A Time Domain Behavioral Model for Oscillators Considering Flicker Noise

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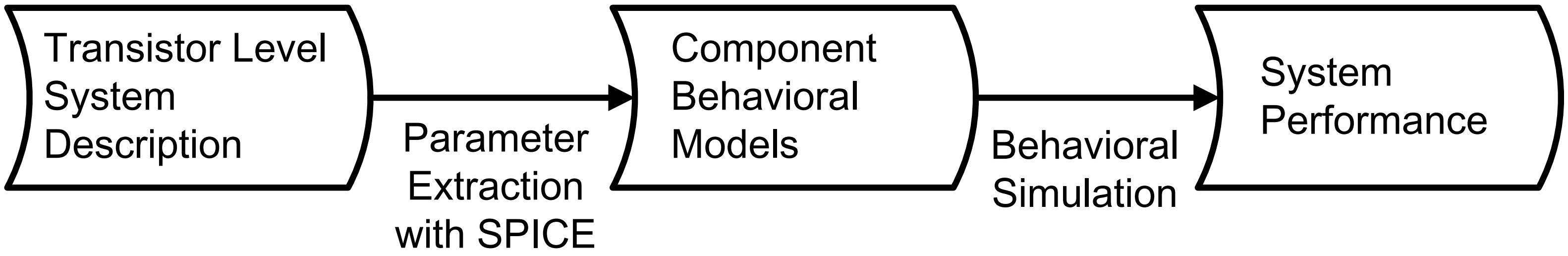
Outline

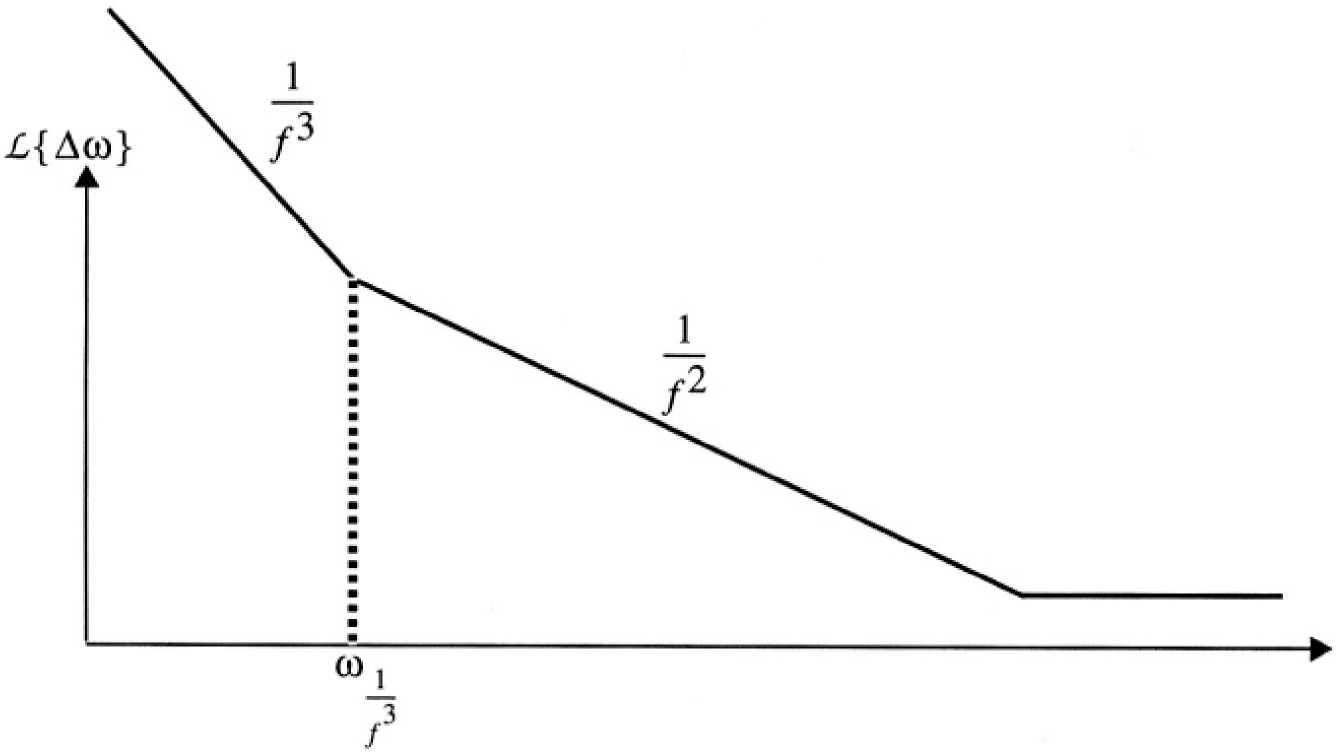
- Introduction
- Relationship between jitter and phase noise
  - The link for thermal noise
  - Discuss and derive the link for flicker noise in detail.
- Model Implementation
- Theory and model verification
  - Theory verification
  - Model verification
    - Comparison of the phase noise
    - Comparison of the period jitter's PSD
- Conclusions

Time Domain Behavioral Model Needed



- Large-signal time domain model is the only suitable model for the circuit without steady-state solution.
  - Fractional-N PLL
  - Bang-bang PLL
  - ……..
- Design space exploration can be done efficiently by the behavioral model.

