HIE-Block Latency Insertion Method for Fast Transient Simulation of Nonuniform Multiconductor Transmission Lines

Takahiro Takasaki
Dept. of Systems Eng., Graduate School of Eng., Shizuoka University

Tadatoshi Sekine
Dept. of Mechanical Eng., Shizuoka University

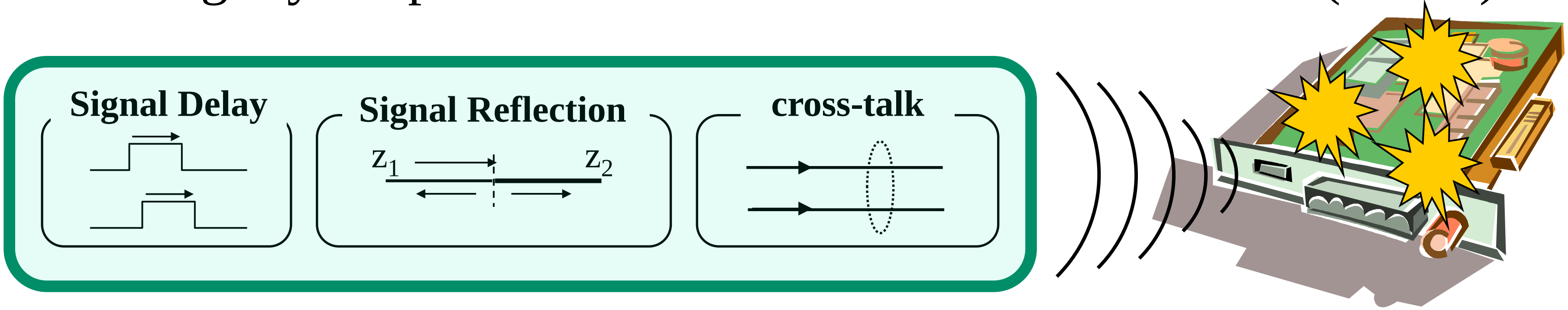
Hideki Asai
Nanovision Research Division, Research Institute of Electronics, Shizuoka University

Outline

- Introduction
- Nonuniform MTLs
- Basic LIM & Block-LIM
- HIE-block-LIM
- Numerical Results
- Conclusion

Introduction

- High-density and high-frequency electronic circuits have been designed
  - Tightly coupled Multiconductor Transmission Lines (MTLs)



- Estimating those effects in a design stage
  - Circuit-based modeling and simulation techniques are adequate in terms of accuracy and efficiency

- SPICE-like simulators
  - Their algorithms with matrix operations are not efficient in transient simulation of a large network

Introduction

- **L**atency **I**nsertion **M**ethod (**LIM**)
  - Fast simulation technique
  - It is based on an explicit leapfrog scheme
- Block-LIM
  - Fast simulation technique
  - It has been proposed to extend LIM to simulations of the MTLs

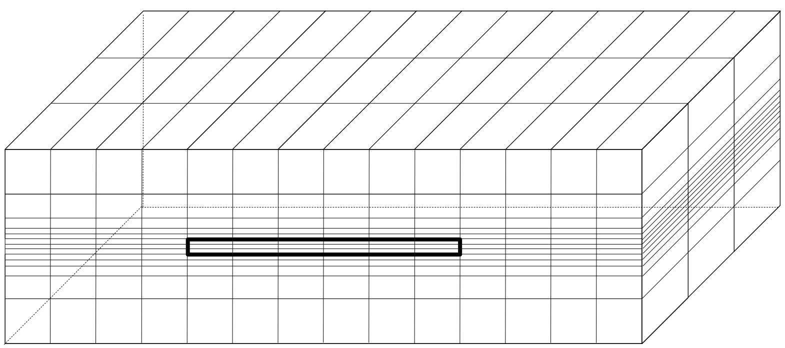
Since both methods are based on the explicit scheme, those have a strict numerical stability condition

Time step size becomes small if there exist small reactive elements in the circuit

Introduction

- **H**ybrid **I**mplicit-**E**xplicit (**HIE**) Finite-**D**ifference **t**ime **D**omain (**FDTD**) Method
  - To alleviate the CFL condition and generate a weakly conditionally stable electromagnetic field solver
  - An implicit difference method is used with respect to the differential equations associated with the one direction

Geometry of the cavity-backed slot

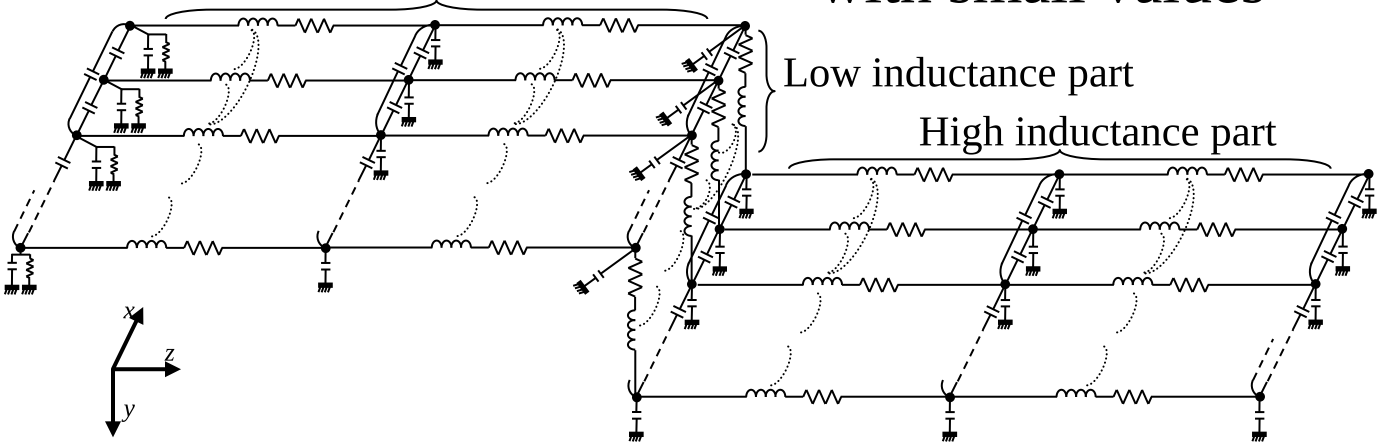


Its model modeled by square mesh

Because the implicit method is unconditionally stable, a time step size depends only on large cell sizes in the horizontal directions

Introduction

- In this presentation….
  - Propose HIE-block-LIM for nonuniform MTLs
    Combing the block LIM and HIE formulation
  - Apply the implicit scheme to a local area
    Includes inductance elements with small values



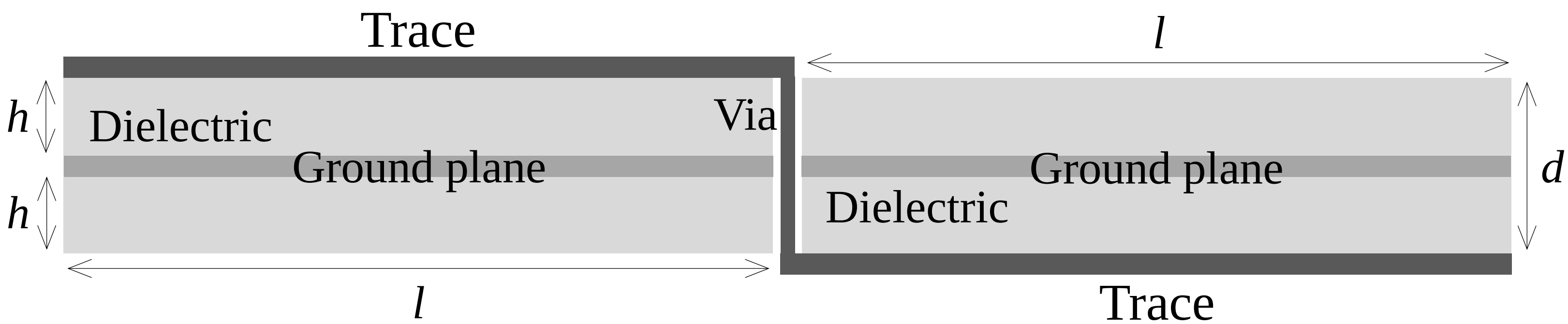
Nonuniform MTLs

  - By using the proposed method

→ The time step size can be chosen depending only on relatively-larger inductance component

