A Novel Data Format for Approximate Arithmetic Computing

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Introduction

- What is Approximate Computing (AC)?
  - Approximate (error) vs. accurate
- Why we need AC?
  - Power/energy efficiency
- Why AC works?
  - Many of the applications are error-tolerable, e.g. Machine Learning, Image/Signal Processing
  - Disable partial computation
- AC at different level
  - Arithmetic, Software, Compiler, Architecture, Memory, and Circuit

Approximate Arithmetic

- Observation:
  - Least Significant Bits (LSB) have much less contribution than Most Significant Bits (MSB) to the overall quality of the result.
- Approach:
  - Compute accurately on MSB
  - Apply approximation on LSB

Approximate Arithmetic

- Example: Compute S = A + B

$$A = 0011\ 1010\ 0001\ 1000_2$$
$$B = 0101\ 1011\ 1011\ 1000_2$$

- Build a 16-bit approximate adder that[1]:
  - 8-bit accurate adder for high 8 bits
  - 8 OR gates for low 8 bits

Error = 0.102% !

$$\begin{array}{ll} & 0111\ 1010\ 0111\ 1001_2 = 31353 \\ + & 0101\ 1011\ 1011\ 1000_2 = 23480 \\ \hline = & 1101\ 0101\ 1111\ 1001_2 = 54777 \end{array}$$

[1] H. R. Mahdiani, A. Ahmadi, S. M. Fakhraie, and C. Lucas, "Bio-inspired imprecise computational blocks for efficient VLSI implementation of soft-computing applications," Circuits Syst. I Regul. Pap. IEEE Trans., vol. 57, no. 4, pp. 850–862, 2010.

Approximate Arithmetic

- However, what if the data is

$$A = 0000\ 0000\ 0101\ 1000_2$$
$$B = 0000\ 0000\ 1011\ 1000_2$$

$$
\begin{array}{rl}
 & 0000\ 0000\ 0111\ 1001_2 = 121 \\
+ & 0000\ 0000\ 1011\ 1000_2 = 184 \\
\hline
= & 0000\ 0000\ 1111\ 1001_2 = 249
\end{array}
$$

Error = 18.4% !

- We need a better approximate adder !

Approximate Arithmetic

- What we have learned:
  - "static" approximate adder vs. "dynamic" data
- Existing solutions:
  - Build additional discriminant circuit inside the approximate adder
- Drawbacks:
  - Fail to deliver significant power savings
  - Less accurate for multiplication

Approximate Integer Format

- Contribution: a novel Approximate Integer Format (AIF) and the corresponding computation mechanisms
- Desired properties of an ideal AIF
  - From "static" to "dynamic"
  - Cut-off the bitwidth of the operands
  - Suitable for all arithmetic operations
  - Provable error bound
  - Applicable to fixed point arithmetic

Preliminary -- Valid Block

- AIF is based on the segmentation of operands.
  - An n-bit positive integer N is segmented into [n/k] blocks with k bits per block.
  - Example: n = 16, k = 4, there are 4 blocks.

$A = 0000\ 1010\ 0001\ 1000_2$

- Definition 1: A valid block in a positive number is a block that has at least one '1' *before or inside* it.

$A = 0000\ 1010\ 0001\ 0000_2$

Preliminary -- Sentinel Bits

- Sentinel bits are used to truncate and round the less important bits to reduce bit-length of the operands
- Definition 2: The i_{th} sentinel bit *st[i]* of a number is defined as

$$st[i] = \begin{cases} 1, & block\ i\ is\ a\ valid\ block \\ 0, & block\ i\ is\ an\ invalid\ block \end{cases}$$

Preliminary --Precision Control

- Definition 3: The precision control 'pc' is the number of valid blocks in the number, from the leftmost one, that will be used in the computation.
- Example:

1500_{10} = 0000 0101 1101 1100_2
800_{10} = 0000 0011 0010 0000_2

  - Both have 3 valid block, st = 0111
  - If pc = 2, 2 blocks of each operand will be selected