Phase Noise in Oscillators



- The -20dB/dec and -30dB/dec regions are up-converted by the thermal and flicker noise respectively.

State-of-the-Art

Predicting the Phase Noise and Jitter of PLL-Based Frequency Synthesizers

Ken Kundert
Designer's Guide Consulting, Inc.

Citation: 169 (based on Google Scholar)

- Most cited paper in modeling PLL and the oscillator behaviorally
- Jitter-based time domain and phase domain model
- To be improved: "This excludes flicker noise."

State-of-the-Art

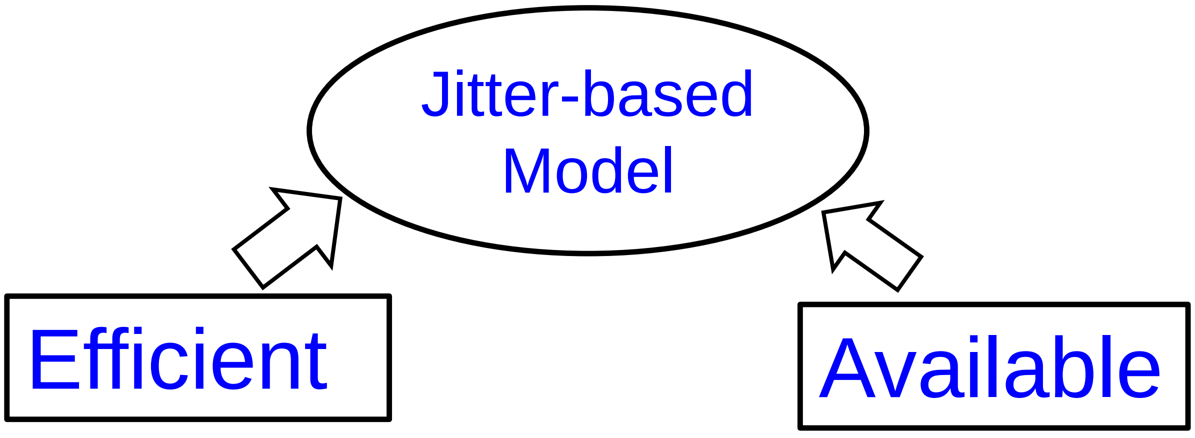
Event-Driven Simulation and Modeling of Phase Noise of an RF Oscillator

R.B. Staszewski et al., *TCAS-I,* 2005

Citation: 101

- Second most cited paper in modeling oscillator behaviorally
- Jitter-based time domain model
- To be improved: “A further correction has to be made” when model the 30dB/dec rolling off region of the phase noise.

Jitter-based Model



- Both of the two top cited models are jitter-based.
  - Efficient: noise is represented only on the timing of the transitions(in the form of jitter)
  - Available: jitter extraction methodology is based on the commercially available simulator such as SpectreRF

Link between Jitter and Phase Noise Considering Only Thermal Noise

- The relationship between the period jitter variance and the phase noise with only the thermal noise is

$$\sigma_{thermal}{}^{2} = L(\Delta f)\frac{\Delta f^{2}}{f_0^{3}} .$$

- The jitter extracted from this formula is proved to be accurate in modeling the -20dB/dec of the phase noise.

Extract the Jitter due to Flicker Noise

Hajimiri(JSSC99), McNeil(ISCAS04) et al.:

$$\sigma^2(\tau) = \frac{2}{(\pi f_0)^2}\int_0^{+\infty} L_\phi(\Delta f) sin^2(\pi \Delta f \tau) d\Delta f$$

☹

- The formula is not closed-form.
- Flicker noise is nonstationary.

R.B. Staszewski(*TCAS-I,* 2005):

$$\sigma_{\Delta T, \frac{1}{f}} = \frac{\Delta f_{c,1}}{f_0} \cdot \sqrt{T_0} \cdot \sqrt{2\mathcal{L}\{\Delta\omega_{c,1}\}}.$$

☹

- Not rigorous
- Further correction to be made

in modeling the -30dB/dec region.