Nonuniform MTLs

- Circuit Structure



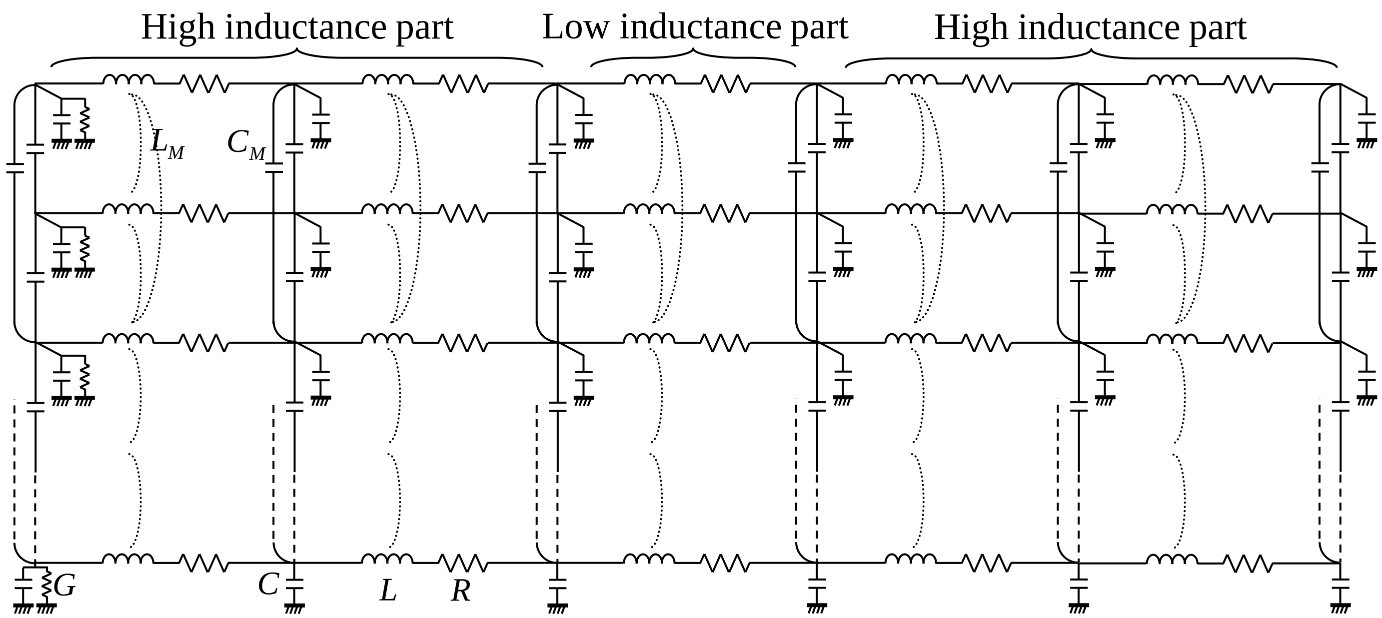
The cross-section view of the nonuniform MTLs

l : Length of each trace in the top and bottom layers [mm]
d : Length of vertical via [mm]
h : Thickness of each dielectric layer [mm]
ε_r : Relative permittivity of the dielectrics is 4.2

  - Three metal layers: two traces, ground plane
  - Dielectric: it is filled between the metal layers
  - Vertical via: provide a signal path from top to bottom

Nonuniform MTLs

- Circuit Structure

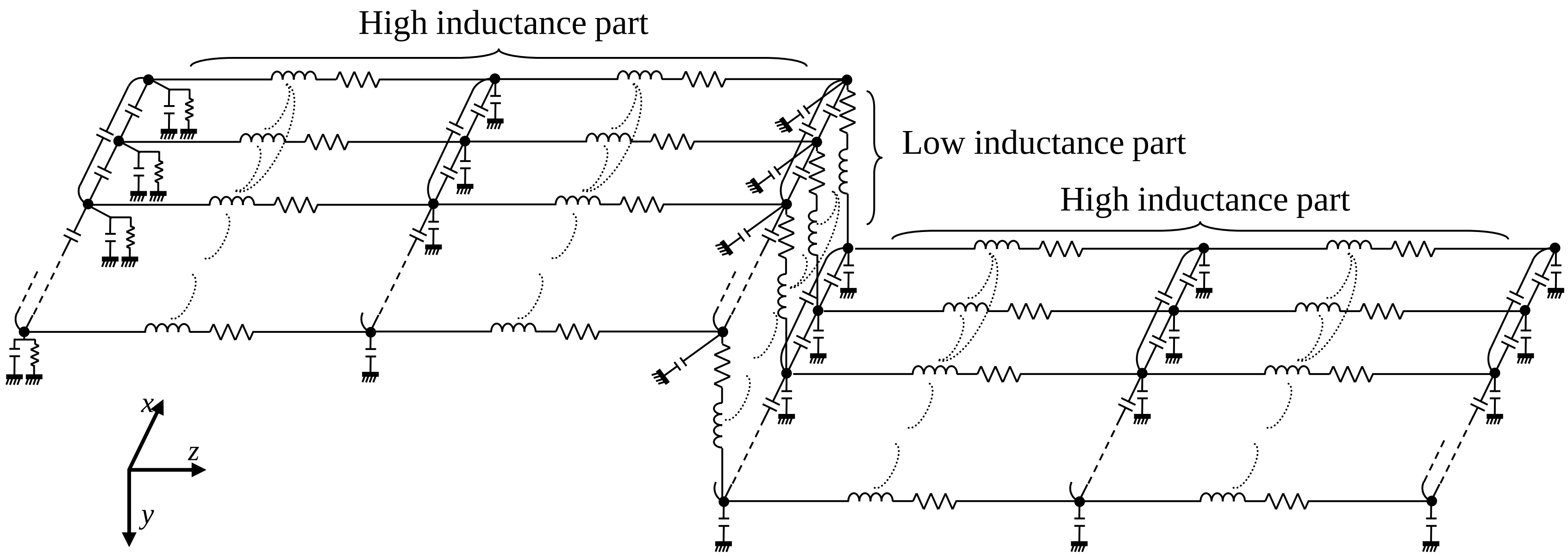


The equivalent circuit model of the entire MTL

R : Resistance
L : Inductance
C : Capacitance
G : Conductance
L_M : Mutual inductance
C_M : Mutual capacitance

Nonuniform MTLs

- Circuit Structure



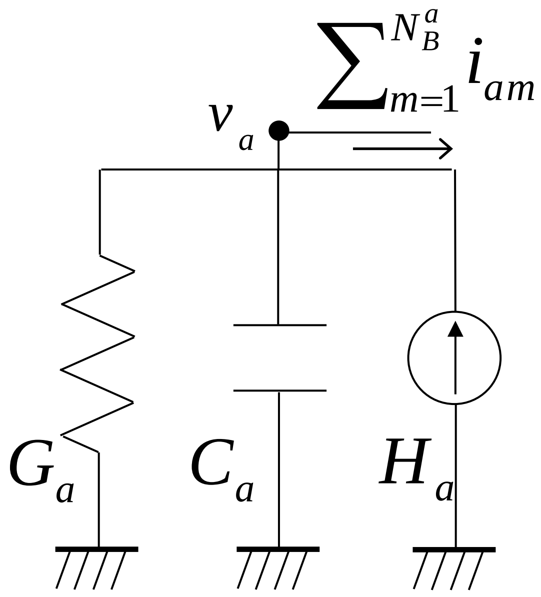
Three-dimensional equivalent circuit of the entire MTL

The high and low inductance parts mean the subcircuits with large and small inductance components, respectively, and correspond to the traces and vias.

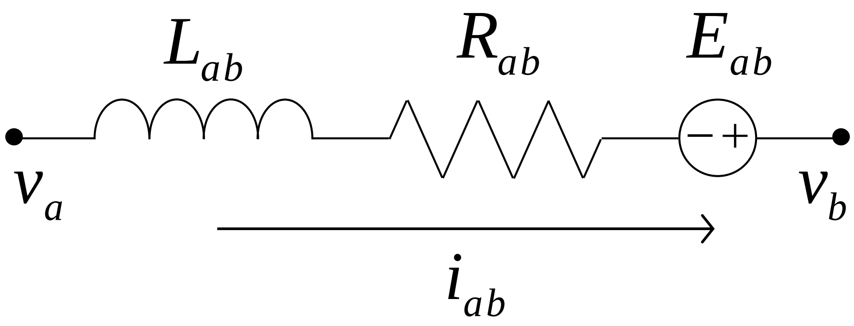
The low inductance part is allocated in the y-direction, and the high inductance parts are allocated in the z-direction

Basic LIM & block-LIM

- Basic LIM Formulation
  - LIM assumes that the circuit to be analyzed is composed of **node topology** and **branch topology**



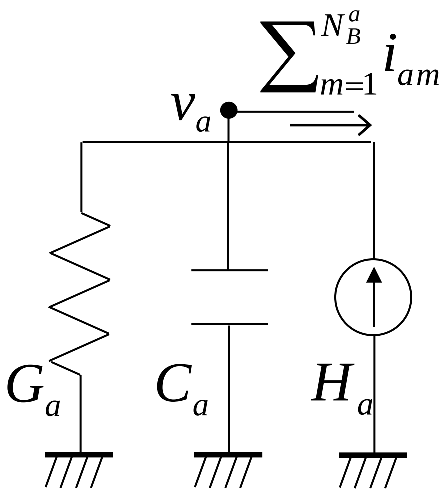
Node topology



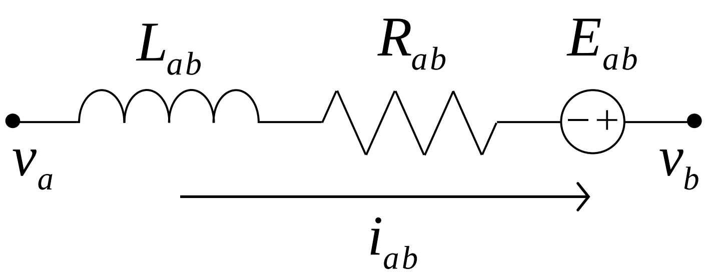
Branch topology

Basic LIM & block-LIM

- Basic LIM Formulation
  - Kirchhoff's current law and Kirchhoff's voltage law are applied to the node topology and the branch topology



