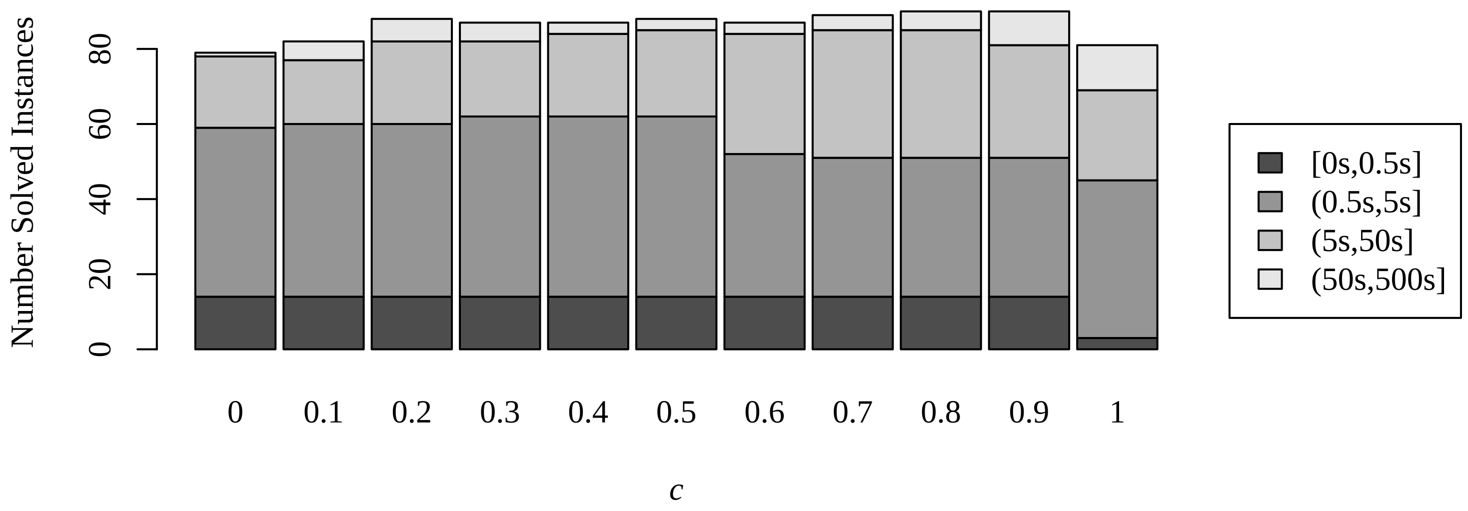
InvGen

[Gupt09]



