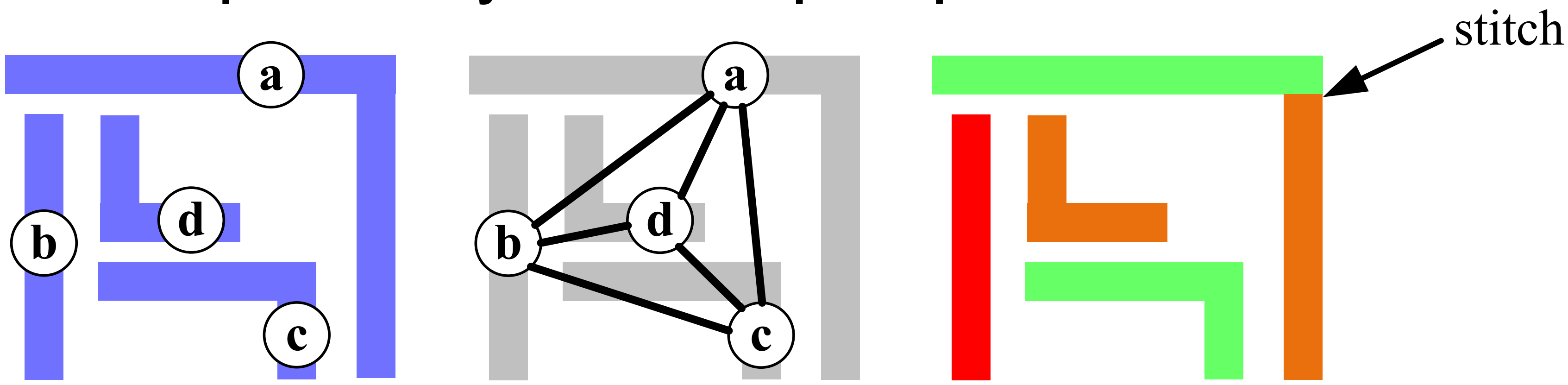
On Coloring Rectangular and Diagonal Grid Graphs for Multiple Patterning Lithography

Daifeng Guo*, Hongbo Zhang^, Martin D. F. Wong*

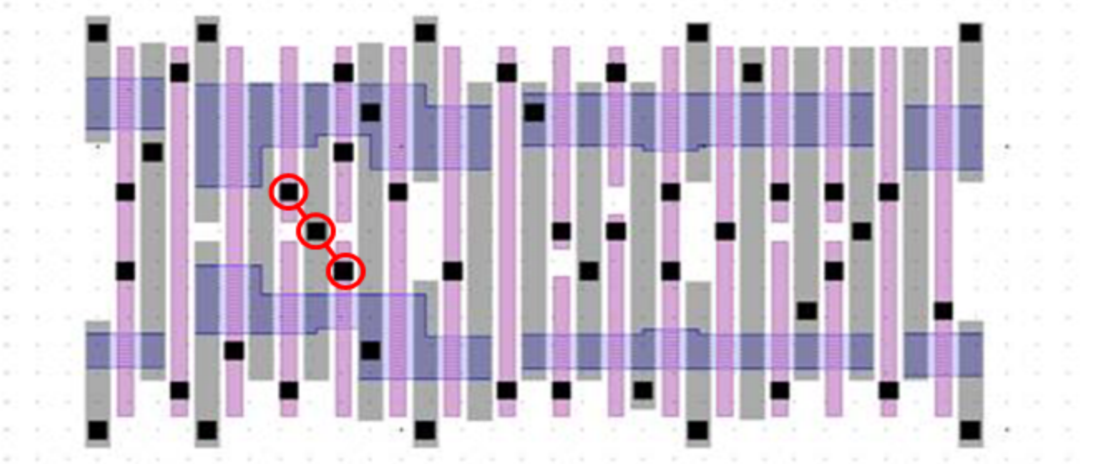
* Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign (UIUC), IL, USA
^ Facebook, USA

Multiple Patterning Lithography

- ❑ **Decompose the layout for multiple exposures**



- ❑ **MPL for contact/via layer**
  - ❑ Very dense
  - ❑ Cannot add stitch
  - ❑ Regular distributed
  - ❑ Conflicts are local