$$C_a \frac{dv_a}{dt} + Gv_a - H_a = -\sum_{b=1}^{N_B^a} i_{ab} \qquad (1)$$



$$L_{ab} \frac{di_{ab}}{dt} + Ri_{ab} - E_{ab} = v_a - v_b \qquad (2)$$

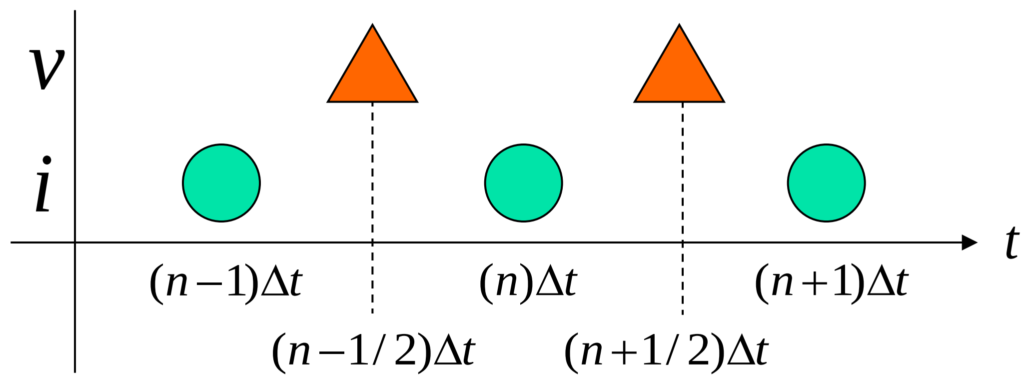
Basic LIM & block-LIM

- Basic LIM Formulation

$$v_a^{n+\frac{1}{2}} = \frac{C_a}{C_a + \Delta t G_a} v_a^{n-\frac{1}{2}} + \frac{\Delta t}{C_a + \Delta t G_a}\left(-\sum_{b=1}^{N_B^a} i_{ab}^{n} + H_a^{n} \right) \quad (3)$$

$$i_{ab}^{n+1} = \frac{L_{ab} - \Delta t R_{ab}}{L_{ab}} i_{ab}^{n} + \frac{\Delta t}{L_{ab}}\left(v_a^{n+\frac{1}{2}} - v_b^{n+\frac{1}{2}} + E_{ab}^{n+\frac{1}{2}} \right) \quad (4)$$

Arranged alternately in every half time step



Matrix solver

→ **Substitution operation**

Basic LIM & block-LIM

- Basic LIM Formulation
  - Since LIM is based on the explicit leapfrog scheme, the maximum time step size $\Delta t_{\max}$ used in LIM is limited by

The number of the nodes in the networks

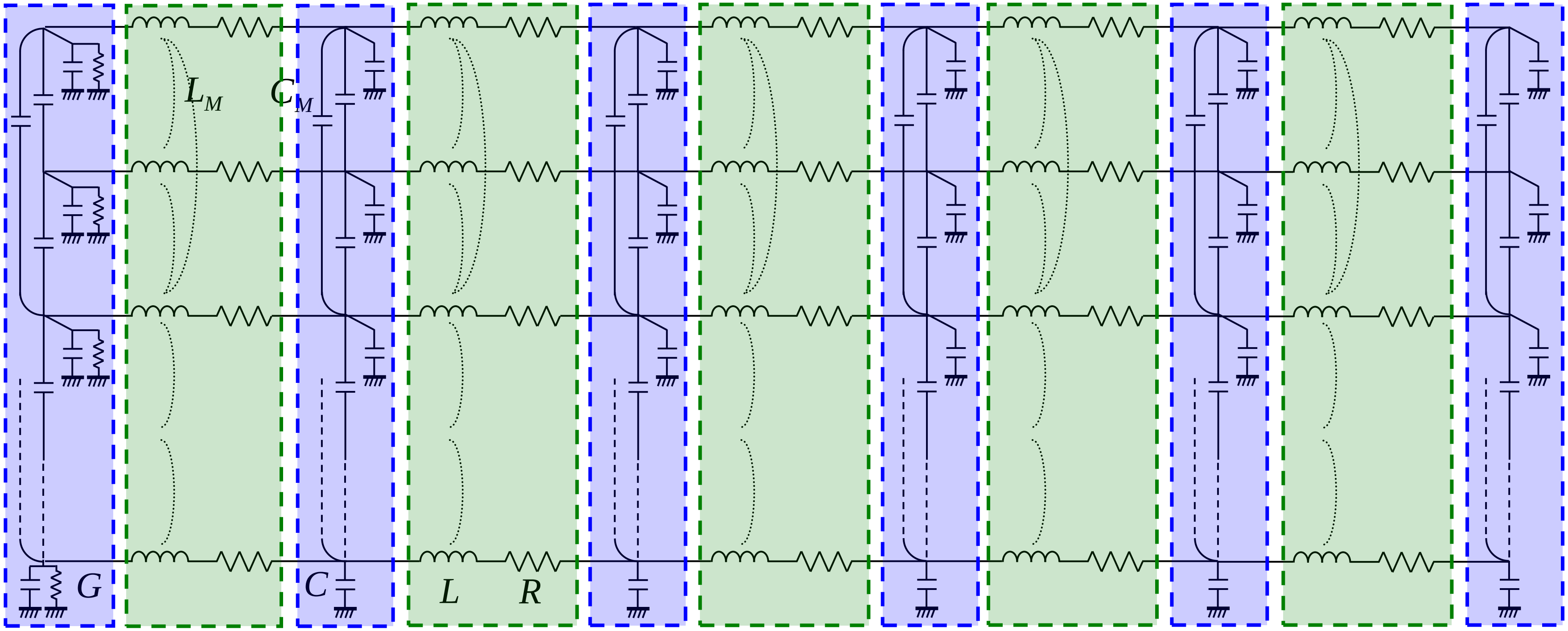
$$\Delta t_{\max} \le \sqrt{2}\min_{a=1}^{N_N}\left(\sqrt{\frac{C_a}{N_B^a}\min_{b=1}^{N_B^a}\left(L_{ab}\right)}\right) \quad (5)$$

The number of the branches connected to the node *a*

If the small reactive elements exist in the circuit, the efficiency of the basic LIM is reduced significantly

Basic LIM & block-LIM

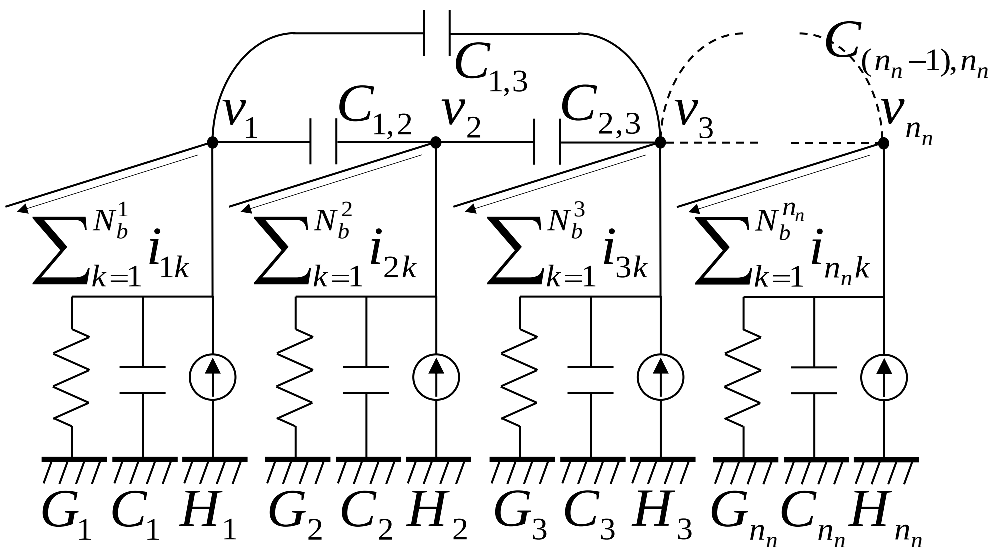
- Block-LIM Formulation
  - Block-LIM is suitable for a fast transient analysis of a tightly coupled circuit constructed by connecting a number of **node blocks** and **branch blocks**



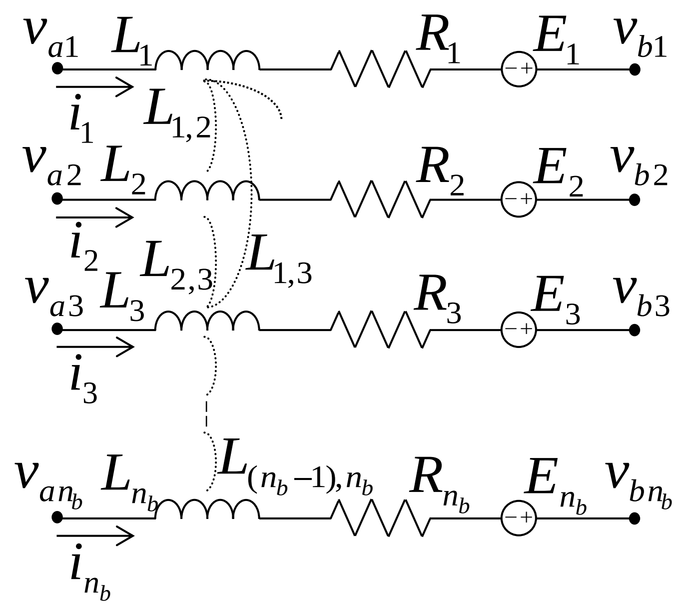
The equivalent circuit model of the entire MTL