1500_{10} = 0000 ***0101 1101*** 1100_2
800_{10} = 0000 ***0011 0010*** 0000_2

Preliminary --Rounding

- When we use sentinel bits to truncate the valid blocks, rounding is needed
- Two rounding techniques:
  - Classic rounding
    - For multiplication and division
  - Efficient rounding
    - For addition and subtraction

Classic Rounding

- Definition 4: The classic rounding of a number N at the i_{th} LSB means
  - adding the i_{th} bit to the $(i+1)_{th}$ bit
  - setting i_{th} bit and bits to its right to zero
- Example: $N = 263_{10} = 0000\ 0001\ 0000\ 0111_2$
  - From the 3rd least significant bit
    - $N = 0000\ 0001\ 0000\ 1000_2$
  - From the 4th bit
    - $N = 0000\ 0001\ 0000\ 0000_2$

Efficient Rounding

- Definition 5: The efficient rounding of A+B at the i_{th} bit is

$$A_{trunc} + B_{trunc} + Cin_{round}$$

  - A_{trunc} and B_{trunc} are obtained by truncating the i least significant bits from A and B
  - $Cin_{round} = (A_i \& B_i)$, AND i_{th} bits of A and B

Efficient Rounding -- Example

To compute $S = A + B$

$A = 0011\ 1010\ 1001\ 1000_2$

$B = 0000\ 1011\ 1011\ 1000_2$

Truncate A and B

$A_{trunc} = 0011\ 1010\ 0000\ 0000_2$

$B_{trunc} = 0000\ 1011\ 0000\ 0000_2$

Compute round-off carry in

$Cin_{round} = A7\ \&\ B7 = 1$

$S' = A_{trunc} + B_{trunc} + Cin_{round}$

$$\begin{array}{rr} & 0011\ 1010 \\ + & 1011 + 1 \\ \hline = & 0100\ 0110 \end{array}$$

S using efficient rounding:

$0100\ 0110\ 0000\ 0000_2 = 17920_{10}$

Accurate S:

$0100\ 0110\ 0101\ 0000_2 = 18000_{10}$

Approximate Integer Format

- Given a 4-block operand $A= b_3b_2b_1b_0$.
  - Only five possible values of A's sentinel bits st_a: 0000, 0001, 0011, 0111, 1111.
  - For the first four cases, the data A will be stored in following format:

st_a	b_2	b_1	b_0

  - For the last case of 1111, A will be stored as:

st_a	b_3	b_2	b_1

AIF Arithmetic --Addition

- Compute the sentinel bits of the result S: $st_s = st_A \mid st_B$
- Truncate i_{th} to $(i\text{-}pc+1)_{th}$ blocks of A and B to obtain A′ and B′, respectively
  - Suppose the leftmost '1' in st_s is in $st[i]$, and we plan to pick pc valid blocks
- Compute S′ = A′ + B′ and C_{out}
- Update st_s by $st_s[i+1] = C_{out}$
- Reformulate S in AIF using st_s and S′. Padding 0's if necessary

AIF Arithmetic --Addition Example

Original data

$A = 0011\ 1010\ 0001\ 1000_2$

$B = 0000\ 1011\ 1011\ 1000_2$

Data in AIF

$A' = 1111\ 0011\ 1010\ 0001_2$

$B' = 0111\ 1011\ 1011\ 1000_2$

Compute st_s:

$$\begin{array}{lr} & 1111 \\ \text{or} & 0111 \\ \hline = & 1111 \end{array}$$

Compute S'

$$\begin{array}{lr} & 0011\ 1010 + 0 \\ + & 1011 + 1 \\ \hline = & 0100\ 0110 \end{array}$$

Reformulate S in AIF:

$1111\ 0100\ 0110\ 0000_2 = 17920_{10}$

Accurate S:

$0100\ 0101\ 1101\ 0000_2 = 17872_{10}$

AIF Arithmetic --Multiplication

- Round the leftmost pc valid blocks of A and B into A' and B′
- Compute S′ = A' * B′
- Compute sentinel bits st_S using st_A and st_B and carry out
- Shift and reformulate S in AIF using S′ and st_S. Padding 0′s if necessary