Mathematical Foundation

$$\sigma^2_{1/f}(\tau,t) = \frac{2}{(\pi f_0)^2}\int_{1/t}^{+\infty}\frac{k}{\Delta f^3} sin^2\,(\pi\Delta f\tau)d\Delta f$$

$$= \frac{2k\tau^2}{f_0^2}\{-\frac{1}{2}[x^{-2}sin^2(x) + x^{-1}\sin(2x)]_{\pi\tau/t}^{+\infty}$$

$$+ \int_{2\pi\tau/t}^{+\infty} \boldsymbol{x^{-1}\cos(x)\,dx}\}$$

$$Ci(z) = -\int_z^{+\infty}\frac{cos(x)}{x}dx$$

$$Ci(z) = \gamma + \ln(z) + \sum_{n=1}^{+\infty}\frac{(-1)^n(z)^{2n}}{(2n)!\,(2\mathrm{n})}$$

- The flicker noise is ‘postulated’ as an stationary stochastic process by introducing a cut-off frequency.
- Solving the integral analytically will establish a link.

Link between Jitter and Phase Noise Considering Only Flicker Noise

- We relate the variance of the period jitter with the phase noise for flicker noise as

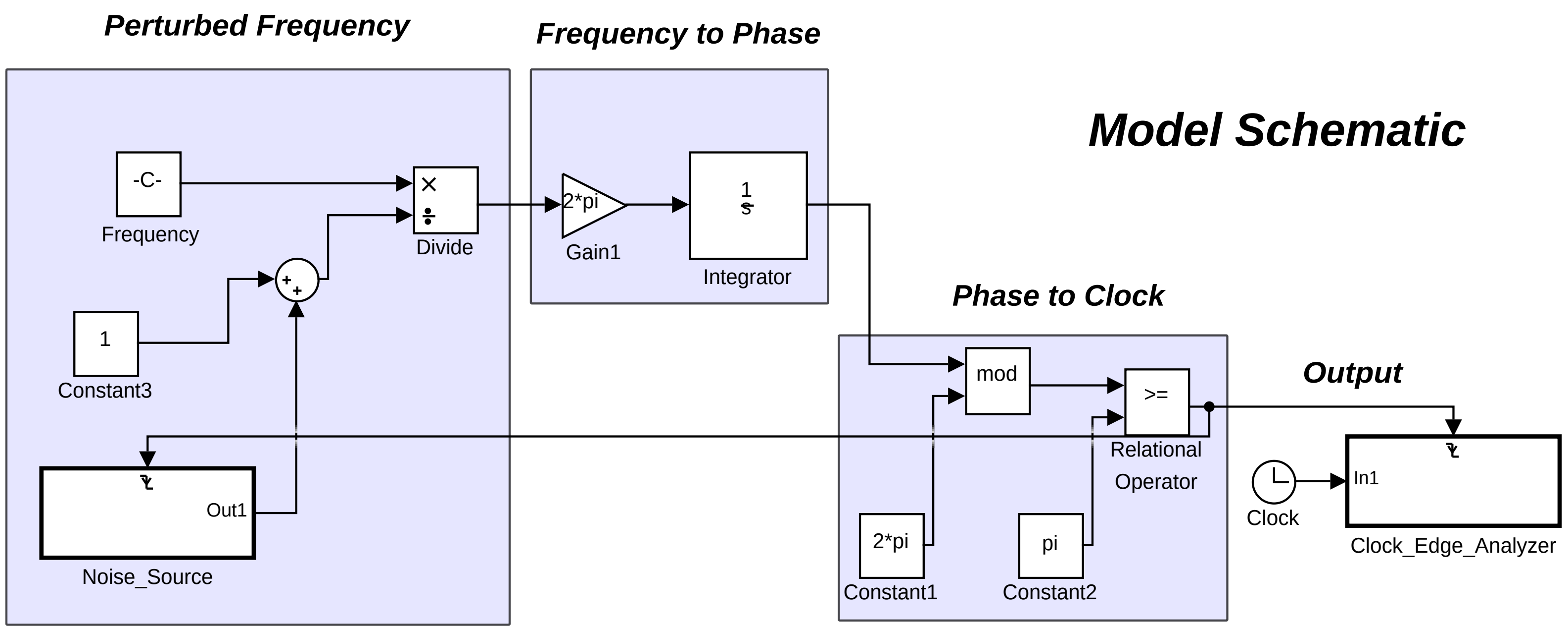
$$\sigma_{1/f}^2(t) = 2\left[ln\left(\frac{t}{T_0}\right) - 0.9151\right]\frac{\Delta f^3}{f_0^4}L(\Delta f)$$

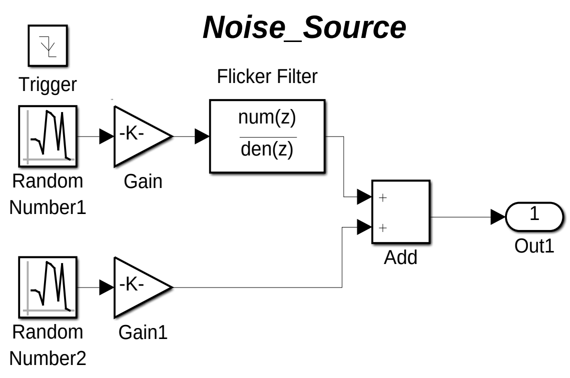
- This expression is CLOSED-FORM and COMPACT.