Basic LIM & block-LIM

- Block-LIM Formulation
  - Node block is composed of node topologies coupled with each other by mutual capacitances
  - Branch block is composed of branch topologies coupled with each other by mutual inductances



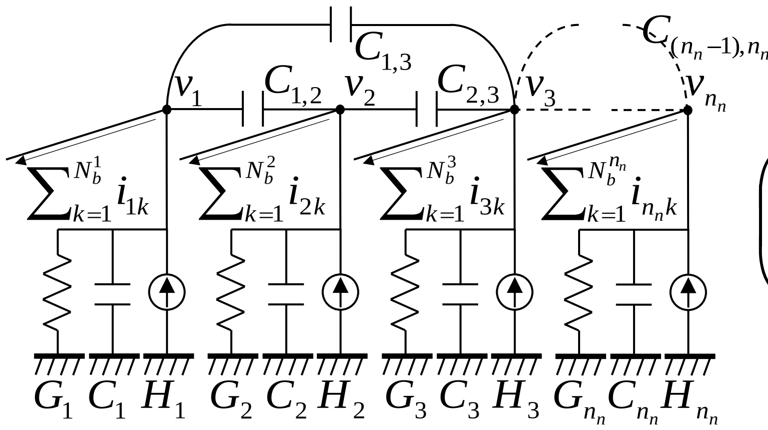
Node block



Branch block

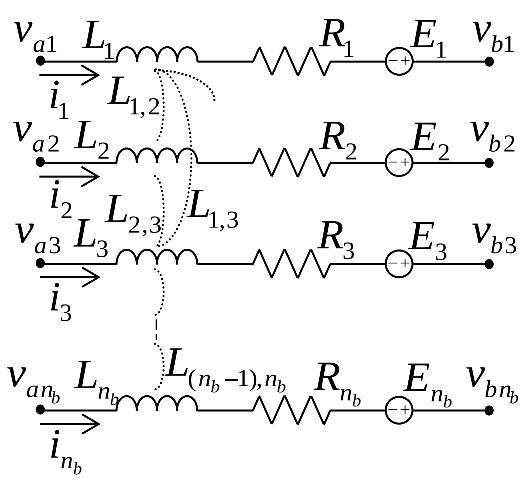
Basic LIM & block-LIM

- Block-LIM Formulation



Node block

$$\left(\frac{1}{\Delta t}\mathbf{C}_a + \mathbf{G}_a\right)\mathbf{v}_a^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_a\mathbf{v}_a^{n-\frac{1}{2}} - \mathbf{i}_a^n + \mathbf{h}_a^n \quad (6)$$

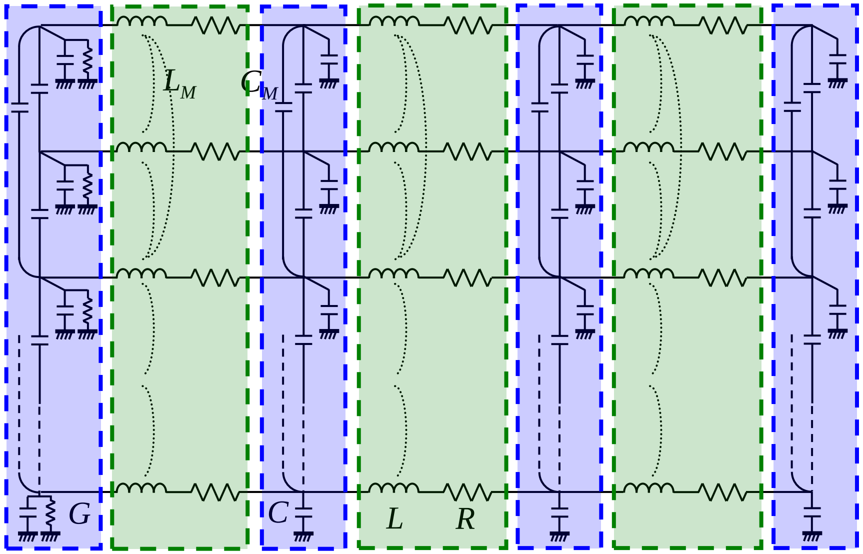


Branch block

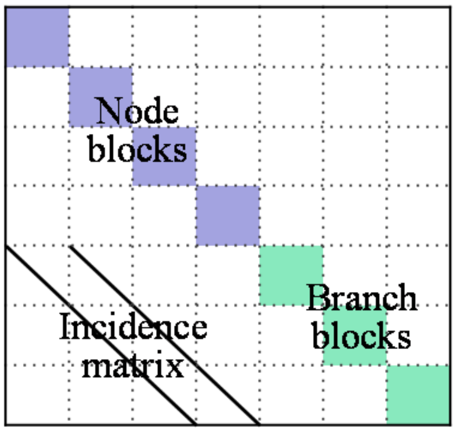
$$\frac{1}{\Delta t}\mathbf{L}_{ab}\mathbf{i}_{ab}^{n+1} = \left(\frac{1}{\Delta t}\mathbf{L}_{ab} - \mathbf{R}_{ab}\right)\mathbf{i}_{ab}^n + \mathbf{v}_{ab}^{n+\frac{1}{2}} + \mathbf{e}_{ab}^{n+\frac{1}{2}} \quad (7)$$

Basic LIM & block-LIM

- Block-LIM Formulation



The structure of the coefficient matrix of unknown voltages and currents



$$\begin{bmatrix} \mathbf{v}_a^{n+\frac{1}{2}} \\ \mathbf{i}_{ab}^{n+1} \end{bmatrix} = \mathbf{b}$$

The coefficient matrix structure of unknown voltages and currents

$$\left(\frac{1}{\Delta t}\mathbf{C}_a + \mathbf{G}_a\right)\mathbf{v}_a^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_a\mathbf{v}_a^{n-\frac{1}{2}} - \mathbf{i}_a^n + \mathbf{h}_a^n \qquad (6)$$

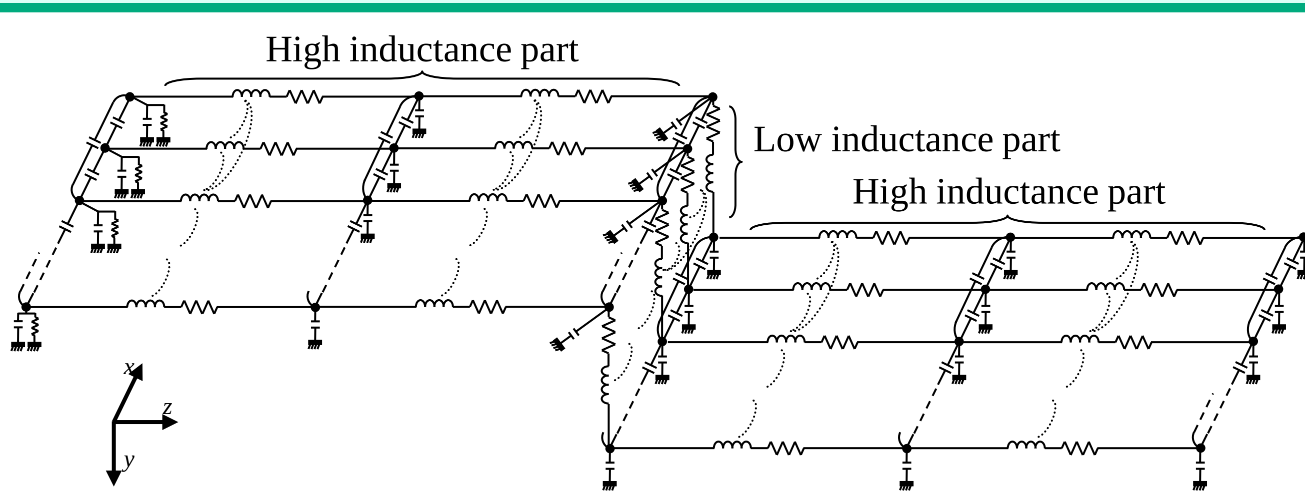
$$\frac{1}{\Delta t}\mathbf{L}_{ab}\mathbf{i}_{ab}^{n+1} = \left(\frac{1}{\Delta t}\mathbf{L}_{ab} - \mathbf{R}_{ab}\right)\mathbf{i}_{ab}^n + \mathbf{v}_{ab}^{n+\frac{1}{2}} + \mathbf{e}_{ab}^{n+\frac{1}{2}} \qquad (7)$$

Basic LIM & block-LIM

- Block-LIM Formulation
  - Since block-LIM is based on the explicit leapfrog scheme, the maximum time step size $\Delta t_{\max}$ used in LIM is limited by

$$\Delta t_{\max} \le \sqrt{2} \min_{a=1}^{N_N} \left(\sqrt{\frac{C_a}{N_B^a} \min_{b=1}^{N_B^a} \left(L_{ab} \right)} \right) \qquad (5)$$