Error analysis

- Let rounding error of A and B are er_A and er_B, respectively.
- Error of AIF based addition:
  - $Er_{add} = 2*\max(er_A, er_B)$
- Error of AIF based multiplication:
  - $er_A + er_B + er_A*er_B$
  - er_A<<1, er_B<<1, $Er_{mul} \approx er_A + er_B$

Negative AIF

- Deal with negative integer
  - Cannot use previous equation to compute st
- Solution:
  - Re-define the valid block
  - A valid block in a negative number is a block that has at least one '0' before or inside it.
  - Replace logic OR with logic AND when computing st
  - Arithmetic operations remains the same

High level programming language

- Introduce appropriate instructions and data type.
- Incorporate it in compiler and ISA
- Example: define apxint as the AIF data type

```
int main(){
        apxint a = 2341;
        apxint b = 546;
        apxint sum = a + b, prod = a * b;
        …
}
```

Compute in Caution

- Condition criterion, e.g. if, while condition
- Data value that the result is very sensitive to
- Functions that have periodical property, e.g. sin, cos, and modulo operation

Experiment Results: HW Cost

- Overhead Comparison of Arithmetic Units and The Sentinel Bits Computing Circuits

	cells	area	power(nW)
8-bit checker	10	23.93	61475.68
16-bit checker	17	39.89	132145.68
8-bit adder	91	212.12	752614.84
16-bit adder	230	322.33	2234652.24
32-bit adder	498	1116.93	4818821.94
8-bit multiplier	377	1037.62	2829745.1
16-bit multiplier	1406	4208.68	10815807.06
32-bit multiplier	4916	15126.01	34033690.3

Fibonacci Sequence: Accuracy

- First 40 elements in Fibonacci Sequence
  - A number is sum of its previous two: 1, 1, 2, 3, 5, 8, 13, 21, 34, …
  - Test the error propagation

#	pc=2	pc=3	pc=4	#	pc=2	pc=3	pc=4	#	pc=2	pc=3	pc=4
1~13	0	0	0	22	-0.02732	3.49E-05	0	31	-0.02291	-0.00035	2.30E-06
14	-0.00984	0	0	23	-0.02692	0	0	32	-0.02267	-0.00059	-8.51E-06
15	-0.01317	0	0	24	-0.02707	1.33E-05	0	33	-0.02276	-0.0005	-4.38E-06
16	-0.01691	0	0	25	-0.02912	0.000404	8.24E-06	34	-0.02273	-0.00053	-5.96E-06
17	-0.01858	0	0	26	-0.03225	0.000336	1.02E-05	35	-0.02274	-0.00052	-5.36E-06
18	-0.01985	0	0	27	-0.0375	0.000362	9.44E-06	36	-0.02274	-0.00052	-5.59E-06
19	-0.02173	-0.00044	0	28	-0.03947	0.000352	9.72E-06	37	-0.0328	-0.0001	1.05E-06
20	-0.02905	0.000183	0	29	-0.04118	0.000356	9.61E-06	38	-0.03621	-0.00046	-5.53E-06
21	-0.02625	-5.65E-05	0	30	-0.04205	0.000354	9.66E-06	39	-0.04003	-0.00064	-3.02E-06
								40	-0.04174	-0.00077	-3.98E-06

Real Life Application --IDCT

Which one is the original image?

PSNR = 41.76

(a)

(b)

PSNR = 89.64

PSNR =116.29

(c)