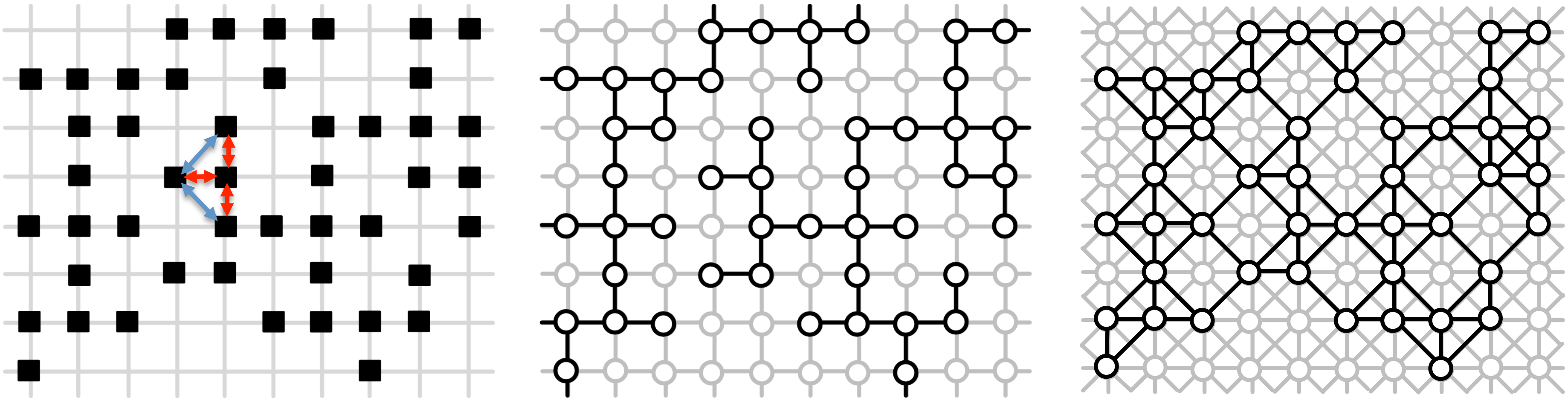


How does the regularity help us?

- ❑ **The conflict graph of vias is special!**
- ❑ **Rectangular grid graph (RGG):**

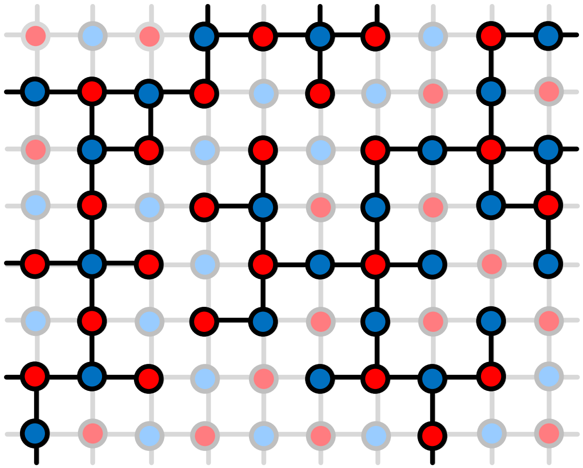
  An induced subgraph of a rectangular grid
- ❑ **Diagonal grid graph (DGG):**

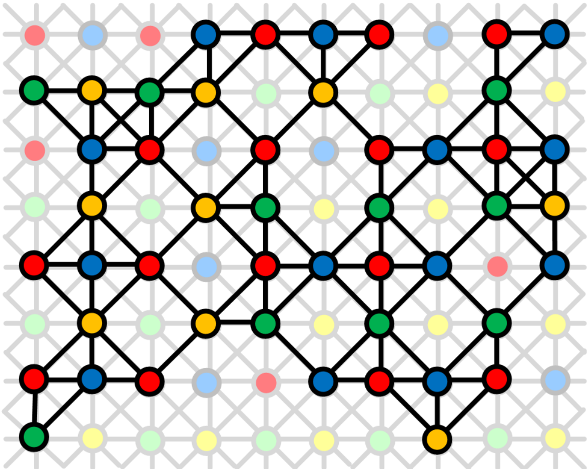
  An induced subgraph of a diagonal grid