Overall Performance

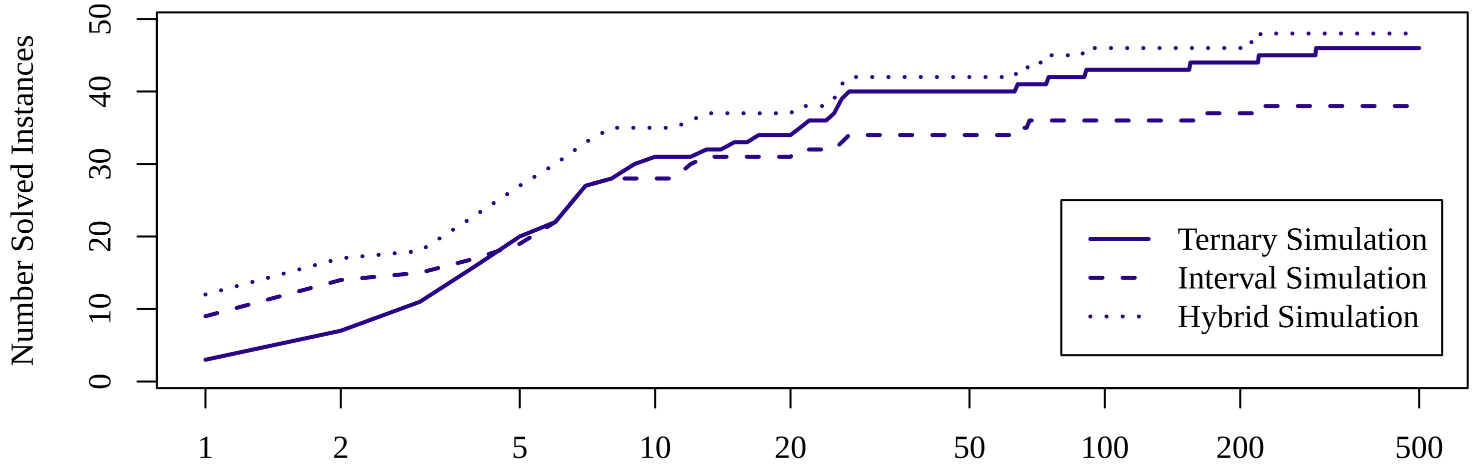
All



Impact of Simulation Type

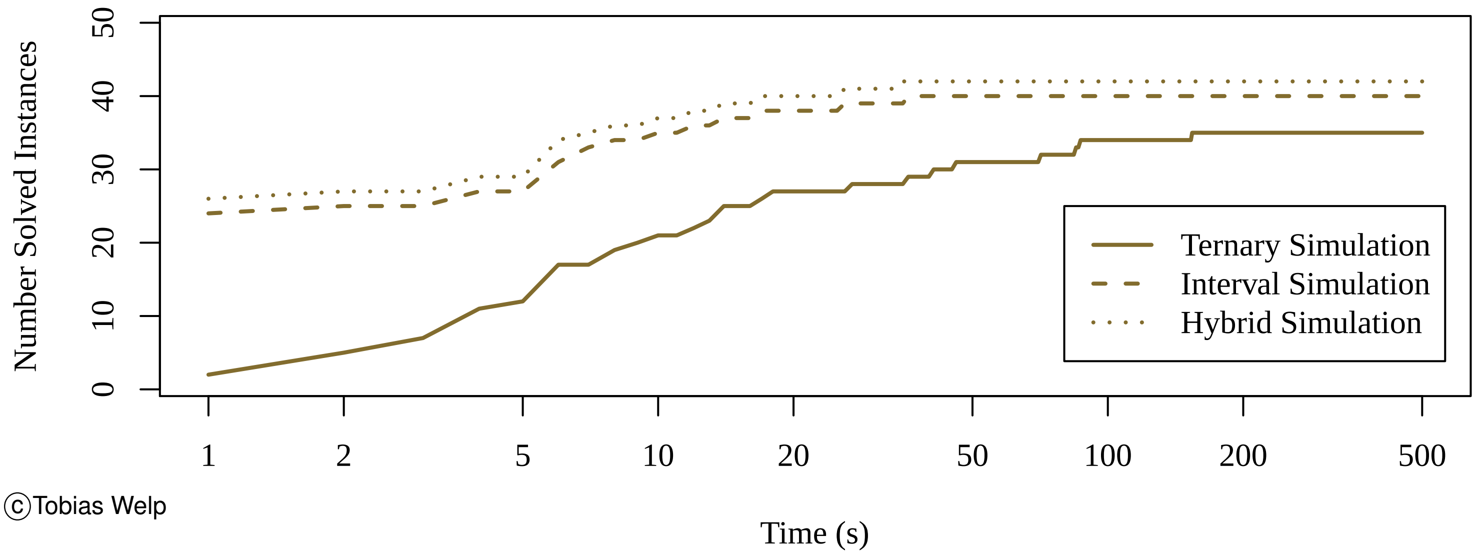
SV-COMP

[Beye12]