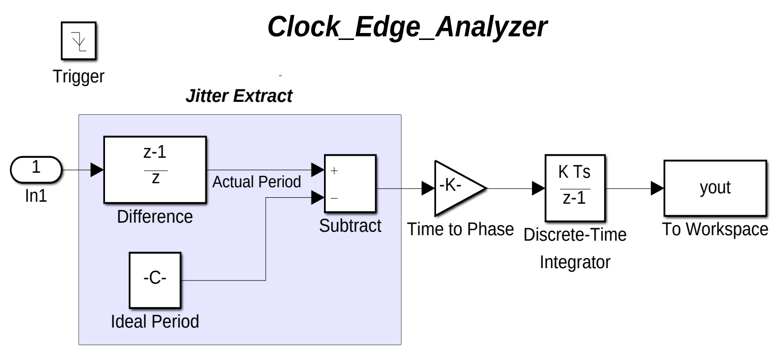
t: the observation time Δf : the offset frequency

$L(\Delta f)$:the single-sided spectral noise density f_0: the nominal frequency

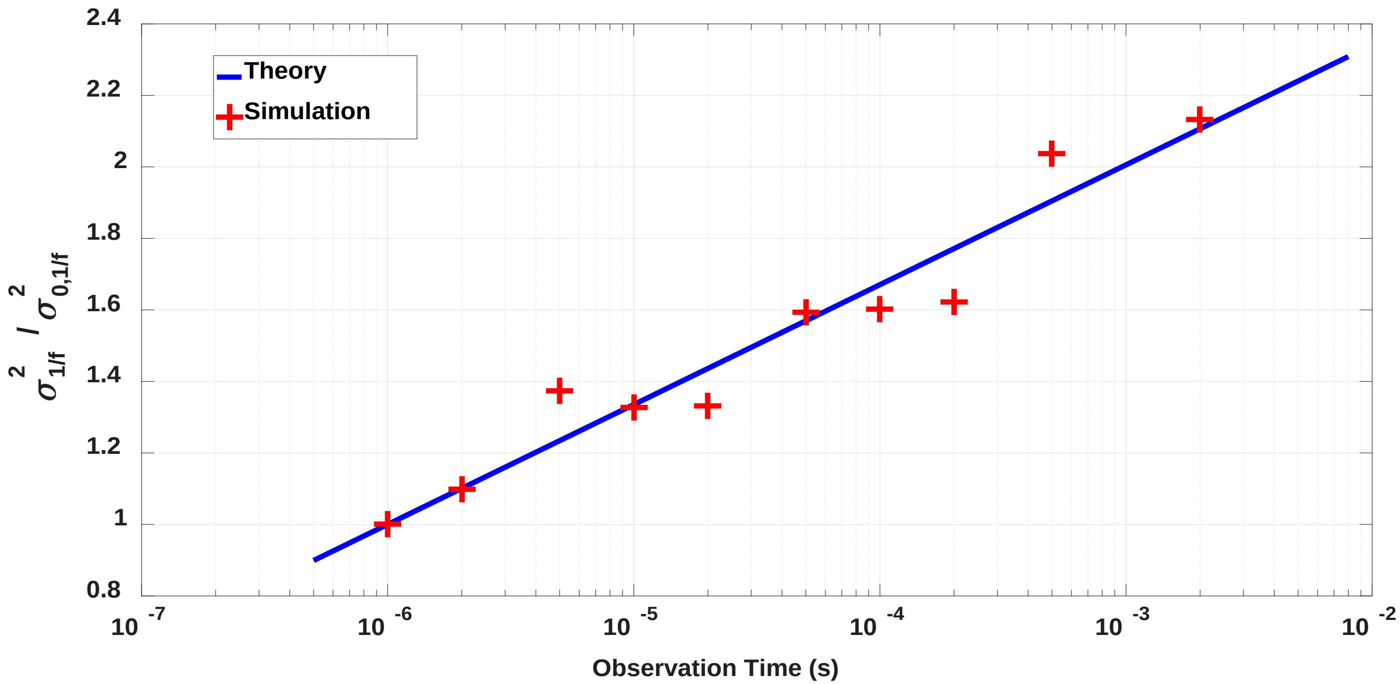
Time Domain Model of Oscillators Including the White and Flicker Noise