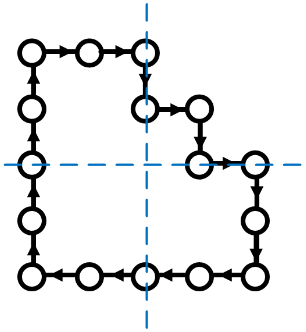


Coloring Grid Graphs: A complete analysis

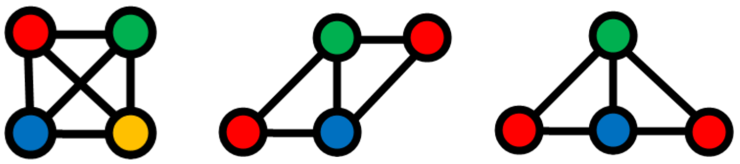
- **Rectangular grid graph:**
  - 2-colorable? Always. Easy to prove by even cycles.
- **Diagonal grid graph:**
  - 2-colorable? Find the odd cycle!
  - 3-colorable?
  - 4-colorable? Always!





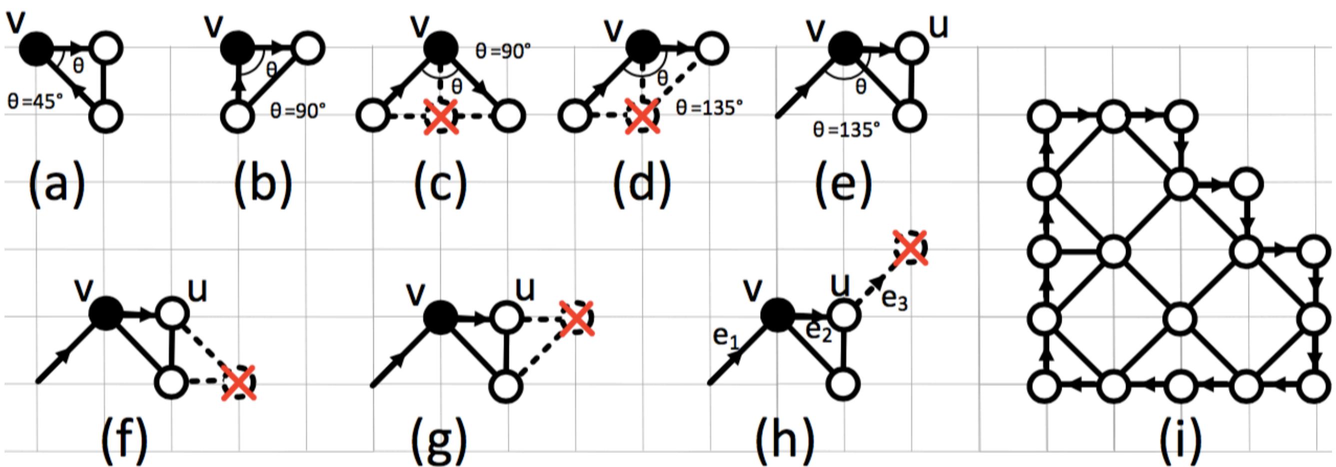


3-coloring Diagonal Grid Graphs

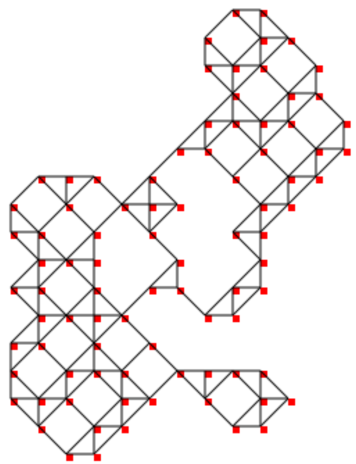


K4 Two diamond examples

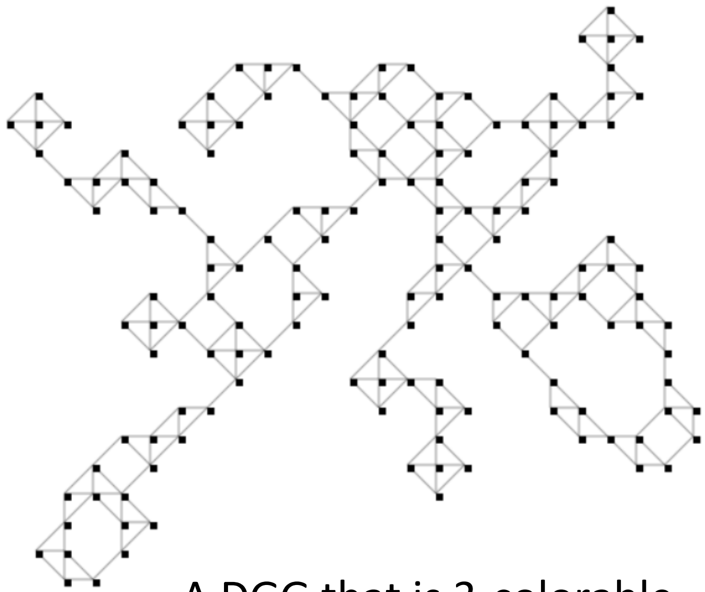
- **Necessary conditions**
  - No K4.
- **Sufficient conditions**
  - Planar graph free of 4-, 5-, 6-, 7-cycle is 3 colorable.
  - Steinberg conjecture: planar graph free of 4- or 5-cycle is 3 colorable. Proved false by Cohen-Addad et al. in 2016.
  - DGG w/ degree < 4 is 3-colorable by Brooks' theorem.