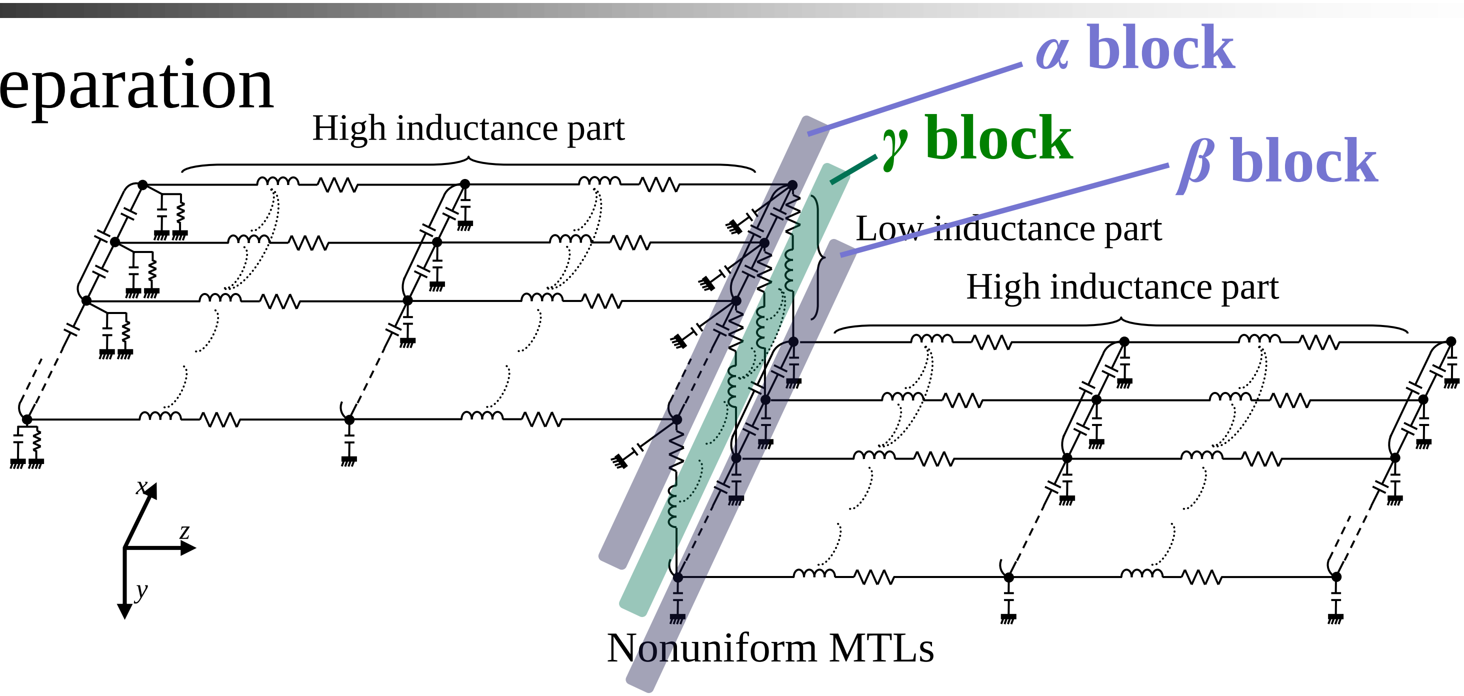
The maximum time step size of block LIM is restricted due directly to the small inductance in the low inductance part



Nonuniform MTLs

HIE-block-LIM

- Preparation



We assume that the number of the signal traces is 128, and the traces and vias are divided into 5 branch blocks and 6 node blocks

The branch block related to the current variables in the y-direction is defined as γ block

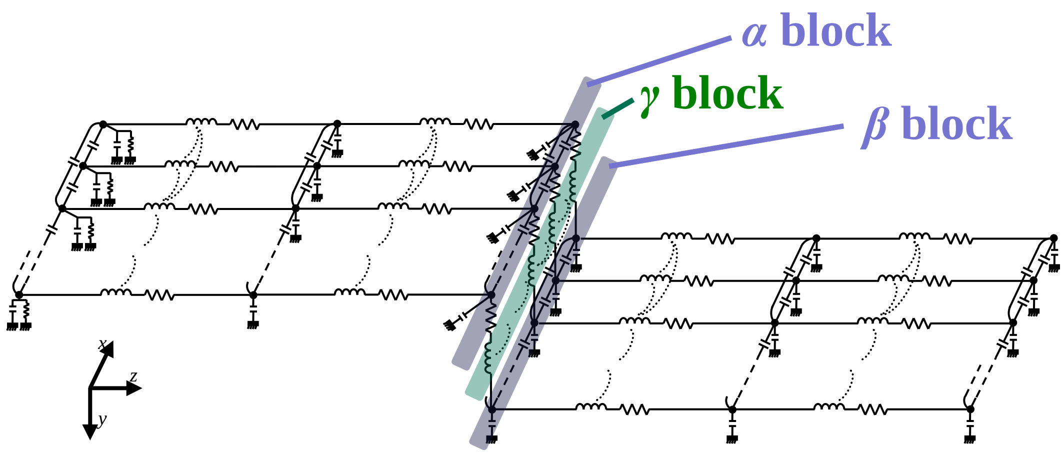
The node blocks connected to the γ block are defined as α and β blocks

HIE-block-LIM

- Formulation

$$\mathbf{C}_\alpha \frac{d}{dt}\mathbf{v}_\alpha + \mathbf{G}_\alpha \mathbf{v}_\alpha - \mathbf{h}_\alpha = -\mathbf{i}_\alpha$$

$$\mathbf{C}_\beta \frac{d}{dt}\mathbf{v}_\beta + \mathbf{G}_\beta \mathbf{v}_\beta - \mathbf{h}_\beta = -\mathbf{i}_\beta \qquad (8)$$



Nonuniform MTLs

The current variables are divided into $\mathbf{i}_\gamma$ and $\tilde{\mathbf{i}}_a$

$$\mathbf{C}_\alpha \frac{d}{dt}\mathbf{v}_\alpha + \mathbf{G}_\alpha \mathbf{v}_\alpha - \mathbf{h}_\alpha = -\left(\tilde{\mathbf{i}}_\alpha + \mathbf{i}_\gamma\right)$$

$$\mathbf{C}_\beta \frac{d}{dt}\mathbf{v}_\beta + \mathbf{G}_\beta \mathbf{v}_\beta - \mathbf{h}_\beta = -\left(\tilde{\mathbf{i}}_\beta + \mathbf{i}_\gamma\right) \qquad (9)$$

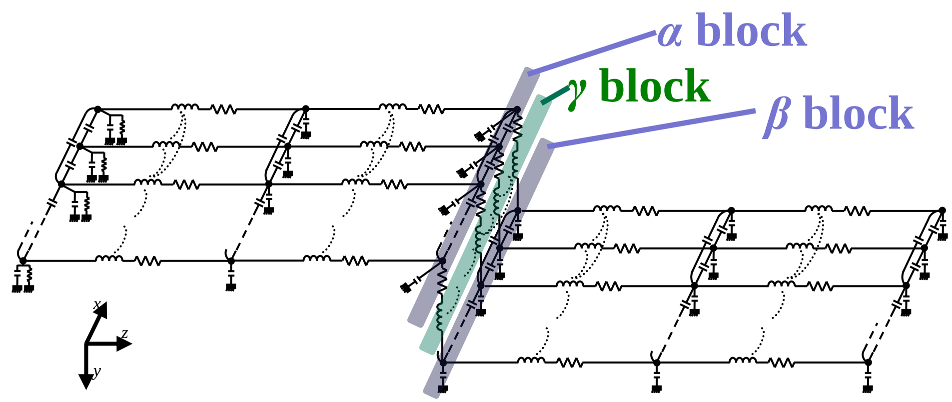
$$\left(\begin{array}{l} \mathbf{i}_a = \tilde{\mathbf{i}}_a + \mathbf{i}_\gamma \\ a = \alpha, \beta \end{array}\right)$$

HIE-block-LIM

- Formulation

$$\mathbf{C}_\alpha \frac{d}{dt}\mathbf{v}_\alpha + \mathbf{G}_\alpha \mathbf{v}_\alpha - \mathbf{h}_\alpha = -\left(\tilde{\mathbf{i}}_\alpha + \mathbf{i}_\gamma\right)$$

$$\mathbf{C}_\beta \frac{d}{dt}\mathbf{v}_\beta + \mathbf{G}_\beta \mathbf{v}_\beta - \mathbf{h}_\beta = -\left(\tilde{\mathbf{i}}_\beta + \mathbf{i}_\gamma\right) \quad (9)$$



Nonuniform MTLs

By applying implicit scheme and leapfrog scheme

The current variables in branch block γ

The others variables

$$\left(\frac{1}{\Delta t}\mathbf{C}_\alpha + \mathbf{G}_\alpha\right)\mathbf{v}_\alpha^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\alpha \mathbf{v}_\alpha^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\alpha^{n} - \mathbf{i}_\gamma^{n+\frac{1}{2}} + \mathbf{h}_\alpha^{n}$$

$$\left(\frac{1}{\Delta t}\mathbf{C}_\beta + \mathbf{G}_\beta\right)\mathbf{v}_\beta^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\beta \mathbf{v}_\beta^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\beta^{n} - \mathbf{i}_\gamma^{n+\frac{1}{2}} + \mathbf{h}_\beta^{n} \quad (10)$$

The node voltages and the branch currents in the block circuit are arranged at the same time step

Simultaneous equations must be solved

HIE-block-LIM

- Formulation
  - T… ted from (10)

Fi… hod to

… γ

Because $\mathbf{L}_\gamma$ and $\mathbf{R}_\gamma$ are the relatively small block submatrices, the calculation cost to derive $\mathbf{i}_\gamma^{n+\frac{1}{2}}$ is not large

$$\mathbf{i}_\gamma^{n+\frac{1}{2}} = \left(\frac{1}{\Delta t}\mathbf{L}_\gamma + \mathbf{R}_\gamma\right)^{-1}\left(\mathbf{v}_\alpha^{n+\frac{1}{2}} - \mathbf{v}_\beta^{n+\frac{1}{2}} + \frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{e}_\gamma^{n+\frac{1}{2}}\right) \quad (11)$$