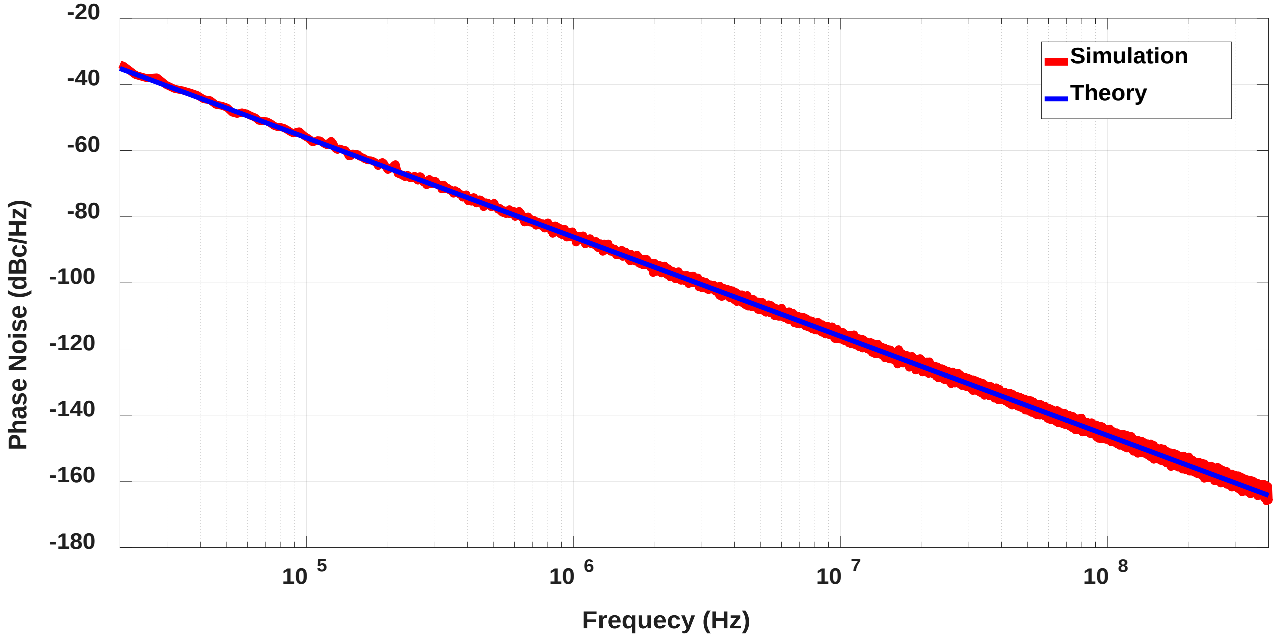


Theory Verification



- The model with only flicker noise is used.
- Jitter's variance grows along $\left[\ln\left(\frac{t}{T_0}\right) - 0.9151\right]$.
- It is redicted by our theory and formula.