  - DGG free of "diamond" is 3-colorable.



3-coloring Diagonal Grid Graphs

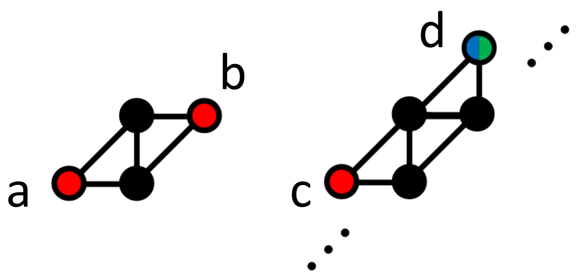


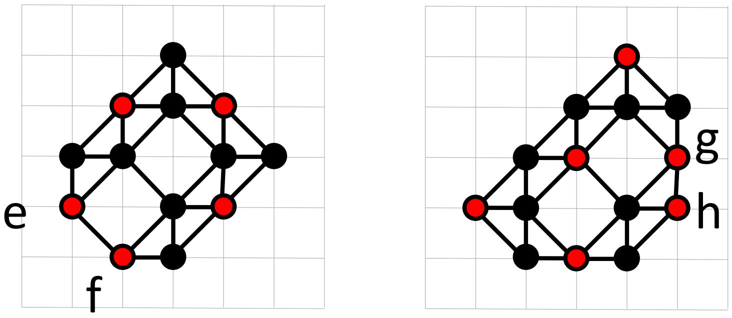
A DGG that is not 3-colorable



A DGG that is 3-colorable

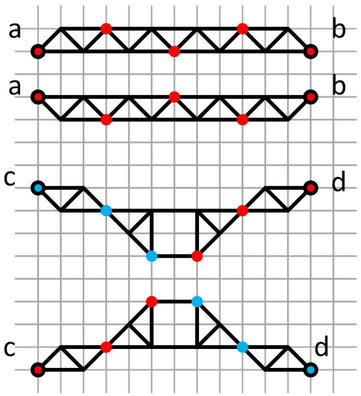
- ❑ **A chain of diamonds can spoil the 3-colorability.**



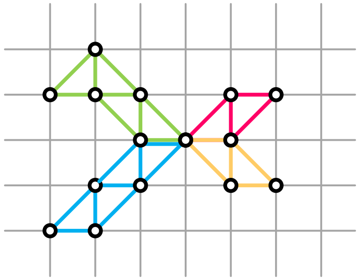


3-coloring Diagonal Grid Graphs

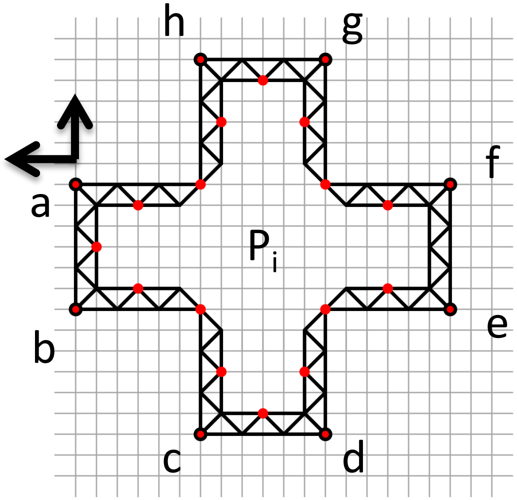
- **Can we embed any planar graph into a DGG and preserves its coloring by using diamond chains?**
  If so, the problem is NP-Hard.
- **To construct a desired DGG embedding**
  - Vertex degree unbounded vs. Vertex degree bounded
  - Any edges vs. Rectilinear diamond chains
  - The embedding has to be polynomial