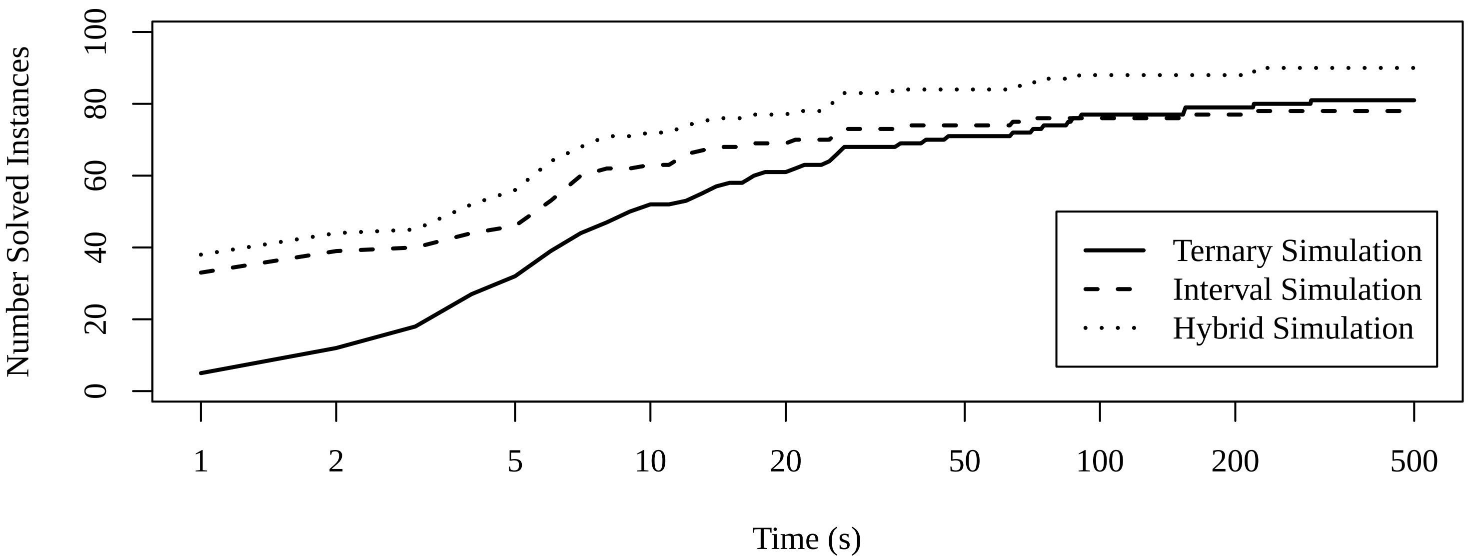
InvGen

[Gupt09]