Theory Verification

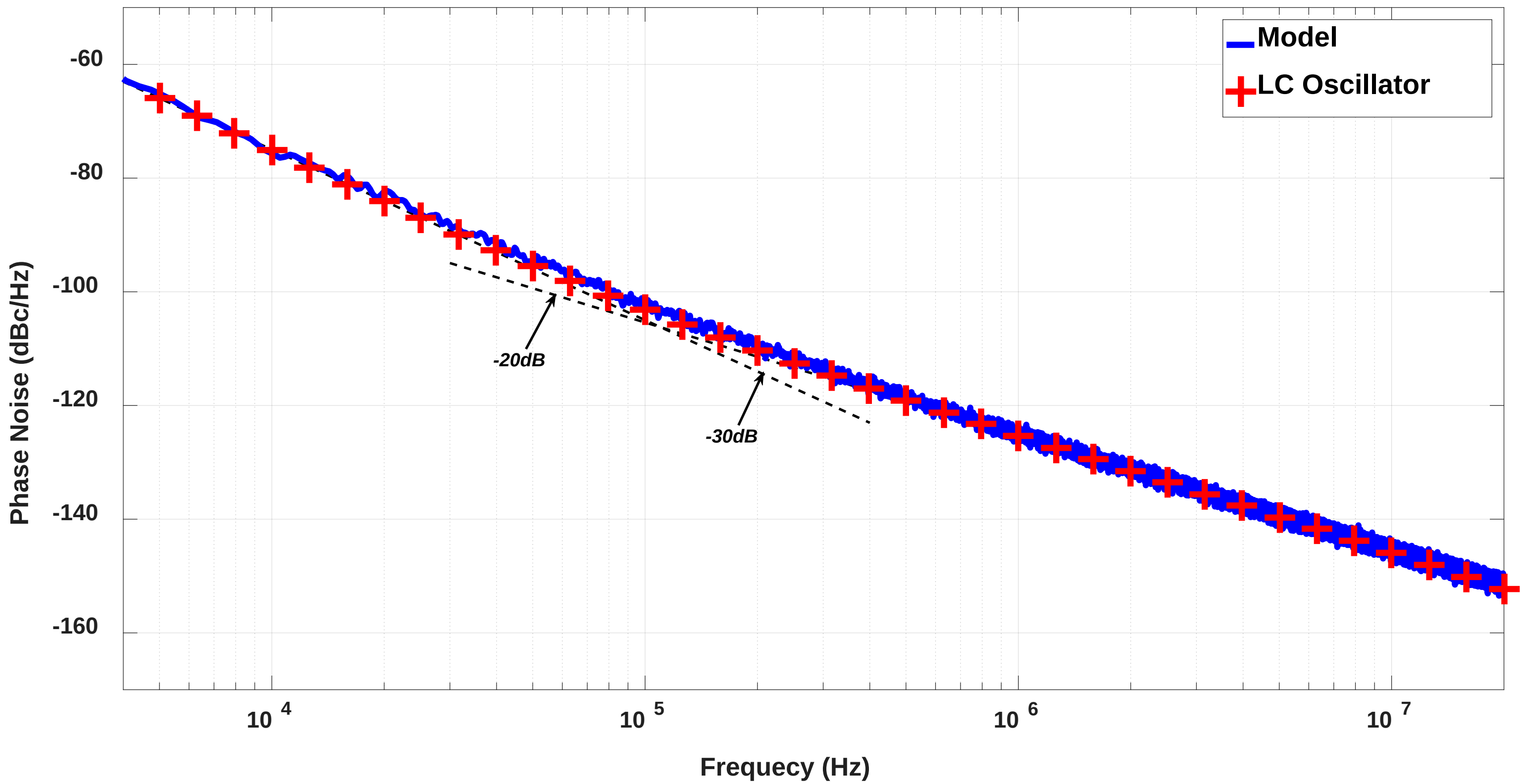


- The variance of the period jitter is fixed.
- The phase noise is predicted by our formula by

$$L(\Delta f) = \frac{\sigma_{1/f}^{2}(T_0)}{2\left[\ln\left(\frac{t}{T_0}\right) - 0.9151\right]} \frac{f_0^4}{\Delta f^3}.$$

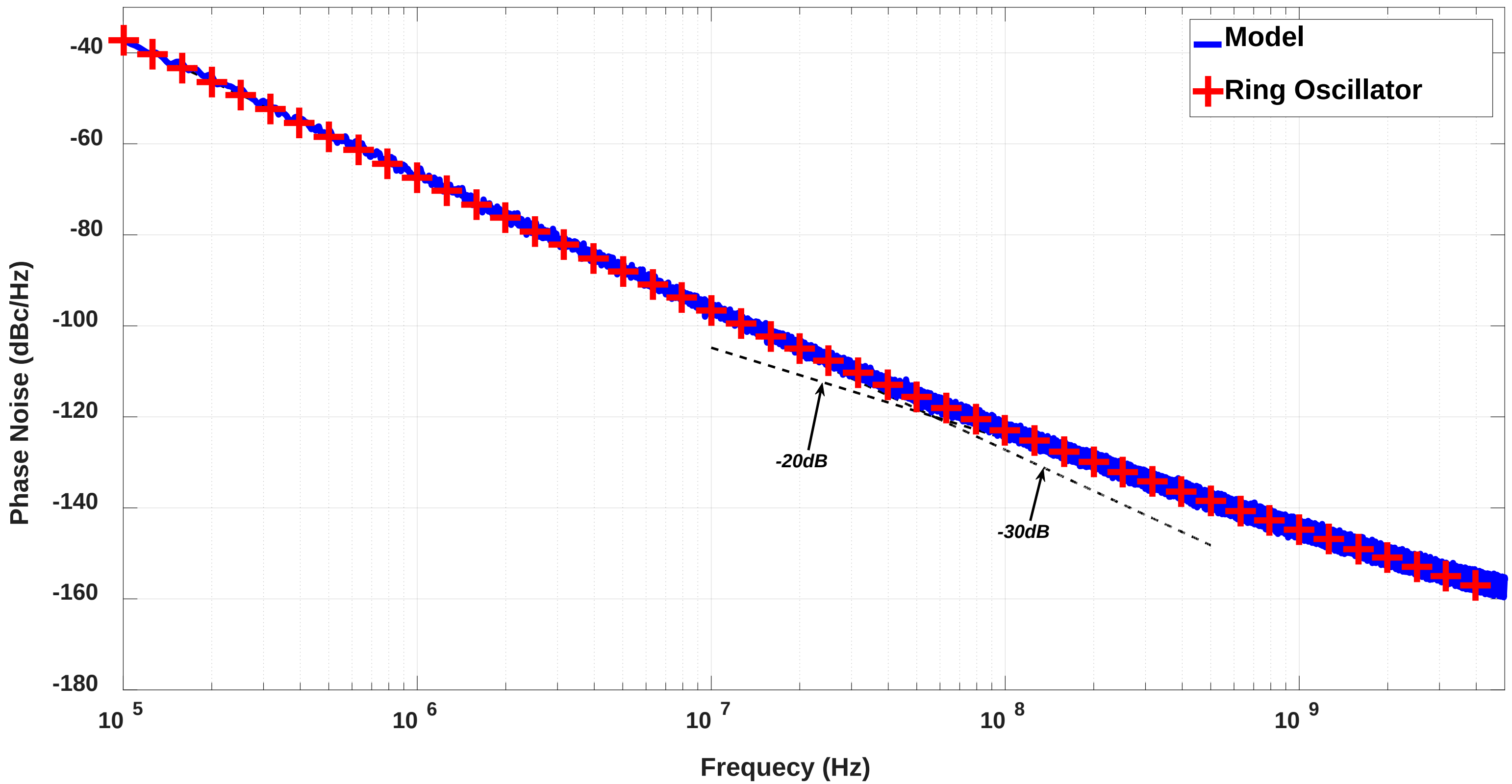
- The simulation results conforms the prediction.