Edge (v_i, v_j) => a diamond chain



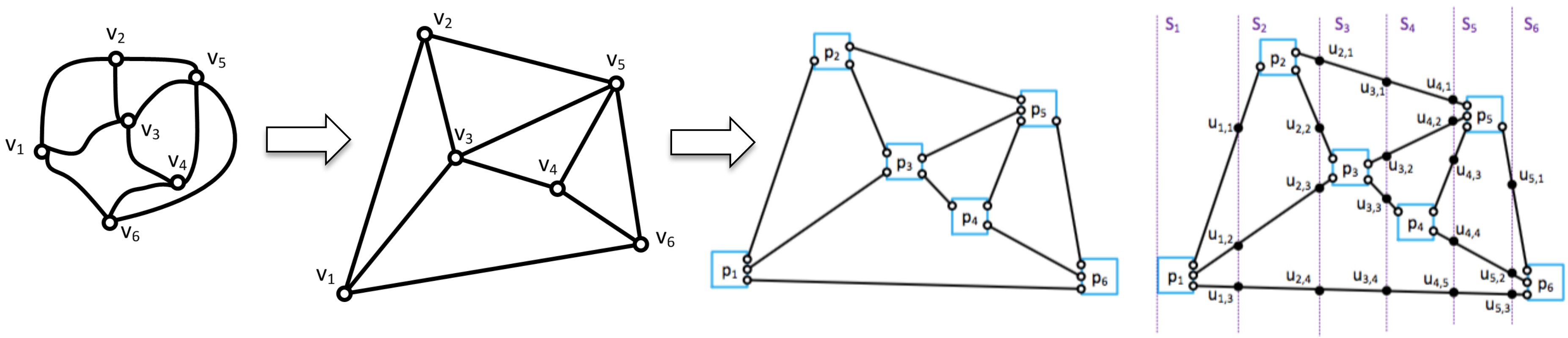
At most 4 diamond chains are connected



Vertex v_i => a diamond chain (loop) P_i

3-coloring Diagonal Grid Graphs

- **Can we embed any planar graph into a DGG and preserves its coloring by using diamond chains?**
  If so, the problem is NP-Hard.
- **To construct a desired DGG embedding**
  - A linear embedding
  - Vertex transformation
  - Edge transformation