Second: (11) is substituted into (10) and rearranging the equations

$$\left(\frac{1}{\Delta t}\mathbf{C}_\alpha + \mathbf{G}_\alpha\right)\mathbf{v}_\alpha^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\alpha \mathbf{v}_\alpha^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\alpha^{n} - \mathbf{i}_\gamma^{n+\frac{1}{2}} + \mathbf{h}_\alpha^{n}$$

$$\left(\frac{1}{\Delta t}\mathbf{C}_\beta + \mathbf{G}_\beta\right)\mathbf{v}_\beta^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\beta \mathbf{v}_\beta^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\beta^{n} + \mathbf{i}_\gamma^{n+\frac{1}{2}} + \mathbf{h}_\beta^{n}$$

(10)

HIE-block-LIM

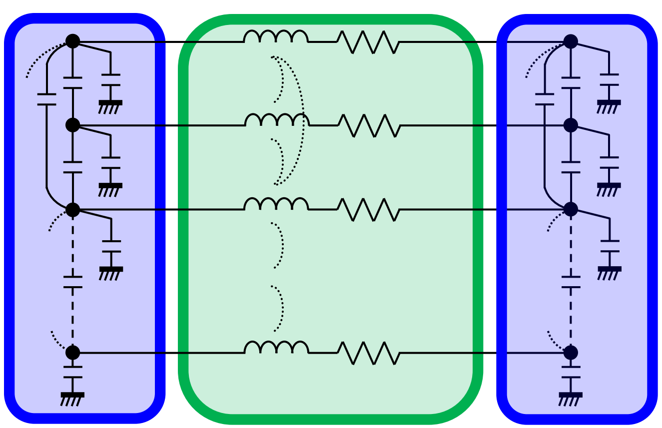
Formulation

$$\mathbf{T}_\alpha \mathbf{v}_\alpha^{n+\frac{1}{2}} - \hat{\mathbf{K}} \mathbf{v}_\beta^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\alpha \mathbf{v}_\alpha^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\alpha^{n} - \hat{\mathbf{K}} \frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{s}_\alpha$$

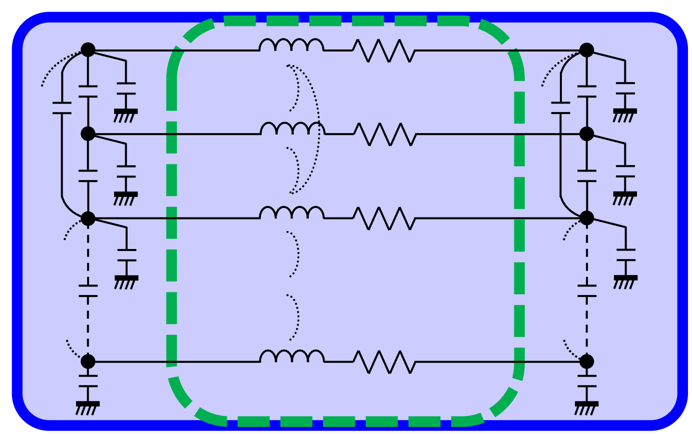
$$\mathbf{T}_\beta \mathbf{v}_\beta^{n+\frac{1}{2}} - \hat{\mathbf{K}} \mathbf{v}_\alpha^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\beta \mathbf{v}_\beta^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\beta^{n} - \hat{\mathbf{K}} \frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{s}_\beta \qquad (12)$$

$$\left(\begin{array}{l} \hat{\mathbf{K}} \equiv \left(\frac{1}{\Delta t}\mathbf{L}_\gamma + \mathbf{R}_\gamma\right)^{-1} \\ \mathbf{T}_a \equiv \frac{1}{\Delta t}\mathbf{C}_a + \mathbf{G}_a + \hat{\mathbf{K}} \\ \mathbf{s}_a \equiv -\hat{\mathbf{K}}\mathbf{e}_\gamma^{n+\frac{1}{2}} + \mathbf{h}_a^{n} \\ a = \alpha, \beta \end{array}\right)$$

- There are $\mathbf{v}_\alpha$ and $\mathbf{v}_\beta$ at the (n+1/2)-th step
- The node blocks α and β are correlated with each other through the branches between them due to the implicit formulation associated with the branch block γ



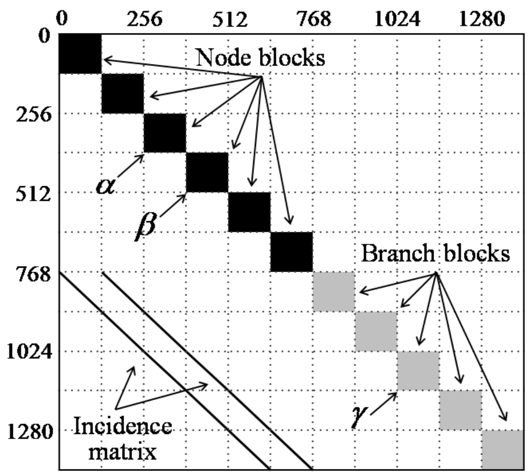
block-LIM



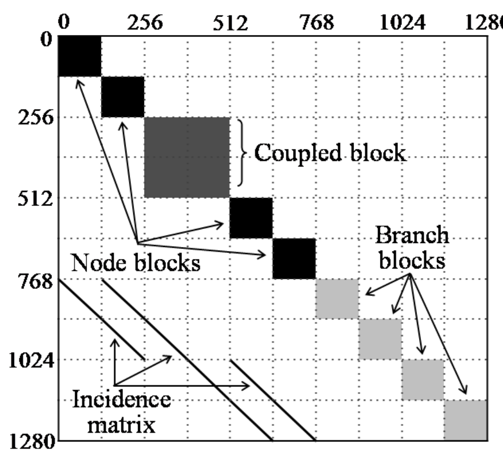
HIE-block-LIM

HIE-block-LIM

Formulation



(a) block LIM



(b) HIE-block-LIM

The structures of the coefficient matrices of the entire circuit equations

- The node blocks connected to the branch block of the low inductance part are united with each other and become the single and locally dense block
- It is defined as **the coupled block circuit**
- The updating formula of the voltage variables in the coupled block circuit

$$\mathbf{T}_\beta \mathbf{v}_\beta^{n+\frac{1}{2}} - \hat{\mathbf{K}} \mathbf{v}_\alpha^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\beta \mathbf{v}_\beta^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\beta^{\,n} - \hat{\mathbf{K}}\frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{s}_\beta$$

$$\mathbf{T}_\alpha \mathbf{v}_\alpha^{n+\frac{1}{2}} - \hat{\mathbf{K}} \mathbf{v}_\beta^{n+\frac{1}{2}} = \frac{1}{\Delta t}\mathbf{C}_\alpha \mathbf{v}_\alpha^{n-\frac{1}{2}} - \tilde{\mathbf{i}}_\alpha^{\,n} - \hat{\mathbf{K}}\frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{s}_\alpha \quad (12)$$

$$\begin{bmatrix} \mathbf{T}_\alpha & -\hat{\mathbf{K}} \\ -\hat{\mathbf{K}} & \mathbf{T}_\beta \end{bmatrix} \begin{bmatrix} \mathbf{v}_\alpha^{n+\frac{1}{2}} \\ \mathbf{v}_\beta^{n+\frac{1}{2}} \end{bmatrix} = \frac{1}{\Delta t}\begin{bmatrix} \mathbf{C}_\alpha & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_\beta \end{bmatrix} \begin{bmatrix} \mathbf{v}_\alpha^{n-\frac{1}{2}} \\ \mathbf{v}_\beta^{n-\frac{1}{2}} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{i}}_\alpha^{\,n} \\ \tilde{\mathbf{i}}_\beta^{\,n} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{K}}\frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} \\ \hat{\mathbf{K}}\frac{1}{\Delta t}\mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} \end{bmatrix} + \begin{bmatrix} \mathbf{s}_\alpha \\ \mathbf{s}_\beta \end{bmatrix} \quad (13)$$

HIE-block-LIM

Formulation

The coupled block circuit

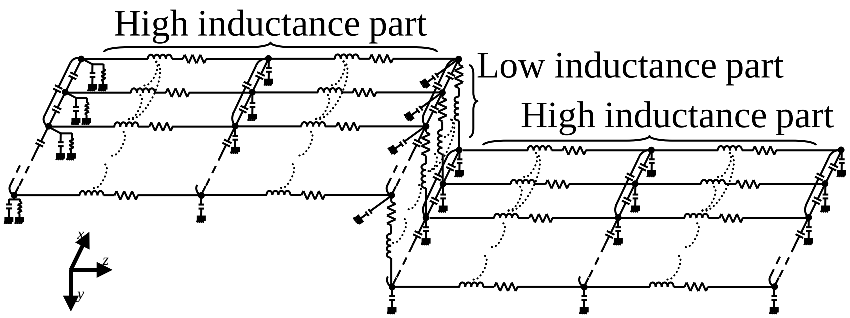
$$\begin{bmatrix} \mathbf{T}_\alpha & -\hat{\mathbf{K}} \\ -\hat{\mathbf{K}} & \mathbf{T}_\beta \end{bmatrix} \begin{bmatrix} \mathbf{v}_\alpha^{n+\frac{1}{2}} \\ \mathbf{v}_\beta^{n+\frac{1}{2}} \end{bmatrix} = \frac{1}{\Delta t} \begin{bmatrix} \mathbf{C}_\alpha & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_\beta \end{bmatrix} \begin{bmatrix} \mathbf{v}_\alpha^{n-\frac{1}{2}} \\ \mathbf{v}_\beta^{n-\frac{1}{2}} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{i}}_\alpha^{n} \\ \tilde{\mathbf{i}}_\beta^{n} \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{K}} \frac{1}{\Delta t} \mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} \\ \hat{\mathbf{K}} \frac{1}{\Delta t} \mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} \end{bmatrix} + \begin{bmatrix} \mathbf{s}_\alpha \\ \mathbf{s}_\beta \end{bmatrix} \quad (13)$$

$$\mathbf{i}_\gamma^{n+\frac{1}{2}} = \left(\frac{1}{\Delta t} \mathbf{L}_\gamma + \mathbf{R}_\gamma \right)^{-1} \left(\mathbf{v}_\alpha^{n+\frac{1}{2}} - \mathbf{v}_\beta^{n+\frac{1}{2}} + \frac{1}{\Delta t} \mathbf{L}_\gamma \mathbf{i}_\gamma^{n-\frac{1}{2}} + \mathbf{e}_\gamma^{n+\frac{1}{2}} \right) \quad (11)$$