Model Verification with Real Oscillator Circuits



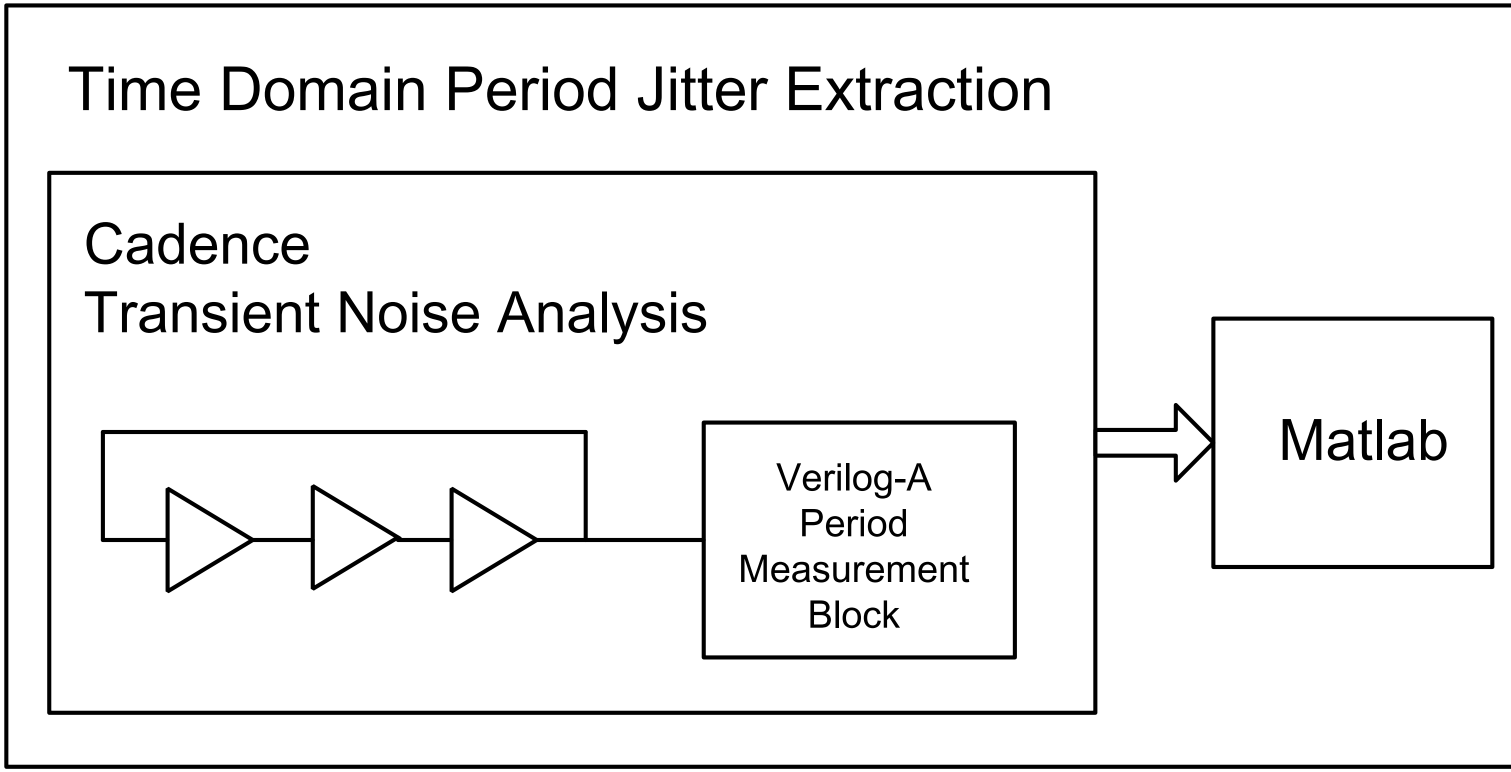
- Parameter Extraction(1.422GHz LC oscillator circuit with about 100KHz noise corner frequency)
  - Thermal noise jitter: $\sigma_{thermal}{}^2 = L(\Delta f)\frac{\Delta f^2}{f_0^3}$
  - Flicker noise jitter: $\boldsymbol{\sigma_{1/f}^2(t) = 2\left[\ln\left(\frac{t}{T_0}\right) - 0.9151\right]\frac{\Delta f^3}{f_0^4}L(\Delta f)}$