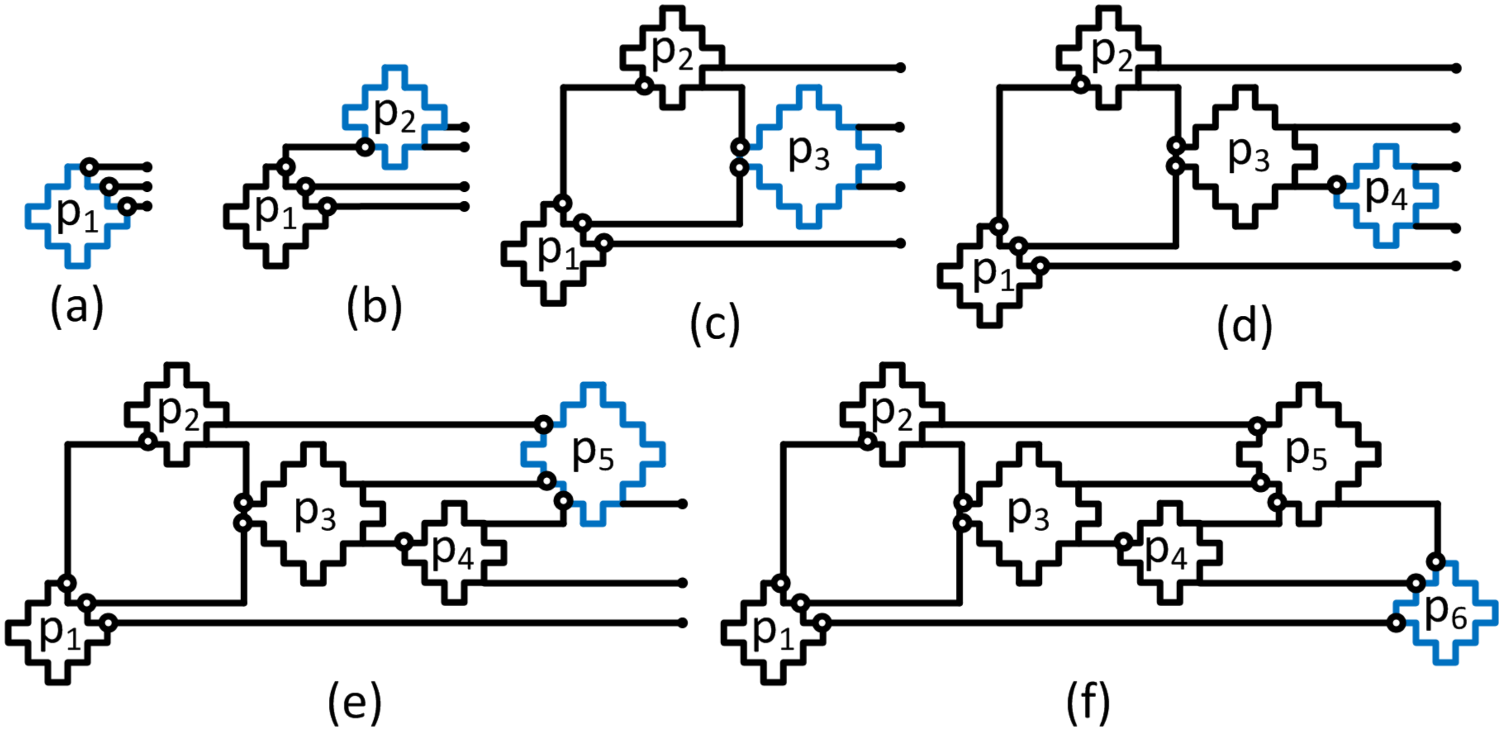


Any planar graph

A linear embedding by Fary's theorem

Vertices are replaced by diamond chains (loops)

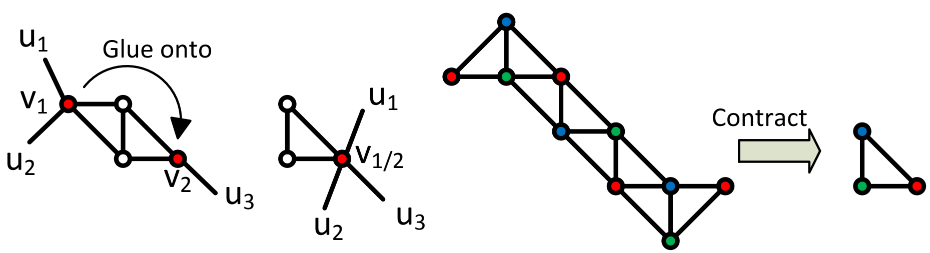
Construct a DGG embedding



- Can we use diamond chains as edges to embed any planar graph into a DGG preserving coloring?
  Yes!
- We prove that the construction is polynomial.
  => 3-coloring DGG is NP-complete.

What can we learn from the proof?

- Can we still determine 3-colorability of DGG efficiently?
- An optimal algorithm potentially has to explore all color enumerations.
  - Returns early in 3-colorable graph and has to exhaust the solution space when the graph is not 3-colorable.
- Sparse DGG is easy to color; Difficult to color DGG has lots of diamonds.



Algorithm 4: DSATUR based Backtracking(V)

Data: A set of vertices V

Result: A valid coloring of V or no coloring exists

if $Sat(v) \geq k$ *for any unassigned* $v \in V$ **then**
  return no coloring exists
if *All* v *have been assigned* **then**
  output coloring
Update saturation $Sat(v)$
while V *is not empty* **do**
  Pick v with largest $Sat(v)$ and assign v with next available color.
  if *no color available* **then**
    return
  Backtracking(V)

- An exact algorithm
  - Diamond contractions
  - Saturation based backtracking
    - Saturation of v is # of distinct colors of its neighbors.