The others block circuit

$$\left(\frac{1}{\Delta t} \mathbf{C}_a + \mathbf{G}_a \right) \mathbf{v}_a^{n+\frac{1}{2}} = \frac{1}{\Delta t} \mathbf{C}_a \mathbf{v}_a^{n-\frac{1}{2}} - \mathbf{i}_a^n + \mathbf{h}_a^n \quad (6)$$

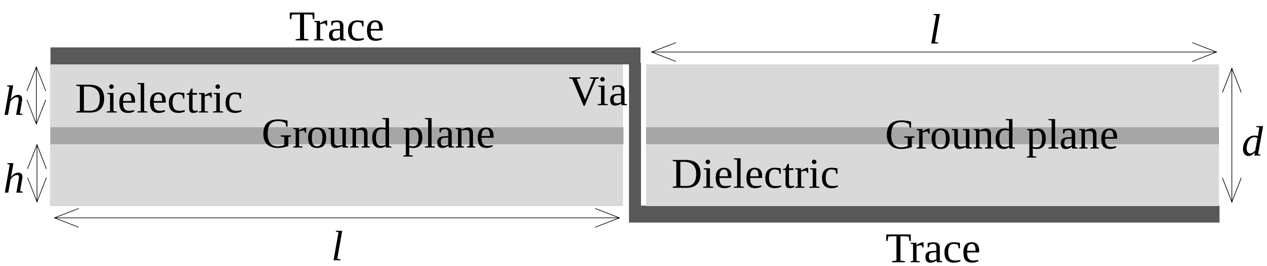
$$\frac{1}{\Delta t} \mathbf{L}_{ab} \mathbf{i}_{ab}^{n+1} = \left(\frac{1}{\Delta t} \mathbf{L}_{ab} - \mathbf{R}_{ab} \right) \mathbf{i}_{ab}^{n} + \mathbf{v}_{ab}^{n+\frac{1}{2}} + \mathbf{e}_{ab}^{n+\frac{1}{2}} \quad (7)$$

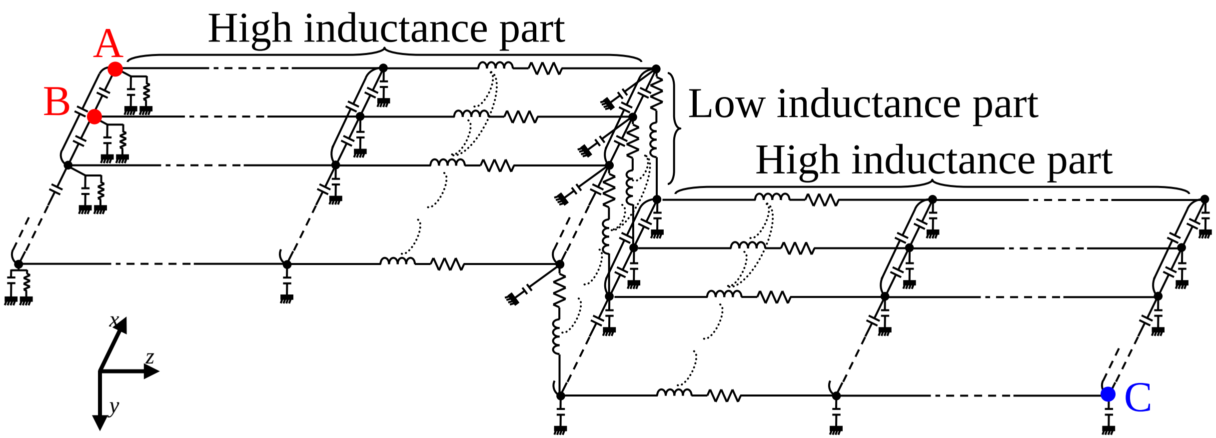


The maximum time step size used in HIE-block LIM depends only on the reactive elements in the high inductance parts

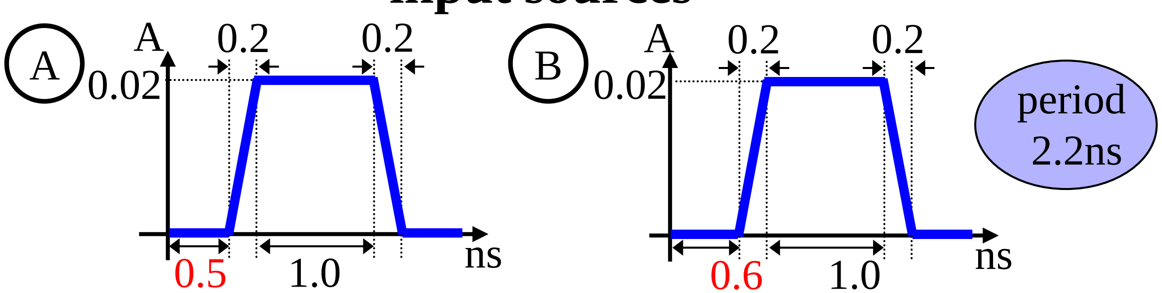
Numerical Results

- Example circuit (Nonuniform MTLs)





input sources

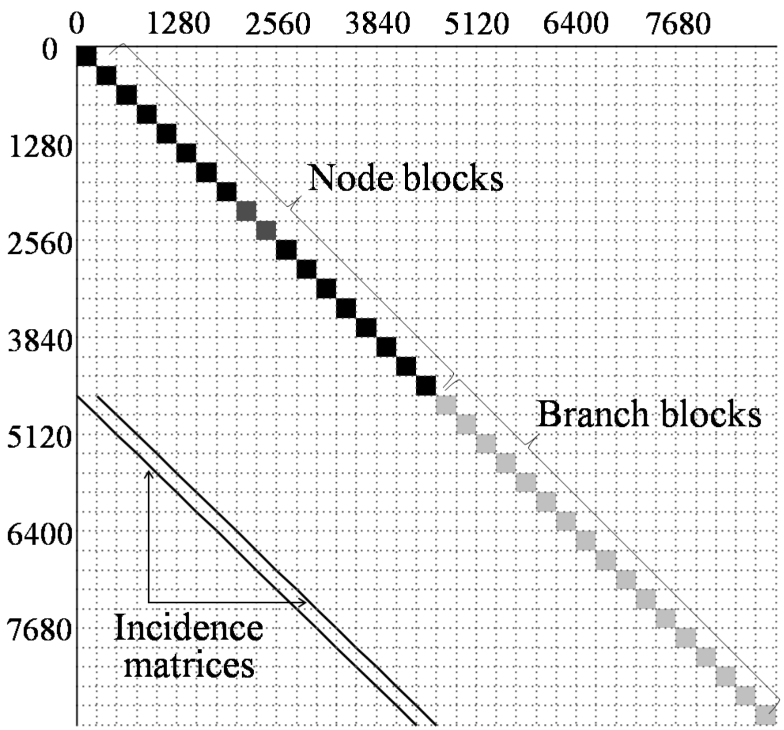


Parameters	
the number of signal traces	256
l	48 [mm]
d	0.221 [mm]
h	0.1016 [mm]
the number of node blocks	18
the number of branch blocks	17
C	0.754 [pF]
C_m	0.543 [fF]
G	0.02 [S]
L_H	0.936 [nH]
R_H	33.1 [mΩ]
L_L	44.0 [pH]
R_L	1.12 [mΩ]
coupling coefficient k	0.015