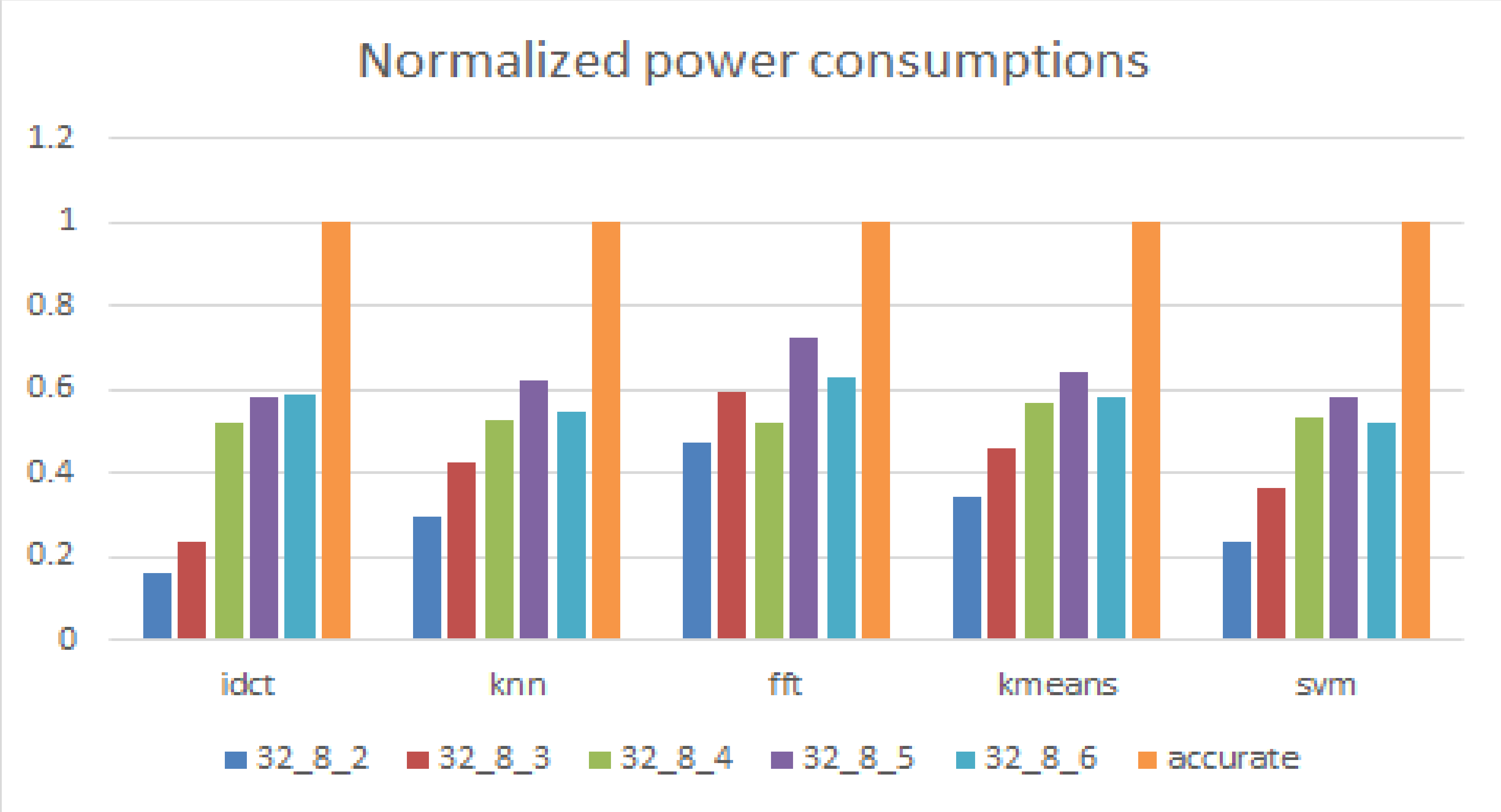
(d)

original image

Real Life Applications --Quality

AIF modules	IDCT	KNN	FFT	Kmeans	SVM
32_8_2	41.7579	92.9%	0.0081	1.125%	60.94%
32_8_3	64.6398	93.2%	2.79E-04	0.125%	85.11%
32_8_4	89.8324	93.1%	1.61E-05	0	85.98%
32_8_5	112.0872	93.2%	3.02E-06	0	85.59%
32_8_6	116.2944	93.2%	7.11E-08	0	86.42%
baseline	116.2949	93.2%	0	0	86.42%
Error Metric	PSNR	Classification accuracy	ARES	miss-clustered%	Classification accuracy

Final Result --Power Savings



Thank you!