Results

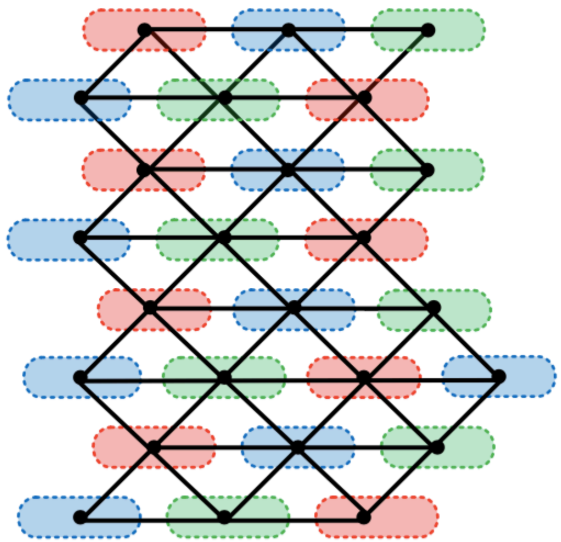
- **Experimental results of 3-coloring DGG**
  - Generate randomly 1000 NxN grids with a certain density.
  - Our algorithm can solve 3-coloring for 400x400 grid graphs.

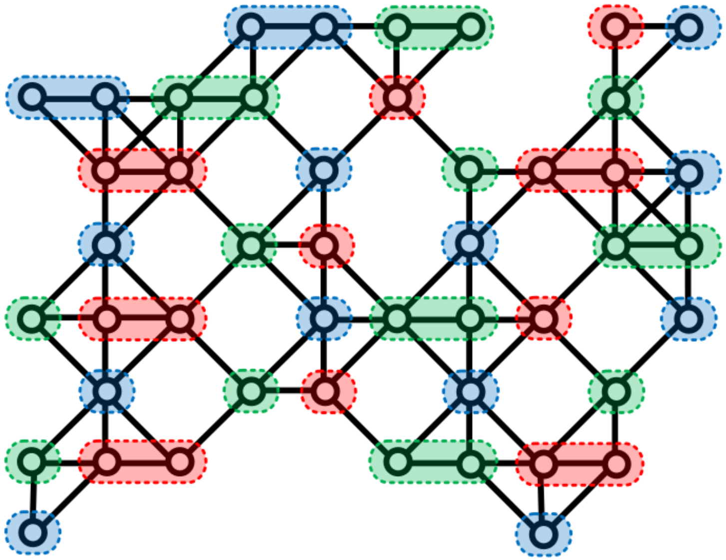
Density = 50%		Our algorithm			UTD			
# of Grids	Grid Size	# of colorable	# of not colorable	Avg. CPU (s)	# of colorable	# of not colorable	Accuracy* %	Avg. CPU (s)
1000	60x60	995	5	0.99	715	285	72.0	1.16
1000	80x80	991	9	0.99	559	441	56.8	1.53
1000	100x100	983	17	1.65	362	621	37.9	1.90
1000	200x200	910	90	23.28	8	992	0.9	6.81
1000	300x300	780	220	118.46	0	1000	0	16.66
1000	400x400	735	265	383.03	0	1000	0	30.86

UTD: Yu, Bei, et al. "Layout decomposition for triple patterning lithography." IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems 34.3 (2015): 433-446.
* Accuracy is the percentage of correctly colored cases over all colorable cases.

DSA-MP hybrid lithography

- ❑ **DSA and Double patterning**
  - ❑ Group-2-coloring DGG is NP-Hard.
  - ❑ Proved by reduction from planar 3SAT problem.
- ❑ **DSA and Triple patterning**
  - ❑ Group-3-coloring DGG is guaranteed.





Summary

- Theoretical results

	2-colorability	3-colorability	4-colorability
RGG	YES	YES	YES
DGG w/ small degree	Polynomial	YES	YES
DGG w/o diamond	Polynomial	YES	YES
DGG	Polynomial	NP-complete	YES

	g-2-colorability	g-3-colorability
RGG	YES	YES
DGG	NP-complete	YES

THANK YOU!