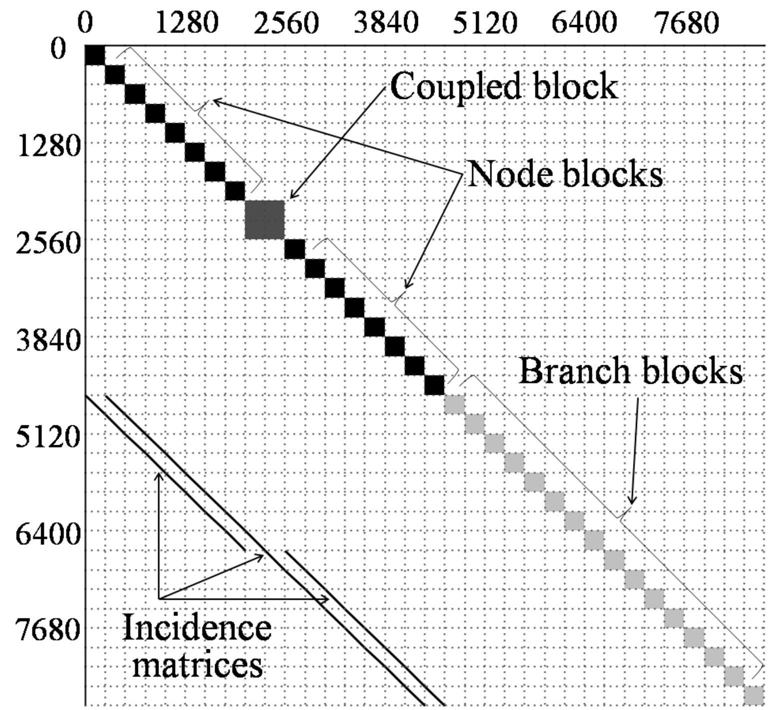
Numerical Results

- Example circuit (Equivalent circuit PDN)

Method	Maximum time step size
block-LIM	4.02 [ps]
HIE-block-LIM	26.6 [ps]



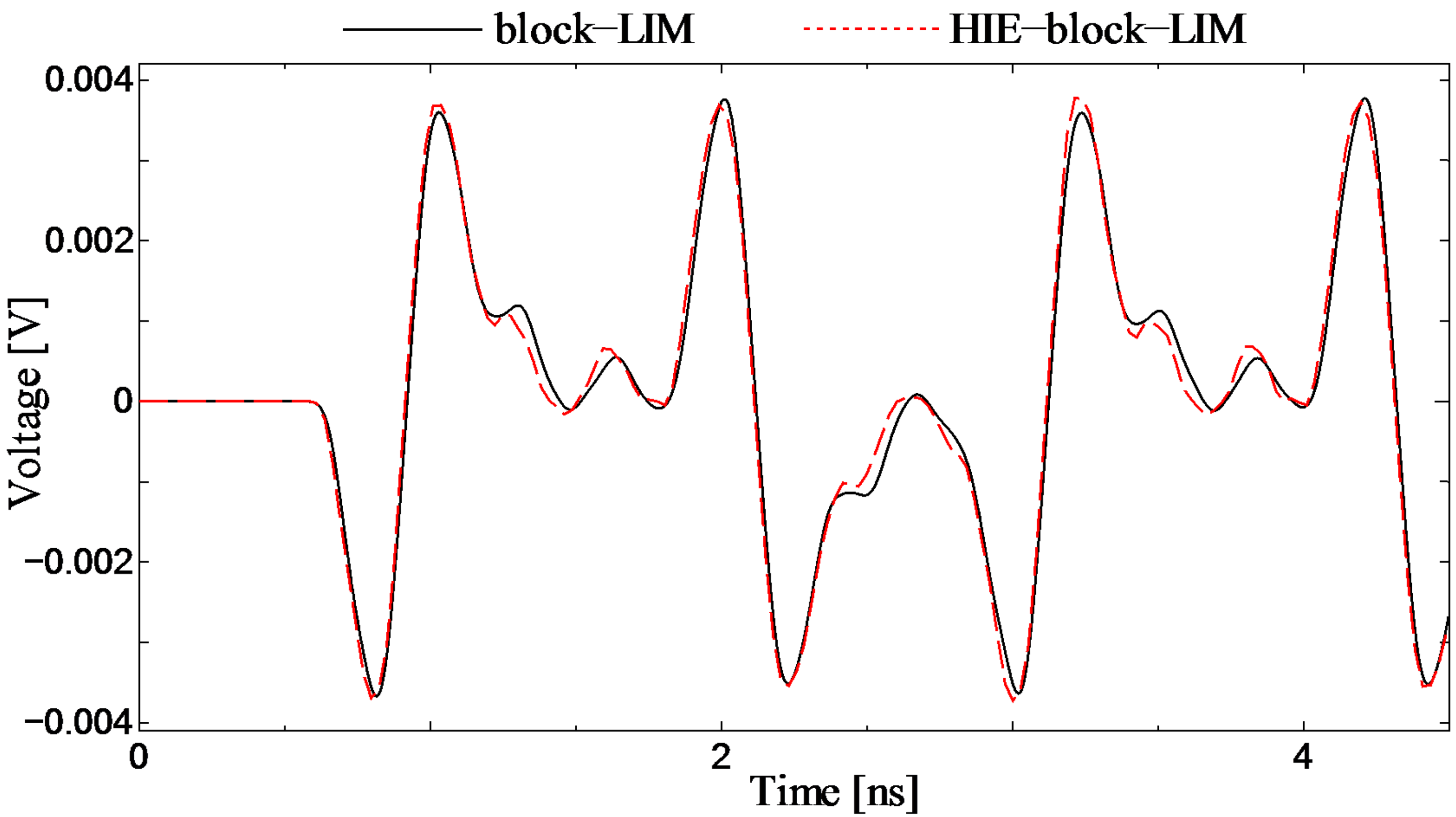
(a) block-LIM



(b) HIE-block-LIM

the structures of the coefficient matrices of the unknown voltages and currents

Numerical Results



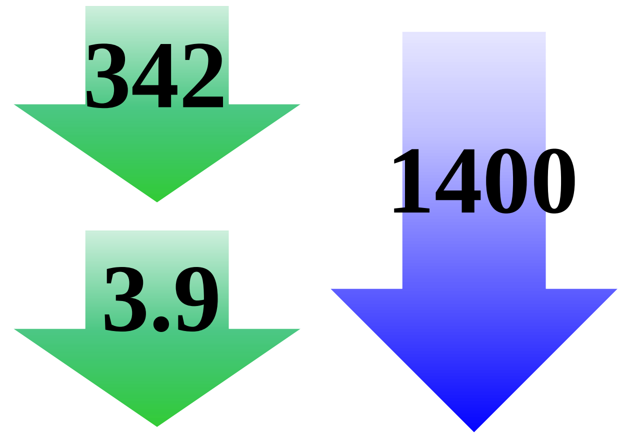
Waveform results of the node voltage

The proposed method can provide numerically stable solution even if the larger time step size is used

Numerical Results

Table 1. Comparison of CPU time

Method	CPU Time [s]	Speed-up
HSPICE	6242.04	
block-LIM	18.21	
HIE-block-LIM	4.66	



The proposed method is **3.9** times faster than block-LIM without losing accuracy

HIE-block-LIM

Supplemental explanation

- It is expected that the proposed method can be used to simulate the power distribution network (PDN)
- It has been proven that locally implicit (LI) LIM [1, 2] and block-LILIM [3], which are similar to the proposed method, are much faster than basic LIM and useful for the analysis of PDN

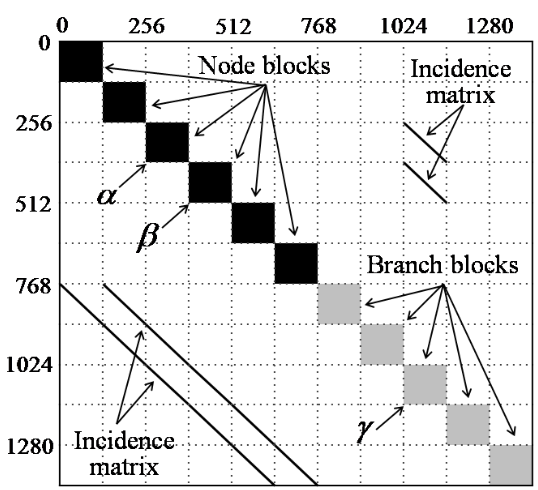
[1] H. Kurobe, T. Sekine, and H. Asai, "Locally implicit LIM for the simulation of PDN modeled by triangular meshes," *IEEE Microw. Wireless Compon. Lett.*, vol. 22, pp. 291–293, Jun. 2012.

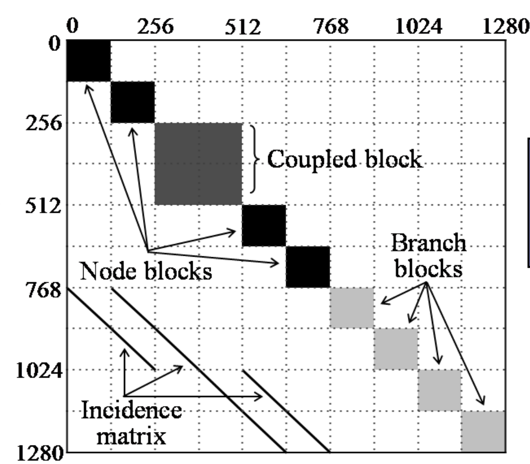
[2] T. Takasaki, T. Sekine, and H. Asai, "Efficient PDN simulaion by locally implicit latency insertion method based on rectangular meshes," in *Proc. IEEE EDAPS 2013*, Dec. 2013.

[3] S. Okada, T. Sekine, and H. Asai, "Locally implicit block-LIM for the simulation of multilayered PDN modeled by triangular meshes," in *Proc. AP-RASC 2013*, Sep. 2013, pp. 1–6.

Conclusion

- HIE-block-LIM
  - It is based on the block-LIM and HIE formulation
  - The limit of the time step size could be alleviated by applying the implicit difference method with respect to the current variables in the y-direction
  - Numerical results showed that the proposed method was faster than the block-LIM with appropriate accuracy





Alleviated Δt

&

Fast Simulation

Thank you for your attention.