Model Verification with Real Oscillator Circuits



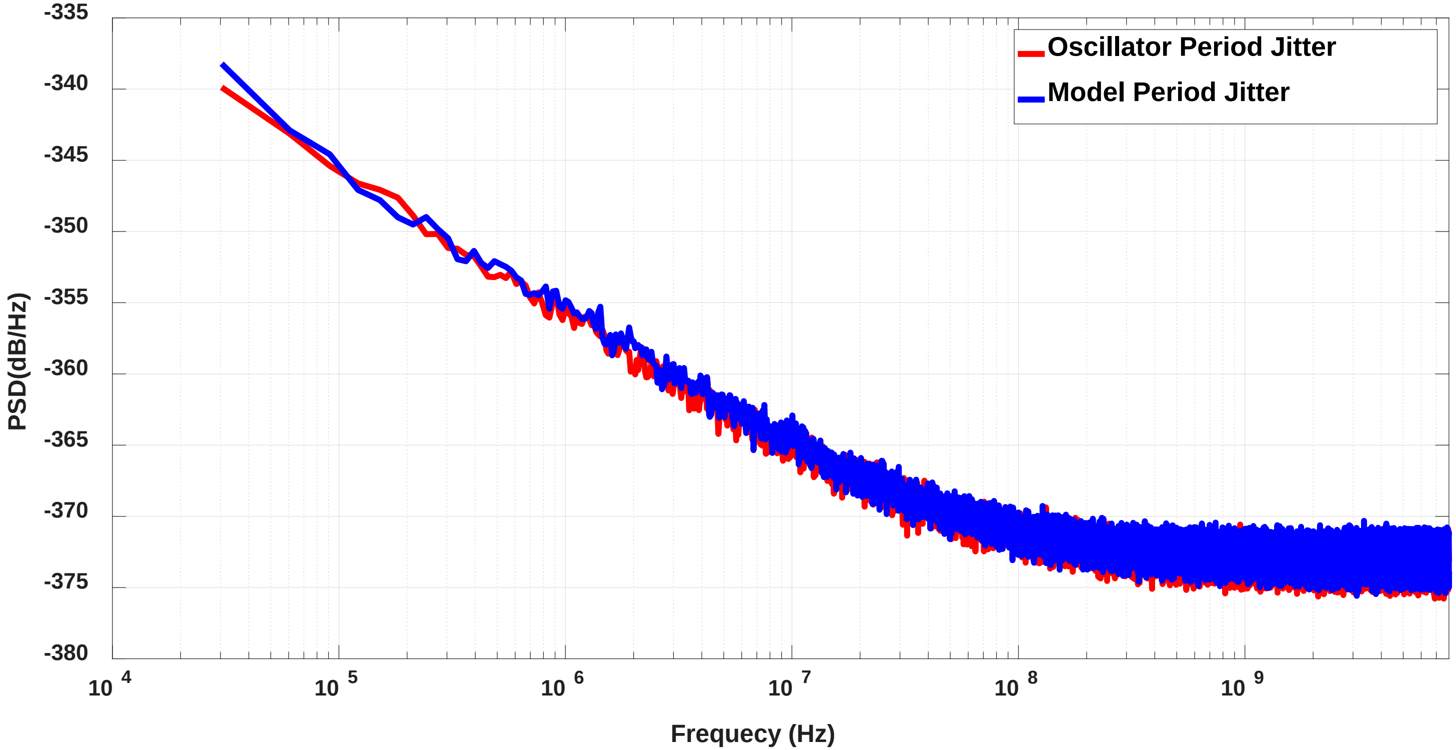
- Parameter Extraction(15.911GHz ring oscillator circuit with about 60MHz noise corner frequency)
  - Thermal noise jitter: $\sigma_{thermal}{}^2 = L(\Delta f)\frac{\Delta f^2}{f_0^3}$
  - Flicker noise jitter: $\boldsymbol{\sigma^2_{1/f}(t) = 2\left[\ln\left(\frac{t}{T_0}\right) - 0.9151\right]\frac{\Delta f^3}{f_0^4}L(\Delta f)}$

Further Model Verification with Real Oscillator Circuits



- Extract the period jitter by transient noise analysis
  - Setup: noisemin is 10K, noisefmax is 500G
  - Runtime: 10 days ☹ to complete 1ms simulation(server with E5 processor and 16G memory)

Further Model Verification with Real Oscillator Circuits



- Comparison of the period jitter spectrum between
  - Our model (extracted by the link between the phase noise and the jitter)
  - That extracted directly in time domain by the transient noise analysis

Conclusions

- We have detailly discussed and derived the link between jitter and phase noise for the flicker noise.
  - A closed-form analytical expression is given without any approximation.
  - Demonstrate the link between period jitter and phase noise by simulation for the first time.
- Present a time domain behavioral model for oscillators considering the flicker noise.
  - The first work to model the up-converted flicker noise region of the phase noise accurately in time domain
  - Universal and accurate for either LC or ring oscillators
- Two different ways are used to verify the model, both observe excellent agreements.

Thank you!

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