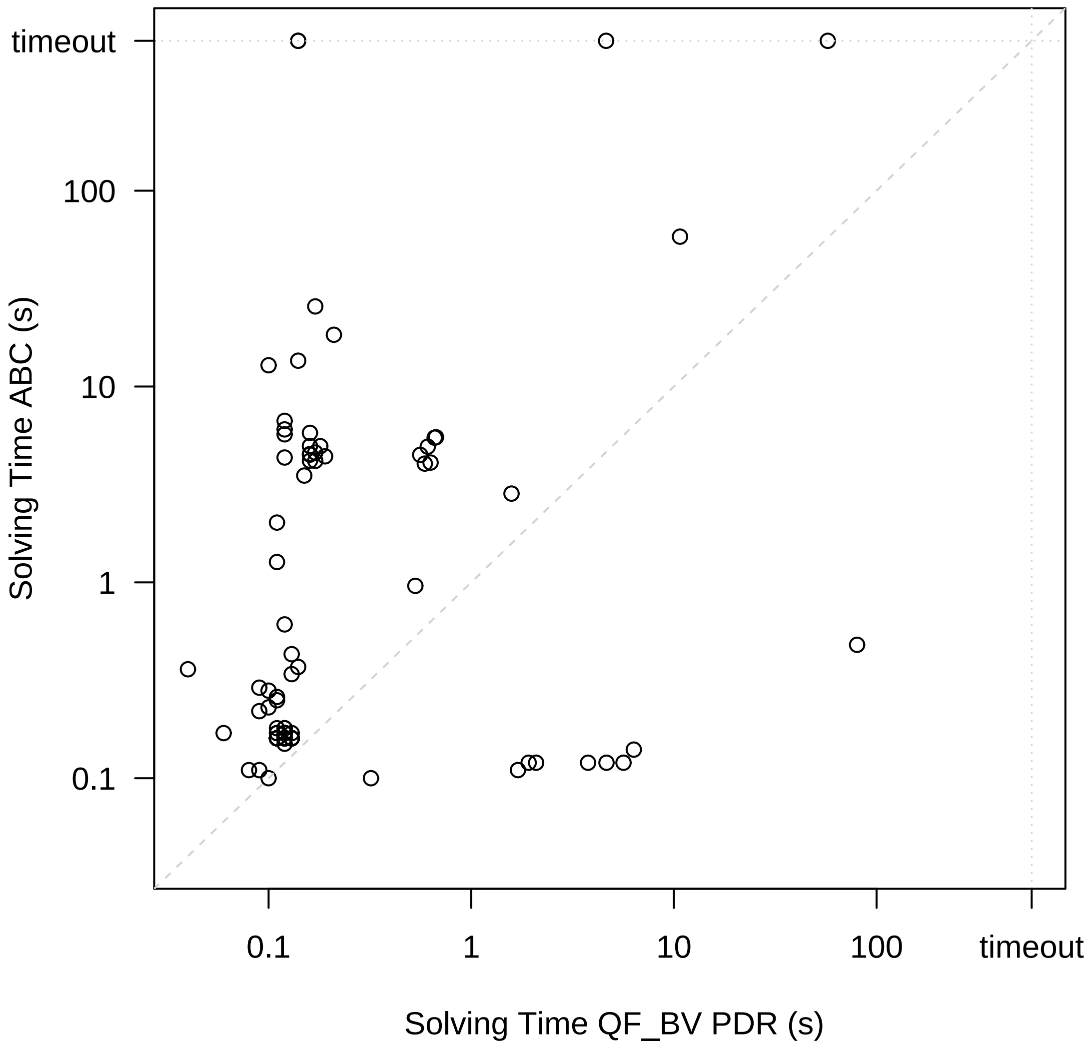


Impact of Simulation Type

All



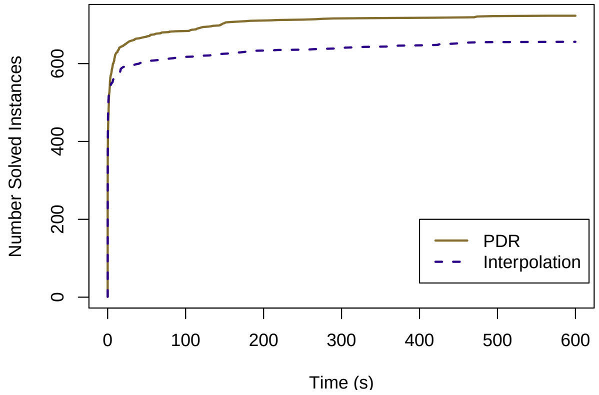
Comparison vs ABC PDR



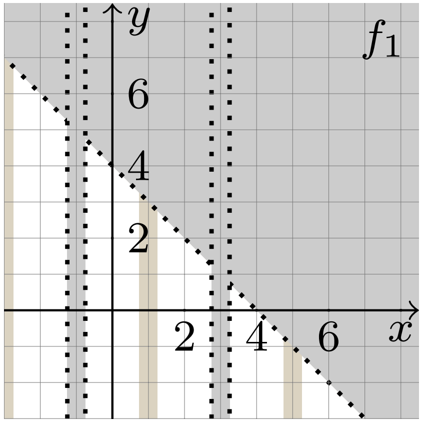
Outline

1. Introduction
2. Property Directed Reachability
3. Generalization of PDR to QF_BV
4. Experimental Results
5. **Summary**

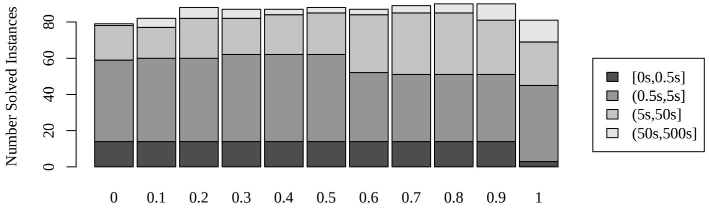
Summary



- PDR is an efficient algorithm for solving model checking problems.



- PDR with Boolean cubes performances poorly with arithmetic invariants.
- PDR with polytopes performances poorly with bit-level invariants.



- The hybrid formulation outperforms the pure versions.

Thank you!

for your

attention

twelp@berkeley.edu

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